INVESTIGATION OF STRATEGIES AND MATHEMATICAL CREATIVITY OF 8TH GRADE GIFTED STUDENTS ON NONLINEAR REALISTIC GENERALIZATION TASKS

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ABSTRACT

INVESTIGATION OF STRATEGIES AND MATHEMATICAL CREATIVITY OF 8TH GRADE GIFTED STUDENTS ON NONLINEAR REALISTIC GENERALIZATION TASKS

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The present study aims to investigate the strategies that 8th-grade gifted students use while solving alternative types of nonlinear realistic generalization tasks and their mathematical creativity while solving these tasks. Four 8th-grade gifted students who attend Science and Art Centers (SAC) in İstanbul and Ankara, Türkiye, constituted the participants of the study. The design of the study was a collective case study design. Data was collected during the 2023-2024 academic year via semi-structured task-based interviews, problem sheets, and the researcher's field notes. The data was analyzed through content analysis. The findings showed that while solving different types of nonlinear realistic generalization tasks gifted students generated diverse and numerous strategies, which are, *counting from a drawing*, *connecting to the known pattern*, *grouping and assigning a variable*, *using the existing generalization*, *identifying regularities by investigating the numerical relationships embedded in the figure*, *identifying regularities by assigning alternative values to the variable and generating the visual representation*, and *identifying regularities by generating the algebraic representation and assigning alternative values*. In addition, in this study, the

mathematical creativity of gifted students was analyzed based on four components of mathematical creativity, namely, fluency, flexibility, originality, and elaboration. Findings revealed that 8th-grade gifted students provided creative responses while solving nonlinear realistic generalization tasks.

Keywords: Gifted Students, Generalization, Mathematical Creativity, Solution Strategies

ÖZ

ÜSTÜN YETENEKLİ 8. SINIF ÖĞRENCİLERİNİN DOĞRUSAL OLMAYAN GERÇEKÇİ GENELLEME GÖREVLERİNDEKİ STRATEJİLERİNİN VE MATEMATİKSEL YARATICILIKLARININ ARAŞTIRILMASI

Tuncer, Zeynep
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Bu çalışma 8. sınıf üstün yetenekli öğrencilerin alternatif türdeki doğrusal olmayan gerçekçi genelleme görevlerini çözerken kullandıkları stratejileri ve bu görevleri çözerken matematiksel yaratıcılıklarını araştırmayı amaçlamaktadır. Çalışmanın katılımcıları, Ankara ve İstanbul, Türkiye'deki Bilim ve Sanat Merkezlerine devam eden dört 8. sınıf üstün yetenekli öğrenciden oluşmaktadır. Mevcut çalışma kolektif bir vaka çalışması tasarımıdır. Veriler, 2023-2024 eğitim öğretim yılında yarı yapılandırılmış görev tabanlı görüşmeler, problem kağıdı ve araştırmacının alan notları yoluyla toplanmıştır. Elde edilen veriler içerik analizi metoduyla analiz edilmiştir. Analiz sonucunda elde edilen bulgular, üstün yetenekli öğrencilerin farklı türdeki doğrusal olmayan gerçekçi genelleme görevlerini çözerken çeşitli ve çok sayıda strateji ürettiğini göstermiştir. Bu stratejiler; *görselden sayma*, *bilinen bir örüntü ile ilişkilendirme*, *gruplama ve değişken atama*, *mevcut genellemeyi kullanma*, *şekildeki sayısal ilişkileri inceleyerek düzenlilikleri belirleme*, *değişkene alternatif değerler atayarak ve görsel gösterimi oluşturarak düzenlilikleri belirleme* ve *cebirsel gösterimi oluşturarak ve alternatif değerler atayarak düzenlilikleri belirlemedir*. Buna ek olarak, bu çalışmada üstün yetenekli

öğrencilerin matematiksel yaratıcılığı, akıcılık, esneklik, özgünlük ve detaylandırma olmak üzere matematiksel yaratıcılığın dört bileşenine dayanarak analiz edilmiştir. Bulgular, 8. sınıf üstün yetenekli öğrencilerin doğrusal olmayan gerçekçi genelleme görevlerini çözerken yaratıcı çözümler ürettiklerini göstermektedir.

Anahtar Kelimeler: Üstün Yetenekli Öğrenciler, Genelleme, Matematiksel Yaratıcılık, Çözüm Stratejileri

To my beloved family

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LIST OF ABBREVIATIONS

ABBREVIATIONS	
MoNE	Ministry of National Education
NAGC	National Association for Gifted Children
NCTM	National Council of Teachers of Mathematics
OECD	Organisation for Economic Cooperation and Development
SAC	Science and Art Center
UNESCO	United Nations Educational, Scientific and Cultural Organization

CHAPTER 1

INTRODUCTION

Education is a vital human right that promotes sustainable growth. Each child has a fundamental right to education, and to uphold this right, nations must guarantee that all people have equal access to inclusive, high-quality education and no one gets left behind (United Nations Educational, Scientific and Cultural Organization [UNESCO], 2016). To achieve this goal, the students' diverse characteristics, abilities, needs, and interests need to be acknowledged (Tomlinson, 2001), and education systems and programmes should be created and executed to cater to the vast range of these characteristics and demands (UNESCO, 2016). Although educational systems around the world strive to enhance outcomes for different learners while also modifying curriculum regulations (Organisation for Economic Cooperation and Development [OECD], 2024), gifted students, who are the promising individuals for society, are provided by limited educational opportunities in Türkiye (Şengil-Akar & Yetkin- Özdemir, 2022).

Gifted students are those who are performing or capable of performing at greater levels in comparison with their peers with the same knowledge and circumstance in any number of domains (National Association for Gifted Children [NAGC], 2019; Marland, 1972; Renzulli, 1978). As the definition implies, those students have some distinct characteristics that distinguish them from their peers. More precisely, gifted students are often characterized by their *persistence* (Clark, 2013; Otto, 2012; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986; Tuttle, 1983), *long attention span* (Callard- Szulgit, 2010; Clark, 2002; Silverman & Waters, 1984; Silverman et al., 1986), *curiosity* (Callard-Szulgit, 2010; Clark, 2002; Otto, 2012; Renzulli & Smith, 1979; Tuttle, 1983), *leadership skills* (Callard-Szulgit, 2010; Clark, 2013; Renzulli & Smith, 1979; Tuttle, 1983), *metacognitive*

abilities (Jaušovec, 1991; Lu et al., 2017; Renzulli & Smith, 1979; Shore & Kanevsky, 1993), *strong memory* (Clark, 2002; Silverman & Waters, 1984; Silverman et al., 1986; Tuttle, 1983), *verbal ability* (Callard-Szulgit, 2010; Clark, 2013; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986; Tuttle, 1983), *recalling facts easily* (Callard-Szulgit, 2010), *rapid learning* (Callard-Szulgit, 2010; Ministry of National Education [MoNE], 2022; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986; Winner, 1996), *generalization ability* (Clark, 2002; Krutetskii, 1976), *abstract thinking* (Clark, 2002; Clark, 2013), *visualization skills* (Clark, 2002), *capability to make relation and synthesis* (Callard-Szulgit, 2010; Clark, 2002; Clark, 2013; Tuttle, 1983), *strong number sense* (Silverman & Waters, 1984; Silverman et al., 1986), *flexible strategy use* (Assmus & Fritzlar, 2022; Erdoğan & Gül, 2023; Gutierrez et al., 2018; Jaušovec, 1991; Montague, 1991; Sternberg, 1981; Syawahid & Prahmana, 2024) and *creativity* (Renzulli, 1978; Sternberg, 2003; Winner, 1996).

The diverse characteristics of gifted students bring about a variety of unique educational needs. More specifically, research revealed that gifted students demand tasks that are *challenging* (Özdemir & Işıksal-Bostan, 2021; VanTassel-Baska & Brown, 2007), *open-ended* (Özdemir & Işıksal-Bostan, 2021; Rotigel & Fello, 2004), *interesting* (Ayebo, 2010) and *have a connection with real-life* and *other disciplines* (Özdemir & Işıksal-Bostan, 2021; Rotigel & Fello, 2004; Sheffield, 1994). The instruction that was designed based on gifted students' needs has the potential to enable gifted students to demonstrate deep learning (Aygün, 2022), increase their self-efficacy and motivation (Siegle et al., 2014), and enable them to make significant contributions to society (Renzulli, 2012). Nonetheless, research (Kaplan, 2004; Karataş–Aydın, 2021; Özdemir & Işıksal–Bostan, 2021; Reis et al., 1993; Reis & Renzulli, 2010; VanTassel-Baska et al., 2021) pointed out that the needs of gifted students are not satisfied; most of them keep to spend time in school doing far below their skills, receiving the same assignments as their peers that are simple and repetitious, and failing to be challenged, particularly in elementary and middle schools. The reason behind that is teachers' lack of awareness of gifted

students' abilities and lack of knowledge about differentiation (Reis & Renzulli, 2010). As a result of overlooking gifted students' potential in the classroom, boredom and even dropout are unavoidable results (Johnson, 2000; Reis & Renzulli, 2010).

A closer examination demonstrates that one area where gifted students' demands are often neglected is the improvement of gifted students' creative potential (Plucker & Callahan, 2014). Creativity, which is one of the twenty-first-century skills (Binkley et al., 2012), is the process of developing something considered creative, as well as the generation of new and appropriate products (Amabile, 1983; Boden, 2004; Bruner, 1962). Creativity has become one of the most highly desired abilities in an era characterized by innovation and digitalization, not just for improved job prospects but also for enhanced social and individual lives (Newton & Newton, 2014; Trilling & Fadel, 2009; Vincent-Lancrin, 2021). As exemplified by the phrase (UNESCO, 2006) "*Creativity is our hope*," which emphasizes its capacity to generate solutions and significant transformation, creativity is frequently regarded as an effective instrument for tackling global issues (Newton & Newton, 2014). Besides, creativity is categorized under the "create" category at the top of Bloom's taxonomy, indicating its complexity and significance in education. People with creative talent are needed to drive the changes in this global age (Keleş, 2022). Gifted students are in a good position to develop into such people because they are frequently distinguished by their unique characteristics, such as flexible thinking and solving problems in a novel way (Clark, 2002; Clark, 2013; Renzulli & Smith, 1979; Tuttle, 1983). In this light, encouraging gifted students' creativity becomes more crucial than ever.

To properly discuss the function of creativity in gifted education, it is necessary to take into account how it appears in subject-specific domains, such as mathematics. Despite its importance as future competence, research revealed that creativity is given less weight in mathematics classes (OECD, 2024). The emphasis on memorization, procedural abilities, and test-taking in traditional education has not been conducive to the growth of mathematical creativity (Reis & Renzulli, 2010).

Conversely, instruction that solely focuses on applying mathematical formulae and procedures may result in students' poor reasoning, which makes it impossible for them to develop their thought processes (Kusaeri & Aditomo, 2019).

Nevertheless, several researchers pointed out that creative talent can be developed through appropriate instruction (Newton & Newton, 2014; Sheffield, 2009). To support gifted students' creative potential in mathematics, they need environments where they are encouraged not only to reach the correct answer but also to produce many solutions (fluency), to think in different ways (flexibility), to produce unconventional ideas (originality), and to justify their responses (elaboration). Therefore, it is emphasized that the nature and type of problem play a role in triggering mathematical creativity; especially unconventional, open-ended, multiple-solution problems that are related to real life make students' creative thinking skills more visible (Bicer, 2021; Lekin, 2009; Shriki, 2008). Stated differently, not every problem-solving task is equally appropriate for fostering mathematical creativity and for investigating mathematical creativity (Leikin, 2009). Several researchers (e.g., Assmus and Fritzlar, 2022; Eraky et al., 2022; Singer & Voica, 2022) highlighted that generalization tasks are accessible for studying creativity and promoting mathematical creativity as the generalization process itself was creativity-directed (Singer & Voica, 2022).

The process of drawing conclusions from given instances is known as generalization (Sriraman, 2003). Generalization has a key role in the growth of reasoning in mathematics education and the advancement of mathematics as a discipline (Rivera, 2013; Sriraman, 2003). Even though some researchers (e.g., Krutetskii, 1976) used generalization abilities as defining characteristics of gifted students and generalization tasks as creativity-directed tasks (Singer & Voica, 2022), mathematical creativity and giftedness are infrequently investigated through generalization tasks (Eraky et al., 2022). Although examining mathematical creativity and the generalization of gifted students offers insightful theoretical information, comprehending how generalization actually happens in practice is just as crucial. To comprehend the nature of generalization and the underlying mental

procedure, it is necessary to look at the strategies that gifted students use. Hence, given the paucity of research about mathematical creativity and giftedness in the scope of generalization, the present study attempts to further the field by examining the strategies of gifted students while solving generalization tasks.

Although some researchers (e.g., Amit & Neria, 2008; Erdoğan & Gül, 2023; Girit–Yıldız & Durmaz, 2021; Gutierrez et al., 2018) investigated the strategies of gifted students while solving different generalization tasks, i.e., linear and nonlinear generalization tasks, the majority of current research tends to concentrate on linear generalizations (Girit–Yıldız & Durmaz, 2021), while nonlinear generalizations remain understudied, especially among gifted middle school students. Such problems are more complex than the linear generalization tasks, thus, it could be more challenging for gifted students. Given that gifted students demand challenging tasks and find those tasks engaging (Özdemir & Işıksal-Bostan, 2021), the use of nonlinear generalization tasks might enable gifted students to uncover their way of thinking and creative potential. Considering the needs of gifted students, the features of creativity-directed tasks, the role of generalization in revealing mathematical creativity, and the gaps in the field, this study involves nonlinear realistic generalization tasks that provide an authentic setting for generalization and mathematical creativity.

Generalization is one of the essential elements of algebraic thinking, and representing is one of the essential elements of generalization (Kaput, 2008). The capacity to create and employ mathematical structures by interacting with several representation types and skillfully switching between them is one of the main traits of gifted students (Assmus & Fritzlar, 2022). To further develop these traits, gifted students need to be encouraged by activities that allow them to employ a variety of representations and interpret and relate these representations to one another (Aygün, 2010). Despite its relevance, less has been done on what types of representation gifted students use in generalization and how representation functions in the generalization processes of gifted students. Considering the gifted students' ability to use and make connections with different representations and the

relationship between representing and generalization, there is room to investigate the types of representations that gifted students use while engaging in nonlinear generalization tasks.

1.1. Purposes and Research Questions of the Study

Building on this background, the aim of the present study was twofold. The study's first aim is to investigate the strategies that 8th-grade gifted students use while solving different types of nonlinear realistic generalization tasks. In addition to the students' strategies, this study also focuses on the mathematical creativity of 8th-grade gifted students while solving these tasks. In line with these aims, the following research questions will be explored in the current study:

1. How do the strategies of 8th-grade gifted students vary when solving alternative types of nonlinear realistic generalization tasks?
2. How creative are 8th-grade gifted students while solving alternative types of realistic nonlinear generalization tasks (in terms of fluency, flexibility, originality, and elaboration)?
    2.1. What types of representations do 8th-grade gifted students use in their responses to nonlinear realistic generalization tasks?

1.2. Significance of the Study

The present study addresses several understudied areas in the subject of mathematics education, which makes it significant both conceptually and practically.

First of all, it expands the current body of knowledge about the well-accepted but infrequently empirically investigated mathematical creativity of gifted students in the context of generalization. The processes of generalization—finding patterns, formulating rules, and expanding mathematical concepts beyond particular instances—naturally call for creative thinking in mathematics (Singer & Voica, 2022). Nonetheless, there is limited research on how mathematical creativity

manifests itself in generalization tasks, particularly for gifted students (Eraky et al., 2022). For gifted children, whose cognitive characteristics indicate a significant capacity for both mathematical creativity and generalization (Krutetskii, 1976; Sriraman, 2005), it is especially crucial to understand how these constructs interact.

To investigate these interactions, the appropriate learning environments are needed. The review of the literature emphasized that gifted students' needs are not met in their mathematics classrooms (Kaplan, 2004; Karataş–Aydın, 2021; Özdemir & Işıksal–Bostan, 2021; Reis et al., 1993; Reis & Renzulli, 2010; VanTassel-Baska et al., 2021) and they received limited educational opportunities in Türkiye (Şengil–Akar & Yetkin–Ozdemir, 2022). The reason behind this is teachers' lack of awareness about gifted students' mathematical abilities and their lack of knowledge about differentiation (Reis & Renzulli, 2010; Rotigel & Fello, 2004). Nonetheless, several researchers argued that students' creativity can be developed if the students are given the opportunity to attend appropriate learning environments (Newton & Newton, 2014; Sriraman, 2005). Therefore, it is necessary to investigate creativity-directed tasks to satisfy the needs of gifted students and enhance their creative potential. Several studies (e.g., Assmus and Fritzlar, 2022; Eraky et al., 2022; Singer & Voica, 2022) highlighted that generalization tasks are appropriate tools to investigate and develop students' mathematical creativity. The present study employs a variety of generalization tasks, including figural, non-figural, near, and far problems, that were designed according to the needs of gifted students and the features of creativity-directed tasks. Thus, it not only enables a more thorough analysis of how problem type affects gifted students' mathematical creativity and generalization, but also provides teachers with useful guidance to recognize gifted students' creative talent in generalization tasks and create tasks that foster mathematical creativity and generalization of gifted students in a variety of relevant ways.

In addition, to give gifted students the chance to engage in generalization in ways that reflect reasoning in real life, the current study includes realistic generalization tasks set in relevant contexts. Since context has a crucial role in students' reasoning

(Leikin &Lev, 2013), the involvement of realistic contexts in this study could broaden the insights about how mathematical creativity and the generalization of gifted students develop in authentic problem-solving settings. In addition to this, since it is critical for students to observe how the mathematics they are studying is used in real-world situations (English & Mousoulides, 2011), the realistic problems used in this study can help students see the power in mathematics, that is, enable them to develop a productive disposition. Therefore, the findings of this study may pave the way for further studies investigating the mathematical creativity of gifted students using realistic problems in different content areas of mathematics.

Additionally, the current study aimed to focus on the strategies of gifted students while solving nonlinear generalization tasks. Examining the strategies used by gifted students provides insight into how gifted students reach that conclusion, rather than what conclusion they reach. Although prior studies provide insights into the generalization process of gifted middle school students, they mainly focus on linear generalization tasks (Girit–Yıldız & Durmaz, 2021). Given that Turkish middle school mathematics curricula involve the linear relationship between the quantities (MoNE, 2018), by specifically examining 8th-grade gifted students' strategies and mathematical creativity in the nonlinear generalization task, the current study extends beyond the curriculum. To be more specific, exploring the strategies gifted students use in different types of nonlinear generalization problems might provide a richer perspective for understanding how gifted students structure the nonlinear generalization problems, reach generalizations, and how their strategy use varies across nonlinear generalization problems. In addition, the present study shows how nonlinear generalization tasks can be an effective and challenging differentiation tool for gifted students and might open the way for future studies to design and explore learning experiences for gifted students involving tasks that extend beyond the traditional curriculum.

In addition to examining the strategies, this study adds to the expanding corpus of research on generalization by examining the representations employed by gifted students. The kinds of representations students use can affect the depth and breadth

of their generalizations since generalization frequently entails recognizing and articulating underlying mathematical patterns (Kaput, 1999; Sriraman, 2003). Mathematics educators may be able to create more engaging lessons and tasks that encourage complex mathematical thinking if they have a better understanding of the kinds of representations gifted students select and how they switch between them.

As a result, the study is significant for enriching theoretical and practical knowledge of mathematical creativity and giftedness within the process of generalization as well as for guiding the development of instructional tasks that will help gifted students reach their greatest potential.

1.3. Definitions of Important Terms

The definitions of some important terms are given below:

Gifted Students: Students who perform or have the potential to perform at higher levels than their peers in a variety of categories are considered gifted (NAGC, 2019; Marland, 1972; Renzulli, 1978). In this study, students who attended Science and Art Centers in Türkiye were defined as gifted students.

Generalization: Generalization is the process of inferring and drawing conclusions from specific circumstances (Sriraman, 2003). In this study, generalization was described as the process of identifying commonalities, extending the commonalities beyond the given instances, and expressing the conclusions or the rules.

Nonlinear Generalization Task: The nonlinear generalization tasks are generalization problems that entail recognizing and articulating the structure of patterns that show non-constant rates of change.

Figural Generalization Task: Figural generalization task is a type of generalization tasks that involve a figure. In the present study, the Honeybee problem was considered as a figural generalization task.

Non-Figural Generalization Task: Non-figural generalization task is a type of generalization task that does not involve any figure. In the present study, the COVID problem was considered as a non-figural generalization task.

Near Generalization Task: Near generalization tasks are tasks that encourage students to apply a rule, pattern, or relationship to a scenario that is comparable to or close to the ones that have already been provided (Stacey, 1989). In the present study, the b option of the Honeybee problem was considered as a near generalization task.

Far Generalization Task: Far generalization tasks are the type of generalization tasks that require developing the general rule and extending beyond the given cases (Stacey, 1989). In the present study, the c option of the Honeybee problem and the COVID problem were considered as a far generalization task.

Creativity: The creation of novel and appropriate products, as well as the process used to develop something deemed creative (Amabile, 1983; Boden, 2004; Bruner, 1962).

Mathematical Creativity: Mathematical creativity is the capacity to come up with original, unconventional, and appropriate solutions to mathematical problems (Chamberlin & Moon, 2005; Sriraman, 2004). In this study, mathematical creativity was investigated in terms of fluency, flexibility, originality, and elaboration components of creativity in students' appropriate responses.

Appropriate Response: The correct and complete response to the nonlinear realistic generalization problems.

Fluency: Fluency is the total number of appropriate solutions or ideas produced for a problem (Guilford, 1959; Kim, 2006; Levenson, 2022). In this study, fluency was considered as the number of appropriate responses generated for a problem.

Flexibility: Flexibility is the number of different categories of responses for a problem (Kim et al., 2016; Rubenstein et al., 2019). In the present study, students' ability to diversify the main idea behind the answers, the way the answer was

represented, and the strategy used to reach the answer were considered as flexibility.

Originality: Originality is the uniqueness of the response (Kim et al., 2016). In the current study, it was defined as the rarity of a response to a problem among all responses.

Elaboration: Elaboration is the degree of detail about specific actions (Guilford, 1959). In this study, elaboration was defined as students' ability to explain his/her thought process and/or provide in-depth reasoning.

CHAPTER 2

LITERATURE REVIEW

The purpose of this study was twofold: to examine the strategies of 8th-grade gifted students while solving nonlinear realistic generalization tasks and to investigate the mathematical creativity of 8th-grade gifted students in solving those tasks. Accordingly, this section was divided into six main sections, namely, the conception of giftedness, characteristics of gifted students, creativity, mathematical creativity, generalization, and lastly, the summary of the literature.

2.1. Conception of Giftedness

Although various conceptions of giftedness existed in the literature, there is no commonly accepted definition of giftedness. In the early years, the conception of giftedness was more conservative (Renzulli, 2002), but over the years, the conception of giftedness has evolved. This section will include diverse conceptions of giftedness along with their historical development.

The traditional conception of giftedness relied on general intelligence. This conception was started with the studies of Spearman (1904) and Terman (1926). Those researchers associated giftedness with general intelligence. More specifically, Spearman (1927) thought that each person had some sort of cognitive power and that people might be classified based on their level of intelligence. According to him, every ability must serve a universal purpose, which is the general ability factor or g factor. Finding relationships between concepts and being able to recognize ramifications or new linkages according to these relationships were the two types of talents Spearman (1923) most closely linked with the g factor. The conception of the researchers influenced their way of defining giftedness and the identification of gifted students. Aligning with their conceptions,

they supported that gifted students could be identified by a standardized intelligence test. Specifically, Terman (1926, p. 43) described giftedness as: “the top 1% level in general intellectual ability, as measured by the Stanford-Binet Intelligence Scale or a comparable instrument”. However, this conservative conception of giftedness has received many criticisms from other researchers (e.g., Feldhusen & Heller, 1985; Sak, 2021; Winner, 2000). One of the criticisms is that this conception ignores the variety of ways that giftedness might appear by supporting that it is unidimensional (Feldhusen & Heller, 1985). Another criticism is that the threshold score was vague and rigid (Feldhusen & Heller, 1985; Sak, 2021).

Compared to Spearman (1904) and Terman (1926), some researchers proposed that giftedness is domain-specific. According to a domain-specific view, giftedness is the extremely high level of intellect displayed within particular areas (Brody & Stanley, 2005; VanTassel-Baska, 2005). A domain-general or global g-factor model of intelligence is likewise incompatible with this viewpoint because, according to the domain-specific conception of giftedness, while high g-factor intelligence may be fulfilling for the individual, it has minimal effect on the society from which it originated if it is not closely associated with a particular area of modern life. Moreover, a domain-specific view of giftedness assumes that rather than being innate to giftedness, task commitment, motivation, and creativity are developed, which in turn catalyze the development of aptitude and are subordinate to the idea of giftedness (VanTessel-Baska, 2005).

Unlike the researchers who supported a domain-specific conception of giftedness, some researchers believed that the psychological constructs are essential components of giftedness (Sternberg & Kaufman, 2018). Renzulli is one of the most well-known among those researchers. Renzulli (1978) proposed a three-ring conception of giftedness. As the name implies, according to the three-ring conception of giftedness, three main factors determine gifted behavior, which are task commitment, creativity, and above-average ability. Figure 2.1 represented the components of the three-ring conception of giftedness.

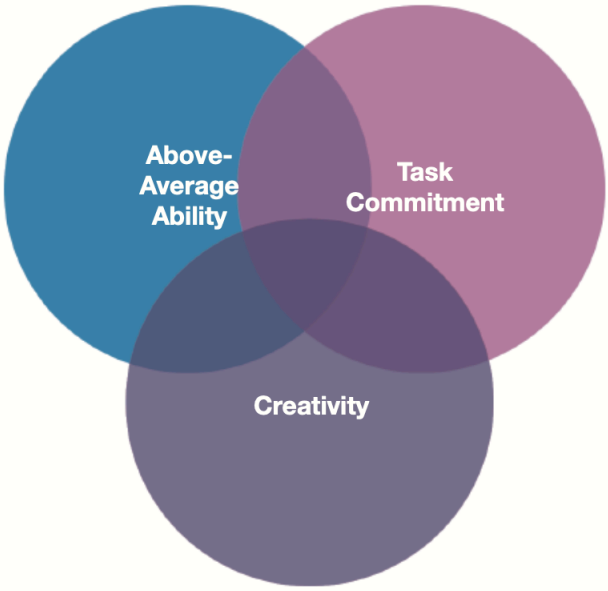


Figure 2.1. Three-Ring Conception of Giftedness (Renzulli, 2011, p.83)

Renzulli (1978, 1999, 2011) highlighted that giftedness is not made by any one cluster; rather, gifted behaviors are the result of interactions between and among those three clusters. This interaction is shown in Figure 2.1 as the shaded part. Renzulli (1978, 2011) also claimed that each component plays an equally important role in giftedness. However, this idea evolved over the years, and in their 2021 study, Renzulli and Reis declared that the above grade level ability component largely influenced the schoolhouse giftedness, one of the types of giftedness, and standardized ability exams are the most straightforward way to measure it (Renzulli, 1999). On the other hand, an interplay between all three components is required for high-level performance in creative/productive giftedness, which is a second type of giftedness, and it covers the creation of novel concepts intended to influence one or more target audiences (Renzulli, 1999). However, when engaging in creative and productive activities, the size of the clusters need not be uniform, nor should they all be of the same size (Renzulli & Reis, 2021). In addition to this revision, Renzulli and Reis (2021) also added two clusters to the three-ring conceptions of giftedness because they believed that these have the potential to contribute to creative productivity. These co-cognitive traits were employing executive function skills to produce action-oriented goods and leverage abilities for

social benefit (Renzulli & Reis, 2021). By incorporating co-cognitive executive function abilities such as self-efficacy, leadership, and teamwork, the Three Ring Conception of Giftedness has evolved beyond the antiquated notion that giftedness is purely intrinsic (Renzulli & Reis, 2021).

Similar to Renzulli (1978), Sternberg (2003) proposed a model of giftedness that includes not only intelligence but also other psychological constructs. In 2003, Sternberg proposed the "WICS" model of giftedness, where W represents wisdom, I represents intelligence, C represents creativity, and S represents synthesis. Like Renzulli (1978, 2011, 2021), Sternberg (2003) claimed that no single attribute generates giftedness; instead, the synthesis of these attributes is necessary to create gifted behaviors. Another similarity between Renzulli's three-ring conception of giftedness and Sternberg's WICS model of giftedness was that neither denied the role of intelligence in the conception of giftedness, and both emphasized the role of creativity in their models. In fact, Sternberg (2003) asserted that creativity was just as crucial as analytical intellectual skills. Furthermore, although Sternberg (2003) did not use the word "creative self-efficacy", he highlights the importance of self-efficacy by stating that creative individuals need to have faith in their capacity to change things. Besides, Sternberg prioritized wisdom in his model because, according to him, someone can be creative or intelligent but not wise (Sternberg, 2003). According to Sternberg (2003), being wise involves more than just maximizing one's own or another's self-interest; it also involves striking a balance between one's interests, those of others, and other facets of one's environment.

Although the WICS model of giftedness stresses the role of other factors, such as environment, many conventional conceptions of giftedness place a greater emphasis on the person. Researchers (e.g., Chowkase, 2022; Plucker & Barab, 2005) criticized placing a firm focus on the individual, and they agreed that a solely individualist way of defining giftedness is limited. Among those researchers, Chowkase (2022) redefined giftedness by proposing the Three C's conception of giftedness. According to the Three C's conception of giftedness, the three categories of human traits must cooperate to produce gifted behavior:

“Competence in one’s action”, “commitment to task”, and “concern for others”. Figure 2.2 depicts the Three C’s conception of giftedness.

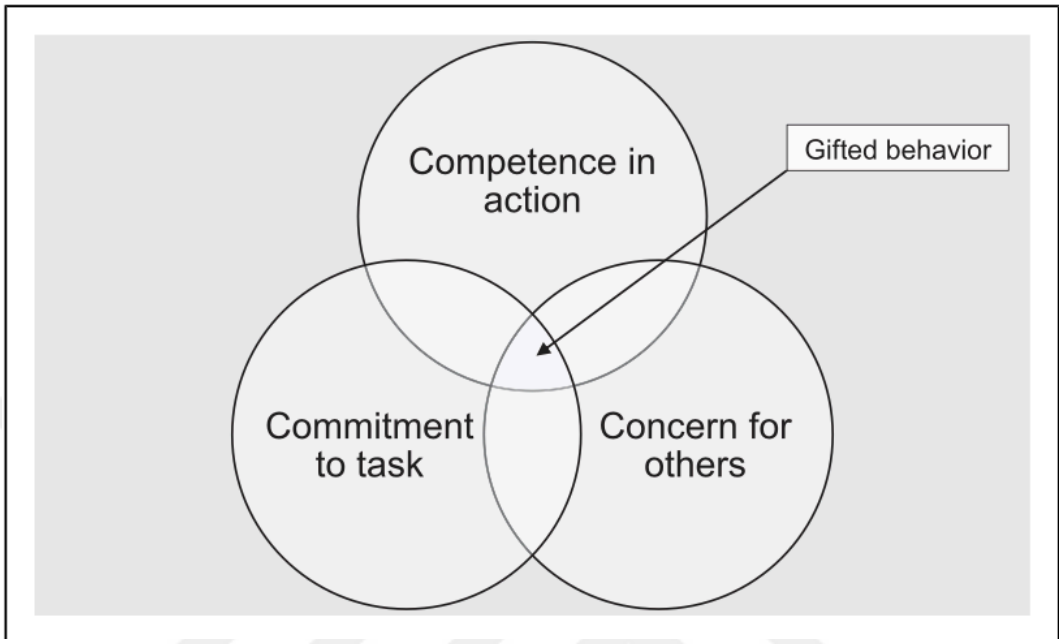


Figure 2.2. The Three C’s Conception of Giftedness (Chowkase, 2022, p.405)

Like Renzulli's three-ring model of giftedness (see Figure 2.1), Chowkase (2022) also showed each cluster that constitutes gifted behavior as a set and placed gifted behavior at their intersection. From this point, it can be said that, just like Renzulli (2011) and Sternberg (2003), in Chowkase's model, the intersection of each component contributes to gifted behavior. Chowkase (2022) related the first two components of his model, which are competence in action and commitment to the task, with the components of Renzulli’s (1978) three-ring conception of giftedness. More specifically, Chowkase (2022) asserted that competence in an action referred to above grade level ability in Renzulli’s conception (1978), and creativity in Renzulli’s model is the outcome of the two c’s, namely competence and commitment to task. Yet, what made this model unique was the last c, concern for others. According to Chowkase and Watve (2022), concern for others is characterized by an awareness of their needs and difficulties, a sense of connection with them, an emotional desire to assist, and the development of motivation to take action.

Due to being a multifaceted concept, an investigation of the characteristics of gifted students might provide insights into the conception of giftedness. Thus, in the following section, the characteristics of gifted students will be presented.

2.2. Characteristics of Gifted Students

As explained in the previous section, researchers have diverse conceptions of giftedness. Aligning with their conceptions of giftedness, their definitions of gifted students showed variation. More specifically, Terman (1926), who associated giftedness with general intelligence, defined gifted students as students with 140 IQ. Furthermore, Renzulli (1978), who associated gifted behavior with above-grade-level ability, task commitment, and creativity, defined gifted students by the means of those components as follows: "Gifted and talented children are those possessing or capable of developing this composite set of traits and applying them to any potentially valuable area of human performance" (Renzulli, 1978, p.261). Marland provided one of the broadest and most cited definitions of gifted students in 1972. Marland's (1972, p.2) definition is given below:

> Gifted and talented children are those identified by professionally qualified persons who by virtue of outstanding abilities, are capable of high performance. These are children who require differentiated educational programs and/or services beyond those normally provided by the regular school program in order to realize their contribution to self and society. Children capable of high performance include those with demonstrated achievement and/ or potential ability in any of the following areas, singly or in combination: general intellectual ability, specific academic aptitude, creative or productive thinking, leadership ability, visual and performing arts, and psychomotor ability.

In recent years, the National Association for Gifted Children defined gifted students as: "Students with gifts and talents perform—or have the capability to perform—at higher levels compared to others of the same age, experience, and environment in one or more domains" (NAGC, 2019, p.1). The commonality of

these definitions was that they all highlighted that gifted students have distinct characteristics. Since understanding the characteristics of gifted students provides insights into gifted students' personality traits and intellectual skills and acts as a guide in the identification of gifted students, those characteristics have been the subject of various research studies. Many researchers (e.g., Clark, 2013; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986) listed unique characteristics of gifted students and provided diverse classifications of those characteristics. In light of the review of the literature, in this section, the characteristics of gifted students were classified and presented as affective and cognitive characteristics.

Affective characteristics involve emotional and motivational features that gifted students possess. One of the most frequently noted affective characteristics of gifted students was persistence. To be more precise, researchers (Clark, 2013; Otto, 2012; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986; Tuttle, 1983) declared that gifted students are perseverant, have goal-directed behavior, possess keen interest and show long attention span in their area of interest. Nevertheless, some researchers (e.g., Renzulli & Smith, 1979) also stated that gifted students get bored easily when they are faced with a situation or subject, they are not interested in. This implies that the level of interest of gifted students affects their persistence and highlights the necessity of engaging gifted students by considering their needs, such as the demand for more challenging, real-life, and open-ended tasks (Özdemir & Işıksal-Bostan, 2021; Rotigel & Fello, 2004). Furthermore, research also pointed out that gifted students have leadership skills (Callard-Szulgit, 2010; Clark, 2013; Renzulli & Smith, 1979; Tuttle, 1983). Leadership skills are closely related to the other characteristics of gifted students depicted in the literature such as responsibility (Renzulli & Smith, 1979), sensitivity (Clark, 2013; Callard-Szulgit, 2010; Silverman & Waters, 1984), compassion for others (Silverman & Waters, 1984), being observant (Callard-Szulgit, 2010), curiosity (Callard-Szulgit, 2010; Clark, 2002; Otto, 2012; Renzulli & Smith, 1979; Tuttle, 1983), moral judgment (Clark, 2013), self-confidence

(Renzulli & Smith, 1979), and high degree of energy (Clark, 2013; Silverman & Waters, 1984). Another defining characteristic is self-awareness. Gifted students show self-awareness about their strengths and weaknesses (Clark, 2013), and they are self-critical (Callard-Szulgit, 2010). Research also highlighted that those students have high expectations of themselves and others (Callard-Szulgit, 2010; Clark, 2013). Lastly, gifted students have self-efficacy (Hoge & Renzulli, 1993), which is interconnected with other affective characteristics of gifted students, such as being self-critical.

