Topology of Arrangements with an Eye to Applications

Sep 1 – 5, 2025
Polo Fibonacci, Building E
Europe/Rome timezone

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Session 1

Monday, September 1st, 17:30, main hall of the Mathematics Department

The weak categorical quiver minor theorem and its applications

Presenter: Luigi Caputi
Affiliation: University of Bologna
Abstract: Building upon previous works of Proudfoot and Ramos, and using the categorical framework of Sam and Snowden, we present an extension of the weak categorical minor theorem from undirected graphs to quivers. As an example, we show consequences on the homology of multipath complexes and magnitude cohomology of quivers; in particular on the growth of the torsion in homology. This is joint work with C. Collari and E. Ramos.

Toric wonderful models

Presenter: Lorenzo Giordani
Affiliation: Ruhr-Universitaet Bochum, Università di Bologna
Abstract: Wonderful compactifications have been of central importance in the study of hyperplane and subspace arrangements. These varieties were introduced by De Concini and Procesi and are obtained by compactifying the ambient space of the arrangement and blowing up a prescribed set of intersections indexed by a "building set". More recently, an analogue for toric arrangements has been developed by De Concini and Gaiffi, and studied in the case of "well connected"

building sets. We remove the well-connectedness hypothesis and we provide a new presentation and a monomial basis for the cohomology rings of toric wonderful compactifications for general building sets. Then, we investigate an operad-like structure on all (cohomology rings of) toric wonderful compactifications via Feynman categories analogous to that established by Coron for matroids. This is joint work with R.Pagaria and V.Siconolfi and work in progress with L.Moci, G.Paolini, R.Pagaria, T.Rossi.

A representation of the holonomy Lie algebra of a matroid

Presenter: Yongho Lee
Affiliation: Louisiana State University
Abstract: Let M be a simple matroid for which the cardinality of each rank two flat is at most 3. For any such matroid M, we produce a representation of the holonomy Lie algebra of M using the combinatorial Laplacian.

Alexander type invariants of line arrangements

Presenter: Manousos Manouras
Affiliation: Nantes Université, Université de Pau et des pays de l' Adour, Universidad de Zaragoza
Abstract: We are interested in the topology of plane algebraic curves and line arrangements. The fundamental group of their complement is a natural invariant, strong enough to show that the combinatorial description of the curve (local type of singularities and incidence relations) may not determine its embedding. The twisted Alexander polynomials provide an accessible invariant of the group, still very sensitive to the topology of the embedding. The twisted Alexander polynomial was introduced by Wada for knots and has been studied thereafter for more general manifolds as the complement of algebraic curves or line arrangements. We will discuss the relation of the twisted Alexander polynomial of the exterior manifold of a line arrangement and the twisted Alexander polynomial of its boundary manifold. We will present how using twisted Alexander polynomials related to metabelian representations, we can extract non-trivial topological information, giving some new results concerning the space of metabelian representations of such groups.

Milnor fibrations and oriented matroids

Presenter: Paul Mücksch
Affiliation: TU Berlin
Abstract: We introduce a combinatorial model for the Milnor fibration of a complexified real arrangement using oriented matroids. It is a poset quasi-fibration whose domain is a natural subdivision of the Salvetti complex. This yields a concrete finite regular CW-complex which is homotopy equivalent to the Milnor fiber of the complexified real arrangement and implies that the homotopy type and hence all homotopy invariants of the Milnor fiber of a complexified real arrangement only depend

on the underlying combinatorial structure given by its oriented matroid. Moreover, our construction works for any oriented matroid, disregarding realizability, so we obtain a notion of a combinatorial Milnor fibration for any oriented matroid. This is joint work with Masahiko Yoshinaga.