Whereas the affective characteristics are related to emotion and attitude, the cognitive characteristics refer to knowledge and intellectual abilities (Baker, 2012). To be more organized, in the following paragraphs, the cognitive characteristics of gifted students were categorized and presented under seven themes, namely, language, thought process, learning, memory, metacognition, mathematics-related characteristics, and creativity.

The review of the literature revealed that gifted students possess language-related cognitive characteristics. More specifically, those characteristics involve a high level of language development (Clark, 2013), verbal ability (Callard-Szulgit, 2010; Clark, 2013; Renzulli & Smith, 1979; Tuttle, 1983), extensive vocabulary (Silverman & Waters, 1984; Silverman et al., 1986), reading ability (Silverman & Waters, 1984; Silverman et al., 1986), and strong communication skills i.e., ability to express ideas with diverse ways of communication (Renzulli & Smith, 1979).

Beyond language, gifted students were also known for their strong memory. More specifically, numerous researchers (e.g., Clark, 2002; Silverman & Waters, 1984; Silverman et al., 1986; Tuttle, 1983) indicated that those students have a large storage capacity. Their storage capacity might be the explanatory factor for their ability to recall facts easily (Callard-Szulgit, 2010) and their capacity for information processing (Clark, 2013), which were their other defining characteristics.

Gifted students not only recall facts easily but also learn faster as compared to their non-gifted peers (Callard-Szulgit, 2010; MoNE, 2022; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986; Winner, 1996). In addition to their learning abilities, their thought process was also distinct. More specifically, they were capable of abstract thinking (Clark, 2002; Clark, 2013) making generalizations (Clark, 2002), visualization (Clark, 2002), relating and synthesizing different ideas (Callard-Szulgit, 2010; Clark, 2002; Clark, 2013; Tuttle, 1983), and utilization of analogies (Clark, 2013).

Besides their advanced thought process, gifted students also have metacognitive skills. To be more specific, gifted students are aware of their knowledge (Lu et al., 2017). They not only know what they know but also know how to utilize their knowledge, and they were able to distinguish the irrelevant information from the relevant one (Shore & Kanevsky, 1993). These metacognitive skills were observed in their problem-solving process and interrelated with their mathematics-related characteristics. More specifically, Renzulli and Smith (1979) indicated that those students can identify what information is necessary to solve the problem, understand how each stage relates to the process as a whole, and give enough time to complete each step. In addition to these, gifted students are aware of whether they have solved the problem accurately or not (Jaušovec, 1991), and they have strong number sense (Silverman & Waters, 1984; Silverman et al., 1986). These characteristics inspired mathematics education researchers to conduct different studies on the problem-solving strategies of gifted students as compared to their non-gifted peers. The studies' remarkable findings showed that, in contrast to their peers who are not gifted, gifted students have a great deal of strategic knowledge and apply diverse strategies to different kinds of problems (Jaušovec,1991; Montague, 1991; Sternberg, 1981). These notable findings suggested that gifted students are flexible, which is one of the components of creativity (Torrance, 1966).

Creativity holds paramount importance in the literature on giftedness and gifted characteristics. As addressed in the conception of giftedness, many researchers

(e.g., Renzulli, 1978; Sternberg, 2003; Winner, 1996) related creativity with giftedness and involved creativity in their conception of giftedness. The review of literature illustrates that gifted students were able to generate numerous and original ideas (Clark, 2002; Clark, 2013; Renzulli & Smith, 1979; Tuttle, 1983), think flexibly (Clark, 2013), and solve problems in novel ways (Winner, 1996), which are essential ingredients of creativity (Torrance, 1966). Due to being an essential ingredient of giftedness, a more thorough investigation of creativity, including definitions, theoretical underpinnings, measurement, and studies with gifted education, is required. Since the present study focused on the gifted students' creativity in mathematics, it would be helpful to review the literature about creativity because it will give the essential groundwork before focusing specifically on creativity in mathematics. Hence, in the following sections, the related literature will be presented.

2.3. Creativity

Similar to the conception of giftedness, there was no commonly accepted definition of creativity. Various definitions and theories about creativity exist in the literature. Those definitions and theories differed in some respects. Therefore, this section covered some essential definitions and theories of creativity.

The interest in creativity began with Guilford's effort in 1950. Guilford (1967) noticed and criticized the lack of research on the nature of creative individuals and the mental processes needed in creative products. Guilford (1950, p. 444) defined creativity as follows: "... refers to the abilities that are most characteristic of creative people. Creative abilities determine whether the individual has the power to exhibit creative behavior to a noteworthy degree". Guilford's pioneering studies paved the way for many researchers to work on creativity in the following years. These studies changed old approaches and beliefs about creativity. Previously, it was believed that creativity is a simple mental process and is innate (Rhodes, 1961). In the upcoming studies, intellectual abilities that affect creativity have been the subject of research. Guilford (1967) categorized those abilities as "divergent

thinking" and "convergent thinking" abilities. More specifically, divergent thinking skills are relevant to idea development and problem-solving, both of which require diversity (Guilford, 1967). On the other hand, convergent thinking is the capacity to choose the greatest option from a variety of viable concepts (Guilford, 1967). Stated differently, based on an initial problem or reference point, divergent thinking is the capacity to investigate a thought space in several different paths (Lubart, 2016). On the one hand, convergent thinking is the capacity to focus on a single concept or solution that best suits the limitations of a task (Lubart, 2016). Guilford (1959) proposed that creative thinking covered four components, which are fluency (the total number of responses), flexibility (the variety of responses), originality (the scarcity of responses), and elaboration (the degree of details about specific actions). Those components are still used today to measure not only general creativity but also domain-specific creativity, i.e., mathematical creativity.

Guilford's research on divergent thinking has served as a crucial foundation for studies on the measurement of creativity. Based on Guilford's divergent thinking components, Torrance (1966) developed the most well-known test for creativity, namely the Torrance Tests of Creativity (TTCT), which was designed to measure divergent thinking skills such as fluency, flexibility, originality, and elaboration. Torrance (1966) defined creativity based on the process:

> A process of becoming sensitive to problems, deficiencies, gaps in knowledge, missing elements, disharmonies, and so on; identifying the difficult; searching for solutions, making guesses or formulating hypotheses about the deficiencies, testing and retesting these hypotheses, possibly modifying and retesting them, and finally communicating the results. (p.8)

Moreover, some researchers (e.g., Boden, 2004; Bruner, 1962; Ochse, 1990; Sternberg & Lubart, 1999) defined creativity by emphasizing the product. They defined creativity as the generation of original and appropriate products or ideas. Stated differently, the commonality of these definitions was that they highlighted the two dimensions of the product to be called creative: novelty and

appropriateness. In fact, Simonton (2011) used the formula of Creativity = Originality x Appropriateness to highlight that both determinants are necessary, and neither is adequate on its own.

Amabile (1983) criticized the product definitions and assessment of products in creativity studies. Hence, Amabile (1983) provided a conceptual definition of creativity focusing on the process and the product: “Creativity can be regarded as the quality of products or responses judged to be creative by appropriate observers, and it can also be regarded as the process by which something so judged is produced” (p. 31). She proposed a componential model of creativity, which was based on two elements and three assumptions. First, the product or the response needs to be novel and appropriate. Second, the task needs to be heuristic, not algorithmic. As addressed above, the novelty and appropriateness of the product were emphasized by many researchers (e.g., Plucker et al., 2004; Simonton, 2012; Sternberg & Karami, 2022). Therefore, the unique element of the componential model of creativity was the heuristic characteristics of the task. According to Amabile (1983), whether a task is deemed "algorithmic" or "heuristic" is determined by the performer's knowledge of the task. The task is regarded as heuristic for that individual if there is an algorithm for solving it, but he or she is unaware of it. Besides, the three assumptions on which the componential model of creativity is grounded are as follows: First, creativity exists on a continuum, and anyone can be creative to some extent in a certain domain. Second, a person can show varying levels of creativity. Third, creative performance can be increased. Based on the two elements and three assumptions mentioned, Amabile (1983) generated a componential model of creativity including three components of creative performance, namely, domain-relevant skills, creativity-relevant skills, and task motivation. Figure 2.3 illustrates the components of creative performance.

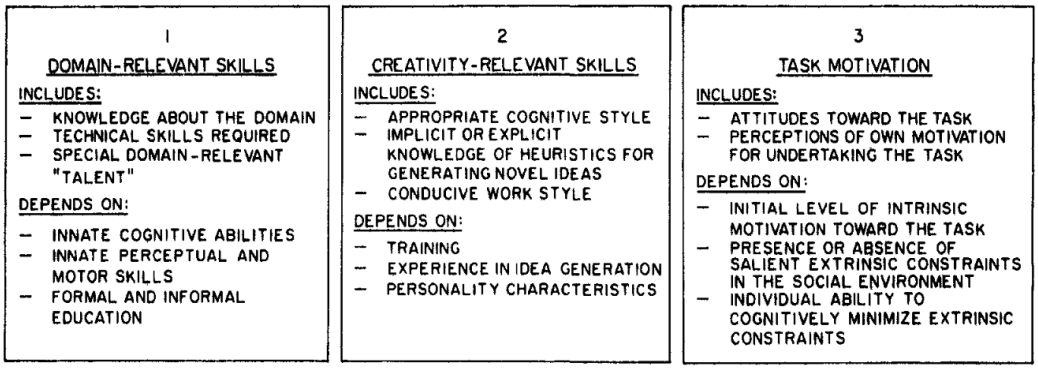


Figure 2.3. Components of Creative Performance (Amabile, 1983, p. 362)

As depicted by Figure 2.3, according to the componential model of creativity, domain-relevant skills, creativity-relevant skills, and task motivation were necessary for creativity to take place. The domain-relevant skills, which are essential for any performance, constitute knowledge about the domain, technical skills required, and special domain-relevant talent. On the other hand, creativity-relevant skills cover appropriate cognitive style, implicit or explicit knowledge of heuristics, and a conducive work style. Lastly, different than other models that have been mentioned so far, the componential model of creativity includes an affective component, task motivation. Task motivation includes attitudes toward the task and perceptions of motivation of self to understand the task. Amabile (1983) claimed that even if the three primary components make up the entire set of general characteristics required for creativity, the list of components within each component may only be completed gradually as creativity research advances.

The Four C model expands on Amabile's point that creativity is a continuum that can be accessed by anyone by classifying creativity into four different levels: mini-c, little-c, Pro-C, and Big-C creativity. Kaufman and Beghetto (2009) proposed the Four C Model of creativity, which was a developmental trajectory of creativity of an individual. There was an ongoing distinction between big-c creativity, which performs at the societal level, and little-c creativity, which functions at the individual level (Glaveanu, 2009; Kaufman & Beghetto, 2009; Simonton, 2013). Different researchers used different names that referred to big-C and little-c

creativity. For instance, Boden (2004) used P-creativity to represent everyday creativity, which refers to little-c creativity, and used H-creativity to stand for creativity for the whole discipline or culture, which refers to big-c creativity. Nevertheless, Beghetto and Kaufman (2007) criticized that little-c is an overly general concept because, according to the definition of little-c creativity, almost everyone is capable of this level of creativity. For that reason, Beghetto and Kaufman (2007) proposed a third c, which is mini-c creativity. In mini-c creativity, an idea needs to be new and appropriate for the person who created it (Kaufman, 2016). Kaufman and Beghetto (2009) argued that there is a lack of a category for people who are professional creators but have not attained a highly renowned rank and do not belong in any suitable category. Therefore, they added a fourth C, which is pro-c creativity, and proposed the Four C model of creativity.

The one-sided/unilateral and narrow definition of creativity was questioned by several researchers (e.g., Amabile, 1983; Rhodes, 1961). Rhodes (1961) criticized the overuse and misuse of the term “creative”, which is only one aspect of the phenomenon, to describe “creativity”. He asserted that regardless of the significance of that aspect, one single aspect alone cannot adequately describe creativity. For this reason, he explored broader, more comprehensive definitions of creativity. In light of his efforts, Rhodes (1961) proposed the "Four P" model of creativity and defined creativity by means of four strands, namely, people, processes, press, and products:

“The word creativity is a noun naming the phenomenon in which a person communicates a new concept (which is the product). Mental activity (or mental process) is implicit in the definition, and of course, no one could conceive of a person living or operating in a vacuum, so the term press is also implicit” (p. 305).

The Four P model of creativity served as a good start (Sternberg & Karami, 2022) and inspired other researchers to propose other theoretical frameworks for creativity. Glaveanu was one of those researchers, and he proposed the 5A model of creativity in 2013. The 5A model covers actor, action, artifact, audience, and

affordances. The actor in the 5A model corresponds to the person in the 4P model, the action corresponds to the process in the 4P model, and the artifact corresponds to the product in the 4P model, but it also incorporates sociocultural context. Nevertheless, audience and affordances were related to the press in the 4P model. The audience in the 5A model was about the relationship between the person's (actor) interactions with other people. Furthermore, affordances mean the relationship between the person (actor) and the material environment (Glaveanu, 2013).

The other theoretical framework, which was proposed by Lubart and Thornhill-Miller in 2019, is the 7C framework. According to the 7C framework, creativity can be conceptualized by the means of 7Cs, which are creators, creating, creations, contexts, collaborations, consumption, and curricula. Creators refer to the person in the 4P model and the actor in the 5A model. Creating refers to the process in the 4P model and action in the 5A model. Context refers to the press in the 4P model. Creations refer to the product in the 4P model and artifacts in the 5A model. Collaborations refer to the gradual procedures that take place when individuals engage in creative products. Moreover, consumptions mean the adoption of creative endeavors. The last component of the 7C model is curricula, and it was not involved in the 4P and 5A models. Curricula refer to the techniques to teach creativity so that people can cultivate it (Lubart & Thornhill-Miller, 2019).

The Four P model of creativity evolved over the years. Many researchers also suggested other p's, such as potential (Runco, 2003), persuasion (Simonton, 1990), and phases (Cropley, 2015). More recently, Sternberg and Karami (2022) extended the Four P model of creativity and proposed the Eight P model of creativity. Before proposing the Eight P model of creativity, they expanded the Four P model of creativity to the 6P framework for intelligence and wisdom (Sternberg and Karami, 2021) by adding two p's, namely, purpose and problem. Then, they extended the 6P framework by incorporating propulsion and public and finalized the Eight P framework of creativity. They explained the rationale behind including propulsion because, unlike intelligence and wisdom, creativity constantly modifies the way

that one thinks. Moreover, they added the public because big-c and pro-c creativity requires public judgment (Sternberg & Karami, 2022).

The studies mentioned so far reveal that creativity is conceptualized in diverse ways in the scope of person, process, and product or combination. Beyond these different approaches, the other enduring debate is about whether creativity is limited to a certain field. Stated differently, there were different views on whether or not creativity is domain-specific or domain-general. Some researchers (e.g., Plucker, 1998) believed that it is a domain-general ability, and regardless of the domain, the creative process is similar. On the other hand, other researchers (e.g., Baer, 2012) argued that it is a domain-specific claiming that creativity in various fields encompasses a range of information, creative thinking abilities, perspectives, or inclinations. In addition to these views, recent studies supported that creativity is both domain-general and domain-specific. More specifically, Sternberg (2010) declared that although there may be certain domain-general aspects to creative potential, an individual must acquire the knowledge and abilities necessary to generate creative endeavors in the specific domain in which one is to do so. Given that mathematical creativity includes unique abilities such as abstract thinking, pattern generalization, proof, etc. (Balka, 1974; Carlton, 1959; Krutetskii,1976; Sriraman, 2004), it is an example of domain-specific creativity. Since this study investigates how creative thinking emerges in the context of mathematics, to better understand and analyze creativity in the particular field that is pertinent to this study, it will be helpful to review the literature on mathematical creativity.

2.4. Mathematical Creativity

Since the present study aimed to investigate the mathematical creativity of gifted students while solving nonlinear realistic generalization tasks, it is important to investigate prior research about how mathematical creativity has been described and assessed. Therefore, to situate the present study within the existing research, this section covers the definition of mathematical creativity, assessment of

mathematical creativity, and the related studies about mathematical creativity and gifted students.

2.4.1. Definition of Mathematical Creativity

Like the study of creativity, the study of mathematical creativity began with one speech. In 1908, an eminent mathematician, Henri Poincaré conducted a speech at the French Psychological Society, and it was regarded as the initial effort to characterize mathematical creativity (Liljedahl, 2008). Poincaré (1948) simply defined mathematical creativity as “choice”. When the history of mathematical creativity is examined, Jacques Hadamard is another mathematician frequently mentioned in addition to Poincaré. Hadamard and Poincaré were influenced by Gestalt psychology (Sriraman, 2004). According to their perspective, creativity is the process where a mathematician chooses between problems that result in something new and those that don't (Sriraman, 2005). They were inspired by the creative problem-solving model of Wallas, which was proposed in 1926 (Sriraman, 2004). According to Wallas (1926), the creative problem-solving process consisted of four stages, namely, preparation, incubation, illumination, and verification. In the preparation stage, the person who is solving the problem is diligently looking for and compiling the information required for the problem (Wallas, 1926). Poincaré named this stage the preliminary period of conscious work, and Hadamard called this stage as preparatory stage (Sririman, 2004). After working diligently on the problem, the problem is purposefully set aside without any conscious effort, which is known as the incubation stage (Wallas, 1926) or the incubatory stage (Hadamard, 1945). After that, an "Aha!" moment, or unexpected insight, happens at the third stage, illumination (Wallas, 1926) or illuminatory stage (Hadamard, 1945), if the incubation stage leads to success. Then, in the last stage, verification, the accuracy of the product is confirmed (Hadamard, 1945; Wallas, 1926).

Although Gestalt psychology influenced several researchers (e.g., Hadamard, 1945; Poincaré, 1948), some researchers (e.g., Ervynck, 1991; Sriraman et al., 2011)

criticized the model because of its drawbacks. More specifically, the model ignores the intriguing process by which the actual questions develop and primarily pertains to problems that mathematicians have stated beforehand. In addition, according to the model, subconscious drives are significantly responsible for the illumination and incubation stage (Sriraman et al., 2011). Ervynck (1991) tried to address the issue of how the problems were developed. He depicted mathematical creativity by the means of three stages, namely, the preliminary technical stage (Stage 0), algorithmic thinking (Stage 1), and creative activity (Stage 1). In the preliminary technical stage, the problem solver applies mathematical rules and procedures with no awareness of the underpinnings. Furthermore, in the algorithmic activity stage, as the name implies, the person performs strategies and algorithms many times. Then, in the creative activity stage, the person applies non-algorithmic decision-making, when the actual mathematical creativity emerges. Sriraman and his colleagues (2011) argued that although the preliminary technical stage and algorithmic thinking stages of Ervynck (1991) partially illustrate the process of mathematicians' reaching a problem, his interpretation of mathematical creativity showed similarity with Poincaré and Hadamard, i.e., all focused on "choice". Sriraman (2004) asserted that the major drawback of these definitions was that they disregarded novelty. In accordance with his criticism, Sriraman (2004) defined mathematical creativity as follows: "the process that results in unusual and insightful solutions to a given problem, irrespective of the level of complexity" (p.20).

Mathematical creativity was studied by mathematicians until the 1950s, but after 1950, researchers in psychology and education became interested in mathematical creativity (Sak et al., 2017). One of the most renowned researchers of this period is the psychologist Krutetskii. Krutetskii (1976) described mathematical creativity in schoolchildren as follows: "The independent formulation of uncomplicated mathematical problems, finding ways and means of solving these problems, the invention of proofs and theorems, the independent deduction of formulas, and finding original methods of solving nonstandard problems". As the definition

implies, Krutetski (1976) defined mathematical creativity within the context of problem-posing and problem-solving. Different than Poincaré's definition of mathematical creativity, Krutetskii's (1976) definition refers to both mathematics and creativity.

When the literature is examined, it is frequently encountered that mathematical creativity is defined in terms of problem-solving and problem-posing. Stated differently, in addition to Krutetskii (1976), many researchers have defined mathematical creativity in the context of problem-solving and/or problem-posing. For instance, Laycock's (1970) definition of mathematical creativity highlights the competency of analyzing a problem in diverse ways, recognizing similarities and contrasts, and noticing patterns. On the other hand, Jensen (1973) described mathematical creativity as the competence to produce various useful problems about mathematical situations in different representations. Besides, Haylock (1987) defined mathematical creativity in terms of two aspects, namely, the capability to overcome fixations while solving mathematical problems and the capability of divergent production. Compared to other researchers, Sriraman's (2005) definition of mathematical creativity involves both problem-solving and problem-posing. Sriraman (2005) defined mathematical creativity as "the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination" (p. 24).

The literature also involves studies examining the characteristics of individuals who possess creative abilities in mathematics. Those studies provide insights into the conceptualization of mathematical creativity. For instance, Carlton (1959) investigated the characteristics of 14 eminent mathematicians who have the capacity for mathematical creativity. Although she could not find a single trait to describe mathematical creativity, she concluded that gifted students with creative abilities in mathematics would exhibit a certain portion of characteristics, such as an intent to enhance a solution's structure or a proof, making connections between concepts and problems, predicting the results of modified hypotheses, enjoy in

dealing with mathematical symbols, tendency to make generalizations by identifying the commonalities, the capacity to understand or visualize an answer or proof intuitively, the conviction that there is always a solution for any problem and persistency.

In addition to Carlton (1959), Balka (1974) also provided six criteria to measure creative ability in mathematics. To identify the criteria, the secondary school mathematics teachers, mathematics educators, and mathematicians were asked to select criteria from the list of 25 criteria, which were used to measure creativity. If at least 80 % of the three groups agreed that it was deemed significant for assessing mathematical creativity, then those criteria were accepted. At the end, based on these responses, Balka (1974) identified six criteria to measure mathematical creativity, which are given below:

> The ability to formulate mathematical hypotheses concerning cause and effect in a mathematical situation (divergent).
>
> The ability to determine patterns in mathematical situations (convergent).
>
> The ability to break from established mindsets to obtain solutions in a mathematical situation (convergent).
>
> The ability to consider and evaluate unusual mathematical ideas, to think through their consequences for a mathematical situation (divergent).
>
> The ability to sense what is missing from a given mathematical situation and to ask questions that will enable one to fill in the missing mathematical information (divergent).
>
> The ability to split general mathematical problems into specific subproblems (divergent).

The mathematical creativity criteria determined by Balka (1974) provided valuable insights into students' creative abilities in mathematics. However, to evaluate students' mathematical creativity, some cognitive components of mathematical

creativity have been emphasized. In the next section, these components will be defined in detail, and how mathematical creativity is assessed in studies using these components will be discussed. Since one of the aims of the present study is to investigate the mathematical creativity of gifted students, having insights into the assessment of mathematical creativity will assist in guiding the study's design and interpretation.

2.4.2. Assessment of Mathematical Creativity

Numerous studies have been conducted to assess the mathematical creativity of an individual. In these studies (e.g., Bicer et al., 2020; Kattou et al., 2013; Kim et al., 2016), the assessment of mathematical creativity was predominantly built on the four components of the divergent thinking model proposed by Guilford in 1959. These components were fluency, flexibility, originality, and elaboration. In the following paragraphs, how each component was described and used to assess mathematical creativity will be described.

Fluency is the number of solutions or ideas produced for a problem (Guilford, 1959; Kim, 2006; Levenson, 2022). This definition of fluency was particularly used in studies about problem-solving. In those kinds of studies, the respondent is asked to solve given problem(s) in as many ways as possible. Then, the problem solver gets a certain point for fluency for each appropriate answer of his/her. However, the key in these studies (e.g., Leikin, 2009; 2018; Jukić Matić & Sliško, 2024) was that the problem is open-ended and has multiple solutions. Several researchers adapted the definition and assessment of fluency based on the purpose of their studies. For instance, in his study with pre-service teachers, in which he aimed to examine the role of multiple representations in enhancing the mathematical creativity of pre-service teachers, Bicer (2021) assessed fluency based on the number of representations generated by pre-service teachers. In this study, each pre-service teacher received certain points for fluency scores for each representation they produced. Besides, in their study, which aimed to investigate the mathematical creativity of sixth-grade students in problem-posing tasks, Sadak

et al. (2022) measured fluency according to the total number of appropriate problems. In this study, each student received a certain point for fluency for each appropriate problem he/she proposed. Compared to those studies, Leikin and Lev (2007) evaluated the fluency of students according to the amount of time the students spent on generating solutions.

The second component of mathematical creativity is flexibility. Flexibility is the total number of categories of responses (Guilford, 1959; Kim et al., 2016; Rubenstein et al., 2019). Compared to fluency, different researchers have defined flexibility with different words. For instance, Wang and Burdina (2024, p.3494) defined flexibility as "the variety of answers". Different than Wang and Burdina (2024), Silver (1997) defined flexibility as follows: "the number of apparent shifts in approaches taken when generating responses to a prompt" (p. 76). However, what was common was that the researchers evaluated flexibility by classifying responses in certain categories. Then, each person receives a certain point for flexibility score based on the total number of categories existing in their appropriate responses.

Originality, which is the third component of mathematical creativity, is the uniqueness or scarcity of the responses (Guilford, 1967; Kim et al., 2016). Different approaches to assessing originality exist in the literature. What is common in these approaches is that researchers (e.g., Kahveci & Akgül, 2019; Kattou et al., 2013; Rubenstein et al., 2019) categorized the responses based on similarities and differences. Then, to determine the originality score, i.e., the uniqueness of the response, they identified the frequency of each category. In the end, the scarcer the response a person gives, the higher the originality score he/she receives. However, their way of identifying the uniqueness of the response shows some variation. More specifically, to determine the uniqueness of a response, some researchers (e.g., Kattou et al., 2013; Leikin, 2009) set certain percentages. For instance, Leikin (2009) considered a response original if it appeared in less than 15% of the responses. Different than Leikin (2009), Kattou and colleagues (2013) identified the solutions that were found in different percentages of given responses.

If the student's answer was found in 1% of the whole responses, then that student receives 1 point for originality. Then, if the student's answer was found between 1% and 5% of the whole responses, then that student receives 0.8 points for originality, etc. They scored the answer in a way that as the percentage of frequency of occurrence within the group increases, the originality scores the student will receive decrease.

On the other hand, to identify the scarcity of answers, some researchers (e.g., Baykal 2009; Kahveci & Akgül, 2019) developed a scoring key for originality based on the rarity of the responses. Table 2.1 represents Baykal's (2009) scoring key.

Table 2.1. Rarity-Based Scoring Key for Originality Scoring (Baykal, 2009, p. 44)

The frequency	The score
1	9
2	8
3-4	7
5-8	6
9-16	5
17-32	4
33-64	3
65-128	2
129-999	1

According to Table 2.1, the more unique the answer is among the answers of the people in the group, the higher the originality score a person receives. For example, in a group of 999 people, if an answer is given by only 1 person, this person gets 9 points for originality. Different researchers (e.g., Kahveci & Akgül, 2019) have adapted this evaluation key to their studies according to the number of people involved in their studies.

Another approach to assess the originality of a response is to use expert evaluation (Kao, 2020; Plucker et al., 2014). For instance, in his study, Kao (2020) asked two raters to assess the novelty of the responses according to a six-point Likert scale. Afterward, the mean of the raters' scores was calculated to identify the originality score for each item.

Different than other studies mentioned, Leikin and Lev (2007) used a different method to assess novelty, i.e., originality. To assess originality, they used three criteria, namely, conventionality, availability, and repetition. Conventionality was assessed by investigating whether the solutions were suggested in the curriculum of students in diverse groups or not. If the solution strategy was suggested by the curriculum, the strategy was considered a conventional strategy, but if it was not, then the solution strategy was evaluated as a non-conventional strategy. Moreover, if the solution strategy was involved in the curriculum but in a different topic, the solution was accepted as a partly non-conventional solution. Furthermore, availability refers to the independent thinking of students. To be more precise, if the students could generate the solution with the help of the hint the researchers provided, then that solution was considered as a potential solution space. Lastly, students are regarded as less creative if they generate repeating solutions.

The fourth component of mathematical creativity is elaboration. The literature involves diverse definitions of elaboration. In simplest terms, Guilford (1967) defined elaboration as the ability to add details. Similarly, Mann (2006) described elaboration as extending the methods. More recently, Blyman et al. (2020) defined elaboration as the ability to describe the mental process in words. Nevertheless, several researchers (e.g., Garaigordobil, 2006; Kozlowski et al., 2019) considered elaboration beyond the ability to add details. More specifically, they defined elaboration as a person's capacity to present justification. To sum up, these definitions highlighted two abilities regarding elaboration: the ability to provide in-depth reasoning behind a response and the ability to give a detailed explanation of an action.

Elaboration is not as prevalently used as the other three components in the assessment of creativity. Stated differently, often the researchers (e.g., Bicer, 2021; Kattou et al., 2013) added the respondent's fluency, flexibility, and originality scores to identify a person's mathematical creativity score. However, as the definition of elaboration implies, elaboration serves as a sign of creative thought as it enables the researcher to have an idea about how the student thinks (Blyman et al., 2020). Therefore, in this study, the mathematical creativity of students will be assessed based on four components, namely, fluency, flexibility, originality, and elaboration.

Researchers (Schindler & Lilienthal, 2017; Silver, 1997) criticized the solely product-oriented assessment of creativity, i.e., focusing only on the final response, and emphasized the value of the process (Joklitsche et al., 2016). Therefore, in the present study, the mathematical creativity of gifted students will be assessed based not only on the product but also on the process, i.e., strategies.

2.4.3. Related Studies about Mathematical Creativity and Gifted Students

In the literature, numerous studies investigated the mathematical creativity of diverse participants from preschool to university and even mathematicians (professionals). As gifted students are known to be creative due to their characteristic features (Chowkase, 2022; Renzulli, 1978; Sternberg, 2021), they have been the subject of several studies on mathematical creativity. Since the aim of this study is to investigate the mathematical creativity of gifted students, studies on the mathematical creativity of gifted students will be presented under this title. Studies on giftedness and mathematical creativity have mainly focused on problem-posing and problem-solving. For this reason, first the studies conducted within the scope of problem-posing, then the studies conducted within the scope of problem solving will be presented.

In 2021, Ayvaz & Durmuş conducted a study to examine the mathematical creativity of 7th-grade gifted students within the context of problem posing. Following their purpose, they conducted a thirty-hour plan, which covered eighteen

problem-posing tasks. They developed the Problem Posing Test based on Stoyanova's (1997) model, including three types of problem-posing situations, namely, structured, semi-structured, and free. The structured problem-posing problem involves a well-defined problem situation and requires students to pose problems about the given situation. Furthermore, in the semi-structured problem-posing type, students were asked to pose problems about a visual or ill-defined situation. Lastly, in the free problem-posing type, no data is given but students are supposed to pose problems about a situation or a topic (Ayvaz & Durmuş, 2021). Fluency was assessed according to the number of meaningful problems posed. To be more precise, each student received 1 point for fluency for each problem they proposed. Besides, for flexibility, each student got 1 flexibility score for each question she/he proposed from each different category. They analyzed gifted students' mathematical creativity in terms of fluency and flexibility. The findings indicate a significant difference between the gifted students' pre-test and post-test scores in terms of fluency, flexibility, and composite creativity. Therefore, Ayvaz & Durmuş (2021) stated that the plan including problem-posing tasks enhanced gifted students' problem-solving and mathematical creativity, which supported the findings of Lewis & Colonnese (2021).

Another study on problem posing and mathematical creativity was the study of Arabacı and Baki (2023), which aimed to compare the mathematical creativity of Turkish gifted and non-gifted middle school students in problem posing. The study consists of 12 middle school students and the researchers collected the data via clinical interviews. Different than Ayvaz and Durmuş (2021), they analyzed the mathematical creativity of students based on three components of creativity, namely, fluency, flexibility, and originality. The results of their study indicated no difference between the fluency of gifted and non-gifted students, but the findings revealed that gifted students' flexibility and originality differed from their non-gifted peers.

Studies focusing on the mathematical creativity of gifted are not limited to problem-posing; instead, numerous research have been carried out to explore the

mathematical creativity of gifted students in problem-solving and comparing gifted and non-gifted students' mathematical creativity in problem-solving tasks. One of the examples of these studies was conducted in 2007 by Leikin and Lev. Leikin and Lev (2007) carried out a study to investigate if and how mathematical creativity is represented by performance on multiple solution tasks. To determine whether and how performance on multiple-solution tasks reflects mathematical creativity, Leikin and Lev (2007) conducted a study with different groups of participants, which were generally gifted students with high achievements in mathematics (Group G), proficient students in mathematics (Group P), and regular students with medium-level mathematics achievement (Group R). There were 6 students in each group, but the grade levels of them were different. To decrease the differences in students' knowledge which stems from mathematics curricula of different levels, they intentionally selected participants from Group G and P from 10th grade students and participants from Group R from 11th grade students. During the individual interviews, students were asked to solve three multiple-solution tasks in as many ways as possible. Within the scope of this study (Leikin & Lev, 2007), they only focused on two algebra tasks, one of which is conventional and the other is non-conventional. They analyzed students' mathematical creativity based on fluency, flexibility, and originality as described in detail above. They found that the differences between the groups were depending on the task. More specifically, gifted students and non-gifted proficient students differed in the non-conventional task while showing similarity in their solutions to the conventional task. Yet, gifted students (Group G) and non-gifted proficient (Group P) students differed from regular students (Group R) in all aspects. Stated differently, gifted students (Group G) got higher scores than both proficient and regular students. In the end, Leikin and Lev (2007) claimed that non-conventional multiple-solution tasks are useful ways to examine school children's mathematical creativity.

In the following years, Leikin and Lev (2009; 2013) continued to conduct successive studies about mathematical creativity over multiple solution tasks with different groups of students. In their 2013 study, Leikin and Lev investigated the

mathematical creativity of three groups of 11th and 12th-grade students in different types of multiple-solution tasks. The purpose of the study was twofold: to examine the relationships between mathematical creativity, general giftedness, and mathematical excellence and to analyze the role of multiple solution tasks in detecting the differences between groups in terms of mathematical creativity. In this study (Leikin & Lev, 2013), the groups were: The G group consists of generally gifted students, The HL group consists of students who receive high-level mathematics instruction, while the RL group consists of students who receive regular-level instruction. The participants were expected to solve the test including five problems. Different than their previous study (Leikin &Lev, 2007), in their study in 2013, the problems varied not only in terms of conventionality but also in terms of the topic and complexity. Yet, similar to the previous study (Leikin & Lev, 2007) the mathematical creativity of students was analyzed in terms of three components, namely, fluency, flexibility, and originality. However, the assessment of these components was different. Leikin and Lev (2007) assessed fluency according to the number of appropriate solutions instead of the amount of time spent on the solution. Moreover, to assess the flexibility they identified the different solution groups. More specifically, they identified two solutions as different if they involve different representations, different properties, or different branches of mathematics. To assess the originality, they set different scores for solutions based on the frequency of a solution in a group. The findings indicated that gifted students have higher performance on all criteria than the high-level mathematics instruction group and high-level mathematics instruction students have higher performance on all criteria than the regular level instruction group. This result supported the results of the previous study (Leikin & Lev, 2007). Similar to the previous results, they found that the findings were dependent on the tasks. To be more precise, giftedness significantly affects the three components of creativity in problems, which promote insight-based solutions and enable students to use non-algorithmic thinking. However, giftedness did not influence the creativity components in algorithmic and less complex tasks (Leikin & Lev, 2013).

In light of the findings of these studies, it can be said that non-algorithmic multiple-solution tasks are effective tools to assess gifted students' mathematical creativity.

Different than the studies of Leikin and Lev (2007; 2013), Kahveci and Akgül (2019) investigated the interaction between creativity and intelligence by analyzing gifted and nongifted middle school students' mathematical creativity. To collect data, they used a survey and Mathematical Creativity Scale, including five items created by Akgul and Kahveci (2016). They analyzed fluency, flexibility, and originality. The findings demonstrated that gifted students, who have an IQ above 130 and participate in a special education program, had higher scores on mathematical creativity than general education students whose IQ is below 130.

Compared to the aforementioned studies, which focus on individual creativity, Şengil Akar and Yetkin Özdemir focused on group creativity in their 2022 study. In this case study, they investigated mathematically gifted middle school students' mathematical creativity as a group during a model-eliciting activity. The model eliciting activity is a Quilt problem in which students were provided a visual of a quilt with the actual lengths and asked to identify the true length of a motif in the given visual of a quilt by using proportional reasoning and generating the pattern of the motif (see Şengil Akar & Yetkin Özdemir, 2022, for more details). The participants of the study were two 7th-grade and two 8th-grade gifted students. Şengil Akar and Yetkin Özdemir (2022) analyzed mathematical creativity based on the process and products. To analyze the process, they focused on fluency, flexibility, graduality, making connections, and constructing. Nevertheless, to analyze the product, they assessed its quality and originality. The findings suggested that the gifted students' cooperative efforts promote their creative thinking (Şengil-Akar & Yetkin-Özdemir, 2022).

In contrast to other studies, recent findings from Assmus and Fritzlar (2022) extended the insights by investigating the primary grades gifted students' mathematical creativity. In their study, Assmus and Fritzlar (2022) focused on the mathematically gifted and non-gifted students' mathematical creativity in patterns

and structures. They conducted interviews with 24 third graders. In those interviews, with the same-sized wooden cubes, pupils were supposed to make as numerous distinct figural patterns as they could, with the first two figures being assigned the numbers 1 and 5. Then, they identified 14 gifted students who got the highest results and 10 students in the lowest group as a comparison group. They focused on fluency, flexibility, and originality to assess the mathematical creativity of the participants. The findings clearly illustrated a high correlation between mathematical creativity and mathematical giftedness. More specifically, although the comparison group's students were also high performing in mathematics, they did not generate any original products, and one was considered to have low flexibility. On the other hand, except for one student, the other mathematically gifted students generated more than one novel pattern, and all had at least moderate flexibility (Assmus and Fritzlar, 2022).

In addition to the pattern creation, Eraky et al. (2022) carried out a study to examine creativity-directed problem-solving of experts in mathematics and gifted students in multiple solutions pattern generalization tasks varying in complexity and representation. 298 students attending two distinct higher education institutions were selected and divided into four groups: generally gifted students with excelling in mathematics (G-EM), generally gifted students not excelling in mathematics (G-NEM), not generally gifted but excelling in mathematics (NG-EM), not generally gifted not excelling in mathematics (NG-NEM). The students were expected to solve different pattern generalization tasks, which were numerically, contextually, and diagrammatically represented. The fluency, flexibility, and originality of the students were assessed. The findings demonstrated that giftedness significantly influences all three components of creativity, regardless of the representation of the task. Besides, the findings indicated that task representation had a significant effect on students' creativity and achievement. More specifically, students have the highest level of creativity and success in multiple solution pattern generalization tasks, which were contextually represented, while the lowest level of creativity and success was in numerically represented multiple solution pattern generalization

tasks. The other notable result of this study was that giftedness has an essential role in solving problems that are unfamiliar and require insight. Lastly, Eraky et al. (2022) demonstrated that there was a significant interaction between excellence in mathematics and giftedness while solving multiple solution pattern generalization tasks. In the end, Eraky et al. (2022) recommended incorporating multiple solution pattern generalization tasks to promote the mathematical creativity and mathematical excellence of students.

Similar to Eraky et al. (2022), Singer and Voica (2022) highlighted the importance of pattern generalization tasks by stating that the pattern-generalization tasks itself was creativity-directed. Since creativity is considered a part of giftedness, and generalization is the essential aspect of creative problem-solving, the present study focused on the strategies and mathematical creativity of gifted students while solving nonlinear realistic tasks. To understand the nature of generalization tasks and strategies used in the present study, it was necessary to understand what generalization means, the alternative types of generalization tasks, and the generalization strategies of students reported in the literature. Therefore, in the following sections, these will be covered.

2.5. Generalization

In simplest terms, generalization refers to a procedure for using a particular argument in a more comprehensive setting (Harel & Tall, 1991). Although generalization is not only applicable in mathematics (Harel & Tall, 1991), many researchers stressed the value of generalization in mathematics teaching and learning. For instance, Mason (1996) used the phrase “heart of mathematics” to describe generalization. Kaput (1999) defined generalization as a process of reasoning extending beyond specific examples to identify the connections between particular situations. Although it relied on the same idea as Kaput’s (1999) definition of generalization, Sriraman (2003, p. 155) provided a broader definition of generalization as:

> Generalization is a process by which students derive or are induced from particular cases. It included identifying commonalities in the structure of the problems and their solutions, making analogies, and specializing from a given set of objects to a smaller one.

Several researchers (e.g., Radford, 2008; Rivera, 2010) proposed a framework to describe the generalization process. Radford (2008) provided a framework to unfold the generalization process. Radford (2008) indicated that the generalization process consists of three stages. The first stage involves noticing the commonalities, the second stage involves expanding the repeated process to distinct terms, and the third stage involves forming a general rule to describe any term (Radford, 2008). On the other hand, Rivera (2010) described the generalization process in terms of two stages: abductive-inductive actions and symbolic actions. Abductive-inductive action means utilizing different counting strategies in an algebraically meaningful way, whereas symbolic action means shifting in algebraic generalization form (Rivera, 2010).

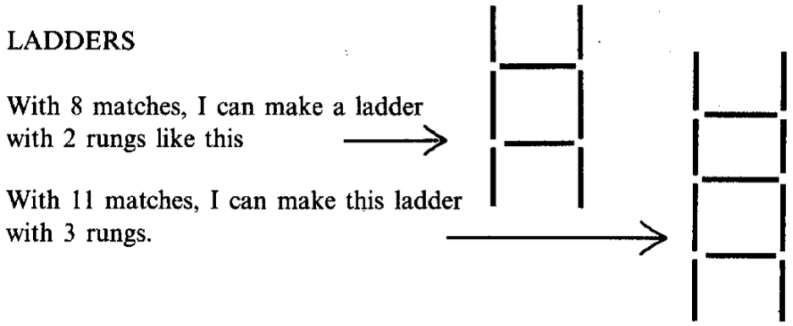
Generalization is the process of recognizing, demonstrating, and representing the task's underlying regularity that links the quantities involved (Ureña et al., 2024). Representing is one of the fundamental components of generalization, which is one of the essential components of algebraic thinking (Kaput, 2008). This type of thinking and generalization is not limited to symbolic representation (Radford, 2018). In addition to symbolic representation, the literature involves other kinds of representation. One of the well-known and frequently used representation frameworks was proposed by NCTM in 2014. NCTM (2014) listed five types of mathematical representations, which are visual, symbolic, verbal, contextual, and physical. Visual representation refers to showing ideas by using pictures, number lines, graphs, and drawings. Moreover, the second type of representation is symbolic representation, which includes the use of numerals, variables, equations, and symbols. The third type of representation is verbal representation, and it entails the use of language. Furthermore, the fourth type of representation, contextual

representation, covers the use of real-life or imaginary situations. Lastly, the physical representation includes the use of concrete materials (NCTM, 2014).

The definitions and frameworks provide a theoretical background about the process of generalization. However, the process might show variation depending on the problem type. For that reason, some researchers (e.g., Chua & Hoyles, 2014; Girit-Yıldız & Durmaz, 2021; Stacey, 1989; Syawahid & Prahmana, 2024) reported different types of generalization tasks. In the following part, related literature about generalization problems will be presented.

2.5.1. Generalization Problems

Lee and Wheeler (1987) defined generalizing problems as problems that may be solved by investigating specific instances, organizing the data, identifying patterns, and applying them to reach a solution. Stacey (1989) was one of the researchers to categorize pattern generalization tasks, and she differentiated between near and far generalization tasks. Near generalization tasks are tasks that require finding the next elements by counting and drawing. On the other hand, far generalization tasks include the identification of the general rule and extend beyond step-by-step ways (Stacey, 1989). The Ladders task (Stacey, 1989) is given below in Figure 2.4 to clarify the near and far generalization problem.



How many matches are needed to make the same sort of ladder with 4 rungs?
How many matches are needed to make a ladder with 5 rungs?
I know that it takes 335 matches to make a ladder with 111 rungs. How many matches would be needed to make a ladder with 112 rungs?
How many matches would you need to make a ladder with 20 rungs?
How many matches are needed for a ladder with 1000 rungs?

Figure 2.4. The Ladders Problem (Stacey, 1989, p. 148)

This task involves both near and far generalization problems. More specifically, the first two problems could be classified as near-generalization problems because these problems require extending the patterns to nearby values and drawing or counting can be employed to solve it. However, the last question can be classified as a far generalization task because it requires extending the pattern to a distant case where the use of drawing and counting was not effective.

Another categorization of pattern generalization tasks was based on the representation of the pattern in the task. More specifically, the review of the literature (Chua & Hoyles, 2014; Girit-Yıldız & Durmaz, 2021; Syawahid & Prahmana, 2024) revealed figural, numerical, and geometric pattern generalization tasks. Syawahid & Prahmana (2024) classified the pattern generalization tasks more broadly; namely, the figural generalization tasks and non-figural generalization tasks. In figural generalization tasks, the pattern was represented in a visual context. On the other hand, non-figural pattern generalization tasks did not involve any pictorial representation (Syawahid & Prahmana, 2024). The famous problem, the Table problem, which was given in Figure 2.5 below is an example of

a figural pattern generalization task because it involves a visual that represents the pattern.

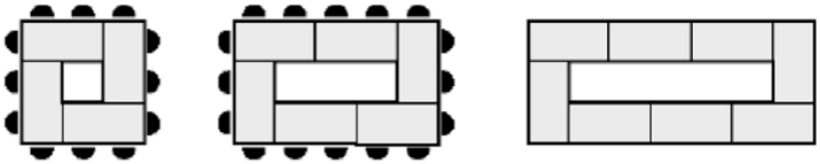
Arranging Tables

A company supplies tables for business meetings.

Each table is a rectangle, and can seat one person on its short edge, and two people on its long edge, like the following figures.

The diagrams below show how these tables can be made into arrangements for different numbers of people. The different arrangements are numbered, like the figures below.

No one sits inside the arrangements of tables.



Size 1
4 tables
12 people

Size 2
6 tables
16 people

Size 3

(1) How many people can sit at a Size 3 arrangement?

(2) How many people can sit around a Size 13 arrangement? Explain how you know your answer is correct.

(3) Write an equation for the number of people *p* who can sit at a Size *S* arrangement. Explain how the parts of your equation relate to the table arrangements.

(4) James is trying to seat 75 people. What size arrangement will he need? Write an explanation that would convince a classmate your answer is correct.

Figure 2.5. Table Problem as an Example of Figural Generalization Task (Lepak &Avieko, 2018, p. 63)

On the other hand, The Plant Growth problem developed by Syawahid and Prahmana (2024) and represented in Figure 2.6 illustrates a non-figural generalization task as it does not involve any visual.

Plant Growth

A researcher was observing the growth of a plant. On first day (H1), the plant height was 4 cm, on second day (H2), the plant height was 6 cm and on the third day (H3) the plant height was 8 cm.

1. Determine the plant height on 4th days (H4) and 5th days (H5)!
2. Determine the plant height on 7th days (H7) and 10th days (H10)!
3. What can you notice about the structure of the plant's growth every day? Write down how you got the height of the plant (T) on a certain day (H).
4. On which day the plant height was 86 cm? Explain how you got your answer!

Figure 2.6. The Plant Growth Problem (Syawahid & Prahmana, 2024, p.98)

In addition to these problem types, generalization problems have also been classified according to the kind of function that underlies the pattern. More specifically, the literature involves linear and nonlinear categorization of generalization problems. The linear generalization problems are described by the means of the linear function, in which the first difference is constant, and the terms are expressed as follows: $f(n)= an + b$, with $b \neq 0$ (Stacey, 1989). On the other hand, a nonlinear generalization problem involves a function that is not linear (Strogatz, 2015), i.e, cannot be described in the form of $f(x) = ax + b$. Common examples of nonlinear terms are powers (e.g., x^2), products (e.g., xy), and other functions of x (e.g., sin(x)) (Strogatz, 2015).

Several studies (e.g., Akkan et al., 2017; Girit-Yıldız & Durmaz, 2021; Stacey, 1989; Syawahid & Prahmana, 2024) were conducted to explore students' generalization strategies in different generalization tasks. Since the present study focused on strategies of gifted students while solving nonlinear realistic generalization tasks, it would be helpful to review the generalization strategies noted in the broader literature. In the following section, those strategies will be described.

2.5.2. Generalization Strategies

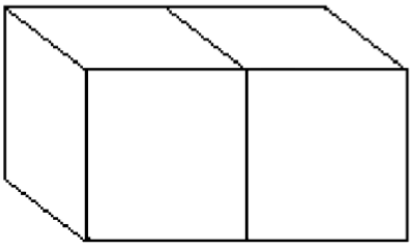
Since generalization was an essential ability, numerous studies were carried out to examine the generalization strategies of different participants. In light of the review of the literature, those strategies can be summarized as follows: *recursive*,

functional, *counting*, *chunking*, *whole object*, *difference* and *guess and check* strategies.

Recursive strategy is an additive strategy in which the person identifies the pattern by relating the term with the previous one, i.e., identifying the change in the dependent variable. (Barbosa & Vale, 2015; Stacey, 1989). For instance, in a task generated by Blanton and Kaput in 2011, students were asked to describe the total number of eyes for an increasing number of dogs. If students are searching for a pattern like "*count by 2's*" or "*add 2 each time*", this is an example of recursive patterning (Blanton & Kaput, 2011) because this means that students focused on the change in the output values (the number of eyes), not the relationship between two quantities (the number of dogs and the number of eyes).

On the other hand, the functional or explicit strategy involves the identification of general rules (Akkan et al., 2017; Barbosa & Vale, 2015) and it focuses on the relationship between the independent variable and the dependent variable(s) (Lannin et al., 2006). For instance, in the Cube Sticker Problem given in Figure 2.7, finding the four rows of stickers that make up the rod's length (4n) and realizing that two more stickers are needed for the ends could result in the explicit rule $S = 4n + 2$, where S is the number of stickers needed for a rod of length n (Lannin et al., 2006). In that rule, the relationship is based on the number of cubes (input values) and the number of stickers (output values).

A company makes colored rods by joining cubes in a row, using a sticker machine to place stickers on the rods. The machine places exactly one sticker on each exposed face of each cube. Every exposed face of each cube must have a sticker, so this length two rod would need 10 stickers.



1. How many stickers would you need for rods of length 7? Length 10? Length 49? Explain your reasoning,
2. Explain how you could find the number of stickers needed for a rod of any length.

Figure 2.7. Cube Sticker Problem (Lannin et al., 2006, p. 300)

The other strategy, which is frequently mentioned in the literature, is counting or counting from a drawing strategy, which means reaching generalization by counting the elements of the figure (Barbosa & Vale, 2015).

Besides, chunking is another strategy that refers to the way of using the difference between the terms and the difference between the number of steps to identify the desired term (Erdoğan & Gül, 2023). To be more precise, in their study with eighth-grade gifted students, Erdoğan and Gül (2023) used The Growing Geometric Pattern Task given in Figure 2.8 and asked students to identify the number of white balls in the 5th figure. Using the chunking approach, a student first determined how many steps there were between the first and fifth figures (which is 4). Then, using the original number of balls down one side (beginning with 3), the student summed the difference he/she got (which is 4) to get 3+4=7. The student interpreted the figure as a square, found the total number of balls by taking the square of this side length (7^2 = 49), and then subtracted the number of black balls (5^2= 25) to discover the number of white balls (24) (Erdoğan & Gül, 2023). This implies that students find the local differences (the difference between the number

of steps between the first and the targeted figure), then based on this difference, make necessary adjustments to find the number of elements.

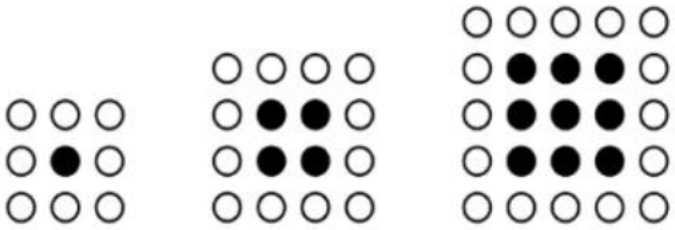


Figure 2.8. The Growing Geometric Pattern Task (Erdoğan & Gül, 2023, p. 372)

Whole-object strategy is another way of reaching a generalization in which the person treats a term as a unit and utilizes multiples of it (Barbosa & Vale, 2015). For instance, Stacey (1989) described the whole object strategy as assuming that M (m, n) = m x M(n), or taking a multiple of the number of matches needed for a smaller ladder in the Ladders problem (see Figure 2.4). More precisely, students using the whole-object strategy realized that 20 equals four times five, therefore, they reasoned that M (20) could be determined by multiplying 17 by 4, which would provide 68, given that M (5) = 17 (Stacey, 1989).

Another generalization strategy is a difference strategy, which concentrates on the differences between consecutive outcomes (Stacey, 1989). According to Stacey (1989), some students suppose a general rule of the type M(n) = d ×n, where d is the common difference, since they recognize the repeated addition of a constant quantity. For instance, students can conclude that the total number of matchsticks is just three times the number of rungs when they see that each more rung requires three more matchsticks (Stacey, 1989).

The last generalization strategy was guess and check or local tactics (Mason, 1996), which is a way of identifying of general rule by guessing a rule and testing the accuracy by attempting several input values (Barbosa & Vale, 2015; Syawahid & Prahmana, 2024).

2.5.3. Related Studies about Generalization and Gifted Students

Due to being a pivotal concept, numerous studies were conducted to examine the generalization process and strategies of students at various grade levels. However, although gifted students are referred to as students with generalization ability (Balka, 1974; Carlton, 1959; Krutetskii, 1976), the number of studies specifically focusing on gifted students' ability to generalize is limited (Erdoğan & Gül, 2023). However, those limited number of studies provides valuable insights into gifted students' ability to generalize, the process of generalization, and the generalization strategies. Since one of the purposes of the current study was to investigate the generalization strategies of gifted students, the related international and national studies will be presented in the upcoming section.

2.5.3.1. International Studies about Generalization and Gifted Students

Several studies were carried out to explore generalization strategies and the process of gifted students. These studies vary in terms of the grade level of the students and the types of problems used in the research. One of the pioneer studies about the generalization of gifted students was conducted by Krutetskii in 1976. In his study, he compared the gifted and non-gifted students' ability to generalize across several experiments. Krutetskii (1976) found that gifted students were capable of forming generalizations in a more rapid and broad way and identifying the overall structure before solving the problems, unlike "average" and "incapable" students.

Kruteskii's (1976) work paved the way for other researchers to conduct studies about the gifted and non-gifted students' generalization ability. In the following years, Sriraman (2003) investigated ninth-grade students' formulation of generalizations and abstractions while solving five non-routine combinatorial problems based on the pigeonhole (Dirichlet) principle. Sriraman defined the pigeonhole principle as "if m pigeons are put in n pigeonholes and if m > n, then at least one pigeonhole will have more than one pigeon" (p. 154). The participants were 9 ninth-grade students, where four of them identified as mathematically gifted. Sriraman was also a full-time teacher of the participants. He integrated

journal writing into the algebra course and consistently asked one nonroutine problem each week and asked students to solve the problem in their journals. While doing so, he expected students to write down all the work they had done, restate the problem, identify what was given and what was asked in the problem, describe their initial attempt to solve the problem and reflect on what went well and what did not. Within the scope of the study, Sriraman (2003) asked the participants to solve five combinatorial journal problems in 7 to 10 days and provide reasoning behind their solutions. Findings revealed that gifted students were able to recognize and formulate the generalization and abstracting of the commonalities in tasks' structure, yet the non-gifted students couldn't notice the generality. The findings confirm the result of Krutetskii's (1976) study, that is, there is a relationship between giftedness and generalization ability.

In addition to student-based comparisons, such as those conducted by Krutetskii (1976) and Sriraman (2003), problem-based comparisons are also encountered in studies focusing on the generalization skills of gifted students. More specifically, in 2008, Amit and Neria explored gifted pre-algebra (sixth and seventh grade) students' generalization strategies in linear and non-linear generalization tasks. In their study, students were expected to solve three daily life pattern tasks, one of which was pictorial-linear, the other one was pictorial non-linear, and the last one was a verbal-nonlinear task. Amit and Neria (2008) stated that data analysis of students' solutions for these tasks indicated two strategies, namely, recursive-local and functional-global. They pointed out that, in the pictorial linear task, all participants started with the use of the additive method. Then, some students continued with a multiplicative approach to reach global generalization. On the other hand, others did not move to the multiplicative approach, instead, they continued to employ additive strategies. These gifted students couldn't answer the pictorial linear task or use recursive generalization. Furthermore, in the pictorial non-linear task, the students used visualization primarily. Lastly, while solving the verbal nonlinear task, they showed a high level of thinking (Amit & Neria, 2008).

More recently, in 2018, Gutierrez and colleagues investigated a nine-year-old gifted student's strategies to solve linear geometric pattern problems in a teaching unit. Gutierrez et al. (2018) reported that the student could solve the problems correctly by using an accurate formulation of the general rule, although he did not have any prior experience in solving geometric pattern tasks before teaching the unit. Gutierrez et al. (2018) interpreted this finding that the ability to recognize patterns and structures and generalize were characteristics of gifted students. Moreover, the findings (Gutierrez et al.,2018) showed that the student used a functional strategy while solving the majority of the problems but used a recursive strategy to find the near term in one problem. Gutierrez et al. (2018) claimed that gifted students were able to shift their strategy from a simple strategy to an efficient strategy flexibly. This finding was aligned with the findings of prior and subsequent studies (e.g., Assmus & Fritzlar, 2022; Erdoğan & Gül, 2023; Jaušovec, 1991; Montague, 1991; Syawahid & Prahmana, 2024).

The comparison of gifted students' generalization processes was not limited to linear and nonlinear tasks. A recent study conducted by Syawahid and Prahmana (2024) examined the functional thinking of two fifth-grade gifted students while solving figural and nonfigural linear pattern tasks. The figural task was a Table task, and the nonfigural task was a Plant Height Growth task, which was described above (see Figure 2.5 and Figure 2.6). Syawahid and Prahmana (2024) noted that gifted students used different strategies while solving the figural linear pattern task: counting, multiplicative method, contextual strategy in near generalization, contextual strategy in far generalization, and multiple difference strategy in formal generalization. On the other hand, while solving the non-figural linear pattern task, the students utilized multiple difference strategies in near, far, and formal generalizations. Like other researchers (e.g., Assmus & Fritzlar, 2022; Erdoğan & Gül, 2023; Gutierrez et al., 2018; Jaušovec, 1991; Montague, 1991) Syawahid and Prahmana (2024) also highlighted the mental flexibility of gifted students by changing one solution to another.

2.5.3.2. National Studies about Generalization and Gifted Students

Compared to the international studies, studies focusing on the generalization of gifted students were limited in Türkiye. One of those studies was conducted by Girit-Yıldız and Durmaz in 2021. Similar to Amit and Neria (2008), Girit-Yıldız and Durmaz (2021) focused on the generalization strategies of gifted students while solving linear and non-linear tasks. Yet, different than Amit and Neria (2008) who worked with middle school students, the participant of Girit-Yıldız and Durmaz's (2021) study was a 10th-grade gifted student. They used the matchsticks problem context, which was a well-known geometric pattern problem reported by Gierdien (2012), in which the student was expected to find the sum of matchsticks involving forming pentagons, squares, and triangles. The findings of the study of Girit-Yıldız and Durmaz (2021) highlighted a difference between the generalization approaches of gifted students in linear and nonlinear patterns. In linear pattern tasks, the gifted student used figural reasoning, while in nonlinear patterns, he used numerical reasoning. The student was informed about Gauss's approach by one of the authors of the study, who is also a teacher of the student at the Science and Art Center he attended. Then, while solving the nonlinear pattern task, he used this approach to identify the general rule. One of the noteworthy findings of Girit-Yıldız and Durmaz (2021) was that gifted students' generalization had two phases in both linear and nonlinear tasks. More specifically, he utilized abduction first to investigate the relation between figures with counting the corresponding increase while solving the problem involving the linear pattern. After that, he moved from a verbal relationship that he found to an algebraic version. Similar to the linear pattern, in a nonlinear pattern, the student utilized the numbers given and investigated the relationships between steps, and at the end, he expressed the general rule algebraically.

Similar to Gutierrez et al. (2018) and Girit-Yıldız and Durmaz (2021), Erdoğan and Gül (2023) studied the generalization strategies of gifted students while solving geometric pattern tasks. Five 8th-grade gifted students made up the participants. They were expected to solve linear and quadratic growing geometric pattern tasks,

including problems asking for the identification of immediate, near, and far terms and the general rule. Compared to the findings of Girit- Yılmaz and Durmaz's study (2021), which reported that a gifted student used figural reasoning in the linear task and numerical reasoning in the nonlinear task, Erdoğan and Gül (2023) illustrated that gifted students used figural and numerical methods to solve both linear and nonlinear problems. However, while solving the nonlinear pattern task, gifted students utilized a functional strategy. Nevertheless, in the linear pattern task, the students use diverse strategies such as recursive, chunking, contextual, and functional strategies.

2.6. Summary of the Literature

The literature review covered the conception of giftedness, characteristics of gifted students, creativity, mathematical creativity, generalization, generalization strategies, and related studies about gifted students and mathematical creativity and gifted students and generalization.

Giftedness evolved over the years from a more conservative approach to a more inclusive one (Renzulli, 2002). Despite no common definition of giftedness existing in the literature, today there is a consensus that general intelligence does not provide the full picture and that gifted students have distinct characteristics that distinguish them from their same-aged peers (Marland, 1972; NAGC, 2019; Renzulli, 1978). Several researchers (e.g., Chowkase, 2022; Renzulli, 1978; Sternberg, 2003) considered creativity as one of these distinguishing characteristics.

Due to being a multifaceted concept, like giftedness, a commonly agreed-upon definition of creativity does not exist. Regardless of the different focus in the definitions, the majority of the research indicated the importance of appropriateness and novelty in the conceptualization of creativity. Therefore, in the present study, the creativity of gifted students will first be examined based on the appropriateness criteria. One ongoing debate is about whether creativity is domain-general or domain-specific. Research (Balka, 1974; Carlton,1959; Krutetskii,1976; Sriraman,

2004) shows that mathematical creativity involves specific mathematics-related abilities such as solving problems in novel ways, making generalizations, and generating proof, therefore, it is domain-specific. This study adopts a domain-specific approach to creativity, specifically in the context of mathematical creativity, because its goal is to examine creativity in the context of generalization tasks.

Review of the literature indicated that creative talent can be developed with the help of appropriate instruction (Newton & Newton, 2014; Sheffield, 2009). Researchers (Bicer, 2021; Lekin, 2009; Shriki, 2008) highlighted that nonroutine, open-ended, multiple-solution problems that are related to real life are the features of creativity-directed tasks, and not all problems are appropriate to investigate mathematical creativity (Leikin, 2009). Generalization tasks stand out as an appropriate tool to investigate mathematical creativity (Assmus and Fritzlar, 2022; Eraky et al., 2022; Singer & Voica, 2022). Although review of the literature shows that generalization ability is one of the characteristics of gifted students (Krutetskii, 1976) and generalization tasks are an effective tool to investigate mathematical creativity (Assmus and Fritzlar, 2022; Eraky et al., 2022; Singer & Voica, 2022), there was limited research combining mathematical creativity and giftedness through generalization tasks (Eraky et al., 2022). Therefore, in light of the review of the literature, the present study aimed to investigate the mathematical creativity of gifted students through generalization tasks that were designed based on the needs of the gifted students and features of the creativity-directed tasks.

While studying mathematical creativity and gifted students through generalization tasks provides useful theoretical insights, understanding how generalization occurs in practice is equally important. To better understand the generalization process, it is crucial to look at the strategies students employ since these strategies show how they think and what conceptual connections they make while generalizing. Even though several studies (e.g., Girit-Yıldız & Durmaz, 2021; Gutierrez et al., 2018) were conducted to explore generalization strategies of gifted students, those studies mainly focused on linear generalization tasks (Girit–Yıldız & Durmaz, 2021).

However, nonlinear generalization tasks are more challenging due to their structural complexity (Strogatz, 2015). Since nonlinear tasks are inherently challenging (Strogatz, 2015), they may present abundant opportunities for challenge. This makes them especially appropriate for gifted students, as the review of the literature shows that gifted students demand challenging tasks (Özdemir & Işıksal-Bostan, 2021). Stated differently, gifted students may be challenged to participate more fully in these tasks, which could provide a more insightful perspective on gifted students' generalization process and their creative potential. Therefore, the current study focused on gifted students' mathematical creativity and strategies while solving different types of nonlinear realistic generalization tasks.

One of the key elements of generalization, which is a crucial part of algebraic reasoning, is representing (Kaput, 2008). The literature (Assmus & Fritzlar, 2022; Aygün, 2010) shows that gifted students are capable of generating several representations, and they need to be supported by the tasks that enable gifted students to use and interpret diverse representations (Aygün, 2010). However, despite its relevance, the number of studies investigating the types of representations gifted middle school students adopted while solving generalization tasks, especially nonlinear generalization tasks, is limited. Therefore, there is room to examine the types of representations that gifted students employed while solving nonlinear generalization tasks. Hence, the present study also aimed to investigate the representations gifted middle school students use while solving nonlinear generalization tasks.

CHAPTER 3

METHODOLOGY

The present study aimed to examine 8th-grade gifted students' solution strategies and mathematical creativity while solving different types of nonlinear realistic generalization tasks. In this regard, this chapter consisted of eight sections, namely, the research design, the context of the study, participants of the study, data collection, data analysis, trustworthiness, researcher's role and bias, and ethics, respectively.

3.1. Research Design

The present study aimed to provide an in-depth investigation of the strategies and mathematical creativity of gifted students while solving alternative types of non-linear realistic generalization tasks. Following this aim, the current study used qualitative research methodology as qualitative research offers a complex and elaborated investigation of the problem or phenomenon that needs to be further investigated (Creswell, 2013; Patton, 1985).

Over the years, many researchers have provided diverse ways to conduct a qualitative inquiry in various disciplines (e.g., Cresswell, 2013; Denzin & Lincoln, 2011; Saldaña, 2011). From those researchers, Creswell (2013) suggested five different systematic approaches for qualitative inquiry among those that are more common in social, behavioral, and health science literature. Those approaches are narrative research, phenomenology, grounded theory, ethnography, and case study. Narrative research is centered on examining a few people, collecting information from their stories, and recounting and arranging the significance of personal experiences. Nonetheless, phenomenology explains the collective significance that multiple people have of their experiences of a subject or phenomenon, as opposed

to a narrative study that describes the experiences of a few people. Different from narrative and phenomenological research, which are descriptive, grounded theory requires the researcher to generate a broad explanation of a process, an activity, or an interaction tailored to the opinions of numerous respondents. Furthermore, in ethnography, the researcher explains and analyzes the common features of the core elements (language, value, etc.) of a group with a common culture. Lastly, the case study was defined comprehensively by Cresswell (2013, p.97) as follows:

> Case study research is a qualitative approach in which the investigator explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audiovisual material, and documents and reports), and reports a case description and case themes.

A review of the literature reveals various definitions of case studies. Specifically, Atkins and Wallace (2012) defined a case study as a small-scale exploration of a research question. Moreover, Stake (1995, p. xi) provided an inclusive definition of a case study as "The study of particularity and complexity of a single case." Case studies were frequently adopted in educational research as they offer a means of meditating on both the detail and the whole while also looking into relationships, trends, and situations (Atkins & Wallace, 2012).

Different researchers offered different categorizations of types of case studies. For instance, Yin (1993) provided three types of case studies, which are exploratory, explanatory, and descriptive. The first type of case study, exploratory studies, aims to determine the questions of a follow-up study. On the other hand, an explanatory case study explains which causes led to which results. Lastly, descriptive studies provide a thorough explanation of a phenomenon about its settings (Yin, 1993). Contrary to Yin's (1993) categorization, case studies were categorized by Stenhouse (1988) according to several facets of instructional action, namely, evaluative, educational, and action research case studies. Cresswell (2013)

classified case studies into three types, namely, single instrumental case study, collective case study, and intrinsic case study. The first one, a single instrumental case study, involves an examination of a certain issue, where one bounded case is chosen to show a particular issue (Creswell, 2013; Stake, 1995). Nevertheless, in collective or multiple case studies, the researcher utilized deliberately selected cases to show the issue. Lastly, in intrinsic case studies, the case itself is the centerpiece (Creswell, 2013; Stake, 1995).

The current study seeks a thorough examination of strategies and mathematical creativity of gifted students (certain groups of people) in different types of generalization tasks (context). Furthermore, it involves extensive data collection from several sources, which enables a thorough explanation of the commonalities in the students' strategies and their creative responses. Hence, based on the definitions provided, the current study is a case study. Since each problem (figural near, figural far, and non-figural far generalization problems) has distinct characteristics and was purposefully selected, each serves as one bounded case. As each problem type is a distinct case, investigation of multiple cases enables a more thorough comprehension of students' strategies and creative responses. Therefore, the present study aligns with the collective case study design.

Among the most essential features of case studies is to determine what the case and unit of analysis are (Miles et al., 2014; Yin, 2014). A design outlines the units of analysis investigated (Patton, 1990). Considering the current study investigates 8th-grade gifted students' strategies and mathematical creativity in distinct types of generalization problems, in this study, each student is the case, and gifted students' responses to different types of generalization tasks are the unit of analysis. In the context of the first research question, we focused on the strategies that students used to solve these problems. In terms of the second research question, students' responses were considered as a whole.

3.2. Context of the study

The participants of this study are 8th-grade gifted students attending the Science and Art Centers (SACs) in Türkiye. Therefore, before presenting the participants' demographics and characteristics, it would be useful to provide information about SACs in Türkiye.

Science and Art Centers (SACs) are educational institutions managed by the Ministry of National Education, aiming to enable gifted students in preschool, primary school, middle school, and high school age to realize their talents and enhance their capacities to the maximum level through interdisciplinary student-centered education (MoNE, 2022). Educational activities in SACs take place outside of school hours on weekdays or weekends. The general aims of SACs are to contribute to raising individuals who are open-minded, leaders, constructive, productive, creative, and aware of values (MoNE, 2022). The identification and placement procedure of gifted individuals to SACs consists of multiple stages (MoNE, 2022). The first step in diagnosis and placement is the nomination of students who are thought to differ from their peers in art, music, or general mental ability areas by their teachers. A student can be nominated in a maximum of two areas. Afterward, the nominated students are subjected to a preliminary evaluation. After the preliminary evaluation, students who pass the scores determined by the ministry are taken into individual evaluation according to their talent areas. Students who are identified as gifted as a result of the individual evaluation have the right to start SACS (MoNE, 2022).

Gifted students follow five programs in stages in SACs: orientation program, support education program, individual talent awareness program, special talent development program, and project production and management program (MoNE, 2022). As the name suggests, the orientation program is a training program that aims to help gifted students who have recently registered with SAC get to know the institution, to facilitate their adaptation to the institution. In this program, the vision, mission, and values of SAC are introduced to the students, and activities

aimed at getting to know the students are organized. Moreover, the second program, the support education program, aims to provide students with communication, learning to learn, research, problem-solving, cooperation, and critical thinking skills, and includes activities that will support the development of these skills in line with this goal. Furthermore, the individual talent awareness program aims to make students whose area of giftedness is general mental ability realize their talents. In line with this purpose, teachers implement group activities to help gifted students realize their talents and skills specific to each area. Besides, the special talent development program includes scientific and art-oriented, student-centered activities related to the student's special talent areas. Lastly, the final stage, the project production and management program, includes training in project development in line with areas of interest, either individually or in groups. Students who continue this program prepare at least one project each year (MoNE, 2022).

3.3. Participants of the study

Developing a thorough investigation of a key phenomenon is the aim of qualitative research instead of making generalizations. Therefore, qualitative researchers deliberately choose people to gain the most insight into this phenomenon (Creswell, 2012). The current study focused on the investigation of 8th-grade gifted students' strategies and mathematical creativity in realistic non-linear generalization tasks. In line with those purposes, the researcher selected the participants purposefully from those who have the potential to provide rich and meaningful information. More specifically, for students to be able to answer the tasks included in the study, they need to have prior knowledge and skills of writing the algebraic expression representing a given situation, stating the rule of the pattern, and knowing the basic rules about exponential expressions. When the middle school mathematics program in Türkiye (MoNE, 2018) is examined, it is observed that an 8th-grade student is expected to have this knowledge and these skills. In addition, since students attending Science and Art Centers (SACs) in Türkiye are identified as gifted, the participants were selected from 8th-grade

students attending SACs. Four (three girls and one boy) 8th-grade gifted students constituted the participants of the study. To ensure confidentiality, the participants were assigned pseudonyms: Amelia, Diana, Luca, and Zena. Table 3.1 depicts the demographic information about the participants.

Table 3.1. Demographic Information About the Participants

Student	Gender	School Type	Grade Level	Duration of attending SAC	Area of giftedness	Mathematics Grade Point Average (out of 100)
Amelia	Female	Private	8th Grade	6 years	General Mental Ability	97
Diana	Female	Public	8th Grade	5 years	General Mental Ability & Art	100
Luca	Male	Private	8th Grade	6 years	General Mental Ability	97
Zena	Female	Public	8th Grade	4 years	General Mental Ability	96

As depicted in Table 3.1, three female and one male student constitute the participants. While two participants are enrolled in private schools, the other students are enrolled in public schools. Furthermore, as can be seen, all students' grade point averages in mathematics were above 95 out of 100, which means they are all high achievers in mathematics. Below is comprehensive information about the participants.