Minimal generating sets of D(A) for graphic arrangements

Presenter: Leonie Mühlherr
Affiliation: Bielefeld University
Abstract: Graphic hyperplane arrangements are an interesting example of arrangements, since they are the subarrangements of the well-studied braid arrangement and have a strong connection to graph theory. This makes it possible to use graph theoretical tools to study them and specifically their module of logarithmic derivations D(A). This poster will introduce the separator poset of a graph and give conditions on this poset in order to find a minimal generating set of D(A).

Isomorphism between standard and dual Artin Groups in free products

Presenter: Sirio Resteghini
Affiliation: University of Pisa
Abstract: Given a Coxeter system with a fixed Coxeter element, there is a surjective group morphism ψ from the standard to the dual Artin groups. We give conditions that are sufficient, necessary or equivalent to ψ being an isomorphism. In particular, we prove that if the Hurwitz action on the reduced words of any element in the noncrossing partition poset is transitive, and if the Hurwitz action on the reduced words of the Coxeter element has the same stabilizer as essentially the same action viewed in the standard Artin group, then ψ is an isomorphism. Both of those conditions are already known in some cases, notably in spherical and affine types. By relating those two conditions to the case of the free groups studied by Bessis, we show that taking the free (or direct) product of groups that satisfy them yields another group that, with a suitable Coxeter element, also satisfies them.

Conjugacy of parabolic subgroups in Dyer groups

Presenter: Marina Salamero
Affiliation: Universidad de Sevilla
Abstract: Dyer groups form a family that generalizes Coxeter groups, RAAGs (Right-Angled Artin Groups) and graph products of finite cyclic groups. They admit a uniform solution to the word problem and allow the definition of parabolic subgroups in a manner analogous to that of Coxeter and Artin groups. These subgroups play a fundamental role in the study of the topological and algebraic properties of said groups. In 1997, Luis Paris, building on the work of Kramer for Coxeter groups, proposed an algorithm that efficiently determines whether two parabolic subgroups are conjugate in the Artin group. In this

poster, we will present an algorithm, based on the works of Paris and Kramer, that decides whether two parabolic subgroups of a Dyer group are conjugate.

This is a joint work with María Cumplido (Universidad de Sevilla) and Mireille Soergel (TU Berlin).

Pencils of curves with 4 or 6 Conic-Line Curves

Presenter: Hasan Suluyer
Affiliation: Middle East Technical University (METU)
Abstract: A pencil of degree d > 2 curves is a line in the projective space of all homogeneous polynomials in $\mathbb{C}[x_0,x_1,x_2]$ of degree d. The k > 2 curves whose irreducible components are only lines in some pencil of degree d curves play an important role for (k,d)-nets. The line arrangement comprised of all these irreducible components has a net structure. It was proved that the number k, independent of d, cannot exceed 4 for an (k,d)-net. When the degree of each irreducible component of a curve is at most 2, this curve is called a conic-line curve and it is a union of lines or irreducible conics in the complex projective plane. The number m of such curves in pencils cannot exceed 6.

We study the restrictions on the number m of conic-line curves in special pencils. We present a one-parameter family of pencils of cubics with exactly 4 conic-line curves while there exists only one known net with k = 4. Moreover, we show the combinatorics of the irreducible components of conic-line curves in odd degree pencils with m = 6.

On Universal derivations for multiarrangements

Presenter: Sven Wiesner
Affiliation: Ruhr-Universität Bochum
Abstract: The study of universal derivations for arbitrary multiarrangements and multiplicity functions was initiated by Abe, Röhrle, Stump, and Yoshinaga, where they focused on arrangements arising from (well-generated) reflection groups. In this paper we provide an alternative criterion for determining whether a derivation is universal along with a characterization of universal derivations for arbitrary 2-multiarrangements. As an application, we present a classification and an alternative proof of the existence of universal vector fields for Coxeter arrangements. Additionally, we give descriptions of universal derivations for several multiarrangements, including the so-called deleted A_3 arrangement. This is the first known example of a non-reflection arrangement that admits a universal derivation distinct from the Euler derivation. This is joint work with Takuro Abe, Shota Maehara, and Gerhard Röhrle.