Amelia was an 8th-grade gifted female student. She lived in Ankara, Türkiye. She attended the SAC for 6 years, and similar to other participants, her area of giftedness was general mental ability. However, different than other students, she had never taken a mathematics course in SAC; instead, she took chemistry and biology courses. Moreover, although she has a high mathematics grade point average (97 out of 100), she thinks that her mathematics is average. In addition, she believed that her ability to solve mathematics problems was moderate. More specifically, she stated that she is confident in her ability to successfully solve the procedural problem, but she sometimes struggles with word problems because of her reading skills. In addition, she believed that her ability to offer multiple solutions to a mathematical problem is moderate. On the other hand, she has low confidence in producing different and unique solutions to a mathematics problem. However, she stated that she is very confident in her ability to detail and justify her answer to a mathematics problem.

Diana was an 8th-grade gifted female student. She lived in İstanbul, Türkiye, and she attended the SAC in İstanbul for five years. Unlike other students, she is gifted in two areas: namely, in general mental ability and art. Moreover, she took mathematics and art courses in SAC. She stated that while in previous years, they had solved brain teasers in her mathematics classes at SAC, this year, in the new SAC, where she transferred, they mostly covered high school mathematics topics, and they had solved basic procedural questions related to these topics. Besides, she has the highest mathematics grade point average (100 out of 100), and she stated that she is good at mathematics and is highly confident in her ability to successfully solve mathematics problems. Furthermore, while she has moderate confidence in her capability to produce multiple solutions to a mathematics problem, she has high confidence in her ability to offer different solutions to a mathematics problem. Moreover, she has low confidence in producing unique solutions to a mathematics problem, but she is very confident that she can explain the reasoning behind her answers.

Luca is the only male student among the participants. He lived in Ankara, and similar to Amelia, he attended the SAC for six years, and his area of giftedness is general mental ability. He took mathematics and information technologies courses in SAC. He stated that last year at the SAC high school subjects were taught, and mostly procedural questions were solved, but this year they will conduct a project. Moreover, he added that he will plan to design a board game that aims to teach square root numbers to 8th-grade students as a project. Besides, his mathematics grade point average was 97 (out of 100), and he thinks that he is good at mathematics and is somewhat confident in solving mathematics problems. Expressly, he stated that he is good at solving procedural problems, yet the same is not true for word problems. Besides, he has moderate to high confidence in producing more than one and different solutions to a mathematics problem. In addition, he expressed that his ability to offer unique solutions to a mathematics problem is moderate. Lastly, he stated that he has moderate confidence in explaining the reasoning behind his answer verbally, but he has high confidence in her ability to provide in-depth reasoning in written language.

Zena was an 8th-grade female gifted student, and like Diana, she lives in İstanbul, Türkiye. She has been going to SAC for four years. Moreover, her area of giftedness is general mental ability, and she took a mathematics course in SAC. She stated that they had previously solved Olympiad questions at SAC, but in the SAC that she started this year, they do high-level mathematical calculations. But she stated that she could not go much this year because of her busy schedule. Besides, her mathematics grade point average was 96 (out of 100), and she stated that she is good at mathematics and mathematics is one of her favorite courses. In addition, she is confident in solving mathematics problems. Furthermore, similar to Luca, she has moderate confidence in her capacity to come up with multiple, diverse, and original solutions to a mathematical problem. On the other hand, she expressed that she is highly confident in providing in-depth reasoning behind her answer.

3.4. Data Collection

In qualitative research, data collection is a sequence of connected tasks intended to obtain high-quality data to address new research questions (Creswell, 2013). To describe the series of actions that were done while collecting the data in the present study, this section will be divided into three main parts: namely, data collection tools, pilot study, and data collection procedure. Each of these parts will be addressed in detail in the following sections.

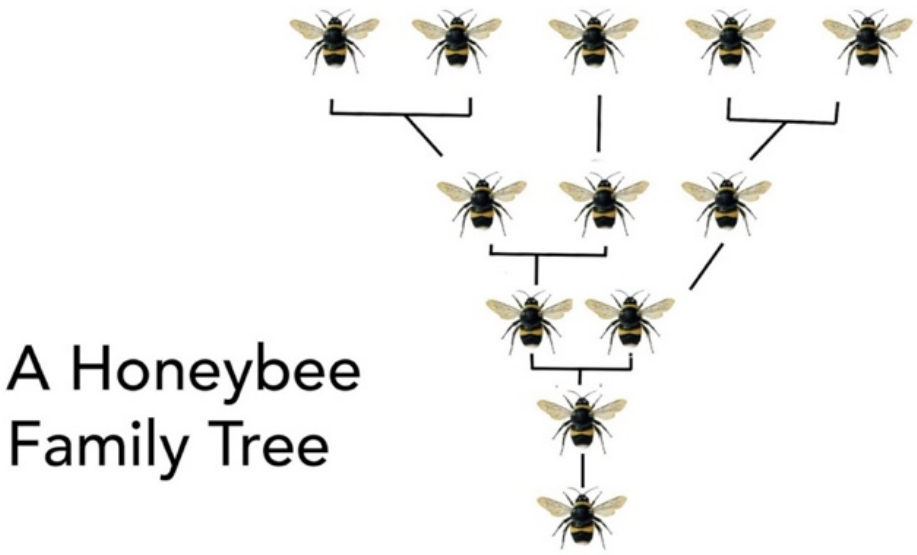
3.4.1. Data Collection Tools

Qualitative researchers use numerous data sources instead of depending on one (Creswell, 2013) to have multiple sources of evidence (Bassey,1999). Hence, in this study, various data collection tools, which were problem sheets that include two realistic non-linear generalization tasks, semi-structured task-based interviews, and the researcher's field notes, were utilized. Each of these tools will be explained in detail.

3.4.1.1. Problem Sheet

The problem sheet included two realistic non-linear generalization tasks. The tasks in the problem sheet were designed according to the gifted students' needs and the creativity-directed tasks' characteristics. To be more precise, the review of the literature (Özdemir, 2016; Özdemir & Işıksal-Bostan, 2021; Rotigel & Fello, 2004) revealed that gifted students need tasks that are open-ended, challenging, demand higher cognitive demand, have multiple solution methods, and real-life connections. Moreover, the characteristics of the creativity-directed tasks are open-ended (Bicer, 2021; Kandemir & Gur, 2007; Kwon, et al., 2006), have multiple entry points (Bicer, 2021; Leikin & Lev, 2013; Silver, 1997), have multiple answers (Tsamir et al., 2010), and have a real-life connection (Shriki, 2008). Therefore, in light of the review of the literature, while selecting the tasks to be involved in the present study, the researcher paid attention to select tasks that include multiple entry and exit points, are open-ended, have real-life connections,

and require higher-order thinking. The process of selecting and adapting each task will be explained below. Figure 3.1 illustrates the original version of the first task in the problem sheet.



Honeybees reproduce according to the following rules:

- If an egg is laid by an unmated female, it hatches a male bee.
- If an egg was fertilized by a male, it hatches a female.

Question 1: **Label the bees in the diagram above as F (female) or M (male).**

Question 2: Draw in more generations of bees above the top row and label those too.

Question 3: **Study the pattern and see what you notice. Keep track of how many bees are in each generation. Keep track of how many males and females are in each generation.**

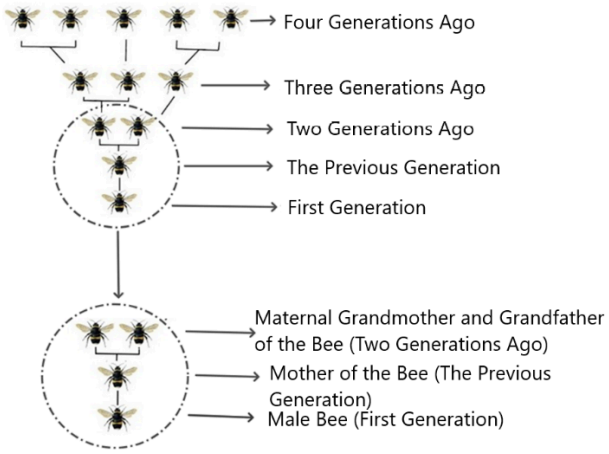
Figure 3.1. The Initial Version of the Honeybee Task (1001 Math Problems, 2016)

The first task in the problem sheet was adapted from a website, 1001 Math Problems, in which the works are licensed under a Creative Commons Attribution 4.0 International License. The context of the task is the reproduction pattern of the bees, and the problem includes three questions. In the first question, the identification of the bees is asked. Moreover, in the second problem, students are

expected to extend the family tree, and lastly, in the third question, the rule of the pattern is asked. Within the scope of the study, the researcher started the adaptation of the task by revising the context with a simple narrative that the target students could understand and that would be interesting to them. Then, each generation was given a name in the figure to make it clearer, and examples from the bottom three generations were depicted to make the context and the figure easier for students to understand and interpret. After those revisions, an expert's opinion was obtained regarding the scientific accuracy of the context and the figure and its suitability for the grade level (target audience). The expert, who is a full professor in science education at one of the state universities in Türkiye, holds master's and doctoral degrees in science education in addition to her bachelor's degree in biology. The expert confirmed the context and the figure of the problem, its scientific accuracy, and its suitability for the grade level. Then, since this study aimed to examine students' strategies and mathematical creativity in different generalization problems, the researcher and her supervisor examined each question in the task and adapted it to serve the purpose of the study better. To put it more clearly, the original form of the task contains three questions, and in the first question, students are expected to determine the gender of the bees in the given family tree. The researcher and her supervisor decided to include this question in the final version of the task because this question could enable the researcher to see whether the students understood the context and the figure accurately. Later, since generalization problems were classified as near and far generalization problems by some researchers (e.g., Stacey, 1989), and since the current study aimed to examine the strategies and mathematical creativity of students in different types of generalization problems, the second question of this task was revised as a near generalization problem. To be more specific, in option b of the final version of the first task, students were expected to identify how many bees there were in the previous six generations. In addition, a bee image was added to this question to observe whether the students think that the situation given in the question is valid, not only when the first bee is male but also when it is female. Furthermore, the

third question in the original task was revised to ask how to express the number of bees in the nth generation in a way that clearly asks for far generalization. In addition to evaluating their creative potential, the students were expected to answer the problems with as many methods as possible. After all these revisions, the final version of the first task is shown in Figure 3.2.

Did you know that not all bees have two parents? Bees have an interesting reproductive system and, therefore, different family trees. Queen bees are female bees responsible for ensuring genetic diversity. Male bees are formed from the unfertilized eggs laid by the queen bees in the hive, and female bees are formed from the fertilized eggs. So male bees have only one parent (mother), while female bees have two parents (mother and father).



a. In the given family tree, indicate female bees with "D" and male bees with "E".



b. Above is the image of a bee. If you accept this bee as the 1st generation, how many bees are there in the six previous generation? Please answer in as many ways as possible.

c. How many bees did you observe in each generation? How do you mathematically express the number of bees in any generation (nth generation)?

Figure 3.2. The Final Version of the Honeybee Task in the Problem Sheet

The second problem was adapted from Bhatnagar's (2020) study about the mathematical modeling of COVID-19 spread. In his study, Bhatnagar aimed to

develop a mathematical modeling of COVID-19 to estimate the number of cases in several countries. Bhatnagar (2020) first proposed the model for an unconstrained scenario and did not consider the virus's incubation period. Then, he took the incubation period into account and finalized the model. Since the part after taking into account the incubation period of the virus requires advanced mathematical operations, that part was not included in the current study. Instead, Bhatnagar's unconstrained scenario, in which the incubation period of the virus is not considered, was used in this study. The original version of the scenario (Bhatnagar, 2020, p.2) is given below:

> We assume that a positive corona patient comes into a country and comes into contact with other people. Since it is an infectious disease, therefore, it spreads in others. We denote the positive corona patient as an active node. Since there is no identification of this active node, therefore, it is in contact with all other nodes. A tree-based structure is shown to clarify the scenario in Fig. 1. It is clear from the figure that node A (an infected person) can infect *a* other nodes (person) in 1 unit time (unit may be day, hour, minute). Now all these new infected nodes are active and each of them can infect other *a* nodes and so on.

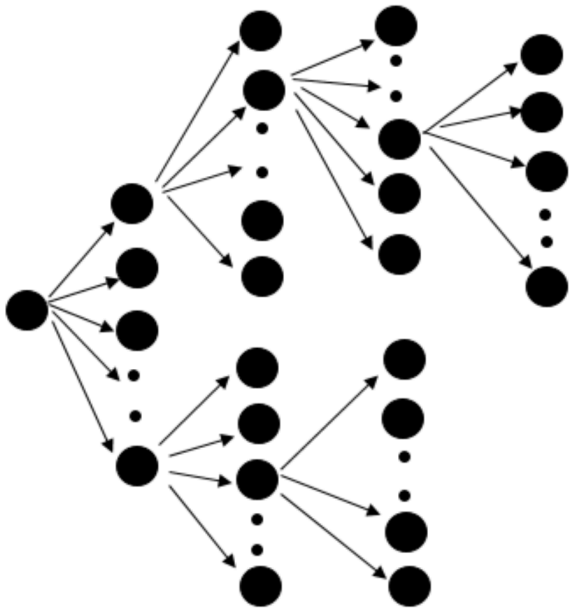


Fig. 1. Proposed model for spread of COVID-19.

Figure 3.3. The Three-Based Structure to Show the Spread of COVID-19 (Bhatnagar, 2020, p.2)

As illustrated by Figure 3.3, Bhatnagar (2020) used a tree-based structure to represent the scenario. The researcher asked two research assistants in mathematics and science education, and one mathematics teacher, to solve the problem. After that, the researcher shared the solution to the problem. However, all stated that they had difficulty in reaching the correct answer because of the figure. To be more precise, they expressed that the figure is misleading because all the people to whom each sick person transmits the disease were not illustrated in the figure. Therefore, the researcher decided not to include the figure in the problem and revised the scenario in a way that had no connection to the figure. Moreover, the researcher wrote the context before the scenario and a note after the scenario to make the problem meaningful, familiar, and clear to the students. Furthermore, like the first question, the researcher asked students to solve the problem in many ways to assess students' creativity. After all these revisions, the second task on the problem sheet became as in Figure 3.4.

The contagious COVID-19 coronavirus disease, first seen in our country in 2020, has affected the whole world. At that time, when it was important to take precautions in advance, a researcher named Manav Bhatnagar developed a mathematical model showing the spread of the COVID-19 virus to accurately predict COVID-19 cases in 2020. In order to observe the effect of measures such as lockdown on the spread of the virus, Bhatnagar first created the model showing the spread of COVID-19 when no precautions are taken, based on the following scenario:

> *Assume that a coronavirus-positive patient arrives in the country and comes into contact with other people. Since coronavirus is an infectious disease, it can also be transmitted to other people. Let us represent the sick person with A. In one unit of time, person A can infect* a *number of other people. Now, all the sick people are active, and each sick person can infect* a *number of other people, and so on.*

Important Note: In this model, the active time of the virus (incubation period) is not taken into account. It is assumed that a sick person is always infected, that is, continues to transmit the disease continuously.

How could Bhatnagar have expressed the total number of sick people mathematically? Try to answer in as many ways as possible.

Figure 3.4. The Final Version of the Second Task in the Problem Sheet

After the revisions were done for both problems, the researcher received opinions from two experts. The first expert was a mathematics teacher with a bachelor's degree in mathematics and five years of mathematics teaching experience. The second expert has a bachelor's degree in elementary mathematics education and a master's degree in mathematics education and is doing a PhD in task design in Northern Europe. In addition, the second expert has previously taken courses on mathematics education for gifted students and has prior knowledge about mathematical creativity. The experts examined and solved the problems, and as a result, they approved the questions in terms of mathematical accuracy, pedagogical appropriateness, and for use in the assessment of mathematical creativity. Besides, since the participants of the study consist of students whose native language is Turkish, the researcher translated the adapted version of the tasks into Turkish. Then, the third expert, who is a mathematics education professor with a C1 English level, compared the English and Turkish versions of the tasks and confirmed the translations.

3.4.1.2. Interviews

Interviews are commonly used in collecting qualitative data (Atkins & Wallace, 2012; Hatch, 2002; Merriam & Tisdell, 2015). In this study, interviewing was used in data collection as it is the most effective method for performing in-depth case studies of a small number of people (Merriam & Tisdell, 2015) and allows the researcher to learn more about participants' thought processes because it invites them to speak (Atkins & Wallace, 2012; Patton, 1990). Interviews can be classified in different ways (Hatch, 2002; Merriam & Tisdell, 2015). Merriam and Tisdell (2015) classified interviews into three types, namely, highly structured interviews, semi-structured interviews, and unstructured interviews. The questions and the sequence of asking the questions are predetermined in highly structured interviews. The less structured form is called semi-structured interviews. Semi-structured interviews consist of a combination of more and less structured questions, or all of the questions are formulated in a flexible manner. Lastly, the unstructured interview does not have any predetermined questions (Merriam & Tisdell, 2015). In the current study, a semi-structured interview was used. To guide the interviews, the researcher generated an interview protocol consisting of two parts, which are presented in Table 3.2 and Table 3.3 below.

Table 3.2. The First Part of the Interview Protocol

Question	Purpose
1. Which school do you go to? (Private/Public)	Aims to have demographic information about the participants
2. Which group are you in at SAC (general ability, music, art)?	Aims to have demographic information about the participants
3. Which courses are you taking at SAC?	Aims to have demographic information about the participants
4. Since which grade have you been going to SAC?	Aims to have demographic information about the participants

Table 3.2. (continued)

5. What have you been doing in SAC?	Aims to have information about participants' SAC experience and background
6. How would you describe your confidence in solving math problems as low, medium, or high?	Aims to determine participants' self-efficacy in problem solving
7. How competent do you consider yourself in producing multiple solutions to a mathematics problem (low- moderate -high)? Why?	Aims to determine participants' creative self-efficacy about fluency
8. To what extent do you see yourself as competent in producing different alternative solutions (low- moderate - high)? Why?	Aims to determine participants' creative self-efficacy about flexibility
9. Do you think that these solutions are new solutions that are rarely or never produced by others in the class? To what extent do you consider yourself competent in this regard (low-moderate-high)? Why?	Aims to determine participants' creative self-efficacy about originality
10. Can you explain the reasoning behind your solutions? How competent do you consider yourself in this regard (low-moderate -high)? Why?	Aims to determine participants' creative self-efficacy about elaboration

Table 3.3. The Second Part of the Interview Protocol

Question	Purpose
1. What do you understand from the problem?	Aims to ensure that the participants understand the problem accurately and check clarity
2. Can you explain each solution method to me as if you were explaining it to a friend who doesn't know it at all? What details did you pay attention to?	Aims to have an insight into students' thought process and their ability to express their thinking.
3. Are there any new or different solutions that you would like to add, other than the ones you have already said?	Aims to provide opportunity to share further responses

As illustrated in Table 3.2, the first part of the interview protocol covered questions about students' demographics, self-efficacy in problem solving, and their mathematical creative self-efficacy. The first five questions in the first part of the protocol were about students' demographics. On the other hand, the sixth question in the first part of the interview protocol was about students' self-efficacy in problem solving, and the rest of the questions in the first part of the interview protocol aimed to evaluate students' mathematical creative self-efficacy. Each question aimed to measure self-efficacy for one of the components of mathematical creativity: fluency (the 7th question), flexibility (the 8th question), originality (the 9th question), and elaboration (the 10th question), respectively.

In comparison, the second part of the interview protocol was task-based, stated differently, consisting of questions related to the task. To be more precise, as can be seen in Table 3.3, each participant was first asked what they understood from the question to see whether they made sense of the problem. Then, the researcher asked each student to explain their solutions. The aim in asking this question is to

evaluate whether students can elaborate on their answers. Lastly, the researcher asked each participant if they had any new answers or not to ensure that each participant showed all the possible solutions they had. In addition to these questions, because of the flexible nature of semi-structured interviews, some probing questions were also asked during the interviews. A few examples of these questions are: What kind of project are you doing right now? Have you taken a mathematics course at SAC before? What does *n* mean here? What was the reason for defining male and female as an *n*? What made you think of that in the question? How did you make sure this was true?

3.4.1.3. Researcher's field notes

Detailed notes that researchers take while observing events or interviewing are described as field notes (Fraenkel et al., 2012). A crucial part of thorough qualitative research is qualitative field notes (Phillippi & Lauderdale, 2018). In the present study, while students were solving the tasks, the researcher took some descriptive and reflective field notes. More specifically, the researcher took notes to describe an answer that needed clarification, special characteristics of the participant that needed attention, or when there were notable situations in students' problem-solving processes that were not visible in the recording (especially in audio-only recordings). Furthermore, the researcher also took some notes when she had insights about the reason behind the student's difficulty or the relationships between different student solutions, etc. While analyzing and interpreting the data, the researcher used field notes, especially when she had difficulty understanding the recordings and student answers. Thus, field notes enabled to have more clear insights and interpretation of the data.

3.4.2. Pilot Study

The pilot study was carried out during the 2023-2024 fall academic year with one seventh-grade gifted student. The pilot study aimed to evaluate if the problems were clear to the student, whether there were unclear questions in the interview protocol, and to determine the grade level for which the problems were appropriate.

Although this student is in 7th grade, she stated that she studied all the 8th-grade subjects with her mother, who is a mathematics teacher, during the summer. The student attended a private school, and her mathematics grade average was 100. In SAC, she took the mathematics and biology courses. The data collection process with the student took place in two sessions. The student was interviewed on the questions in the first part of the interview protocol in the first session. Then, she solved the first problem. In the pilot study, the student was asked about the number of bees in the seven previous generation in option b of the first problem. However, the student took too long to draw up to step 7, and also confused the number of bees. Although the student realized her mistake while explaining her solution, she lost a lot of time. Apart from that, the student answered the first question correctly. At the end of the first session, the researcher received the student's opinion about the clarity of the question and asked the student if she had any suggestions. The student stated that the question was clear and suggested that the number of bees in the previous generations could be asked instead of the seventh one. To prevent the loss of time and motivation that could occur in other interviews, the researcher took this suggestion into consideration and revised option b of the first question to ask the number of bees in the previous six generation. Also, in the first version of the problem sheet, the researcher used the phrase "solve as many different ways as possible". However, during the interview, the student stated that she solved the question in another way, but she did not write it in her solutions because it was similar to one of her other solutions. Since this situation could cause data loss, especially in the fluency dimension of mathematical creativity, the researcher revised that statement in the problem sheet as “solve in as many ways as possible” and asked the students to show all the methods they could think of. Furthermore, in the second problem, the student initially misunderstood the problem; to be more specific, she overlooked the fact that each person continued to infect the disease that was written in the important note. The student suggested that it would be useful to underline this information in the important note section of the question. The researcher considered this information and revised the problem paper in line

with this suggestion. In addition, although the student correctly expressed the general rule of the pattern verbally, she had some difficulty expressing it algebraically because she did not reinforce the information she learned during the summer about exponential expressions and identities. This made the researcher confident that it would be more appropriate to conduct the study with 8th graders.

3.4.3. Data Collection Procedure

Data were collected during the fall semester of 2023. Before collecting data, the researcher applied for ethical permission from the METU Human Research Ethics Center. Upon the approval of the application, the researcher started collecting data. First, the participants of the study and their parents were informed about the study, and their permission was obtained. Since two of the participants resided in Ankara and the other two in Istanbul, the data-collecting process was conducted online with the participants in Istanbul and in person with the participants in Ankara. The participants were first asked the questions in the first part of the interview protocol, as shown in Table 3.2. Then, the students were asked to solve the first task in the problem sheets, which included options a, b, and c, respectively. After the students read each question, the researcher asked each student the first question in the second part of the interview protocol (see Table 3.3). Then, the students were expected to solve the problems and note down all the answers they could think of for each question. Since giving students ample time is one of the educational practices that will encourage mathematical creativity (Bicer, 2021), the researcher did not set any time limits for the students to solve the problems. On the contrary, she told the students that they could think as much as they wanted. Additionally, the researcher reminded the students that they could take a break or end the session if they got tired. With each student, except Luca, the process of gathering data was finished in two sessions. In each session, students were focused on one task. Each session lasted approximately two to two and a half hours. However, with Luca, in a single two-hour session, the data collection procedure was finished. Besides, to avoid any limitations in the online interviews, the researcher asked the students to solve the problems in the way they felt comfortable. Specifically, the student who

felt comfortable solving on the screen solved on the screen. On the other hand, if they wanted to work on paper, the researcher asked the students to position the camera so that she could see the papers they were working on in detail. Moreover, at the end of the interview, the researcher asked for photographs of the papers the students were working on. In addition to the voice recording during the face-to-face interviews, the researcher took notes of her observations to avoid data loss. Immediately after the data collection was done, the researcher started data analysis in January 2024. Figure 3.5 represents the sequential steps of the data collection and data analysis process and corresponding dates.

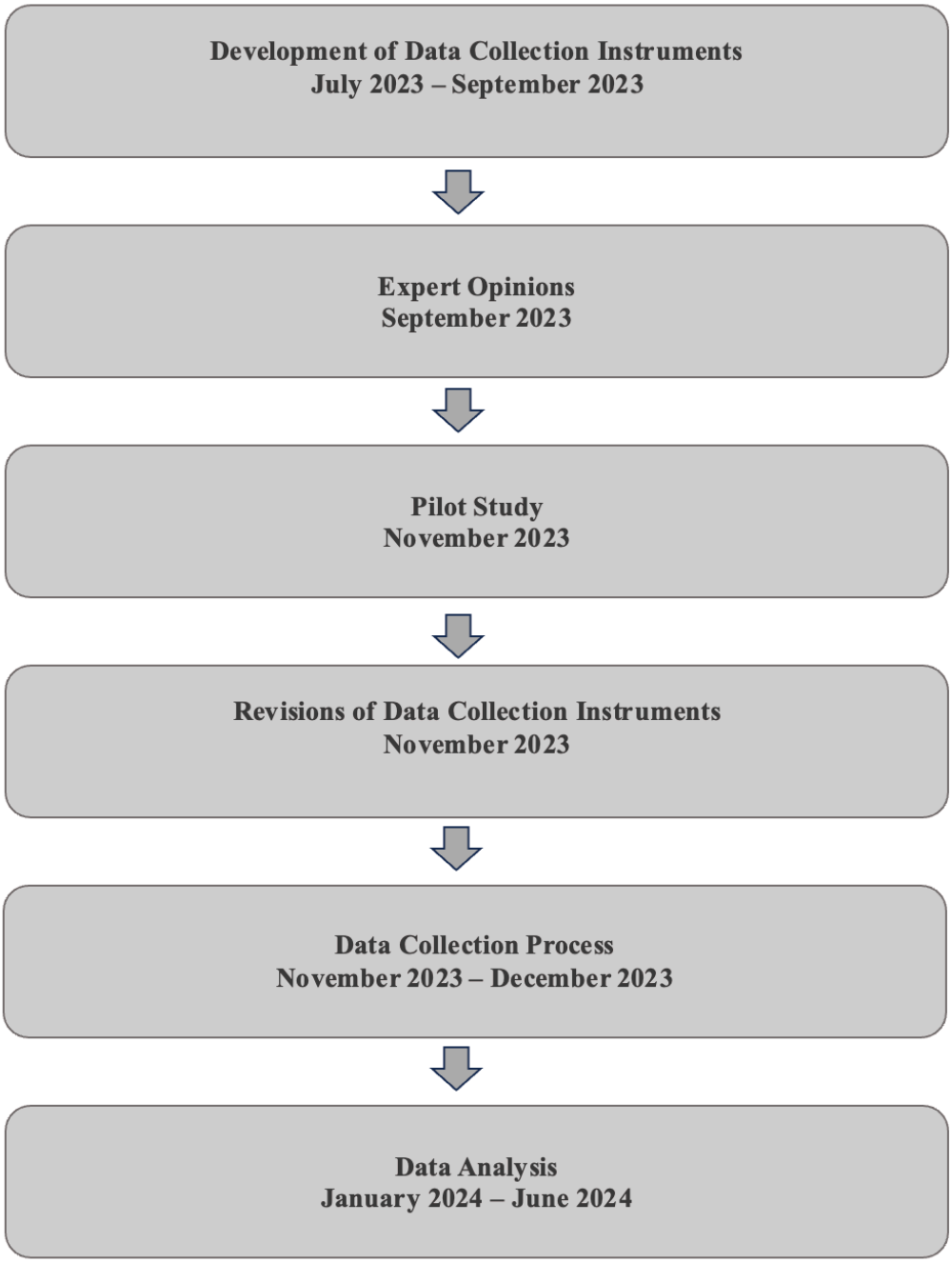


Figure 3.5. Timeline of the Data Collection and Data Analysis Process

The data analysis process will be discussed in the next section.

3.5. Data Analysis

Data analysis is a complex process of finding meaning and insight in the data (Merriam & Tisdell, 2015). Qualitative data analysis entails preparing and organizing the data, simplifying it into themes via systematic coding, and representing the data (Creswell, 2013). In the present study, those steps were followed. More specifically, all the interviews were transcribed to prepare the data for analysis. Then, the researcher organized the data by dividing the transcripts into parts, namely, demographics and problem-solving parts, and identified students' responses. Afterward, in the process of coding, several coding cycles were conducted, and ultimately, meaningful codes were generated from the data. In the coding process, although both inductive and deductive analyses were utilized, the process was predominantly inductive. Inductive analysis is a process of identification of the patterns and classifications in the data by the results of the interaction between the researcher and the data, while in deductive analysis, data is examined based on a pre-established framework (Patton, 1990). After the coding process was done, the data was converted into findings. In the following parts, the analysis of data will be elaborated. Table 3.4 depicts the research questions, data collection tools, and data analysis.

Table 3.4. The Summary of Research Questions, Data Collection Tools, Data Analysis

Research Questions	Main Data Collection Tools	Supporting Data Collection Tools	Data Analysis
1. How do the strategies of 8th-grade gifted students vary when solving alternative types of non-linear realistic generalization tasks?	Interviews Problem sheet	Researcher's field notes (for clarification)	Inductive and deductive content analysis

Table 3.4. (continued)

2. How creative are 8th grade gifted students while solving alternative types of realistic nonlinear generalization tasks (in terms of fluency, flexibility, originality, and elaboration)?	Interviews Problem sheet	Researcher's field notes (for clarification)	Inductive and deductive content analysis
2.1. What types of representations do 8th-grade gifted students use in their responses to nonlinear realistic generalization tasks?	Interviews Problem sheet	-	Deductive content analysis

3.5.1. Data Analysis of the First Research Question

The first research question of the current study was: How do strategies of 8th-grade gifted students vary when solving alternative types of non-linear realistic generalization tasks? In accordance to investigate this research question, first, all students' responses were identified. Then, it was noted that some of the answers were accurate, some were incorrect, and some were incomplete. The incomplete and incorrect answers were categorized as "inappropriate answers," and they were not used in further analysis. On the other hand, the accurate answers were categorized as "appropriate answers." Appropriate answers have the potential to provide clearer insights about effective strategies that lead students to solve the task accurately and have a deeper understanding of the concept being investigated. Therefore, in this study, students' strategies in those responses were investigated. When the identification of appropriate answers was done, the researcher coded each appropriate response as the first letter of the student's name and the number of

answers. For instance, the third appropriate answer of Diana is coded as "D3". Afterward, the researcher determined the students' strategies in the appropriate responses by the use of both inductive and deductive coding. To be more precise, while identifying the strategies, the related transcripts were read several times, and the names of the strategies were derived from the data by noting down the clear descriptions of the strategies. Then, the researcher read and wrote the existing strategies reported in the literature and compared those strategies with the strategies utilized by the 8th-grade gifted students involved in the current study. Strategies that were like the strategies noted in the literature were coded with their names in the literature, while strategies that were not similar were coded in a way that can be clearly expressed and reflect the strategy in its entirety. This process was repeated in the same way for all problem types. In the end, students' strategies in figural-near, figural-far, and non-figural-far strategies were determined. Data analysis revealed that while solving these tasks students used the following seven strategies: *counting from a drawing* (Stacey, 1989), *connecting to the known pattern*, *grouping and assigning a variable*, *using the existing generalization*, *identifying regularities by investigating the numerical relationships embedded in the figure*, *identifying regularities by assigning alternative values to the variable and generating the visual representation*, and *identifying regularities by generating the algebraic representation and assigning alternative values*.

3.5.2. Data Analysis of the Second Research Question

The second research question that guided the present study was how creative gifted students are while solving alternative types of realistic nonlinear generalization tasks (in terms of fluency, flexibility, originality, and elaboration). The students' mathematical creativity in those tasks was analyzed based on five components of mathematical creativity: appropriateness, fluency, flexibility, originality, and elaboration. First and foremost, students' answers were investigated in terms of appropriateness because, as explained in detail in the literature review, appropriateness serves as a core element in mathematical creativity (Amabile, 1983; Simonton, 2012). Therefore, the appropriate answers that satisfied the first

component of mathematical creativity were analyzed in terms of the other four components of mathematical creativity. Then, the literature (e.g., Baykal, 2009; Bicer et al., 2020; Blyman et al., 2020; Guilford, 1959; Kattou et al.,2013; Kim et al., 2016; Kozlowski et al., 2019; Plucker et al., 2014; Rubenstein et al., 2019) about creativity was reviewed to investigate how the components of creativity were defined and analyzed in different studies. However, each of the reviewed studies has its characteristics and limitations due to differences in sample size, study design, and nature of the tasks used. Therefore, a ready-made analysis framework was not directly utilized in the present study. Instead, the approach to defining and analyzing the components was created by synthesizing various definitions and assessment methods reported in the literature, and by taking into account the nature of the data in this study. Hence, how each component of mathematical creativity was analyzed will be explained one by one below.

Fluency score was defined as the total number of correct answers (Kattou et al., 2013). Following this definition, each student received 1 point for each correct answer, and the sum of those points made up the student's fluency score. More specifically, if the student provided five appropriate answers for one question, his/her fluency score for that question was 5.

The other component of mathematical creativity, flexibility, was scored according to the total number of different categories of answers (Kattou et al., 2013). Analysis of students' responses indicated that students' responses differed in terms of the underlying idea behind the response, representations of the response, and the solution strategies that are used to reach that response. Thus, in the present study, the different categories consist of different ideas, different representations, and different strategies. To clarify the categories, the students' appropriate answers were examined in terms of the main idea of the answer, the type of representation of the answer, and the strategy that led to the answer. The researcher identified different ideas, and strategies with open coding, the inductive process of creating a codebook for content analysis or investigating potential patterns (Patton, 1990). Then, if the strategies she described and named had a corresponding name in the

literature, she coded them with the names in the literature, otherwise, she used the names she gave herself. Moreover, while coding the representation type of the answer, the mathematical representations framework of the National Council of Teachers of Mathematics (NCTM, 2014), which was explained in the literature review section, was used. In this study, students used three types of representations to show their answers, namely, visual, verbal, and symbolic representations.

Originality refers to the rarity of responses. As explained in the literature review, researchers adopted diverse (subjective and objective) methods to assess originality (Reiter-Palmon et al., 2019). Plucker and her colleagues (2014) recommended using a combination of subjective and objective scoring approaches to obtain more meaningful results in originality assessments. For this reason, both objective and subjective techniques were used in the evaluation of originality in this study. First, in line with the definition in this study, the originality of students' answers was analyzed based on the rarity-based evaluation approach, which is frequently used in the literature (e.g., Baykal, 2009; Kaufman, 2016; Kim et al., 2004; Leikin, 2009; Shamay-Tsoory et al., 2011). According to this approach, originality is scored and evaluated according to how rarely a given answer is seen by the participants when the same question is asked. Some researchers (e.g., Beketayev & Runco, 2016; Dumas & Dunbar, 2014) described this method as more objective. In the present study, the answers given by the participants were classified according to the categories created, and the total number of answers given to each category and the number of people who responded were determined. Then, the researcher generated the rubric to assess originality. Table 3.5 depicts the scoring of the originality.

Table 3.5. Rubric for Assessing Originality of Students' Responses

Number of students who gave that response	Score that will be assigned to that response
1	4
2	3
3	2

4	1

The table illustrates that students are given originality scores based on the number of students' answers in that category. To be more precise, first, the number of students whose answers belonged to each category was investigated. Tables showing the categorization of students' answers for each problem type were used to find the number of student answers in each category. Then, based on the number of students in that category, students were given originality scores according to Table 3.5. To be more precise, each student was given points for the rarity of each answer: if a student's answer was one given by all four participants, that student received 1 point for originality, two points for an answer given by three of the four participants, three points for an answer given by two of the four participants, and four points for an answer given by herself/himself alone. However, the important point here is that each student receives only one originality score for their answers in the same category. In other words, the number of students who answered in that category is taken into account when scoring, rather than the number of answers in that category. Levenson and her colleagues (2018) emphasized that the identification of what is original depends on the researcher. In addition, Beketayev and Runco (2016) highlighted that the use of "local norms" in originality assessments is not a limitation in terms of sample dependency, but rather an advantage in terms of contextual validity, and that it would be fairer to make originality assessments among students who took the same test under the same conditions. In this study, originality scores were assigned based on comparisons made among students who answered the same questions under the same conditions.

Another approach recommended in the literature for originality assessment, especially in small samples, is expert opinion (Silvia et al., 2008). For this reason, in this study, in terms of the applicability and validity of the scoring system developed, opinions and approval were obtained from an expert who is a full professor and has worked on creativity and interested in measurement and evaluation and a researcher who has worked on the problem-solving skills of gifted students in her master's thesis and who currently doing her PhD in gifted and

creative education in the USA. In addition, a researcher who took a course on gifted students and mathematical creativity and is pursuing a PhD on task design also scored originality, and as detailed in the dependability section, the researcher and she reached 100% agreement. Finally, an expert opinion was obtained from a mathematics education professor who has done many studies with gifted students, and the expert found the scoring appropriate.

To proceed in a more systematic and holistic perspective, a table containing the ideas, the representation type of the answer, and the strategy was created. The researcher followed the same procedure for each question (options b and c of the 1st question and the 2nd question). Stated differently, each question had its table filled out by the researcher. Table 3.6, which represents the categorization of students' responses in the figural near generalization task, is given below as an example. In addition, below, how students' answers to the figural near problem are analyzed within the scope of creativity's fluency, flexibility, and originality components using this table is explained with examples.

Table 3.6. Categorization of Students' Responses for Figural Near Generalization Task

Category	Underlying Idea	Representation Type	Strategy	Student's Answer
1st Category	If the 1st generation is male, the number of bees in the six previous generation is 13.	Visual	Counting from a drawing	D1, A2, Z2, L2
2nd Category	If the 1st generation is female, the number of bees in the six previous	Visual	Counting from a drawing	D2, A1, Z1

	generation is 21.			
3rd Category	The number of bees in a generation is the sum of the number of bees in the two previous generations.	Verbal	Identifying regularities by investigating the numerical relationships embedded in the figure	D3, D4, A3, A4, L3
4th Category	The reproductive pattern of the bees is an example of the Fibonacci sequence.	Verbal	Connecting to the known pattern	L1

Each row in Table 3.6 represents one category. For instance, the answers D1, A2, Z2, and L2 in the first row are in the same category (1st category) because they contain the same underlying idea, the same representation, and the same strategy. On the other hand, in addition to the answers in the same category, there are also answers in different categories. For instance, some responses (e.g., A1 and A2) differed in terms of one aspect, such as main ideas, and some responses differed in terms of more than one aspect, such as A1 and A4. If a student's answer differs from another answer in at least one way, the student receives 1 point for flexibility for that answer. To determine each student's flexibility score, the table showing the categorization of students' answers was examined, and the number of students' answers in different categories was determined. For instance, according to Table 3.6, Amelia has four appropriate responses; this means that her fluency score for the figural near generalization task was four. On the other hand, three of Amelia’s four answers belonged to different categories, which implies that her flexibility

score for the near-generalization task was three. Furthermore, as can be seen in Table 3.6, there were three students' (Diana, Amelia, and Zena) responses in the second category. According to Table 3.5, each of those students received 2 points in terms of originality for their responses in the second category. To be more precise, Amelia received 1 originality score for her answer in the first category, 2 originality points for her answer in the second category, and 2 originality points for her answers in the third category. In total, she got 5 originality scores for her responses to the figural near generalization task.

The literature shows various definitions of elaboration. The two most common of these definitions are the capability to explain the thinking process (Blyman et al., 2020) and justify the response (Kozlowski et al., 2019). Thus, in this study, students' ability to explain their thinking processes or explain the reasoning behind their answers was used as an indicator to assign their elaboration scores. During the interview, each student was asked to explain each of their answers as if they were explaining them to someone unfamiliar with it, and was asked to justify their reasoning. In light of the students' answers to these questions, it was evaluated whether the students could elaborate on their answers. To make this evaluation, the relevant transcripts with each appropriate student response were read several times. In the end, if the student was able to explain the reasoning behind their answer or their thinking process, he/she received 1 point for elaboration. Below is an example of how a student elaborated his answer to a figural far generalization question:

> Luca: The sum of the number of females and the total number of bees equals the number of bees in the previous generation.
>
> Researcher: What do you think might have been the reason for this?
>
> Luca: Because each bee necessarily has one mother. That is why there will be the same number of females in the previous (ancestor) generation. Moreover, there are as many males as there are females.
>
> Researcher: Why are there as many males as females?

Luca: Because females have fathers, males do not. Every female bee will necessarily have one father.

As illustrated by the dialogue, Luca explained the reason why the total number of bees in the current generation would equal the number of female bees (mothers) in the previous generation, as each bee has a mother. Moreover, he justified that since only female bees have fathers, the number of female bees would equal the number of male bees (fathers). Therefore, since the number of bees in any generation can be found by the sum of the number of female and male bees, he added the total number of bees in the current generation (which equals to the number of female bees in the previous generation) and the number of female bees (which equals to the number of male bees in the prior generation). Consequently, he justified why the sum of the number of females and the total number of bees in a generation is equal to the number of bees in the prior generation. Therefore, he received 1 elaboration score for that response.

In this study, mathematical creativity was assessed in four dimensions: fluency, flexibility, originality, and elaboration. As explained above, each dimension reflects different aspects of students' creative thinking skills. These dimensions differ from each other not only in terms of content but also in terms of their measurement properties, and therefore, the relationships between these dimensions may not be linear. Hence, adding students' fluency, flexibility, originality, and elaboration scores linearly may not produce a meaningful total score. In other words, summing up students' scores across these dimensions to form a single total mathematical creativity score may overlook the diversity in students' creative profiles and their strengths. For example, a student with a high originality score but a low elaboration score may have a relatively low total creativity score, which may make the student's creative potential seem more limited than it is. Since each dimension score has a distinct meaning, Torrance (1966, 1974 as cited in Kim et al., 2006) cautioned against using composite scores and said that doing so could be misleading. Therefore, in order to provide a more in-depth and clear assessment of students' mathematical creativity, it was preferred to present each dimension

separately instead of using the total score when analyzing the data and reporting the findings.

3.6. Trustworthiness

Meticulous consideration about the conceptual framework of the study, data collection, analysis, and precise and understandable presentation of findings is necessary to ensure validity and reliability, no matter the type of research it is (Creswell, 2013; Merriam & Tisdell, 2015). Since qualitative research is based on different worldviews and assumptions about reality, several researchers contend that validity and reliability should be assessed in ways that are consistent with the philosophical underpinnings of qualitative research (Merriam & Tisdell, 2015). One of the most well-known of these researchers is Lincoln and Guba (1985). They used "trustworthiness," which refers to validity, reliability, and objectivity in qualitative research. To ensure the trustworthiness of naturalistic research, the researcher needs to consider four criteria, namely, credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985). In the following paragraphs, each of these criteria will be explained.

Credibility, which refers to internal validity, addresses the question of whether the conclusions are credible according to given data (Lincoln & Guba, 1985). To ensure credibility, Lincoln and Guba (1985, p. 301) suggested the following strategies: "*prolonged engagement, persistent observation, triangulation, peer debriefing, negative case analysis, referential adequacy,* and *member checks*". In the current study, triangulation was used to ensure credibility. More specifically, the application of multiple methods when examining a single phenomenon is known as triangulation (Alexander, 2018). Four different ways of triangulation, namely, the use of multiple methods, multiple data sources, multiple investigators, and multiple theories, were offered by Denzin in 1978. In the present study, multiple data sources triangulation was used. To be more precise, while analyzing data, the researcher utilized students' written work, interview transcripts, video recordings of the interviews, and field notes to verify and complement her

interpretations. Having multiple sources of data enabled the researcher to have a more credible and rich interpretation.

The second criterion to ensure trustworthiness in qualitative research is transferability. Transferability, which refers to external validity, deals with the question of the extent to which the conclusions of this study are applicable to other settings (Lincoln & Guba, 1985). Lincoln and Guba (1985) asserted that the way that qualitative researchers demonstrate transferability differs greatly from how conventionalists construct external validity. Qualitative researchers are unable to define the external validity of an inquiry; they can only offer the thick description required for someone who is considering a transfer to determine if it is feasible or not (Lincoln & Guba, 1985). A thick description means a thorough explanation of the study's participants and settings (Merriam & Tisdell, 2015). In the current study, to ensure transferability, the researcher described the characteristics of the participants, the context, the data collection and data analysis processes, and the researcher's role in detail, as well as provided a detailed explanation of the results supported by quotes and written works of the participants.

Dependability, which refers to reliability, concerns the stability, consistency, and accuracy of the study (Lincoln & Guba, 1985). To ensure dependability, a third criterion to ensure trustworthiness, code-recode strategy, and inter-coder agreement were used. The researcher codes the same data twice using the code-recode technique, allowing at least one or two weeks between the coding process (Anney, 2014). In the present study, the researcher coded the data many times, i.e., more than twice, at different periods. Then, she compared her categorization. If there were differences in coding, she reread the transcripts and/or watched video recordings and noted down her final decision with the rationale. This process enabled her not only thorough comprehension of the patterns existing in the data but also to have more consistent coding. In addition to the code-recode strategy, inter-coder agreement was also used in the current study to promote dependability. Inter-coder agreement is the process by which several coders examine and then contrast their coding and categorization to determine how reliable the data analysis

procedure is (Creswell, 2013). O'Connor and Joffe (2020) suggested that 10% to 25 % of data should be multiply coded to have a reliable measure of inter-coder agreement. Furthermore, they asserted that this portion of the data needs to be chosen at random or based on a rationale (O'Connor & Joffe, 2020). Therefore, in the current study, the second coder coded randomly selected one student's transcript, which corresponds to 25 % of the data. The second coder is also the coder from whom the expert opinion was received. This coder first solved the questions herself in more than one way and shared her solutions with the researcher. In other words, the second coder is a coder who is familiar with the aims of the study and the tasks in the study. Then, the researcher had a meeting with the second coder. In this meeting, the researcher introduced the rubric, which was explained in the data analysis, to the second coder, explained how each component of mathematical creativity was defined in the present study, and gave some examples about coding and scoring. Afterward, the researcher shared the tasks in the interview, the rubric, the entire transcript of that student, and the description of the analysis process that the researcher had with the second coder. In this way, both coders went through the same analysis process. A few days after the second coder started analyzing the data, the researcher and the second coder had a meeting in which the second coder asked the researcher the questions she had during the coding. In this way, some points were clarified. After the second coder completed the coding, the researcher and the second coder had another meeting and compared their coding. The initial agreement rate was approximately 90%. In cases where there were disagreements, the transcripts were reviewed again, and both the researcher and the second coder explained their reasons for coding and discussed them. These discussions continued until the researcher and the co-coder reached a 100% agreement.

Confirmability, which refers to objectivity, examines the degree to which the conditions and participants of the study, instead of the researcher's prejudices, influence the study's conclusions (Lincoln & Guba, 1985). Confirmability, which is the fourth criterion of trustworthiness, was ensured in this study by the use of the

reflexive journal. During the data collection and analysis process, the researcher reflected on her experiences, insights, and rationales about her decisions. More specifically, after conducting the pilot study, the researcher reflected on the positive aspects and areas for improvement in the interview experience. Then, she discussed her reflections with her supervisors to plan the actual interviews. In addition, during the data analysis, as explained above, she used a code and recode strategy. In each coding cycle, she wrote detailed notes about her coding and the rationale for her coding, which enabled her to enhance transparency and consistency in data analysis.

3.7. Researcher's Role and Bias

The researcher is a "primary instrument" in collecting and analyzing data in qualitative research (Cresswell, 2013; Merriam & Tisdell, 2015). Merriam and Tisdell (2015) listed the competencies of the qualitative researcher. First, the qualitative researcher should be curious and questioning. In addition, the researcher needs to be flexible and comfortable. Also, the researcher should have systematic monitoring and inductive thinking skills. Lastly, the researcher should have an ease of writing. Throughout the current study, the researcher tried to apply those competencies as much as possible. Moreover, she had active roles and responsibilities in planning the research design, preparing the data collection tools, and collecting, analyzing, and reporting the data. The researcher first tried to find gaps by scanning the relevant literature and clarifying the aims and the research questions of the study with her supervisor. Then, in light of the purpose and research questions, they decided on the appropriate research design that could answer these questions. Afterward, the researcher tried to construct the problem sheet, which was tailored to gifted students' needs and served the purpose of the study. Furthermore, for the participants to feel comfortable in data collection, she emphasized that the researcher did not aim to assess their knowledge but was interested in their thinking processes and strategies. In addition to these, the researcher did not intervene or ask any questions while the participants were solving the problems; she just observed the students and took notes when

necessary. When the students were ready to share their solutions, she listened carefully and asked the questions she wanted to ask while the students were sharing their solution process. While doing this, she took care to ask focusing questions that would reveal students' thinking rather than asking funneling questions that would direct students to a particular answer. Also, after each interview (pilot study and subsequent interviews), the researcher reflected on what went well and what could be improved. Furthermore, in data analysis, she read the transcripts and investigated students' written works several times. Moreover, after many inductive coding cycles, she finalized the coding. Sometimes, there were parts in the coding that she had difficulty with. To clarify these parts, she kept a journal and noted down the reasoning when she made a change. These notes helped the researcher to analyze the data more consistently. Finally, while reporting the data, the researcher tried to provide quotes and students' written works to provide rich descriptions.

3.8. Ethics

To address ethical considerations, the researcher followed some procedures. First, an ethical permission application was made to the METU Human Research Ethics Board, and ethical approval was granted to the study (see Appendix). After that, the researcher informed the participants and their parents about the study and participants' rights. More specifically, students and their parents were informed about the purpose of the study, the procedure to be followed, and what was expected from students within the scope of this study. In addition, the participants were informed that solving the problems in the study was not for testing or grading purposes. Furthermore, the participants were ensured that they could take a break when they got tired and leave when they wanted to finish the study. In addition to these, participants were guaranteed to experience no physical or psychological harm. Afterward, the participants and parents were asked to read and fill out an informed consent form about voluntary participation. Moreover, before the interviews, the researcher also obtained permission from the participants to record the audio and video interviews. Lastly, to protect the participants' identities and

ensure confidentiality, they were all given pseudonyms, and special attention was taken to make sure that none of the data would cause them shame or harm.

CHAPTER 4

FINDINGS

The aim of the present study was twofold. The study's first aim was to investigate the strategies of 8th-grade gifted students while solving different types of realistic non-linear algebra tasks. In addition to the students' strategies, this study also focuses on the mathematical creativity of 8th-grade gifted students in these tasks. In accordance with these aims, this study was framed by the following two research questions:

1. How do the strategies of 8th-grade gifted students vary when solving alternative types of nonlinear realistic generalization tasks?
2. How creative are 8th-grade gifted students while solving alternative types of realistic nonlinear generalization tasks (in terms of fluency, flexibility, originality, and elaboration)?

   2.1. What types of representations do 8th-grade gifted students use in their responses to nonlinear realistic generalization tasks?

The findings will be organized and presented in line with the aforementioned purposes and research questions. Specifically, the findings will be presented in two main sections, each focusing on one of the research questions. First, gifted students' strategies in nonlinear realistic algebra tasks will be shared. Then, students' mathematical creativity in those problems will be elucidated. Instead of reporting the findings case by case, the findings will be structured and presented within problem contexts to enable a more comprehensive insight into gifted students' strategies and mathematical creativity across alternative types of generalization problems.

4.1. Gifted students' strategies in realistic non-linear generalization tasks

The first aim of this study was to investigate 8th-grade gifted students' strategies while solving realistic nonlinear generalization tasks. Aligned with this aim, students were asked to solve two realistic nonlinear pattern generalization tasks, one of which is figural (near and far), and the other is non-figural (far). Analysis of students' correct responses revealed that while solving these tasks, students used the following seven strategies: *counting from a drawing*, *connecting to the known pattern*, *grouping and assigning a variable*, *using the existing generalization*, *identifying regularities by investigating the numerical relationships embedded in the figure*, *identifying regularities by assigning alternative values to the variable and generating the visual representation*, and *identifying regularities by generating the algebraic representation and assigning alternative values*. Table 4.1 demonstrates the students' strategies for solving figural-near, figural-far, and non-figural-far generalization tasks.

Table 4.1. Students' Strategies to Solve Non-Linear Realistic Generalization Tasks

Strategies		
1. Strategies in Figural Generalization Tasks		2. Strategies in Non-Figural Generalization Task
1.1. Strategies in Figural Near Generalization Task	1.2. Strategies in Figural Far Generalization Task	2.1. Strategies in Non-Figural Generalization Task
Counting from a drawing Identifying regularities by investigating the numerical relationships embedded in the figure	Using the existing generalization Identifying regularities by investigating the numerical relationships embedded in the figure	Using the existing generalization Identifying regularities by assigning alternative values to the variable and generating the visual representation

Table 4.1. (continued)

Connecting to the known pattern.	Connecting to the known pattern Grouping and assigning variable	Identifying regularities by generating the algebraic representation and assigning alternative values

As depicted by Table 4.1, gifted students used different strategies while solving figural-near, figural-far, and non-figural far generalization tasks. In addition, analysis of student responses demonstrated that the number of students using these strategies differed. Table 4.2 depicts how often students used each strategy. As explained in detail in the data analysis section, students' answers were coded with the initials of their names and the answer number. For example, "Z2" in the students' answers column in Table 4.2 refers to Zena's second answer.

Table 4.2. Strategies and Students Using Them

Strategies in Figural Near Generalization Task	
Strategy	Students' Answers
Counting from a drawing	D1, D2, A1, A2, Z1, Z2, L2
Identifying regularities by investigating the numerical relationships embedded in the figure	D3, D4, A3, A4, L3
Connecting to the known pattern	L1
Strategies in Figural Far Generalization Task	
Strategy	Students' Answers
Using the existing generalization	D1, A1, A4, L3, L5, Z3
Connecting to the known pattern	L1, L2
Identifying regularities by investigating the numerical relationships embedded in the figure	D2, A3, Z1, Z2, L4, L6

Table 4.2. (continued)

Grouping and assigning variable	A2
Strategies in Non-Figural Far Generalization Task	
Strategy	Students' Answers
Using the existing generalization	A2, A3, L2
Identifying regularities by assigning alternative values to the variable and generating the visual representation	A1, Z1, A4
Identifying regularities by generating the algebraic representation and assigning alternative values	L1, Z2

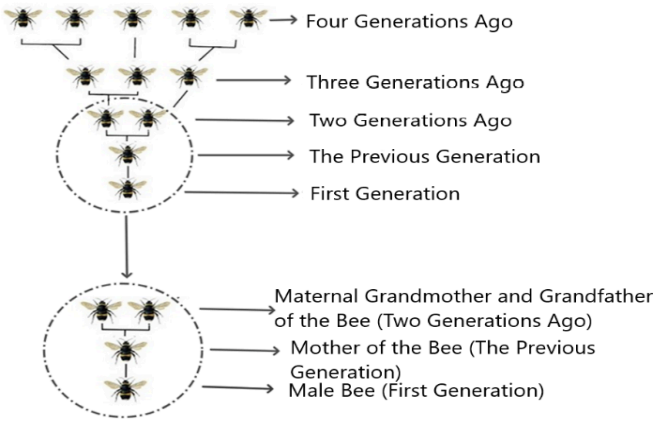
As illustrated in Table 4.2 in the figural near generalization problem, gifted students most commonly utilized counting from a drawing strategy, and it was used by all students at least once. On the other hand, in the figural far generalization problem, *using the existing generalization* and *identifying regularities by investigating the numerical relationships embedded in the figure* were the predominant strategies utilized by gifted students. Lastly, in the non-figural far generalization task, the students mostly used *the existing generalization* and *identifying regularities by assigning alternative values to the variable and generating the visual representation* strategies. In brief, as evidenced by Table 4.2, students' strategy use varied depending on the problem type. Therefore, in the following sections, students' strategies for each problem type will be presented in detail.

4.1.1. Gifted students' strategies in the figural generalization task

The first task was a figural non-linear generalization task consisting of three sub-questions. The first one (a) aimed to investigate if the students make sense of the context and the figure. In this question, the students were expected to complete the identification of male and female bees in the given pedigree. Besides, the

researcher asked each of the participants whether they were familiar with the information given in the question before. Even though none of the students were familiar with the context given in the question, all of them identified the gender of the bees accurately, indicating they understood the context and the figure. Although the responses to question a were not used in the analysis, it would be helpful to provide one example of a student's response to shed light on the context of the problem. Below is the figural generalization task and the answer given by one student to question a.

Did you know that not all bees have two parents? Bees have an interesting reproductive system and, therefore, different family trees. Queen bees are female bees responsible for ensuring genetic diversity. Male bees are formed from the unfertilized eggs laid by the queen bees in the hive, and female bees are formed from the fertilized eggs. So male bees have only one parent (mother), while female bees have two parents (mother and father).



a. In the given family tree, indicate female bees with "D" and male bees with "E".

b. 

Above is the image of a bee. If you accept this bee as the 1st generation, how many bees are there in the six previous generation? Please answer in as many ways as possible.

c. How many bees did you observe in each generation? How do you mathematically express the number of bees in any generation (nth generation)?

Figure 4.1. The Figural Generalization Task

Researcher: Could you explain how you identify the gender of the bees?

Zena: This one (the male bee in the first generation) only has a mother; it has one parent, so it's a male. This one (the female bee in the previous generation) has two parents, so it's female…If the parents of the bees in the four previous generation were visible, I could also determine their genders clearly. There is a fifty percent chance that each of them is either male or female.

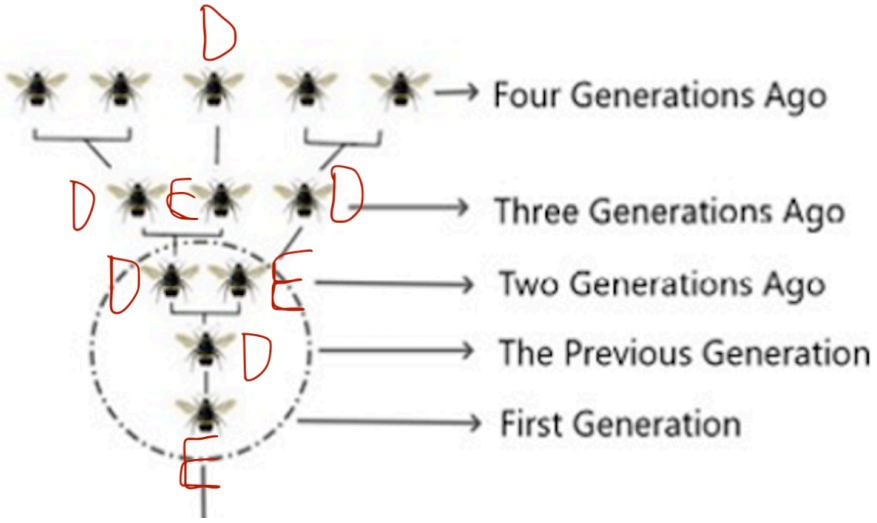


Figure 4.2. Zena's Identification of Bees in the Pedigree

As shown by Figure 4.2 and the interview excerpt, using the information given in Figure 1 that male bees have only one parent (mother) while female bees have two parents (mother and father), the student correctly identified the gender of each bee by examining its parents.

After the students showed a clear understanding of the problem context and figure, they were asked to solve the option b and option c questions of the first task. Option b of the first question was a near-generalization task, and it required students to find the number of bees in the six previous generation. Nevertheless, the option c was a far generalization task, in which students were asked to express the number of bees in the nth generation. The strategies students use in options b and c

of the first task, that is, figural near and far generalization problems, will be addressed in detail.

4.1.1.1. Gifted students' strategies in the figural near-generalization task

The second sub-question of the figural task was a near-generalization task where students needed to figure out the number of bees in the six previous generation. As seen from Tables 4.1 and 4.2, while solving the near generalization task, students used *counting from a drawing*, *identifying regularities by investigating the numerical relationship embedded in the figure*, and *connecting to the known pattern* strategies. Among those strategies, the one that is predominantly used is *counting from a drawing* strategy, and it was used in seven of the thirteen responses. All students used that strategy at least once. The drawing made by a student and her explanation in the interview were given to exemplify *counting from a drawing* strategy:

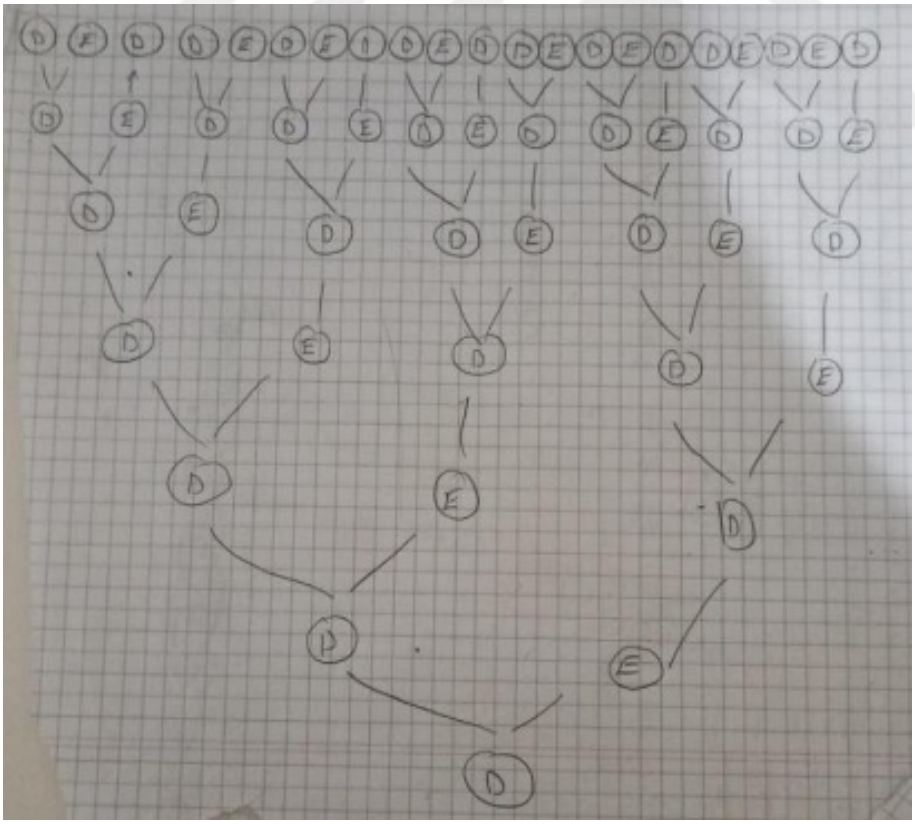


Figure 4.3. Zena's Use of Counting from a Drawing Strategy

Researcher: How many bees did you find in the six previous generation? Could you explain how you found it?

Zena: The female bee (the bee in the first generation) has two parents, so I called the first parent female and the second parent male. The male bee (in the previous generation) has one parent, the female bee. The female bee (in the previous generation) has two parents, the female and the male. This female bee (in the two previous generation) also has two parents. Here are the female and the male parents. The male (in the two previous generation) has one parent, which is the female bee. The female bee (in the two previous generation) has two parents, which are the female bee and the male bee [continue in this way] ... Let me count; there are 21 bees.

As shown in Figure 4.3 and supported by the interview excerpt, Zena drew the pedigree of the bees using the information provided in the question: the female bee has two parents (a female and a male), while the male bee has a single parent (a female). She represented the female bees as "D" and the male bees as "E". Then, she counted the number of bees in the six previous generation and found that there were 21 bees.

Remarkably, although option b of the first task was a near-generalization task, three out of four students could identify the pattern's general rule. Those students found the explicit rule of the pattern using *identifying regularities by investigating the numerical relationships embedded in the figure* and *connecting to the known pattern* strategies. As shown in Table 4.2, three students used *identifying regularities by investigating the numerical relationships embedded in the figure*, and this strategy was used in five of the 13 total responses to this question. Nonetheless, only one student used *the connecting to the known pattern strategy* once. The students who used *identifying regularities by investigating the numerical relationships embedded in the figure* strategy investigated the pedigree, wrote the number of bees in each generation, and then identified the commonalities. Here, one student's written work and the interview excerpt were given to shed light on the student's use of that strategy:

Diana: I realized that the sum of the number of bees in the first generation and the number of its parents gives us the number of bees in the two previous generation. Then the sum of one and two gives us three in the third...There exists a pattern.

Researcher: What kind of pattern have you noticed?

Diana: So, this was the first generation (shows in the pedigree). We had two bees. Three here (in the two previous generation), five here (in the three previous generation), and so on. The sum of these (2 and 3) gives this (5). The sum of these (3 and 5) gives this (8), and the sum of these (5 and 8) gives this (13), and the sum of these (8 and 13) gives this (21). The sum of the number of bees in the previous step and the number of bees in that step gives us the number of bees in the next step, that is, the previous actually.

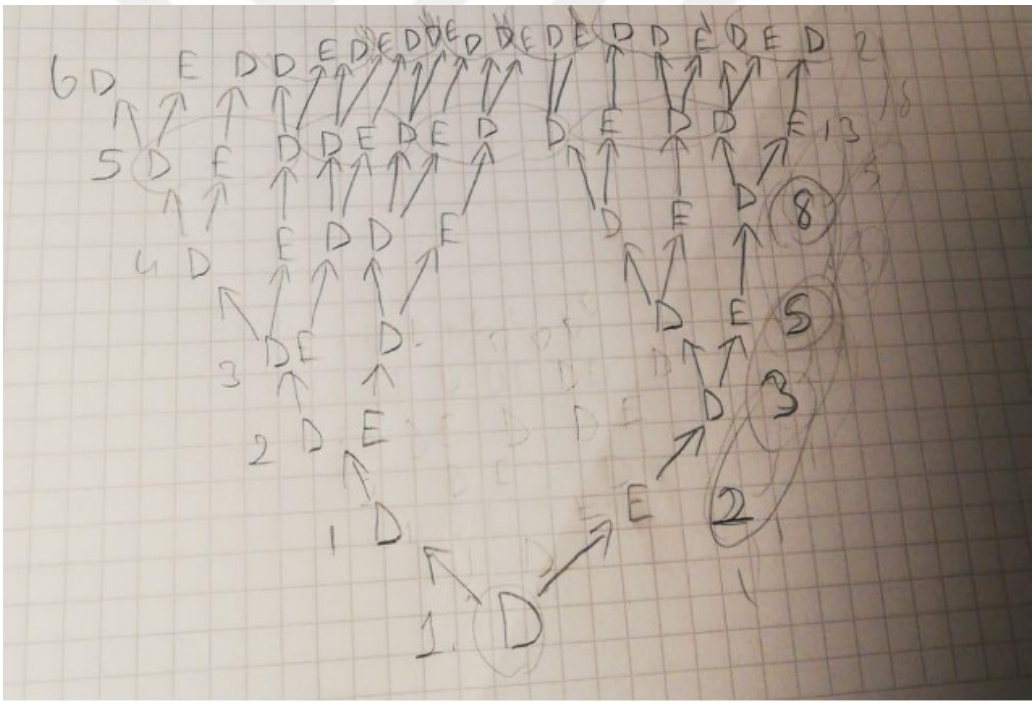


Figure 4. 4. Diana's Use of Identifying Regularities by Investigating the Numerical Relationships Embedded in the Figure

As seen in Figure 4.4, the student wrote the number of generations on the left-hand side of the pedigree and the number of bees in each generation on the right-hand side of the pedigree. Furthermore, it can also be seen that she marked the number

of bees in each generation in pairs. The dialogue clarifies the rationale behind marking the numbers. To be more precise, the excerpt shows that the numbers Diana marked in pairs represent the pattern where the sum of the two numbers in a pair equals the number immediately following them. For instance, she marked 2 and 3 and explained that the sum of 2 and 3 gives 5. She followed the same procedure for other generations as well. The written work of the student and the excerpt illustrate that the student noticed the rule by exploring the numerical relationship, i.e., the number of bees embedded in the pedigree.

The third strategy was *connecting to the known pattern* strategy, and it was used only by one student at once. In that strategy, as far as the student started to investigate the problem, he noticed that the pattern was the same as the previously known pattern, then connected the previously known pattern to the problem. The interview excerpt indicates the student's use of the *connecting to the known pattern* strategy.

> Luca: Well, it could be Fibonacci. There is zero here (showing on the pedigree); add zero and one, and you get one. Add one and one, and you get two. Add two and one, and you get three. Add three and two, and you get five. Five and three is eight. Six generations ago, it was thirteen.

The excerpt shows that Luca noticed that the pattern was the Fibonacci pattern. Stated differently, he connected the relationship between the number of bees in the family tree with the Fibonacci sequence and added the number of bees two generations ago to find the number of bees in a generation.

4.1.1.2. Gifted students' strategies in the figural far generalization task

Contrary to option b of the first question, option c of the first question was a figural far generalization task. In that question, the students were expected to find the number of bees in the nth generation. While solving this problem, the students used four strategies: namely, *identifying regularities by investigating the numerical relationships embedded in the figure*, *using the existing generalization*, *grouping*

and assigning a variable, and *connecting to the known pattern*. While *identifying regularities by investigating the numerical relationships embedded in the figure*, and *using the existing strategies* employed by all students, *grouping and assigning a variable,* and *connecting to the known pattern* were only used by one student. Two responses of one student involved *connecting to the known pattern* strategy. Lastly, the *grouping and assigning variable* strategy was only used once by one student. On the other hand, *identifying regularities by investigating the numerical relationships embedded in the figure* and *using the existing generalization* strategies were both used in 6 of 15 responses.

Figure 4.5 and the interview excerpt shed light on *identifying regularities by investigating the numerical relationships embedded in the figure*.

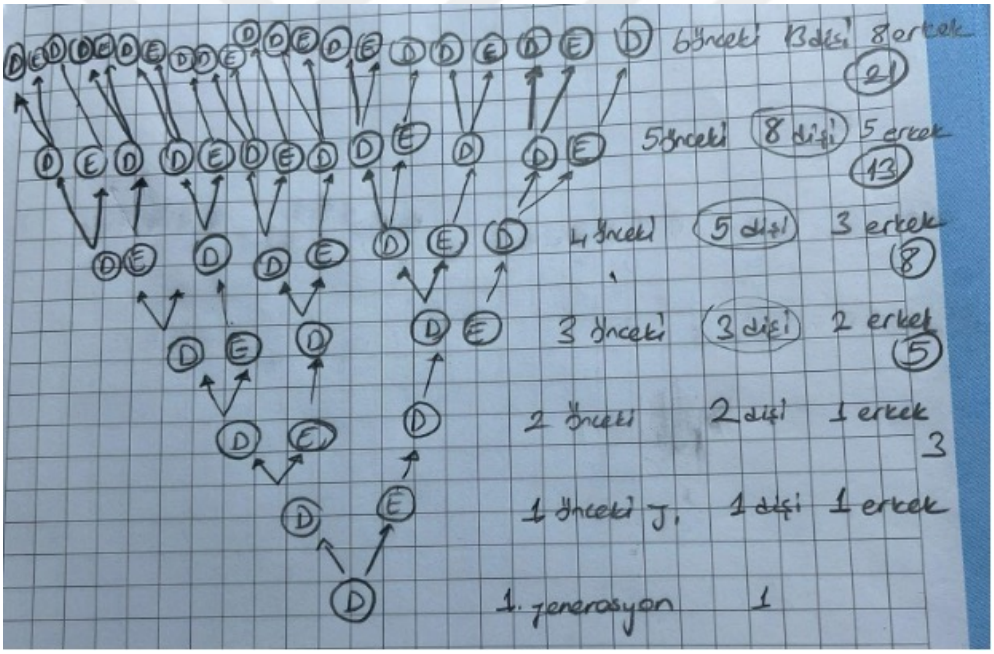


Figure 4.5. Zena's Use of Identifying Regularities by Investigating the Numerical Relationships Embedded in the Figure

Researcher: Could you explain what you have done?

Zena: There is a female, so I added it up with the total. There is a female; I added it with two of them, making three. There are two females; I added two and three together to make five females. There are three females; I

> added three and five together to make eight. There are five females; I added eight and five together to make thirteen. We had eight females; I added thirteen and eight together and got twenty-one. There were twenty-one bees in total. In other words, when I add the number of females and the total number of bees in that step, I find the total number of bees in the previous generation.

Zena first wrote down the number of bees in the pedigree. However, as seen in the pedigree, Zena noted the number of male and female bees in each generation as well as the total number of bees. The interview excerpt shows that Zena examined the family tree and the number of male and female bees in each generation and noticed the rule that when the total number of bees and the number of female bees in a generation are added together, the total number of bees in the next generation is found.