Session 2

Tuesday, September 2nd, 17:30, main hall of the Mathematics Department

A Toric Analogue for Greene's Rational Function of a Poset

Presenter: Elise Catania
Affiliation: University of Minnesota
Abstract: Given a finite poset, Greene introduced a rational function obtained by summing certain rational functions over the linear extensions of the poset. This function has interesting interpretations, and for certain families of posets, it simplifies surprisingly. Greene evaluated this rational function for strongly planar posets in work on the Murnaghan-Nakayama formula.

In 2012, Develin, Macauley, and Reiner introduced toric posets, which combinatorially are equivalence classes of posets (or rather acyclic quivers) under the operation of flipping maximum elements into minimum elements and vice versa. Geometrically, a toric poset corresponds to a toric chamber in the complement of a graphic toric hyperplane arrangement. In this work, we introduce a toric poset analogue of Greene's rational function and study its properties. In addition, we use toric posets to show that the Kleiss-Kuijf relations, which appear in scattering amplitudes, are equivalent to a specific instance of Greene's evaluation of his rational function for strongly planar posets. Also in this work, we give an algorithm for finding the set of toric total extensions of a toric poset.

Topology of real matroid Schubert varieties

Presenter: Leo Jiang
Affiliation: University of Toronto
Abstract: Matroid Schubert varieties are closures of linear spaces in products of projective lines. When the linear space is over the real numbers, we show that the topology of the variety is controlled by the combinatorics of real hyperplane arrangements. More precisely, we exhibit homeomorphisms from real matroid Schubert varieties to quotients of zonotopes. Further, this combinatorial model can be generalised to define a topological space for any oriented matroid. As a consequence, we are able to compute the fundamental group and integral cohomology of these spaces, obtaining virtual Coxeter groups and signed analogues of the graded Möbius algebra respectively. This is joint work with Yu Li.

On the cohomology ring of real k-parabolic subspace arrangements

Presenter: José Luis León Medina
Affiliation: CIMAT Mérida
Abstract: Severs and White introduced k-parabolic real subspace arrangements, generalizing the real k-equal arrangements. Previously, Y. Baryshnikov described the cohomology ring of real k-equal arrangements using Poincaré duality. In this work,

we apply the same method to k-parabolic arrangements. The resulting description is expressed in terms of graphs, and we will discuss whether this approach can be extended to determine the cohomology ring of more general real arrangements.

ψ complete graphical arrangements and MAT-free arrangements

Presenter: Koki Maeda
Affiliation: Kyushu University
Abstract: This research aims to characterize the MAT-freeness of a class of ψ graphical arrangements, which are arrangements close to graphical arrangements, in terms of graphs. The classical result of characterizing the freeness of graphical arrangements is the characterization in terms of graphs. Also, MAT-freeness was characterized in terms of graphs in 2023. On the other hand, the freeness of ψ graphical arrangements was characterized in terms of graphs in 2019, but the MAT-freeness has not yet been characterized, let alone discovered an arrangement that is free but not MAT-free. As a result of this year's research, I have shown the equivalence of ψ graphical arrangements and MAT-freeness in complete graphs, and also given an example of a ψ graphical arrangement that is free but not MAT-free.

Free multiarrangements of not having any free extension

Presenter: Shota Maehara
Affiliation: Kyushu University
Abstract: At once Masahiko Yoshinaga defined the word extendability in his paper, which reflects the idea whether a free multiarrangement can be obtained as a natural restriction of some free simple arrangements of one more higher dimension. The idea of extendability should be important for a very famous conjecture by Hiroaki Terao, thanks to the theory of multiarrangement started by Günter M. Ziegler, but it seems that there are still few results known about extendability except for the first work by Yoshinaga. In this poster, we obtain an infinite set of non-extendable multiplicities on the Coxeter arrangement of type B_2, that would be the first example of free multiplicities on Coxeter arrangement with not having any free extension. In addition, we can also obtain some free Coxeter multiarrangements of type B_n, whose dimension are higher than 2, not having any free extensions as an easy corollary. This is a joint work with Torsten Hoge and Sven Wiesner.