The second predominantly applied strategy was *using the existing generalization* strategy. This strategy involves two cases: if the student used the generalization that he/she arrived at in the previous questions, or if the student applied the generalization that he/she identified in that question. The commonality of these cases is that students generated the generalization on their own; this is the distinction between *using the existing generalization* and *connecting to the known pattern* strategy. Below, one of the students' responses was given to shed light on *using the existing generalization* strategy:

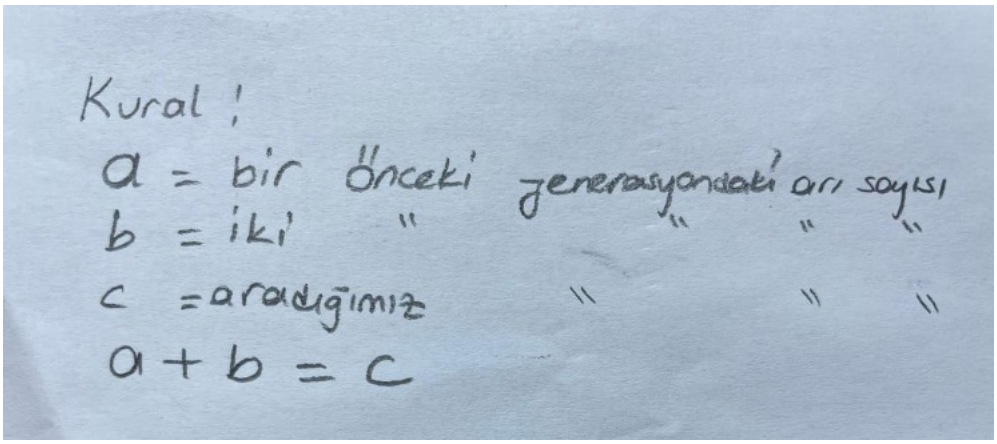


Figure 4.6. Luca's Use of the Existing Generalization Strategy

> Researcher: Could you explain what you have done?
>
> Luca: That's what I had found. The sum of the number of bees in the previous generation (a) and the number of bees in the two previous generation (b) equals the number of bees in the generation we are looking for (c).

The student identified the pattern and expressed it verbally in this question. Then, he utilized his prior answer to arrive at the new answer. The student's written response (see Figure 4.6) indicates that he found the explicit rule of the pattern and expressed it symbolically. In the interview, saying, "That's what I have found," supports that he used the generalization that he had already found out to express the rule symbolically. Thus, the new symbolic generalization was built on the prior generalization.

The third strategy was *grouping based on the variable* strategy, which was used by only one student. This strategy involves considering the entities of the problem as specific groups (grouping) and identifying a general rule by assigning a variable to the particular group (assigning a variable). As seen from Figure 4.7 and the dialogue, the student expressed the rule of the pattern by grouping the female and male bees and assigning a variable, n, to the pair of female and male bees.

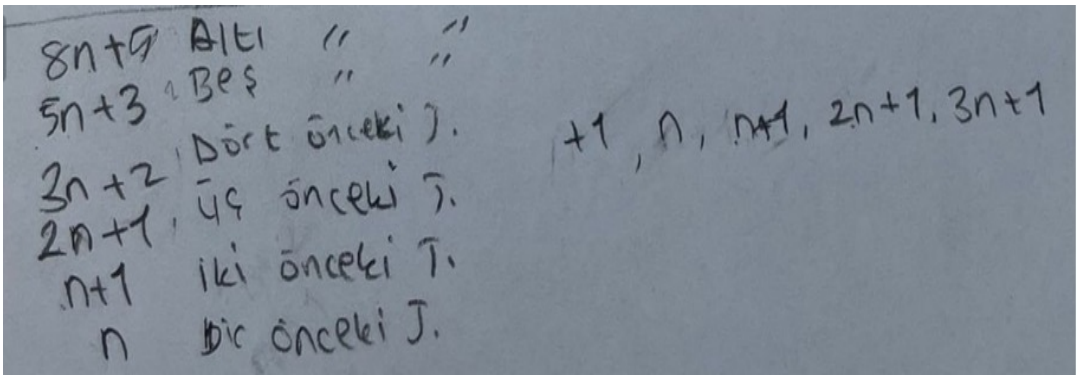


Figure 4.7. Amelia's Use of Grouping Based on the Variable Strategy

> Researcher: Could you tell me how you wrote these expressions?
>
> Amelia: I considered the male and female pair as n. There is no chance of remaining an extra male because the male and female have to be together anyway. That's why the female bees remain. I wrote the number of females as a plus. For example, we have a male and female group in the two previous generation; I called this pair n. Then, one female left, so I wrote this female as a plus one (explains this by showing the bees in the pedigree).

When I asked Amelia how to express the number of bees in any step, she grouped the male and female bees as a pair and named the pair of male and female bees in any step as n and expressed the remaining bees as plus. The dialogue and Figure 4.7 also show that she expressed the number of bees in several generations by using this strategy. For instance, by examining the family tree, the student noticed that there was a female and male bee pair in the two previous generation and named this male and female bee pair as n. Then, she wrote the remaining female bee as plus one and expressed the number of bees in the two previous generation as n+1. Thus, by applying the rule to several cases, she stated that it was the general rule of the pattern.

The last strategy was *connecting to the known pattern* strategy, and it was applied by one student, who also used it in option b of the first question. The excerpt and the written response of the student are given below:

Luca: (Writes “Fibonacci Numbers” as an answer)

Researcher: You wrote Fibonacci numbers. Could you explain what you have done?

Luca: For example, let's say n is 9. The 9th Fibonacci number is 55. If we know the Fibonacci numbers, we can do so.

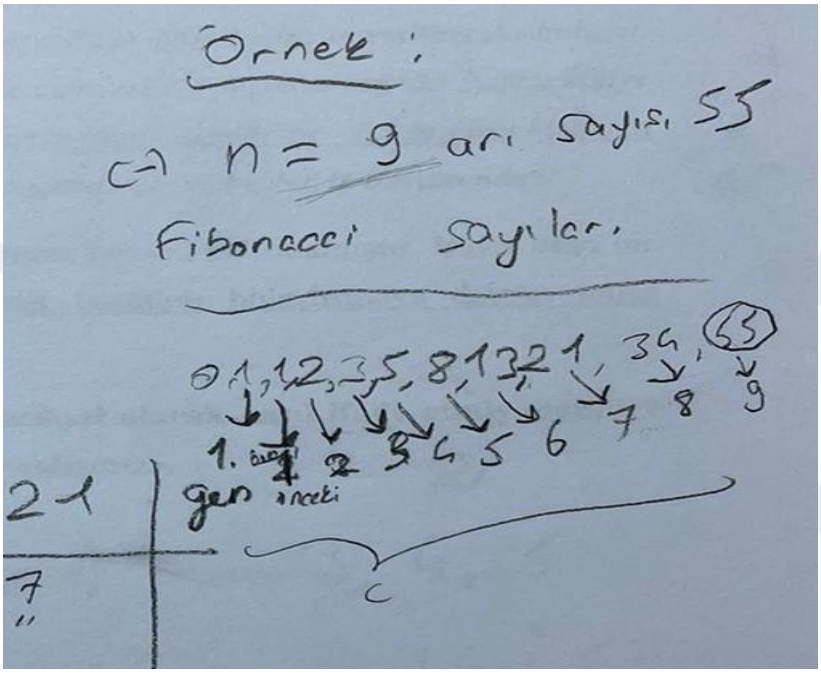


Figure 4.8. Luca’s Use of Connecting to the Known Pattern Strategy

The student said the number of bees in any generation can be expressed in terms of Fibonacci number. Moreover, as illustrated by Figure 4.8, Luca associated the Fibonacci numbers and the number of bees in each generation.

In conclusion, analysis of students’ responses to figural near and figural far tasks indicated that gifted students used more strategies (*using the existing generalization*, *identifying regularities by investigating the numerical relationships embedded in the figure*, *connecting to the known pattern*, and *grouping and assigning variable*) to achieve far generalization than they did to answer the near generalization task (*counting from a drawing*, *identifying regularities by investigating the numerical relationships embedded in the figure*, and *connecting to the known pattern*). Moreover, while students mostly used the *counting from a*

drawing strategy in the near generalization question, they predominantly applied *identifying regularities by investigating numerical relationships embedded in the figure* and *using the existing strategies* to reach the far generalization.

4.1.2. Gifted students' strategies in the non-figural generalization task

The second task is a non-figural far generalization problem. In this problem, by using the given scenario, students were expected to identify the number of infected people in n units of time. The problem is given below:

The contagious COVID-19 coronavirus disease, first seen in our country in 2020, has affected the whole world. At that time, when it was important to take precautions in advance, a researcher named Manav Bhatnagar developed a mathematical model showing the spread of the COVID-19 virus to accurately predict COVID-19 cases in 2020. In order to observe the effect of measures such as lockdown on the spread of the virus, Bhatnagar first created the model showing the spread of COVID-19 when no precautions are taken, based on the following scenario:

> *Assume that a coronavirus-positive patient arrives in the country and comes into contact with other people. Since coronavirus is an infectious disease, it can also be transmitted to other people. Let us represent the sick person with A. In one unit of time, person A can infect* ***a*** *number of other people. Now, all the sick people are active, and each sick person can infect* ***a*** *number of other people, and so on.*

Important Note: In this model, the active time of the virus (incubation period) is not taken into account. It is assumed that a sick person is always infected, that is, continues to transmit the disease continuously.

How could Bhatnagar have expressed the total number of sick people mathematically? Try to answer in as many ways as possible.

Figure 4.9. The Non-Figural Far Generalization Task

Analysis of students' responses demonstrated that gifted students used the following three strategies: *identifying regularities by assigning alternative values to the variable and generating the visual representation*, *identifying regularities by*

generating the algebraic representation and assigning alternative values, and *using the existing generalization*. The first two strategies were unique to this problem type. Each strategy was used by two of the participants. However, while the first and the third strategies were used three times, the second one was used twice.

The first strategy that was used to solve the non-figural nonlinear pattern generalization task was *identifying regularities by assigning alternative values and generating the visual representation*. This strategy was used by all students except Luca. The students who used that strategy were first assigned different values to the unknown. Afterward, they generated a visual representation that represents the given context. Then, by investigating the visual representations and the numerical relationships, they noticed the regularities and identified the rule. The main difference between this strategy and the strategy of *identifying regularities by investigating the numerical relationships embedded in the figure* strategy is that although there was no visual in this question, the students visualized the situation themselves. Here, one student's response was given to elucidate the *identifying regularities by assigning alternative values and generating the visual representation* strategy.

> Amelia: I will assign values. A is one. I gave two to *a*. Now, A has infected two people (since *a* is two). Then these two people will infect two people again, and this (A) will infect two people… (she continued to draw the infected people and note the total number of patients at each step) ... Aha! It goes by powers of three.

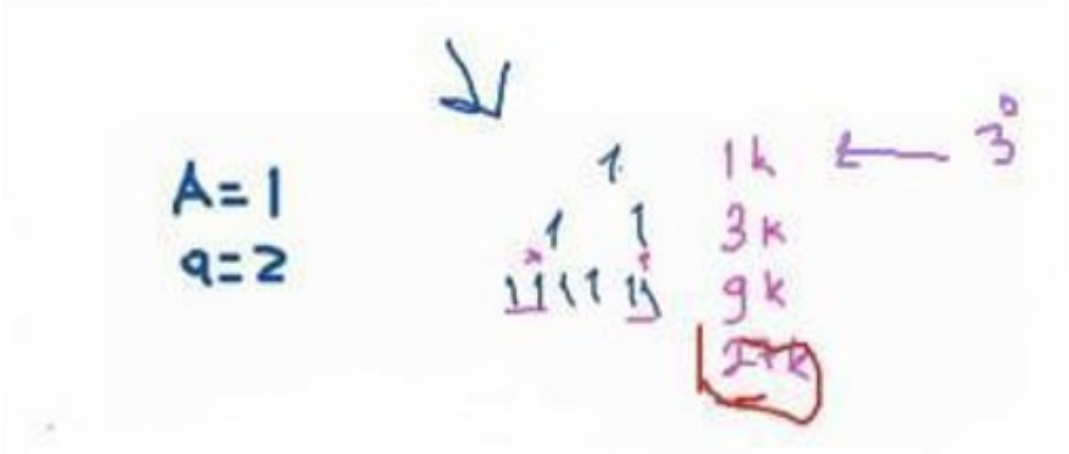


Figure 4.10. Amelia's Written Work for the Non-Figural Far Generalization Task Assuming *a* To Be Two

Amelia: I assumed *a* to be two. However, for example, if we assumed it to be one, there would be one (patient) from A. In the third step, one will come from A. Another one will come from the one infected in the second step. Thus, there will be two new patients. There were already two; now there are four (patients). This time, it was a power of two.

Researcher: Could you draw any inference? How would you express the total number of patients in x units of time?

Amelia: One plus *a* to the power of x.

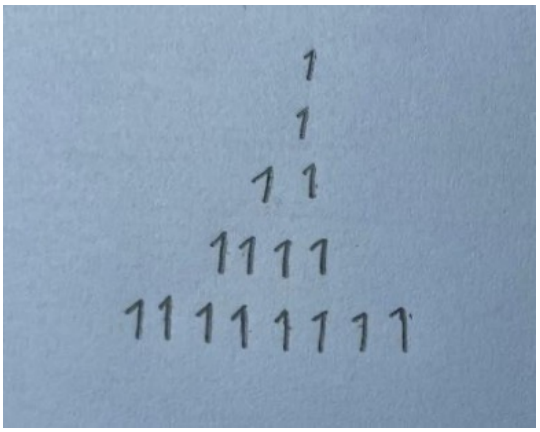


Figure 4.11. Amelia's Written Work for the Non-Figural Far Generalization Task Assuming *a* To Be One

In this question, the unknown is *a*; that is, the number of people each infected person infects with the disease in one unit of time. Here, the student first assigned different values to *a* and visualized the number of patients at each time based on these values. In Figure 4.10, the "1" drawn at the beginning represents person A, who brought the disease to the country. Then, since the student assumed *a* to be two, she drew two sick people infected by A just below the first 1. Then, since each sick person continued to infect people, in the third step, she drew the people infected by the already ill people. Amelia then noted the total number of sick people in each step and noticed that the number of patients in each step was a power of three. However, she has not expressed the general rule of the pattern. Afterward, she assumed that *a* was one, that is, the number of people infected by each infected person at each step, and visualized this situation. Looking at Figure 4.11, the 1 at the top represents person A, who brought the disease to the country. Then, since she assumed *a* to be 1, she drew the person A infected below, and then, the people infected by each sick person. She continued this process until the fifth step. Moreover, it is evident from the dialogue that the student noticed that when *a* is assumed to be one, the number of patients in each step becomes a power of two. In sum, she found that when *a* is two, the number of persons in each step was powers of three, and when *a* is one, the number of persons in each step was powers of two. From these regularities, she noticed that the number of infected patients at each time continues as a power of one more than *a*. Therefore, she expressed the rule as one plus *a* to the power of x, where x refers to the unit of time.

The second strategy gifted students used to solve the non-figural far generalization task was *identifying regularities by generating the algebraic representation and assigning alternative values*, and it was used by two students. Students using this strategy first wrote the algebraic expression that describes the situation at each step. Later, they assigned different values to the unknown and found the general rule from the regularities. Excerpts and written work of one student were given to demonstrate the student's use of *identifying regularities by generating the algebraic representation and assigning alternative values* strategy.

Luca: This is one unit of time (wrote "1+ a "). In two time periods, there is one and a at the beginning, plus a, plus a times a, which is a square because a number of people also infects a different a number of people. So, let's assume a is two. Each of those two people infects two people. So, two times two is four, a square of two (he checked whether a square is logical and accurate or not). What happened here is now one plus two a, plus a square (followed the same procedure for three units of time and expressed the number of patients as $a^3+3a^2+ 3\ a\ +1$, as seen in Figure 4.12).

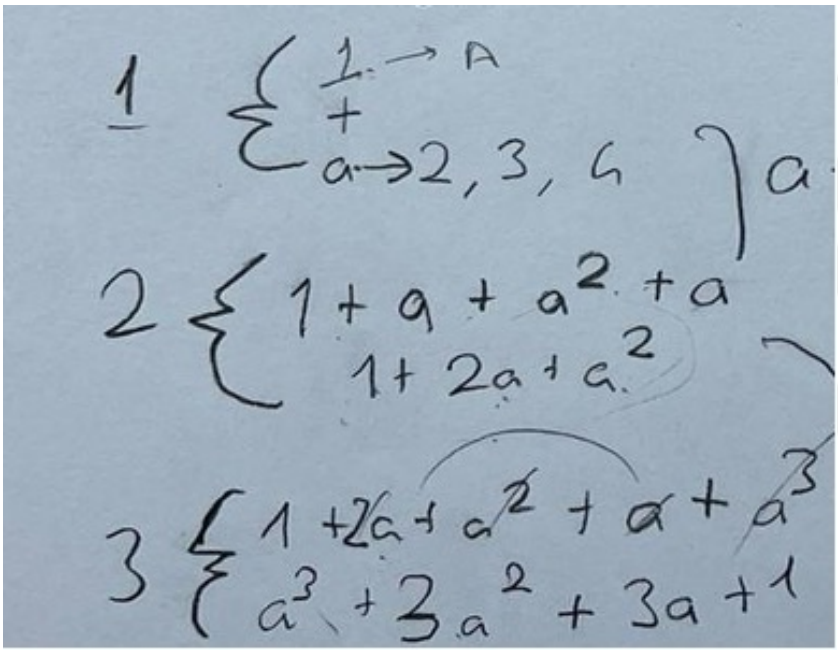


Figure 4.12. Luca's Representation of the Number of Patients in One, Two, and Three Time Units with Algebraic Expression

Luca: Now I have found the number of patients in three time periods. I will assign different values to a because it seems easier to me. I will assign more than one number. Then, I will find the similarities between them. Let's say a is two, the easiest one. If this a is two, those (number of patients) become three, nine, and twenty- seven. What came to my mind was the powers of three.

Researcher: Why might the number of patients be a power of three?

Luca: I don’t know; I'll take a look at the other values as well. This time, let's say *a* is equal to three. Those (number of patients) became four, sixteen, and sixty-four, the powers of four. Let's say *a* is four. Then, it goes like five, twenty-five, and one hundred and twenty-five; these are the powers of five. What's going on? What kind of relationships exist? In all cases, the first number represents the first power. For instance, three means *a* plus one to the first time period power. What about nine or sixteen when *a* equal two and three? This is again the time period power of *a* plus one. Therefore, the time period power of *a* plus one fits all.

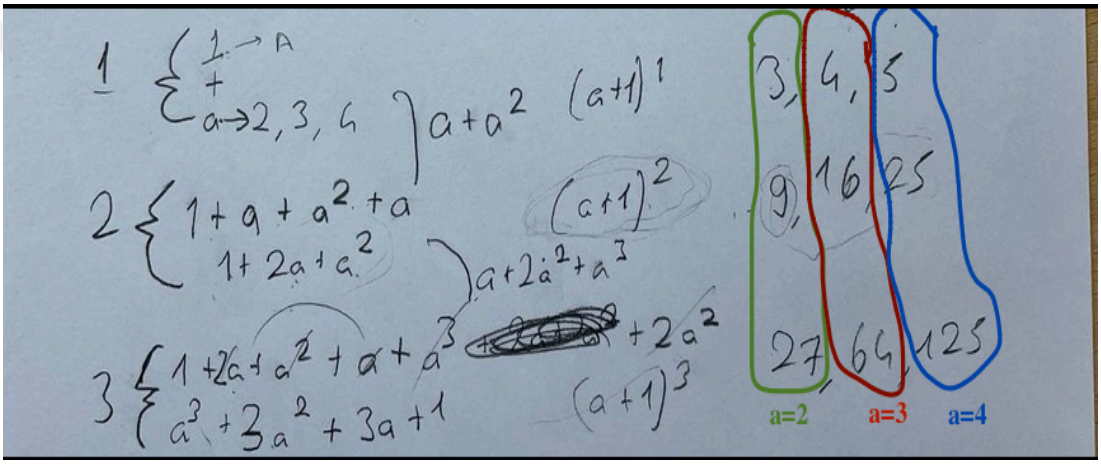


Figure 4.13. Luca’s Use of Identifying Regularities by Generating the Algebraic Representation and Assigning Alternative Values

Figure 4.12 shows that the student wrote the algebraic expression expressing the total number of patients in one, two, and three units of time. The student found the number of patients in the first unit of time as 1+ *a*. While 1 represents the person A, who initially brought the disease to the country, *a* is the number of people A infected with the disease within one unit of time. Then, to express the total number of patients in two units of time, he added the number of patients that A would infect (1* *a* = *a*), the number of people infected by each *a* person (*a* * *a* = a^2), to the number of patients already present (1+ *a*). Therefore, he expressed the total number of patients in two units of time as $1 + 2a + a^2$. Then, to find the total number of patients in three units of time, he added the number of existing patients

($1 + 2a + a^2$), the number of people that A will infect in the third time period ($1 * a = a$), the number of people each a^2 number of people will infect ($a^2 * a = a^3$) and the number of people each of $2a$ number of people would make infected ($2a * a = 2a^2$). As a result, he wrote the total number of patients in three units of time as $a^3 + 3a^2 + 3a + 1$, as can be seen in Figure 4.12. Then, as depicted by Figure 4.13, the student wrote down the total number of patients in each time period by assigning different values to *a* and substituting the assigned values into the algebraic expressions he had previously written ($1 + a$; $1 + 2a + a^2$; $a^3 + 3a^2 + 3a + 1$). The student first assigned the value of two to *a* and realized that the total number of patients was a power of three. However, after realizing this, he did not make any judgment about the general rule. To be more precise, like Amelia, Luca was reluctant to express the rule based on a single case. On the contrary, he said, "I will examine other values as well," as seen in the dialogue. Then, he assumed *a* to be three and realized that this time, the total number of patients was a power of four. However, as seen from the dialogue, he still refrained from commenting on the rule. Then, the student repeated the same process, assuming *a* to be four. Luca indicated that the total number of patients continued as powers of five this time. Afterward, the student reflected on whether there was a relationship between the total number of patients when he assigned different values to *a*. As can be seen from the dialogue, he noticed that in all cases where he gave different values to *a*, the total number of patients in the first time period was one more than *a*. Furthermore, when the student examined the total number of patients in other time periods, he noticed that the number of patients in all cases progressed as a power of $a + 1$ and stated that the time period power of *a* plus one fit all cases. To sum up, he first generated an algebraic representation of the situation, then he assigned different values, and, in the end, he noticed and expressed the rule accurately.

The last strategy applied by the two students was *using an existing generalization*. The excerpt depicts Amelia's utilization of the *existing generalization* strategy.

> Amelia: … *a* times the number of already sick people plus the total number of already ill people in the previous step.

Researcher: How would you express the number of patients in x units of time?

Amelia: X units of time...If I want the previous step, I do x-1. Then, the number of people in x units of time would equal *a* times the number of patients in x-1 units of time, plus the number of patients in x-1 units of time.

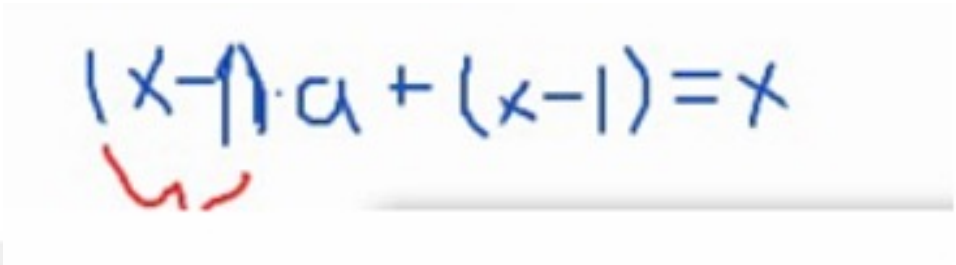


Figure 4.14. Amelia's Use of Existing Generalization Strategy

It is evident that the student first responded verbally and stated that multiplying *a* by the number of existing patients and adding the number of existing patients gives the total number of patients. Then, the researcher asked the student how to express the number of patients in x units of time. Using the generalization expressed verbally, the student described the number of patients in x units of time symbolically. To be more precise, Amelia assumed that the number of patients in the desired step is x and related her verbal expression to the algebraic representation by saying that if I wanted to know how many patients were in the prior step, I would set it to x-1. Then, she expressed the rule in a way that *a* times the number of patients in x-1 units of time (which refers to *a* times the number of already sick people in her first response), plus the number of patients in x-1 units of time (which refers to the total number of already ill people in the previous step).

4.1.3. Summary of the findings of the first research question

To sum up, gifted students utilized various strategies to solve non-linear realistic algebra problems. Besides, as shown in Table 4.2, students tended to utilize the strategies they had used before more than once. In addition, their use of strategies differed in different types of problems. Specifically, if students' strategies were examined regarding near and far problem types, it was observed that while students

employed the *counting from a drawing* strategy to find the unknown in a single case, they used more complicated strategies, such as *identifying regularities by assigning alternative values to the variable and generating the visual representation* and *grouping and assigning variable* strategies to reach the rule of the pattern. Furthermore, regarding figural and non-figural questions, the findings revealed that in figural tasks, students mostly used *counting from a drawing* and *identifying regularities by investigating the numerical relationships embedded in the figure* strategies. On the other hand, in the non-figural task, one of the predominantly employed strategies was *identifying regularities by assigning alternative values to the variable and generating the visual representation* strategy. Moreover, *identifying regularities by assigning alternative values to the variable and generating the visual representation* strategy, and *identifying regularities by generating the algebraic representation and assigning alternative values* strategies were used exclusively in response to the non-figural task. Stated differently, these two strategies were unique to the non-figural problem type. Therefore, it can be concluded that in the problem containing the figure, students investigated the figure and benefited from it in the generalization process. Nevertheless, in the non-figural task, students tended to demonstrate the given situation with different representations, such as algebraic expressions or visual representations.

4.2. Mathematical creativity of gifted students in realistic non-linear generalization tasks

The second aim of the present study was to investigate 8th-grade gifted students' mathematical creativity in realistic nonlinear algebra tasks. Following this aim, students were asked to solve figural near, figural far, and non-figural far tasks in as many ways as possible and elaborate on their responses. As explained in detail in the data analysis section, students' responses were first examined regarding appropriateness. Then, appropriate answers were analyzed in terms of the other four components of mathematical creativity: fluency, flexibility, originality, and elaboration. Students' mathematical creativity in each question will be investigated separately based on the four aforementioned components. Therefore, in the

following sections, the mathematical creativity of the students in figural near, figural-far, and non-figural far generalization tasks will be presented one by one.

4.2.1. Mathematical creativity of gifted students in the figural near generalization task

Students first were asked to solve the figural near generalization task in as many ways as possible. In the following sections, 8th-grade gifted students' mathematical creativity in the figural near generalization task was addressed regarding fluency, flexibility, originality, and elaboration.

4.2.1.1. Fluency of gifted students in the figural near generalization task

Analysis of students' responses for the figural near generalization task revealed 13 appropriate answers. The fluency of the gifted students was investigated individually according to the number of their appropriate responses to the figural near-generalization task, and they received 1 fluency score for each of their appropriate answers. Students' appropriate responses and fluency scores for the figural near generalization task will be presented below.

Amelia provided 4 appropriate answers to the figural near generalization task. Table 4.3 depicts Amelia's responses for the figural near generalization task.

Table 4.3. Amelia's Appropriate Responses for Figural Near Generalization Task

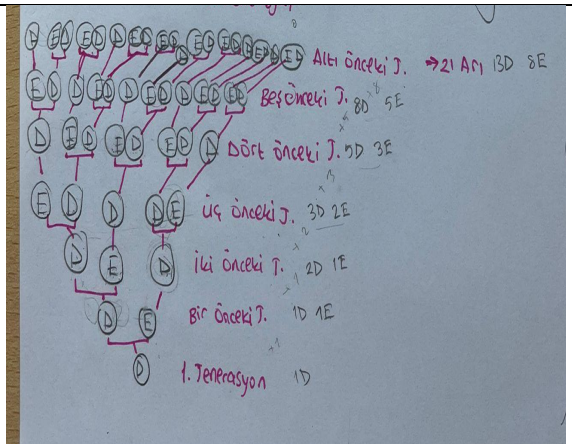
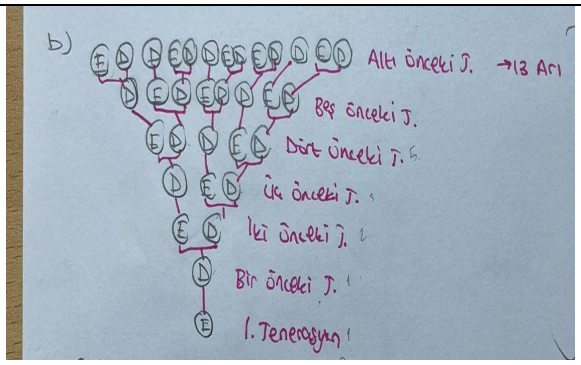
Description	Response
A1: Amelia's first responses to the figural near generalization task	

Table 4.3. (continued)

A2: Amelia's second response to the figural near generalization task	
A3: Amelia's third response to the figural near generalization task	The sum of the number of bees in two and three previous generations gives the number of bees in four previous generation, the sum of the number of bees in the three and four previous generations gives the number of bees in five previous generation, and four and five gives the number of bees in six previous generations, which is 21. So, like this, once we figured out one and two, we could do them all.
A4: Amelia's fourth response to the figural near generalization task	I wrote the male version again to see if the situation for the female bee is also valid for the male bee. It goes like one, one, two, three. It had the same thing. When we add the number of bees in the four and five previous generation, it comes out to 13.

It is evident from Table 4.3 that Amelia provided four appropriate answers for the figural near generalization task. In her first response (A1), she assumed that the first generation is a female bee, and in line with this assumption, she drew the pedigree and found the number of bees by counting as 21. Then, in her second response, A2, she assumed that the first generation is a male bee, and based on this assumption, she drew the pedigree and found the number of bees by counting as

13. Moreover, A3 represents the third response of Amelia in the figural near generalization task. As can be seen from the dialogue, she identified the rule of the pattern as the number of bees in each generation is the sum of the number of bees in the two previous generations by investigating the pedigree where the first generation is female. Afterward, in her fourth response, A4, she assumed the first generation was a male bee and expressed that the rule she had already found was also valid if the first generation was a male bee. Thus, since she provided four appropriate responses, A1, A2, A3, and A4, she received 1 point for each of them, making her fluency score 4.

Like Amelia, Diana also gave four accurate responses to the figural near generalization task, which were similar to Amelia's responses (see Table 4.3). Thus, they have the same fluency score, which is 4.

Luca provided three appropriate answers while solving the figural near generalization task. In his first response, he made the connection between the reproductive pattern of the bees and the Fibonacci sequence. His first response was given below:

> Luca: Well, it could be Fibonacci. There is zero here (showing on the pedigree); add zero and one, and you get one. Add one and one, you get two. Add two and one, you get three. Add three and two, you get five. Five and three is eight. Six previous generation, it was thirteen.

Moreover, Luca's second answer was comparable to Amelia's second response, and his third answer was comparable to Amelia's third response (see Table 4.3). Hence, Luca provided three appropriate responses to the figural near generalization task, which implies that his fluency score is 3.

Notably, Zena insisted that there could only be two solutions to this problem since there were two possibilities for the gender of a bee: male or female. Therefore, she gave two appropriate answers to the figural near generalization task, which were

similar to Amelia's first two responses. For this reason, she had the lowest fluency score in the figural near generalization task, which is 2.

To sum up, students provided different numbers of appropriate responses for the figural near generalization task, so their fluency scores varied. Table 4.4 depicts the fluency scores of the students.

Table 4.4. Fluency Score of Each Student in the Figural Near Generalization Task

Student	Fluency Score
Amelia	4
Diana	4
Luca	3
Zena	2

4.2.1.2. Flexibility of gifted students in figural near generalization task

As stated previously and evident in Table 4.4, data analysis revealed that students provided 13 appropriate answers for the figural near generalization task. Nevertheless, students' responses differed in terms of the underlying idea behind the response, the representation of the response, and the strategy that was used to reach the response. Thus, as explained in detail in data analysis, the idea, the representation type, and the strategy constituted the categories. For this reason, each of the 13 appropriate answers was examined in terms of the underlying idea, representation type, and strategy used, and answers that were similar in terms of these aspects were evaluated in the same category.

When the answers given by the students were examined, four main ideas existed in total responses:

1. If the 1st generation is male, the number of bees in the six previous generation is 13.
2. If the 1st generation is female, the number of bees in the six previous generation is 21.

3. The number of bees in a generation is the sum of the number of bees in the two previous generations.
4. The reproductive pattern of the bees is an example of the Fibonacci sequence.

Upon evaluating the answers given by the students to the figural near generalization task in terms of representation types, it was observed that two forms of representation were used, namely, verbal and visual representations.

Examining the strategies used by the students in the figural near generalization question revealed that the students used three strategies, as discussed in detail in 4.1. These strategies were *counting from a drawing*, *identifying regularities by investigating the numerical relationships embedded in the figure*, and *connecting to the known pattern*.

Table 4.5 shows the categorization of students' answers in terms of the idea behind the answer, the way of representation, and the strategy used to arrive at the answer.

Table 4.5. Categorization of Students' Responses for Figural Near Generalization Task

Category	Underlying Idea	Representation Type	Strategy	Student's Answer
1st Category	If the 1st generation is male, the number of bees in the six previous generation is 13.	Visual	Counting from a drawing	D1, A2, Z2, L2
2nd Category	If the 1st generation is female, the number of bees in the six previous generation is 21.	Visual	Counting from a drawing	D2, A1, Z1

Table 4.5. (continued)

3rd Category	The number of bees in a generation is the sum of the number of bees in the two previous generations.	Verbal	Identifying regularities by investigating the numerical relationships embedded in the figure	D3, D4, A3, A4, L3
4th Category	The reproductive pattern of the bees is an example of the Fibonacci sequence.	Verbal	Connecting to the known pattern	L1

Students' flexibility scores were determined individually based on the categorization of their responses, as depicted in Table 4.5. Students received 1 flexibility score for each of their appropriate answers in different categories. A thorough explanation of how each student's flexibility score was determined will be addressed below.

As evidenced by Table 4.3, Amelia provided four appropriate responses (A1, A2, A3, and A4) to the figural near-generalization task. Her first response belonged to the second category. Specifically, her first response was based on the idea that if the 1st generation is female, the number of bees in the six previous generation is 21. Moreover, she used *counting from a drawing* strategy and visual representation in that response. Furthermore, her second response belonged to the first category. The underlying idea behind that response was that if the 1st generation is male, the number of bees from the six previous generation is 13. In her second response, she applied *counting from a drawing* strategy and visual representation. Although she

used the same strategy and the same representation in her first and second responses, since the idea behind her first and second responses differed, her first and second responses belonged to different categories. In addition, as Table 4.5 depicts, Amelia's third and fourth responses convey the idea that the number of bees in a generation is the sum of the number of bees in the two previous generations. She used verbal representation and *identifying regularities by investigating numerical relationships embedded in the figure* strategy in her third and fourth responses. Since the idea, the representation, and the strategy were the same in her third and fourth responses, those responses were classified in the same category. In the end, Amelia's four responses belong to three different categories, which implies that her flexibility score was 3.

The other student, Diana, provided four responses (D1, D2, D3, and D4) to the figural near generalization task. Diana's responses belonged to the same categories as Amelia's responses. To be more precise, as revealed in Table 4.5, her first response fell under the first category, the second response was classified in the second category, and the third and fourth responses were classified in the third category. Thus, similar to Amelia's responses, the categories of her three responses differ among her four responses, so she received three points for flexibility.

Luca provided three responses to the figural near-generalization task. The idea underlying Luca's first response was that the reproduction pattern of the bees is an example of the Fibonacci sequence. Moreover, he used verbal representation and *connecting to the known pattern* strategy in his first response. Furthermore, his second response was based on the idea that if the 1st generation is male, the number of bees in the six previous generation is 13. He applied a visual representation and *counting from a drawing* strategy in that response. Since the idea, representation, and strategy differed, his first and second responses fell under different categories. Lastly, the idea behind his third response was that the number of bees in a generation is the sum of the number of bees in the two previous generations. The representation type of his third response was verbal representation. Moreover, he employed *identifying regularities by investigating the*

numerical relationships embedded in the figure strategy in his third response. When Luca's three answers were examined, it was seen that each answer contained a different idea, and each answer was solved with a different strategy. Pertaining to representation type, although the first and third answers use the same representation type, verbal representation, these answers were in different categories because they differ in terms of idea and strategy. Thus, since each of Luca's three responses belonged to a different category, his flexibility score was 3.

Zena gave two relevant answers for the figural near generalization task. Her first response conveyed the idea that if the 1st generation is female, the number of bees in the six previous generation is 21. Moreover, the representation type of her first response was a visual representation, and the strategy used in that response was *counting from a drawing*. On the other hand, although she used visual representation and *counting from a drawing* strategy in her second response, the ideas behind her first and second responses were different. Expressly, her second response was based on the idea that if the 1st generation is male, the number of bees in the six previous generation is 13. Thus, since the idea behind her first and second responses differed, they belonged to different categories. Therefore, as her responses were placed in two distinct categories, her flexibility score was 2.

To sum up, as explained in detail above, flexibility scores of the students were determined individually according to the categories of their appropriate responses to the figural near-generalization task. Table 4.6 shows the flexibility scores of the students.

Table 4.6. Flexibility Score of Each Student in the Figural Near Generalization Task

Student	Flexibility Score
Amelia	3
Diana	3
Luca	3
Zena	2

4.2.1.3. Originality of gifted students in the figural near generalization task

Originality was evaluated according to the frequency of the responses, as explained in detail in the data analysis section. To conduct this evaluation, Table 4.5 was utilized. In the last column of Table 4.5, the answers of the students in each category were given. Each cell in that column was examined individually; then, the number of people who gave that response was identified for each response in a particular category. Afterward, the students' originality scores were determined according to Table 4.7, which is given below. However, when scoring, the number of students whose answers were placed in a particular category was taken into account rather than the number of answers in that category. As detailed in the data analysis section, expert opinion and approval were obtained about scoring.

Table 4.7. Rubric for Assessing Originality of Students' Responses

Number of students who gave that response	Score that will be assigned to that response
1	4
2	3
3	2
4	1

Upon reviewing the responses in the first category in Table 4.5, it was observed that four students' answers were in this category. According to Table 4.7, for the answers in the first category, each student who has an answer in that category received 1 point for originality. Furthermore, the answers in the second category belong to three different students. For this reason, each student with an answer in this category got an originality score of 2, according to Table 4.7. Similarly, there were again three different students' answers in the third category. Thus, each student with an answer in the third category was given 2 points for originality. Finally, since there was only one student's answer in the fourth category, this student received 4 points for originality, according to Table 4.7. Consequently,

originality scores obtained by each student from the different categories were added, and each student's originality score was determined. The process of determining each student's originality scores will be explained in detail below.

It was evident in Table 4.5 that Amelia's first answer was in the second category, her second answer was in the first category, and her third and fourth answers were in the third category. Amelia received an originality score of 1 for her answer in the first category, an originality score of 2 for her answer in the second category, and an originality score of 2 for her third and fourth answers in the third category. As a result, her total originality score was 5.

As illustrated by Table 4.5, Diana’s first response belonged to the first category; her second response was located in the second category, and her third and fourth responses were in the third category. She got a 1 point for her response in the first category, 2 points for her answer in the second category, and 2 points for her third and fourth answers in the third category. Hence, in sum, her originality score was 5.

Table 4.5 shows that Luca’s first response was in the fourth category, the second response was in the first category, and the third response was in the third category. He received an originality score of 1 for his response in the first category and an originality score of 2 for his response in the third category. Moreover, Luca got an originality score of 4 for his response in the fourth category. In conclusion, Luca’s total originality score for the figural near generalization task was 7. Luca’s first response, which belonged to the fourth category, was the most original response for the figural near generalization task. Luca’s first response is given below:

> Luca: Well, it could be Fibonacci. There is zero here (showing on the pedigree); add zero and one, and you get one. Add one and one, and you get two. Add two and one, and you get three. Add three and two, and you get five. Five and three is eight. Six previous generation, it was thirteen.

As seen in the excerpt, he connected the reproductive pattern of the bees and the Fibonacci sequence. He was the first and only student to notice this relationship, which made this answer the most original one.

As shown in Table 4.5, Zena's first response was in the second category, and her second response was in the first category. Therefore, she received 1 point for her response in the first category and 2 points for her response in the second category, which made her total originality score 3.

To sum up, students had different originality scores according to the uniqueness of their answers. Table 4.8 summarizes the originality scores of each student.

Table 4.8. Originality Score of Each Student in the Figural Near Generalization Task

Student	Originality Score
Amelia	5
Diana	5
Luca	7
Zena	3

4.2.1.4. Elaboration of gifted students in the figural near generalization task

As described in detail in the Methodology chapter, if the student was able to explain his/her thinking process or justify his/her answer during the interview, he/she received one point for elaboration of his/her response. The student's total elaboration score was equal to the number of answers she/he elaborated.

As shown by Table 4.3, Amelia provided four appropriate responses to the figural near generalization task. Nevertheless, Amelia was able to elaborate on three of her four answers. Thus, her elaboration score was 3. Here is an example of a response Amelia was able to elaborate on.

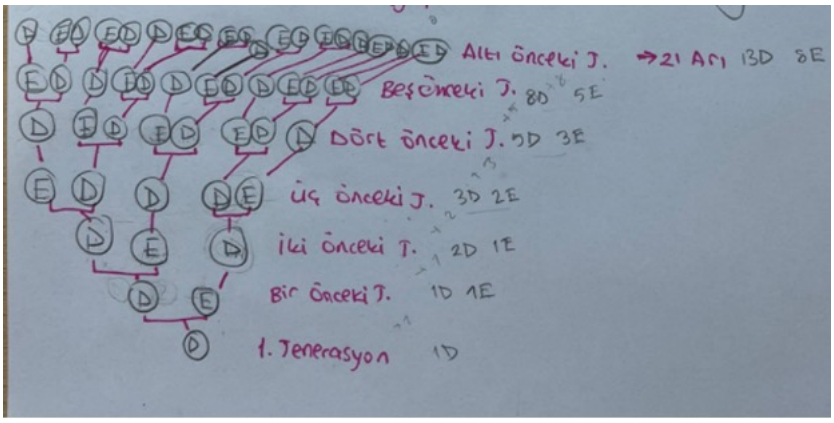


Figure 4.15. Amelia's First Response for the Figural Near Generalization Task

> Researcher: Could you explain this solution as if I were unfamiliar with it? What did you pay attention to, and how did you start to solve it?
>
> Amelia: The gender of the bee must be either male or female. So, we have two options, two possibilities. The question didn't say whether the bee was male or female. Thus, I first assumed the gender of the bee in the first generation to be female. If we accept this bee as a female, it will have two parents, so in the previous generation, there will be two bees, one female and one male. This female will also have two parents up there. Since the male will have only female parents, there will be three bees in the two previous generation. So, I started from the bottom and progressed to the top of the family tree. I completed the family tree in that way. Then, I counted and found 21 bees.

As can be seen from the dialogue, she justified the reasoning behind accepting the bee in the first generation as female, as the gender was not specified in the question. Furthermore, she explained how she progressed in drawing the family tree (bottom to top) and how she arrived at 21 bees. To sum up, she justified her reasoning and explained her thought process.

Diana gave four appropriate responses to the figural near generalization task (see Table 4.4), and she elaborated on all of her responses to the figural near generalization task. Thus, her elaboration score was 4.

Luca provided three appropriate answers while solving the figural near generalization task (see Table 4.4). Among those responses, he could elaborate on two of them, which made his elaboration score 2.

Zena provided two appropriate responses to the figural near generalization task and elaborated on both of her responses. Since she received 1 point for each of her responses that she was able to elaborate on, her elaboration score was 2.

Table 4.9 summarizes the elaboration scores of each student.

Table 4.9. Elaboration Score of Each Student in the Figural Near Generalization Task

Student	Elaboration Score
Amelia	3
Diana	4
Luca	2
Zena	2

4.2.1.5. Summary of mathematical creativity scores by components in the figural near generalization task

There were 13 appropriate responses for the figural near generalization task. These responses were evaluated in terms of fluency, flexibility, originality, and elaboration components of mathematical creativity. Table 4.10 illustrates each student's fluency, flexibility, originality, and elaboration scores.

Table 4.10. Mathematical Creativity of Gifted Students in the Figural Near Generalization Task

Students / MCC	Amelia	Diana	Luca	Zena
Fluency Score	4	4	3	2

Table 4.10. (continued)

Flexibility Score	3	3	3	2
Originality Score	5	5	7	3
Elaboration Score	3	4	2	2

MCC: Mathematical Creativity Components

As seen in Table 4.10, when the students' fluency scores are examined, it can be seen that all students' fluency score is more than one, which means that they were able generate more than one responses for the figural near generalization task However, among all the students, Amelia and Diana were the ones who produced the most answers to this question, so they had the highest fluency scores.

Furthermore, when the students' flexibility scores are examined, it can be said that the students diversified their answers to the figural near generalization question because the flexibility score of each student was greater than 1. Moreover, it is also seen that Amelia, Diana and Luca's flexibility scores are higher than Zena's. In addition, Luca and Zena's fluency and flexibility scores are equal, which shows that all of Luca and Zena's answers are in different categories, in other words, they can diversify all of their answers.

Table 4.10 shows that Luca has the highest originality score among all students, which shows that Luca produced the most original solutions to the figural near generalization problem. The person with the lowest originality score in the figural near generalization question is Zena, who also has the lowest scores in the other components.

When the students' elaboration scores are examined, it is observed that all students' elaboration scores are greater than 1 and close to their fluency scores. This shows that the gifted students in this study were able to justify and elaborate most of their answers in the figural near generalization problem. Diana and Zena's fluency scores

and elaboration scores are equal, which means that Diana and Zena were able to elaborate all of their answers to the figural near generalization question. The student with the highest elaboration score was Diana, followed sequentially by Amelia, Luca, and Zena.

4.2.2. Mathematical creativity of gifted students in the figural far generalization task

Fluency, flexibility, originality, and elaboration of gifted students in the figural far generalization task will be shared in the upcoming parts.

4.2.2.1. Fluency of gifted students in the figural far generalization task

Students provided 15 appropriate answers while solving the figural far generalization task. Based on their appropriate responses to the figural far-generalization task, the fluency of the gifted students was investigated individually and will be shared below.

Amelia provided four appropriate answers to the figural far generalization task. Table 4.11 presents Amelia's responses for the figural far generalization task.

Table 4. 11. Amelia's Appropriate Responses for Figural Far Generalization Task

Description	Response
A1: Amelia's first responses to the figural far generalization task	If we want to find the number of bees in the nth generation, we will add the number of bees in the prior generation to the number of bees in the two previous generation, and that will give the result.
A2: Amelia's second response to the figural far generalization task	I considered the male and female pair as n. There is no chance of remaining an extra male because the male and female have to be together anyway. That's why the female bees remain. I wrote the number of females as a plus.

Table 4.11. (continued)

A3: Amelia's third response to the figural far generalization task	The number of male bees in a generation is the same as the number of female bees in the next generation (in the generation of their own children). Besides, to find the number of females, we add the number of females and males, which gives the number of females in the previous generation (generation of ancestors).
A4: Amelia's fourth response to the figural far generalization task	If I add up the total number of bees and female bees in the n-1 previous generation, it gives the number of bees in the nth generation.

Table 4.11 summarizes four responses of Amelia for the figural far generalization task. She used existing generalization in her first response (A1). Moreover, in her second response (A2), she grouped the female and male bees and named them as n. Then, she stated that the number of bees in any generation can be expressed as the female and male bee pairs in that generation plus the number of male bees. On the other hand, in her third response, Amelia identified the rule that the total number of bees and female bees in a particular generation gives the number of bees in the previous generation. Moreover, in her fourth response, she adapted the rule that she identified in her third response to the nth generation. Hence, since she provided four appropriate responses for the figural far generalization task, her fluency score was 4.

Diana provided two responses to the figural far generalization task. Her first response was similar to Amelia's first response. On the other hand, Diana's second response conveyed the idea that the number of female bees in a generation is the sum of the number of bees in the two previous generations. Moreover, the number of male bees in a generation is the sum of the number of bees in the two previous

generations. Diana's second response for the figural far generalization task is given below:

> Diana: The number of females in the nth step equals the total number of female bees found in the two previous steps. The same goes for males. The number of males found in the nth step equals the total number of males found in the two previous steps…Thus, the sum of the previous two steps gives that step.



Figure 4.16. Diana's Written Work from Her Second Response for the Figural Far Generalization Task

As can be seen from Figure 4.16, Diana noted down the number of female and male bees in each generation based on the pedigree. Then, she identified the regularities and expressed the rule. In conclusion, Diana gave two appropriate responses for the figural far generalization task, meaning her fluency score was two.

Luca was the student who produced the highest number of answers to the first question, and he provided six appropriate answers to the figural far generalization task. His fifth response was similar to Amelia's and Diana's first responses, and his

sixth response was similar to Amelia's third response (see Table 4.11). However, his first four responses were different than other students' responses. Table 4.12 shows Luca's first four responses.

Table 4.12. Luca's First Four Responses for Figural Far Generalization Task

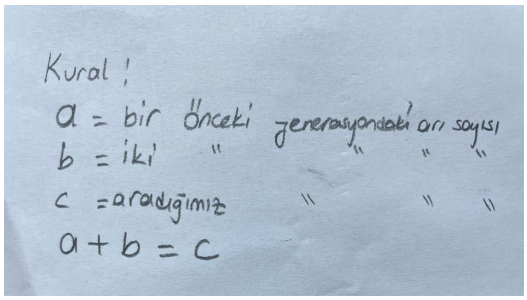
Description	Response
L1: Luca's first responses to the figural far generalization task	We can find the number of bees in any generation using the Fibonacci rule. We can find it by adding the number of bees in the previous two steps.
L2: Luca's second response to the figural far generalization task	(Writes "Fibonacci Numbers" as an answer). For example, let's say n is 9. If we know the Fibonacci numbers, we can do so.
L3: Luca's third response to the figural far generalization task	The sum of the number of bees in the previous generation (a) and the number of bees in the two generations ago (b) equals the number of bees in the generation we are looking for (c). 
L4: Luca's fourth response to the figural far generalization task	The number of males in the previous generation (n-1st generation) plus the number of male bees in the nth generation gives the number of females in the nth generation. The number of females in the previous generation is equal to the number of males in the next generation. When we add them, we can find the number of bees in any generation.

Table 4.12 illustrates that in his first response, Luca found that the reproductive pattern of the bees has the rule of the Fibonacci sequence. Moreover, in his second response, Luca stated that the number of bees in the nth generation can be found if the nth Fibonacci number is known. Furthermore, in his third response, he used symbolic representation to express the general rule: the sum of the number of bees in the previous generation (a) and the number of bees in the two generations ago (b) equals the number of bees in the current generation (c). Lastly, he identified the rule regarding the number of female and male bees, as depicted in Table 4.12. As a result, since each appropriate response received 1 fluency score and he provided six appropriate answers, his fluency score was 6.

Moreover, Zena gave three appropriate responses to the figural far generalization task. Her first two responses showed similarity with Amelia's third response (see Table 4.11). Furthermore, her third response was similar to Amelia's fourth response (see Table 4.11). Therefore, since she provided three appropriate responses, this implies that her fluency score was three.

In conclusion, students' fluency scores for the figural far generalization task differed. Table 4.13 illustrates the fluency scores of the students in the figural far generalization task.

Table 4.13. Fluency Score of Each Student in the Figural Far Generalization Task

Student	Fluency Score
Amelia	4
Diana	2
Luca	6
Zena	3

4.2.2.2. Flexibility of gifted students in the figural far generalization task

Table 4.13 indicated that students gave 15 appropriate answers for the figural far generalization task. Each of those appropriate answers was evaluated based on the

underlying idea, representation style, and strategy used. Based on the evaluation, the ideas, representation types, and strategies will be shared below:

Analysis of students' responses revealed seven main ideas in total for the figural far generalization task:

1. The number of bees in a generation is the sum of the number of bees in the two previous generations.
2. The reproductive pattern of the bees is an example of the Fibonacci sequence.
3. The number of bees in the nth generation equals the nth Fibonacci number.
4. The number of female bees in a generation is the sum of the number of bees in the two previous generations. The number of male bees in a generation is the sum of the number of bees in the two previous generations.
5. The number of bees in any generation can be expressed by representing each female and male bee pair as n and adding the remaining bees.
6. The sum of the total number of bees and the number of female bees in a generation is equal to the total number of bees in the previous generation (ancestor generation).
7. The number of males in the n-1st generation plus the number of males in the nth generation equals the number of females in the nth generation. The number of females in the n-1st generation equals the number of males in the nth generation. When added together, the number of bees in any generation is found.

Moreover, evaluating the students' responses to the figural far generalization question in terms of representation types revealed the use of both verbal and symbolic modes of representation.

Lastly, regarding strategies, analysis of students' responses indicated that they employed the following four strategies: *using the existing generalization*, *connecting to the known pattern*, *identifying regularities by investigating the numerical relationships embedded in the figure*, and *grouping and assigning a variable*.

The categorization of the students' responses according to the underlying idea, representation type, and strategy used to reach the answer is displayed in Table 4.14.

Table 4.14. Categorization of Students' Responses to Figural Far Generalization Task

Category	Underlying Idea	Representation Type	Strategy	Student's Answer
1st Category	The number of bees in a generation is the sum of the number of bees in the two previous generations.	Verbal	Using the existing generalization	D1, A1, L5
2nd Category	The number of bees in a generation is the sum of the number of bees in the two previous generations.	Symbolic	Using the existing generalization	L3
3rd Category	The reproductive pattern of the bees is an example of the Fibonacci sequence.	Verbal	Connecting to the known pattern	L1
4th Category	The number of bees in the nth generation is equal to the nth Fibonacci number.	Verbal	Connecting to the known pattern	L2

Table 4.14. (continued)

5th Category	The number of female bees in a generation is the sum of the number of bees in the two previous generations. The number of male bees in a generation is the sum of the number of bees in the two previous generations.	Verbal	Identifying regularities by investigating the numerical relationships embedded in the figure	D2
6th Category	The number of bees in any generation can be expressed by representing each female and male bee pair as n and adding the remaining bees.	Verbal	Grouping and assigning variable	A2
7th Category	The sum of the total number of bees and the number of female bees in a generation is equal to the total number of bees in the previous generation (ancestor generation).	Verbal	Identifying regularities by investigating the numerical relationships embedded in the figure	A3, Z1, Z2, L6
8th Category	The sum of the total number of bees and the number of female bees in a generation is equal to the total number of bees in the previous generation (ancestor generation).	Verbal	Using the existing generalization	A4, Z3

Table 4.14. (continued)

9th Category	The number of male bees in the n-1th generation plus the number of male bees in the nth generation equals the number of female bees in the nth generation. The number of female bees in the n-1th generation equals the number of male bees in the nth generation. When added together, the number of bees in any generation is found.	Verbal	Identifying regularities by investigating the numerical relationships embedded in the figure	L4

Students' flexibility scores were determined according to the categorization of their answers shown in Table 4.14. A detailed explanation of how each student's flexibility score was calculated will be covered below.

As reflected in Table 4.13, Amelia provided four appropriate responses to the figural far generalization task. Amelia's first response was in the first category. Expressly, the idea behind her first response was that the number of bees in a generation is the sum of the number of bees in the two previous generations. Furthermore, she used *existing generalization* strategy, and verbal representation in her first response. Besides, her second response was in the sixth category. Her second response conveyed the idea that the number of bees in any generation can be expressed by representing each female and male bee pair as n and adding the remaining bees. Moreover, Amelia used *grouping and assigning variable* strategy and verbal representation in her second response. Although the representation type was the same in her first and second responses, since the underlying idea and the

strategy were different, her first two responses belonged to different categories. Furthermore, Table 4.14 showed that Amelia's third response conveyed the idea that the sum of the total number of bees and the number of female bees in a generation is the same as the total number of bees in the previous generation (ancestor generation). She used verbal representation and *identifying regularities by investigating the numerical relationships embedded in the figure* strategy. Since the underlying idea and strategy differed from her previous responses, her third response belonged to a different category. Lastly, the idea behind Amelia's fourth response was that the sum of the total number of bees and the number of female bees in a generation is the same as the total number of bees in the previous generation. In her fourth response, she employed verbal representation and *using the existing generalization* strategy. Though the idea and the representation type were the same in her third and fourth responses, since the strategy was different, her third and fourth responses were in different categories. Thus, since Amelia's responses belonged to four different categories, her flexibility score was 4.

Diana provided two appropriate answers to the figural far generalization task. Her first response belonged to the first category, which implies that the underlying idea was that the number of bees in a generation is the sum of the number of bees in the two previous generations, the representation type was verbal, and the strategy was *using an existing generalization*. Moreover, her second response was based on the idea that the number of female bees in a generation is the sum of the number of bees in the two previous generations, and the number of male bees in a generation is the sum of the number of bees in the two previous generations. In addition, she used verbal representation and *identifying regularities by investigating the numerical relationships embedded in the figure* in her second response. Thus, since her two responses differed regarding the underlying idea and strategy, they belonged to two categories. Therefore, Diana's flexibility score was two.

Luca gave six appropriate responses to the figural far generalization task. Luca's first response conveyed the idea that the reproductive pattern of the bees is an example of the Fibonacci sequence. Furthermore, he employed verbal

representation and *connecting to the known pattern* strategy. Moreover, his second response was based on the idea that the number of bees in the nth generation equals the nth Fibonacci number. In that response, he applied verbal representation and *connecting to the known pattern* strategy. Although the representation and the strategy were the same in his first and second responses, since the ideas differed, those responses were classified into different categories. Moreover, Luca's third response is based on the idea that the number of bees in a generation is the sum of the number of bees in the previous two generations. He applied an *existing generalization strategy* and symbolic representation in his third response. Since his third response differed from his previous two responses regarding the underlying idea, representation, and strategy, his third response was in a different category. Furthermore, the underlying idea behind Luca's fourth response was as follows: The number of males in the n-1st generation plus the number of males in the nth generation equals the number of females in the nth generation. The number of females in the n-1st generation equals the number of males in the nth generation. When added together, the number of bees in any generation is found. Moreover, he used *identifying regularities by investigating numerical relationships embedded in the figure* strategy and verbal representation in his fourth response. Since his fourth response differed from his previous responses regarding the idea and strategy, it belonged to a different category. Besides, Luca's fifth response conveyed the idea that the number of bees in a generation is the sum of the number of bees in the two previous generations. In that response, he employed *using the existing generalization* strategy and verbal representation. As can be seen from Table 4.14, though his third and fifth responses had the same underlying idea and the same strategy, since the representation types were different, they belonged to different categories. Lastly, Luca's sixth response was based on the idea that the sum of the total number of bees and the number of female bees in a generation equals the total number of bees in the previous (ancestor) generation. Moreover, in his sixth response, he employed *identifying regularities by investigating the numerical relationships embedded in the figure* and verbal representation. Since the idea

behind his sixth response differed from his previous responses, his sixth response belonged to a different category. Thus, as Luca provided six appropriate answers to the figural far generalization task and Luca's responses belonged to different categories, his flexibility score for the figural far generalization task was six.

Zena gave three appropriate answers. Her first response was based on the idea that the sum of the total number of bees and the number of female bees in a generation is the same as the total number of bees in the next generation. She used *identifying regularities by investigating the numerical relationships embedded in the figure* strategy and verbal representation. Similarly, her second response was also based on the same idea, and she employed the same strategy and the same representation in her second response. Therefore, her first and second responses belonged to the same category. Moreover, her third response conveyed the same idea as her first and second responses. Also, she used verbal representation in her third response as she did in her first two responses. However, in her third response, she applied *the existing generalization* strategy, which differed from the strategy she used in her first and second responses. As a result, since her responses belonged to two distinct categories, Zena's flexibility score for the figural far generalization task was two.

In conclusion, according to the categories of the students' responses, the flexibility scores of the students were determined. Table 4.15 illustrates the flexibility scores of the students in the figural far-generalization task.

Table 4.15. Flexibility Score of Each Student in the Figural Far Generalization Task

Student	Flexibility Score
Amelia	4
Diana	2
Luca	6
Zena	2

4.2.2.3. Originality of gifted students in the figural far generalization task

To evaluate the originality of gifted students' responses, each cell in the students' answers column of Table 4.14 was examined, the frequency of each response in the same category was determined, and the students' originality scores were assigned based on the previously set criteria (see Table 4.7) and expert opinion.

When the answers in the first category were investigated, it was observed that the first category included three students' answers. Therefore, according to Table 4.7, each student whose answer belonged to that category received 2 points for originality. Moreover, the second, third, fourth, fifth, sixth, and ninth categories included only one student's answer. Therefore, each student with an answer in these categories received 4 points for originality. Furthermore, in the seventh category, there were answers belonging to three different students. Therefore, each student with an answer in this category received 2 originality score. Lastly, when the eighth category was examined, there were two students' responses. Thus, each of these students got 3 points for originality, according to Table 4.7. A thorough explanation of how each student's originality scores were determined will be addressed below.

It was evident in Table 4.14 that Amelia's first answer was in the first category, her second answer was in the sixth category, her third answer was in the seventh category, and her fourth answer was in the eighth category. Amelia received an originality score of 2 for her answer in the first category, an originality score of 4 for her answer in the sixth category, an originality score of 2 for her answer in the seventh category, and an originality score of 3 for her answer in the eighth category. As a result, her total originality score was 11.

Table 12 showed that Diana's first response belonged to the first category, while her second was in the fifth category. She got 2 points for her response in the first category and 4 points for her answer in the fifth category. Thus, her total originality score was 6.

As shown by Table 4.14, Luca's first response was in the third category, the second response was in the fourth category, the third response was in the second category, the fourth response was in the ninth category, the fifth response was in the first category, and his sixth response was in the seventh category. He received an originality score of 2 for his response in the first category, an originality score of 4 for his responses in the second, third, fourth, and ninth categories, and an originality score of 2 for his response in the seventh category. Therefore, his total originality score was 20.

Table 4.14 depicts that Zena's first and second responses belonged to the seventh category, and her third response belonged to the eighth category. She received a 2 point for her responses in the seventh category and 3 points for her response in the eighth category, which made her total originality score 5.

In conclusion, the most original answers to the figural far generalization task, that is, the answers with the highest originality scores, are as follows: A2 (see Table 4.11); D2 (see Figure 4.16); and L1, L2, L3, L4 (see Table 4.12). Table 4.16 demonstrates the originality scores for each student.

Table 4.16. Originality Score of Each Student in the Figural Far Generalization Task

Student	Originality Score
Amelia	11
Diana	6
Luca	20
Zena	5

4.2.2.4. Elaboration of gifted students in the figural far generalization task

The students' ability to evaluate their responses was assessed based on their answers to the probing questions, such as "How could you explain your response to someone who did not know the answer at all?" and "What could be the reasoning

behind your response?". The student's total elaboration score was determined according to the number of answers she/he elaborated.

As illustrated by Table 4.11, Amelia gave four appropriate responses to the figural far generalization task and elaborated on all her answers. Thus, her elaboration score was 4. Notably, Amelia's elaboration of her second answer led her to give her third answer. The dialogue below illustrates this situation.

> Amelia: I considered the male and female pair as n. There is no chance of remaining an extra male because the male and female have to be together anyway. That's why the female bees remain. I wrote the number of females as a plus. (Amelia's third response)
>
> Researcher: Why do the male and female bees have to be together anyway? What was the reasoning behind this?
>
> Amelia: Males come from females; males do not have male parents. They go up as females. Aha, the number of male bees in a generation is equal to the number of female bees in the next generation.
>
> Researcher: Could you explain what conclusion you reached?
>
> Amelia: The number of male bees in a generation is equal to the number of female bees in the next generation (in the generation of their own children). Besides, to find the number of females, we add the number of females and males, which gives the number of females in the previous generation (generation of ancestors) (Amelia's fourth response).

As can be seen from the dialogue, while elaborating on her third answer, Amelia noticed that the number of male bees in a generation is equal to the number of female bees in the next generation (in the generation of their own children), which helped her arrive at her fourth answer.

The other student, Diana, provided two appropriate responses to the figural far generalization task, yet she could elaborate on one of her responses to the figural far generalization task. Thus, her elaboration score was 1.

Luca provided six appropriate answers while solving the figural far generalization task. He elaborated on four of his six responses, which implies that his elaboration score was 4. Here is one example of a response (L6) that Luca could elaborate on.

> Luca: The sum of the number of females and the total number of bees equals the number of bees in the previous generation.
>
> Researcher: What do you think might have been the reason for this?
>
> Luca: Because each bee necessarily has one mother. That is why there will be the same number of females in the previous (ancestor) generation. Moreover, there are as many males as there are females.
>
> Researcher: Why are there as many males as females?
>
> Luca: Because females have fathers, males do not. Every female bee will necessarily have one father.

As shown by the dialogue, Luca explained that since each bee has a mother, the total number of bees in the current generation would equal the number of female bees (mothers) in the previous generation. Moreover, he pointed out that since only female bees have fathers, the number of female bees would equal the number of male bees (fathers). Thus, since the number of bees in any generation can be found by adding the number of female and male bees, he added the total number of bees in the current generation (which equals to the number of female bees in the previous generation) and the number of female bees (which equals to the number of male bees in the prior generation). To sum up, he provided in-depth reasoning behind why the sum of the number of females and the total number of bees in a generation is equal to the number of bees in the prior generation.

Zena gave three appropriate responses to the figural far generalization task and elaborated on two of her responses, which made her elaboration score 2.

Table 4.17 summarizes the elaboration scores of each student.

Table 4.17. Elaboration Score of Each Student in Figural Far Generalization Task

Student	Elaboration Score
Amelia	4
Diana	1
Luca	4
Zena	2

4.2.2.5. Summary of mathematical creativity scores by components in the figural far generalization task

Analysis of students' responses revealed that 15 responses existed to the figural far generalization task. These responses were evaluated concerning the four components of mathematical creativity: fluency, flexibility, originality, and elaboration. Each student's fluency, flexibility, originality, and elaboration scores are shown in Table 4.18.

Table 4.18. Mathematical Creativity of Each Student in Figural Far Generalization Task

Students / MCC	Amelia	Diana	Luca	Zena
Fluency Score	4	2	6	3
Flexibility Score	4	2	6	2
Originality Score	11	6	20	5

Table 4.18. (continued)

Elaboration Score	4	1	4	2

MCC: Mathematical Creativity Components

As depicted by Table 4.18, all students' fluency scores are more than one, which implies that gifted students in the present study were able to generate more than one response for the figural far generalization task. In contrast to the figural near generalization task, Luca was the student who produced the most solutions to the figural far generalization problem, followed by Amelia, Zena, and Diana, respectively. When students' fluency scores in the figural near and figural far generalization questions are compared, some variability is observed. Amelia has the same fluency score in both questions. Diana's fluency score in the figural far generalization question is two points lower than her fluency score in the figural near generalization question. Luca and Zena's fluency scores are higher in the figural far generalization question than in the figural near generalization question. To be more specific, Luca's fluency score in the figural near generalization question is 3, while his fluency score in the figural far generalization question is 6, meaning that he was able to produce twice as many answers to the figural far generalization question. Lastly, Zena's number of appropriate answers in the figural far generalization question is one more than in the figural near generalization question, which caused her fluency score in the far generalization question to be one higher than her fluency score in the figural near generalization question.

When the students' flexibility scores are examined, it is seen that all students' flexibility scores are greater than one and that all students' fluency scores and flexibility scores are equal, except for Zena. This shows that gifted students produce answers in different categories in the figural far generalization question and diversify their answers. Luca has the highest flexibility score among all students, Amelia has the second highest flexibility score, and Diana and Zena have the lowest flexibility score, with equal flexibility scores. When the students'

flexibility scores in the figural near and figural far generalization questions are compared, it is observed that Amelia and Luca's flexibility scores are higher in the figural far generalization question, Zena's flexibility scores are equal in both questions, and Diana's flexibility score in the figural far generalization question is one point lower than her flexibility score in the figural near generalization question.

Similar to the figural near generalization task, Luca is the one with the highest originality score. He was followed by Amelia, Diana, and Zena, respectively. Among the components of creativity, the difference between the figural near and figural far generalization questions is most obvious in the originality component. All students' originality scores are higher in the figural far generalization question than in the figural near generalization question. To be more precise, all students' originality scores in the figural far generalization question were at least twice as high as their originality scores in the figural near generalization question. This shows that students were able to generate more novel responses while solving the figural far generalization task.

When the students' elaboration scores are examined, it is observed that all students' elaboration scores are at least 1, which shows that gifted students in this study were able to justify and elaborate most of their answers in the figural far generalization problem. In the figural far generalization question, Amelia and Luca received the highest elaboration scores, followed by Zena. And Diana. Diana, who had the highest elaboration score in the figural near generalization question and elaborated all of her answers, had the lowest elaboration score in the figural far generalization question. When the students' elaboration scores in the figural near and figural far generalization questions are compared, it is seen that Amelia has one more elaboration score, Luca has two more elaboration score, Zena has the same score, and Diana has three score lower elaboration score in the figural far generalization question compared to the figural near generalization question.

4.2.3. Mathematical creativity of gifted students in the non-figural far generalization task

The following sections cover the fluency, flexibility, originality, and elaboration of gifted students in the non-figural far generalization task. However, since Diana did not provide any appropriate responses to the non-figural far generalization task, her answers were not evaluated regarding the other four creativity components because they did not meet the first condition of mathematical creativity, appropriateness. Therefore, unlike other sections, this section will address three students' mathematical creativity.

4.2.3.1. Fluency of gifted students in non-figural far generalization task

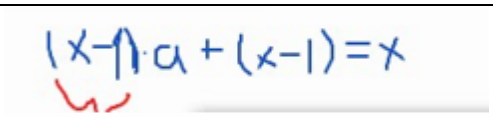
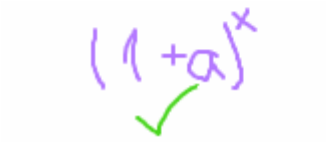
While solving the non-figural far generalization task, students produced eight appropriate answers in total. Furthermore, students' appropriate answers to non-figural far generalization tasks were analyzed, and their fluency scores were determined.

Amelia provided four relevant responses for the non-figural far generalization task. Table 4.19 illustrates those responses:

Table 4.19. Amelia's Appropriate Responses for Non-Figural Far Generalization Task

Description	Response
A1: Amelia's first responses to non-non-figural far generalization task	a times the number of already ill people plus the total number of already ill people in the previous step gives us the total number of ill people.
A2: Amelia's second response to the non-figural far generalization task	We can call the number of ill people as n. Then, the answer would be n times a plus n.

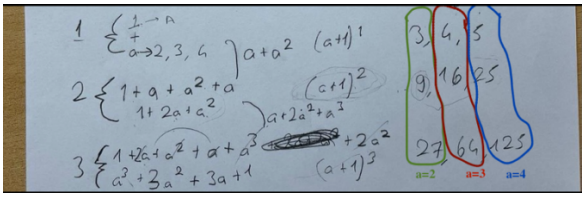
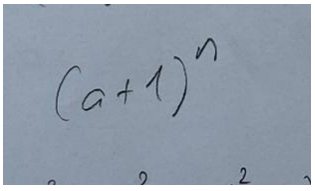
Table 4.19. (continued)

A3: Amelia's third response to non-non-figural far generalization task	
A4: Amelia's fourth response to the non-figural far generalization task	

As evidenced by Table 4.19, Amelia stated verbally that the total number of patients in any step can be found by multiplying a by the number of already ill people and adding it to the total number of already ill people in the previous step. Moreover, her second and third responses were based on the same idea as her first response. However, he used symbolic representation in her second and third responses. Furthermore, in her last response, Amelia found the explicit rule as $(1+a)^x$, where x represents the unit of time. Hence, as she generated four appropriate responses for the non-figural far generalization task, her fluency score was 4.

Luca gave two appropriate answers while solving the figural far generalization task. His responses are given below:

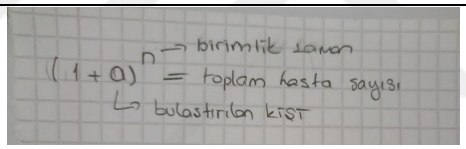
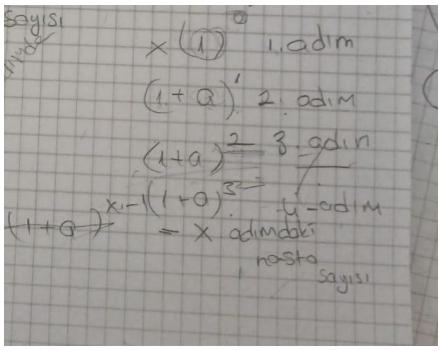
Table 4.20. Luca's Appropriate Responses for the Non-Figural Far Generalization Task

Description	Response
L1: Luca's first responses to the non-figural far generalization task	The time period power of a plus one fits all. 
L2: Luca's second response to the non-figural far generalization task	

In her first response, Luca wrote algebraic expressions to represent the number of infected individuals in one, two, and three units of time. Then, he assigned different values (2, 3, and 4) to *a* and recorded the number of infected individuals in one, two, and three units of time. Afterward, he noticed the regularities and expressed the rule verbally as "the time period power of *a* plus one fits all". Moreover, contrary to his first response, in his second response, Luca used symbolic representation. In conclusion, as depicted by Table 4.20, he provided two appropriate responses for the non-figural far generalization task. Therefore, his fluency score was two.