q-deformation of chromatic polynomials and graphical arrangements

Presenter: Tongyu Nian
Affiliation: Osaka University
Abstract: We first observe a mysterious similarity between the braid arrangement and the arrangement of all hyperplanes in a vector space over the finite field $\mathbb{F}_q$. These two arrangements are defined by the determinants of the Vandermonde and the

Moore matrix, respectively. These two matrices are transformed to each other by replacing a natural number n with q^n (q-deformation).

In this paper, we introduce the notion of “q-deformation of graphical arrangements” as certain subarrangements of the arrangement of all hyperplanes over $\mathbb{F}_q$. This new class of arrangements extends the relationship between the Vandermonde and Moore matrices to graphical arrangements. We show that many invariants of the “q-deformation” behave as “q-deformation” of invariants of the graphical arrangements. Such invariants include the characteristic (chromatic) polynomial, the Stirling number of the second kind, freeness, exponents, basis of logarithmic vector fields, etc.

Arithmetic conditions for non-very generic arrangements

Presenter: Takuya Saito
Affiliation: Hokkaido University
Abstract: A discriminantal hyperplane arrangement introduced by Manin and Schechtman is constructed from a given hyperplane arrangement. An intersection in discriminantal arrangement is non-very generic if it arises when the original arrangement is non-very generic. In this paper, we give some arithmetic conditions for non-very generic intersections in discriminantal arrangements. We especially correct the result by Libgober and Settepanella on rank two intersections in discriminantal arrangements.

Magnitude homology of metric fibrations

Presenter: Yu Tajima
Affiliation: National Institute of Technology, Sasebo College
Abstract: T. Leinster introduced the notion of metric fibration, and proved that the magnitude of a metric fibration with finitely many points is equal to product of those of the base and the fiber. We proved the same is true for the magnitude homology using Algebraic Morse theory. This is joint work with Yasuhiko Asao and Masahiko Yoshinaga.

Extracting Sparse Eilenberg-MacLane Coordinates via Principal Bundles

Presenter: Xiaochen Xiao
Affiliation: Northeastern University
Abstract: Let X be a finite data set sampled from an unknown metric space ($\mathbb{X}$,d). The problem this project seeks to address is to develop methods for generating “Eilenberg-MacLane Coordinates”, i.e. functions f : X → K(G,n) characterizing the non-trivial persistent cohomology classes in $PH^n(R(X);G)$. Using the theory of principal bundles, soft sheaves, and Cech Cohomology, we aim to explicit formulas, a stability theory, and algorithms to generate such ”Eilenberg-MacLane

Coordinates" for any discrete Abelian group G. A complete proof chain of a one-to-one correspondence connecting $PH^n(R(X);G)$ and $f : X \to K(G,n)$ is presented, with an explicit formula for computing these coordinates, and as the following work, an explicit algorithm is being extracted from this proof chain in ongoing research.

Categorification of polynomial invariants of matroids

Presenter: So Yamagata
Affiliation: Fukuoka University
Abstract: Khovanov introduced a bigraded cohomology theory of links, whose graded Euler characteristic is the Jones polynomial. Analogously, several constructions of the Khovanov-type (co)homology theories have been provided beyond the knot theory, such as the chromatic cohomology of graphs and the characteristic homology of hyperplane arrangements. A matroid is a structure that reflects the notion of abstract dependency, including cycles in graphs and linear dependency of vectors. In particular, we can obtain matroids from both graphs and arrangements of hyperplanes. In this poster, we provide (co)homology groups associated to polynomial invariants of matroids as a generalization of the chromatic cohomology and characteristic homology of hyperplane arrangements. This poster is based on the joint work with Takuya Saito.

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