Zena provided two appropriate responses to the non-figural far generalization task. Her responses are shown in Table 4.21:

Table 4.21. Zena's Appropriate Responses for Non-Figural Far Generalization Task

Description	Response
Z1: Zena's first responses to the non-figural far generalization task	
Z2: Zena's second response to the non-figural far generalization task	 In the first step, one person. In the second step, one plus *a* person. In the third step, one plus *a* to the power of two people. In the fourth step, one plus *a* to the power of three, Frankly, the total number of infected people in the xth step would be *a* plus one to the power of x-1.

In her first response, Zena assigned different values to a and generated a visual representation to demonstrate the situation and she identified the regularities and expressed the rule symbolically as $(1+a)^n$. Furthermore, in her second response, she wrote the algebraic expression representing the number of patients in different steps. Unlike her first responses, she assumed that the first step was the arrival of the person who brought the disease to the country. Therefore, she expressed the rule as $(1+a)^{x-1}$. As a result, since she provided two appropriate responses for the non-figural far generalization task, her fluency score was two.

Table 4.22 summarizes the fluency scores of each student for the figural far generalization task.

Table 4.22. Fluency Score of Each Student in the Non-Figural Far Generalization Task

Student	Fluency Score
Amelia	4
Diana	-
Luca	2
Zena	2

4.2.3.2. Flexibility of gifted students in non-figural far generalization task

As can be seen from Table 4.22, there were six appropriate answers for the non-figural far generalization task. The students' answers to the non-figural far generalization question were categorized under these two main ideas:

1. If the number of people infected at each step is multiplied by the number of patients in the prior step and added to the number of patients in the prior step, the total number of patients is found.

2. The time period power of a plus one gives the total number of patients.

Besides, analysis of students' responses indicated the use of two types of representation: verbal and symbolic. Furthermore, while solving the non-figural far

generalization task, students utilized three strategies: *identifying regularities by assigning alternative values and generating the visual representation*, *identifying regularities by generating the algebraic representation and assigning alternative values*, and *using an existing generalization*. As a result, students' answers for the non-figural far generalization task were classified under six different categories. Table 4.23 depicts the classification of the students' responses regarding the idea, representation type, and strategy.

Table 4.23. Categorization of Students' Responses to the Non-Figural Far Generalization Task

Category	Underlying Idea	Representation Type	Strategy	Students' Answers
1st Category	If the number of people infected at each step is multiplied by the number of patients in the previous step and added to the number of patients in the previous step, the total number of patients is found.	Verbal	Identifying regularities by assigning alternative values and generating a visual representation	A1
2nd Category	If the number of people infected at each step is multiplied by the number of patients in the previous step and added to the number of patients in the previous step, the total number of patients is found.	Symbolic	Using the existing generalization	A2, A3

Table 4.23. (continued)

3rd Category	The time period power of *a* plus one fits all gives the total number of patients.	Verbal	Identifying regularities by generating the algebraic representation and assigning alternative values	L1
4th Category	The time period power of *a* plus one fits all gives the total number of patients.	Symbolic	Identifying regularities by assigning alternative values and generating the visual representation	Z1, A4
5th Category	The time period power of *a* plus one fits all gives the total number of patients.	Symbolic	Using the existing generalization	L2
6th Category	The time period power of *a* plus one fits all gives the total number of patients.	Symbolic	Identifying regularities by generating the algebraic representation and assigning alternative values	Z2

Table 4.22 displayed that Amelia provided four appropriate responses while solving the non-figural far generalization task. Her first response was located in the first category. To be more precise, her first response lay on the idea that if the number of people infected at each step is multiplied by the number of patients in the prior step and added to the number of patients in the prior step, the total number of patients is found. Moreover, in her first response, she used verbal representation and *identifying regularities by assigning alternative values and generating a visual representation* strategy. Similarly, her second and third responses conveyed the same idea as her first response. However, she used symbolic representation and *using the existing generalization* strategy in her second and third responses. Since her second and third responses were based on the same idea, including the same type of representation, and solved with the same strategy, her second and third responses belonged to the same category. Nevertheless, as the representation type and strategy were different from her first response in the first category, her second and third responses belonged to a different category than her first response. Lastly, the underlying idea behind Amelia's fourth response was that the time period power of *a* plus one gives the total number of patients. Furthermore, she applied *identifying regularities by assigning alternative values and generating the visual representation* strategy and symbolic representation in her fourth response. Although the strategy she used and the way she presented her answer were similar to her answers in other categories, her fourth response was placed in a different category because it contained a different idea. Ultimately, her answers were classified into three different categories, so her flexibility score for the non-figural far generalization task was three.

Luca produced two appropriate responses while solving non-figural far generalization task. The underlying idea behind Luca's first response was that the time period power of *a* plus one gives the total number of patients. He utilized *identifying regularities by generating the algebraic representation and assigning alternative values* strategy and presented his response verbally. Moreover, although his second response conveyed the same idea as his first response, he used a

different strategy and representation. Expressly, he used *existing generalization* and symbolic representation in his second response. As the strategy and representation were not the same as his first response, his second response was in a different category. Hence, since Luca's responses belonged to two distinct categories, his flexibility score was two.

Zena gave two responses to the non-figural far generalization task. Similar to Luca's first response, the idea behind Zena's first response was that the time period power of *a* plus one gives the total number of patients. In her first response, she used *identifying regularities by assigning alternative values and generating the visual representation* strategy and symbolic representation. Furthermore, while her second response has the same underlying idea and representation type as her first response, she used a different strategy in her second response: *identifying regularities by generating the algebraic representation and assigning alternative values* strategy. Therefore, as her two responses were in different categories, her flexibility score for the non-figural far generalization task was two.

Table 4.24 summarizes the flexibility scores of each student for the non-figural far generalization task according to the categorization of their responses.

Table 4.24. Flexibility Score of Each Student in the Non-Figural Far Generalization Task

Student	Flexibility Score
Amelia	3
Diana	-
Luca	2
Zena	2

4.2.3.3. Originality of gifted students in the non-figural far generalization task

In the nonfigural far generalization question, Diana could not give any appropriate responses. Therefore, the rarity of appropriate answers was determined based on

three participants who gave appropriate answers (Amelia, Luca, and Zena). Hence, for the nonfigural far generalization question, a new originality scoring table, Table 4.25, was created based on three participants. Then, expert opinion and approval were obtained about scoring.

Table 4.25. Rubric for Assessing Originality of Students' Responses

Number of students who gave that response	Score that will be assigned to that response
1	3
2	2
3	1

Investigating the scarcity of students' responses revealed that the number of responses in each category was diverse. More specifically, there was one student's answer in the first, second, third, fifth, and sixth categories. According to Table 4.25, students received three points for originality in their responses in those categories. Moreover, the fourth category consisted of two students' responses. Hence, each student with an answer in those categories got an originality score of two.

As shown by Table 4.23, Amelia's first answer was in the first category, her second and third answers were in the second category, and her fourth answer was in the fourth category. She got an originality score of three for each of her answers in the first and second categories and an originality score of two for her answers in the fourth category. Therefore, her total originality score for the non-figural far generalization task was 8.

It was reflected in Table 4.23 that Luca's responses belonged to the third and fifth categories. He received three points for his response in the third category and three points for his response in the fifth category. Thus, his total originality score was 6.

As displayed by Table 4.23, Zena's first response was in the fourth category, while her second response was in the sixth category. Therefore, she got an originality score of two for her response in the fourth category and an originality score of three for her response in the sixth category. Hence, her total originality score was 5.

To sum up, the most original answers to the non-figural far generalization task are A1 (see Table 4.19), L1, L2 (see Table 4.20), and Z2 (see Table 4.21). Originality scores of students are illustrated in Table 4.26.

Table 4.26. Originality Score of Each Student in the Non-Figural Far Generalization Task

Student	Originality Score
Amelia	8
Diana	-
Luca	6
Zena	5

4.2.3.4. Elaboration of gifted students in non-figural far generalization task

As explained in detail in the prior sections, gifted students' total elaboration scores were determined based on the number of answers they elaborated on.

Amelia could elaborate on two of the four responses to the non-figural far generalization task. Therefore, her elaboration score was two. One (A1) of the responses of Amelia that she elaborated on was given below:

> Amelia: *a* times the number of already ill people plus the total number of already ill people in the previous step gives us the total number of patients.
>
> Researcher: What could be the reasoning behind this?
>
> Amelia: If we add the number of new patients and those who were already infected, we get the total number of patients. Since each person makes *a* number of people infected, multiplying the total number of people by *a*

gives us the new patients. Then, to find the total number of patients, we need to add the new patients and the ones who were already infected.

The interview excerpt shows that she could explain her thought process in words and justify multiplying the number of patients by *a* and adding this result to the number of already infected people, which gives the total number of patients. Specifically, Amelia first stated that the total number of patients can be found by adding the already infected people and the new patients. Then, she explained that since each infected person infected *a* number of people, the number of new patients can be found by multiplying the number of already infected people by *a*.

Luca provided two appropriate answers to the non-figural far generalization task. Yet, he elaborated on one of his responses. As a result, his elaboration score was one.

Zena gave two appropriate responses to the non-figural far generalization task and elaborated on both responses, implying that her elaboration score was two.

Table 4.27 demonstrates the elaboration scores of each student.

Table 4.27. Elaboration Score of Each Student in the Non-Figural Far Generalization Task

Student	Elaboration Score
Amelia	2
Diana	-
Luca	1
Zena	2

4.2.3.5. Summary of mathematical creativity scores by components in the non-figural far generalization task

Students' fluency, flexibility, originality, and elaboration scores for the non-figural far generalization task were displayed in Table 4.28. As noted above, Diana did not provide any appropriate answers to the non-figural far generalization question, i.e.,

none of her responses satisfied the first component of mathematical creativity. This implies that she did not have any response to be evaluated in terms of the other four components of mathematical creativity. For this reason, Diana's score on each component is shown with a dash. Therefore, the fluency, flexibility, originality, and elaboration scores of the other three students in the figural far generalization question will be addressed in Table 4.28 below.

Table 4.28. Mathematical Creativity of Gifted Students in Non-Figural Far Generalization Task

Students / MCC	Amelia	Diana	Luca	Zena
Fluency Score	4	-	2	2
Flexibility Score	3	-	2	2
Originality Score	8	-	6	5
Elaboration Score	2	-	1	2

MCC: Mathematical Creativity Components

As can be seen in Table 4.28, students' fluency scores are greater than one, indicating that the gifted students in this study produced more than one appropriate answer to the figural far generalization question. In this question, the student with the highest fluency score is Amelia, and Luca and Zena follow Amelia with equal fluency scores. When the students' fluency scores in the figural near and non-figural far generalization questions are compared, it is seen that Amelia and Zena's fluency scores in both questions are equal, while Luca's fluency score in the non-figural far generalization question is one point lower than his fluency score in the figural near generalization question. When the students' fluency scores in the figural far and non-figural far generalization questions are compared, it is seen that

Amelia's fluency scores in both questions are equal, while Luca and Zena's fluency scores in the non-figural far generalization question are lower than their fluency scores in the figural far generalization question.

Table 4.28 shows that students' flexibility scores are higher than one in the non-figural far generalization question and are close to students' fluency scores. This shows that students diversified their answers in the non-figural far generalization question as in other questions. Among the students, Amelia has the highest flexibility score, followed by Luca and Zena with equal flexibility scores. When we make a problem-based comparison, it is seen that while Amelia and Zena's flexibility scores are equal in the figural near generalization question and the nonfigural far generalization question, Luca's flexibility score in the nonfigural far generalization question is one point lower than his flexibility score in the figural near generalization question. On the other hand, the flexibility scores of Amelia and Luca in the non-figural far generalization question are lower than their flexibility scores in the figural far generalization question, while Zena’s flexibility score in the non-figural and figural far generalization tasks is equal.

Table 4.28 depicts that the student with the highest originality score in the non-figural far generalization question is Amelia, followed by Luca and Zena, respectively. Since one student (Diana) could not give an appropriate answer to the nonfigural far generalization question, and the answers of three students were analyzed, the originality scores for the nonfigural far generalization question differed from the figural near and figural far generalization questions. For this reason, it is not appropriate to compare the originality scores of the students in the nonfigural far generalization question with their scores in other questions.

Table 4.28 shows that the students' elaboration scores in the non-figural far generalization question are at least one, indicating that the students were able to elaborate on at least one of their answers in the non-figural far generalization question. In the nonfigural far generalization question, Amelia and Zena received the highest elaboration scores, while Luca's elaboration score was one point lower

than theirs. When the students' elaboration scores are compared within the questions, it is seen that Luca and Amelia's elaboration scores in the nonfigural far generalization question are lower than their elaboration scores in the figural near and figural far generalization questions, while Zena's elaboration score in all questions is the same.

To sum up, students' mathematical creativity varied in different subdimensions of creativity on a problem-by-problem and students-by-student basis.

CHAPTER 5

DISCUSSION

The current study aimed to investigate strategies that 8th-grade gifted students use while solving alternative types of nonlinear realistic generalization tasks and their mathematical creativity while solving these tasks. In accordance with these purposes, this section will consist of three main parts. First, the conclusion and discussion of the findings with reference to previous studies will be addressed. Second, the implications of the present study will be presented. Third, limitations and recommendations for future research will be addressed.

5.1.Discussion of Findings

Aligning with its purposes, the findings of the present study will be discussed under two main headings: strategies and mathematical creativity of gifted students. To be more precise, in the first part, strategies of gifted students will be discussed. In the second part, the mathematical creativity of gifted students will be discussed.

5.1.1. Strategies of Gifted Students

As stated above, the present study's aim was twofold. One of the present study's aims was to investigate the strategies of gifted students while solving alternative types of nonlinear realistic generalization tasks. In line with this aim, in this section, the strategies of gifted students will be discussed.

In the current study, gifted students were expected to solve one figural task, which included one near and one far generalization question, and one nonfigural far generalization task. Before delving into the strategies of gifted students, it was worth highlighting that gifted students could solve nonlinear realistic generalization tasks correctly, although nonlinear generalization tasks were not covered in the

middle school mathematics textbooks in Türkiye, and gifted students in the present study were not familiar with these types of tasks. Although most studies focus on the performance of students while solving linear pattern generalization tasks (Amit & Neria, 2008), the findings of the studies about the performance of middle school students in nonlinear tasks were controversial. More specifically, some studies (e.g., Kerbs, 2003) showed that students have difficulty in generalizing nonlinear generalization tasks. However, this finding of the present study aligned with the findings of Amit & Neria's (2008) study reporting that gifted students were successful in nonlinear generalization tasks, and Eraky and her colleagues' study (2022) showing that giftedness, i.e., their insight-based problem-solving skills (Eraky et al., 2022; Leikin & Lev, 2013), was essential while solving unfamiliar tasks.

Despite all students' success in solving nonlinear generalization tasks, in this study, only one student, Diana, could not reach a correct answer for the nonfigural far generalization task. Therefore, her responses for nonfigural generalization tasks were not used in further analysis as they did not satisfy the appropriateness criteria. The reason behind Diana's inability to solve the nonfigural generalization task might be the nature of the problem type. Since the non-figural task did not involve any visuals, Diana tried to visualize the given situation in pictorial and symbolic representations. However, she did not succeed in the visualization of the situation, thus, she did not identify the general rule. Indeed, in the interview conducted at the end of the study, Diana stated that she solved questions that included visuals more easily and that she usually created a visual if there was no visual in the question. This shows that Diana might have difficulty visualizing abstract relationships in her mind and therefore resorted to visual supports to make sense of the problem. The fact that she could not visualize the situation in the nonfigural generalization question accurately may have caused her not to produce the correct answer to the question. This finding supports the findings of the prior research (Amit & Neria, 2008; Eraky et al., 2022; Kerbs, 2003) claiming that visuals had a critical role in reaching the accurate solution and enhancing generalization.

Additionally, while solving the near generalization task, where gifted students did not ask to find the general rule, most of the students (three out of four) noticed and expressed the general rule of the pattern. Stated differently, the gifted students in the present study not only identified the commonalities in given instances but went beyond. Mitchelmore (2002) named this action as an abstract general process of generalization, by which relational understanding takes place. Therefore, this result highlighted that gifted students in the present study were able to detect the commonalities and formalize generalizations with a deep understanding of the underlying structure. This might originate from their abstract thinking and synthesis ability (Callard-Szulgit, 2010; Clark, 2002; Clark, 2013; Sriraman, 2003; Tuttle, 1983). To be more precise, abstract thinking can be defined as the manipulation of one's ideas and involves focusing on the relationships between data rather than superficial features (Dumontheil, 2014). In this study, the fact that gifted students not only focused on solving the near generalization problem but also tried to understand the underlying structure of the information given and to reach far generalizations from this structure shows that these students have abstract thinking and synthesis skills.

Moving to the primary findings related to the first research question, analysis of students' responses indicated that while solving nonlinear realistic generalization tasks gifted students in the present study used a variety of strategies such as *counting from a drawing*, *connecting to the known pattern*, *grouping and assigning a variable*, *using the existing generalization*, *identifying regularities by investigating the numerical relationships embedded in the figure*, *identifying regularities by assigning alternative values and generating the visual representation*, and *identifying regularities by generating the algebraic representation and assigning alternative values*. The fact that the students in this study could produce different strategies for the questions may be because these problems were prepared according to their needs, i.e., they were related to real life, they were challenging, open-ended, and non-routine. In other words, the fact that the questions were not routine, that is, they were different from the standard

problem types that the students had encountered in their schools, may have enabled them to develop various strategies.

Another notable finding of this study is that the strategies gifted students used while solving alternative types of nonlinear realistic generalization problems differ based on the problem type. To be more precise, it was observed that certain strategies were preferred more frequently for particular problem types. More specifically, while solving the figural near generalization task, gifted students mostly used *counting from a drawing strategy*. On the other hand, for the figural far generalization task, they mostly used two strategies, namely, *using existing generalization* and *identifying regularities by investigating numerical relationships embedded in the figure.* Lastly, while solving the non-figural far generalization task, the two strategies were used predominantly: *using existing generalization* and *identifying regularities by assigning alternative values to the variable and generating a visual representation.* In addition, it was also noticed that some strategies utilized by students were specific to each problem type. To be more precise, *counting from a drawing* strategy was used only in solving the figural near generalization task, while *grouping and assigning a variable* strategy was exclusively used in solving the figural far generalization task. On the other hand, *identifying regularities by assigning alternative values to the variable and generating a visual representation,* and *identifying regularities by generating the algebraic representation and by assigning alternative values* strategies were observed only in responses to the nonfigural far generalization task.

The nature of the problem type and characteristics of gifted students might affect their strategy use. More precisely, the reason behind using *counting from a drawing* strategy exclusively and frequently while solving the figural near generalization task might be that those tasks require students to identify specific instances of a pattern and can be solved by step-by-step counting (Stacey, 1989).

Nevertheless, all students were aware that the *counting from a drawing* strategy was not an effective strategy in far generalization tasks as it is impractical with

large numbers, and therefore, none used this strategy in questions requiring far generalization. Instead, they investigated the regularities between numbers embedded in the figure. Since the far generalization task requires students to go beyond the given instances, it can be said that the far generalization task requires higher cognitive demands and is more abstract compared to the near generalization task. In this case, students may have turned to the relationships embedded in the figure to make the problem more concrete. Stated differently, as the problem type became more complex, gifted students in the present study tended to use more complex strategies.

Furthermore, the use of strategies such as *counting from a drawing*, *identifying regularities by investigating numerical relationships embedded in the figure* in figural questions that include visuals, reveals that gifted students used the visuals and benefited from them, especially in solving the figural generalization task. This might be related to gifted students' visualization ability (Van Garderen, 2006). Visualization has been defined as a crucial element of reasoning and problem-solving, which could aid students in comprehending the tasks (Arcavi, 2003). In the interviews conducted at the end of this study, all students stated that the visual facilitated their understanding of the problem. This viewpoint explained their strategy use, i.e., *identifying regularities by investigating numerical relationships embedded in the figure,* and supports their visualization abilities (Van Garderen, 2006).

On the other hand, in the non-figural generalization task, which does not include any visual, gifted students used strategies such as *identifying regularities by assigning alternative values to the variable and generating visual representations* and *identifying regularities by generating the algebraic representation and assigning alternative values*, indicating that they felt the need to represent the given situation with algebraic expressions or pictorial representations. These findings reveal that visualization plays an essential role in gifted students' problem-solving strategies and that they tend to use symbolic and pictorial representations when visuals are not available.

The diversity in the frequency of use of strategies and uniqueness of strategy use support and extend the existing literature (Assmus & Fritzlar, 2022; Erdoğan & Gül, 2023; Gutierrez et al., 2018; Jaušovec, 1991; Montague, 1991; Sternberg, 1981; Syawahid & Prahmana, 2024) that gifted students are flexible strategic thinkers, i.e., develop and use efficient strategies according to problem types in the context of a generalization of non-linear realistic tasks as well. This cognitive flexibility of gifted students might be related to their metacognitive skills. To be more precise, as gifted students were aware of their knowledge (Lu et al., 2017), identify which data is needed to solve the question (Renzulli & Smith, 1979), discriminate the relevant and irrelevant information, and know how to use their knowledge (Shore & Kanevsky, 1993), they might identify whether their strategy is effective or not, observe their process and shift their strategies according to the problems.

Another remarkable finding of this study is that gifted students in the present study can adapt their knowledge and experience to other situations. More specifically, the use of *existing generalization* and *connecting to the known pattern* strategies indicated that gifted students were able to utilize previously used strategies to generate new responses and make connections between the given problem and the known sequence, such as the Fibonacci sequence. The ability to make connections might be caused by gifted students' metacognitive abilities because making connections between situations and concepts requires students to monitor and evaluate the appropriateness and applicability of their previous knowledge and strategies.

5.1.2. Mathematical Creativity of Gifted Students

The second aim of the present study was to examine the mathematical creativity of gifted students in nonlinear realistic generalization tasks.

One of the groundbreaking findings of the current study was that although gifted students did not have any prior experience with nonlinear problems and were not familiar with the problems involved in the current study, they were able to provide

creative responses to nonlinear realistic generalization tasks. This finding implies that the mathematical creativity of gifted students in the present study was independent of their familiarity with problem types and that they could adapt their reasoning to unfamiliar problems. Moreover, gifted students' ability to generate creative responses to unfamiliar problems lends credence to the notion that mathematical creativity involves not just the application of past knowledge but also the ability to investigate, rationalize, and generate novel concepts when confronted with unfamiliar problems.

In addition to the problem type, gifted students in the present study were also unfamiliar with the context of the figural tasks. Nevertheless, they were able to solve these tasks and produce creative responses. This finding suggests that the mathematical creativity of gifted students in the present study is context-independent, which was one of the salient findings of the present study. The findings of studies were controversial about whether familiarity nurtures creativity or not. To be more precise, some research (e.g., Crisp & Turner, 2011) showed that familiarity with a domain resulted in an increase in creativity. However, others (e.g., Frensch and Sternberg, 1989; Tien et al., 2019) declared that experience limits creativity. The findings of the current study were in line with the second view because gifted students could provide creative answers despite their unfamiliarity with the tasks. This might result from the realistic and non-routine characteristics of the tasks that were used in the present study. To be more precise, as gifted students demand challenging and engaging tasks (Özdemir & Işıksal-Bostan, 2021) and have insight-based problem-solving skills (Eraky et al., 2022; Leikin & Lev, 2013), they might find those tasks more engaging, thus generating creative responses. Thus, it can be said that providing opportunities for gifted students to engage in non-routine, open-ended, realistic tasks could help enhance their mathematical creativity.

In the present study, the mathematical creativity of gifted students was analyzed in terms of fluency, flexibility, originality, and elaboration. Therefore, this section

will consist of four headings, and the findings will be discussed under these headings: fluency, flexibility, originality, and elaboration.

5.1.2.1. Fluency

In this study, gifted students' fluency was evaluated according to the number of appropriate answers they provided. Stated differently, each student received a fluency score for each appropriate answer to a question. As discussed above, except for Diana, all students were able to generate more than one appropriate answer to the nonlinear realistic generalization tasks. This finding supports the studies (e.g., Leikin, 2009; Şengil-Akar & Yetkin- Özdemir, 2022) in the literature emphasizing that gifted students are fluent in problem solving. The fact that students produced more than one solution to the problems may be due to the fact that the gifted students in this study were high achievers in mathematics. In addition, in the interview conducted at the end of the study, the students stated that the problems used in this study allowed for producing multiple solutions because they contained uncertainty, were thought-provoking, and were open-ended. Thinking in this way, i.e., finding these tasks conducive to multiple responses, may have encouraged the students to produce a variety of solutions to these questions.

However, the number of responses provided by students for each question varied according to the problem type. More specifically, gifted students provided 13 appropriate responses for the figural near generalization task, 15 responses for the figural far generalization task, and 8 responses for the nonfigural far generalization task in total. The reason why the total number of answers in the non-figural task was less is that Diana could not give any appropriate answer to this question. Another reason why students produced more answers to the figural near and figural far generalization questions may be that these questions contain visuals. Given that gifted students produced more creative answers to the problem that included visuals aligned with the findings of prior research (e.g., Amit & Neria, 2008; Eraky et al., 2022; Kerbs, 2003), indicating that visuals play a significant role in encouraging mathematical creativity. Gifted students' visualization ability (Clark,

2002), i.e., the ability to manipulate data involved in the visuals in creative approaches, might be the possible reason behind their ability to produce more creative responses to figural generalization tasks. On the other hand, the question that received the most responses among these three problem types is the figural far generalization question, which means that gifted students were more fluent while solving the figural far generalization task. This might stem from the nature of the figural far generalization task, i.e., it was more complex than the near generalization task, and it involved a visual. More specifically, since far generalization tasks require students to go beyond the given instances, it can be said that they demand more abstraction and higher cognitive demand. The complexity of the figural far generalization task may have appealed to gifted students' desire for challenging problems (Özdemir & Işıksal-Bostan, 2021), and thus it might stimulate them to generate multiple responses.

5.1.2.2. Flexibility

Another component used in the assessment of mathematical creativity in this study is flexibility. In the current study, students' flexibility scores were the number of their answers in different categories. While identifying the categories, students' responses were categorized according to the idea behind the response, the representation type of the response, and the strategy used to reach a response. Findings indicated that gifted students in the current study were able to differentiate their solutions. Research (Assmus & Fritzlar, 2022; Erdoğan & Gül, 2023; Gutierrez et al., 2018) has shown that gifted students exhibit cognitive flexibility, i.e., develop and adapt different strategies specific to certain types of problems. Since gifted students' strategy use varied across different generalization tasks involved in this study, it can be said that the findings of the current study support and enrich the existing literature in a way that 8th-grade gifted students in the present study exhibit flexible strategic thinking in their processes of generalizing nonlinear patterns as well.

In addition to students' responses varying in terms of strategy use, they also varied in terms of representation use. Stated differently, the other remarkable finding of the current study was that gifted students in this study represented their responses with different representations, namely, verbal representation, pictorial representation, and symbolic representation. Aligning with prior research (e.g., Renzulli & Smith, 1979), this finding supports that gifted students were able to demonstrate ideas through diverse ways of communication. Nevertheless, although gifted students in the present study adopted different types of representations to demonstrate their responses, they predominantly used verbal representation. The frequent use of verbal representation might rely on gifted students' verbal abilities such as communication skills and extensive vocabulary often reported in the literature (e.g., Callard-Szulgit, 2010; Clark, 2013; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986; Tuttle, 1983).

Furthermore, students' flexibility scores across different types of generalization tasks show a small variation. When the fluency and flexibility scores of the students in each question were compared and the extent to which the students could diversify their answers was examined, it was observed that the students in the present study diversified their answers at similar rates in all questions. Therefore, the findings of the current study revealed that 8th-grade gifted students in the present study were able to generate not only a high number of responses but also a variety of responses for the nonlinear realistic generalization tasks.

5.1.2.3. Originality

The third component of mathematical creativity is originality. Originality scores of students were determined based on the rarity of their responses in different categories. Data analysis revealed that gifted students in the present study were able to provide original responses to nonlinear realistic generalization tasks. This finding is in line with the findings of previous studies (e.g., Chamberlin & Moon, 2005; Şengil-Akar & Yetkin- Özdemir, 2022), claiming that gifted students were able to generate novel responses to problems. The fact that gifted students in the

present study produced original solutions reveals that these students follow unconventional paths in their problem-solving processes and develop different perspectives. This might stem from students' flexible thinking skills, i.e., the ability to develop different ways of thinking and to change strategies, when necessary (Jaušovec, 1991), as flexible thinking may enable students to generate novel and a diversity of ideas (Erbas & Bas, 2015). In addition, the curiosity of gifted students may have directed them to different solutions.

Furthermore, findings revealed that students provided more original responses to the figural far generalization task than the figural near generalization task. This might result from gifted characteristics and/or task characteristics. More specifically, gifted students are known for their curiosity (Callard-Szulgit, 2010; Clark, 2002; Otto, 2012; Renzulli & Smith, 1979; Tuttle, 1983), and divergent thinking ability (Chamberlin & Moon, 2005), which is related to generating a variety of novel solutions. In line with these characteristics, several researchers (e.g., Özdemir & Işıksal-Bostan, 2021; Rotigel & Fello, 2004) pointed out that these students need challenging and unconventional problems and engage with problems that require higher-order thinking. Since far generalization tasks demand students to extend the relationship beyond the given cases, it can be said that they require deeper investigation. Aligning with the gifted students' needs and characteristics, these features of the far generalization task might cause gifted students to find those tasks more engaging, thus enabling them to put more effort into coming up with original answers.

Nevertheless, in this study, the originality of students was identified for each problem based on the rarity of the response. While all students produced appropriate answers to the figural near and figural far problems, one participant could not give any appropriate answer for the nonfigural far generalization task, and the originality scoring for this problem was done based on three participants. This difference limits the direct comparability of the originality scores obtained for the figural generalization problems and the nonfigural far generalization problem. Therefore, the originality scores obtained by the students from the figural

generalization problems and the nonfigural far generalization question should be interpreted in a relative context and cannot be compared.

5.1.2.4. Elaboration

The fourth component of mathematical creativity that was investigated in the present study was elaboration. In this study, gifted students' elaboration scores were determined according to the number of responses the students explained their thought process and provided justifications. Findings indicated that gifted students in the current study were able to elaborate on most of their responses to all problems. This result might have stemmed from gifted students' verbal proficiency (Callard-Szulgit, 2010; Clark, 2013; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986; Tuttle, 1983), which might enable them to explain their thought process in words. Stated differently, the ability to express ideas or concepts in words, both orally and in writing, is referred to as verbal ability (Andrew & Giampietro, 2005). In this study, students who could elaborate on their answers demonstrated their mathematical knowledge and their ability to communicate this knowledge through language. This makes the thinking processes and conceptual understanding of gifted students more visible, while also reflecting their language skills. Indeed, when the students were asked about their perception of competence in elaborating their ideas/responses at the beginning of the interview, all of the students stated that they have high confidence in their ability to detail and justify their answer to a mathematics problem. While solving the problems in the study, the students were able to justify their answers and express their thought process in a way that supported these perceptions. This harmony shows that the gifted students in the present study not only have strong verbal competence but also high cognitive awareness, which supports the existing literature claiming that gifted students have verbal proficiency (Callard-Szulgit, 2010; Clark, 2013; Renzulli & Smith, 1979; Silverman & Waters, 1984; Silverman et al., 1986; Tuttle, 1983), and self- awareness (Clark, 2013).

In addition to verbal proficiency, research showed that gifted students are curious (Callard-Szulgit, 2010; Clark, 2002; Otto, 2012; Renzulli & Smith, 1979; Tuttle, 1983) and have metacognitive abilities (Lu et al., 2017). To be more precise, since gifted students are aware of what they do and monitor their thinking process (Lu et al., 2017; Shore & Kanevsky, 1993), and they might seek to explore the underlying reasons behind the issues or concepts, and these hallmarks of giftedness might engage them to provide in-depth reasoning behind their responses.

Another reason might be the open-ended, non-algorithmic, and realistic features of problems that were used in the present study. To put it more clearly, since those tasks were nonroutine, which means that they demand reasoning beyond algorithmic thinking, these tasks might enable gifted students in the present study to monitor their cognitive process, thus allowing them to explain and justify their responses.

One of the compelling findings of the current study was that gifted students in this study were able to elaborate their responses more on the figural generalization tasks. The reason behind this might be the nature of figural generalization tasks. More precisely, since figural generalization tasks include a visual, and gifted students in the present study indicated that the visual facilitates their understanding of the problem, it might help them to identify the commonalities, which stimulates them to identify and express the in-depth reasoning behind their answers.

5.2. Implications of the Study

The present study aimed to examine strategies that 8th-grade gifted students use while solving alternative types of nonlinear realistic generalization tasks and their mathematical creativity while solving these tasks. The findings of this study provided valuable insights and implications for researchers, teachers, and curriculum developers. In the following paragraphs, those implications will be presented.

First, the findings pointed out that gifted students in the present study were able to solve non-linear generalization tasks accurately and generate creative responses for those tasks, despite non-linear generalization tasks were not involved in the Turkish middle school mathematics textbooks and the students in the present study stated that they were not familiar with these types of tasks. This implies that gifted students in the present study were able to show performance beyond the curriculum and expand their knowledge and experience into unfamiliar situations. Therefore, this suggests and supports the necessity to differentiate the curriculum and learning practices for gifted students in a way to includes non-routine tasks that challenge them to go beyond the taught curriculum.

In addition to their performance in solving the nonlinear realistic generalization tasks, the present study provided notable insights into gifted students' problem-solving abilities and mathematical talents. To be more precise, the findings of this study indicated that gifted students in the present study were able to notice the patterns, formalize generalizations, use different representations and a variety of strategies, adapt their previous knowledge to unfamiliar situations, shift their strategies effectively across tasks, generate novel responses, and provide in-depth reasoning behind their responses. By providing illustrative instances and detailed descriptions of gifted students' creative problem-solving processes and those abilities, the findings of the present study may be helpful for teachers to not only differentiate the instruction but also notice gifted students' mathematical and creative talent in their classes.

Moreover, the problems used in this study were realistic, had multiple entry and exit points, and were non-algorithmic. Given that gifted students in the present study provided a high number and diversity of novel responses while solving those problems provided evidence that these problems were appropriate for them to show their mathematical talent and creative performance. Therefore, the tasks used in this study can serve as an example for researchers to design creativity-directed tasks to assess the mathematical creativity of gifted students. In addition to

researchers, teachers can adapt and implement these tasks into their classrooms to engage gifted students and promote their creative talent.

Furthermore, considering the finding that gifted students in the present study frequently use verbal representations despite their potential to use different representations (Assmus & Fritzlar, 2022) and the role of different representations in conceptual understanding (Van de Walle et al., 2020), it can be said that teachers should encourage gifted students to use diverse representations and establish relationships between representations.

Last but not least, in this study, the mathematical creativity of gifted students was evaluated based on four components, namely, fluency, flexibility, originality, and elaboration. To assess the fluency of gifted students, the researcher identified the number of appropriate responses of each gifted student. Stated differently, the researcher focused on the final product, which was the appropriate response. On the other hand, while assessing the flexibility and originality of gifted students, the researcher categorized gifted students' responses. While making this categorization, the researcher examined the way the student represented her answer (which was product-oriented), the idea behind the answer (which was process-oriented), and the strategy the student used to reach the answer (which was also process-oriented). Furthermore, while assessing the elaboration of gifted students, the researcher asked students to explain their solution process. Since the focus was on the way gifted students think, it was process-oriented. Given that researchers criticized the solely product-oriented assessment of creativity (Schindler & Lilienthal, 2017; Silver, 1997) and emphasized the value of the process (Joklitsche et al., 2016), it is worth highlighting that this study adopted an integrative approach focusing on both the product and the process. This dual way of assessing mathematical creativity enabled the researcher to provide more comprehensive insights into the depth of students' cognitive creative thinking process involved in their creative product and their creative problem-solving abilities. Therefore, the assessment approach adopted in this study might serve as a framework in the upcoming studies. By building on this method, i.e., incorporating the creative

process and product, further studies might investigate the mathematical creativity of gifted or nongifted students in different content areas. In addition to the researchers, teachers can also use the product and process integrated approach that was used in this study to encourage students with creative potential, even if their answers were not complete.

5.3. Limitations and Recommendations

Despite the valuable findings the present study offered, it also had some limitations. In this part, the limitations of this study and recommendations for future research will be presented.

The current study sought to provide a thorough investigation of strategies and mathematical creativity of gifted students while solving nonlinear realistic generalization tasks. Following this aim, this study adopted a case study design, which enables researchers to have in-depth data (Cresswell, 2013). Therefore, aligning with the design of the study, this study shed light on gifted students' problem-solving process within the scope of alternative types of nonlinear realistic generalization tasks. However, the findings of the present study were not generalizable due to the small number of participants, which was intentionally set as four because of the nature of qualitative inquiry and case study design. If generalization of the study's findings is intended, a quantitative study with large samples of 8th-grade gifted students can be carried out to confirm and extend the insights the current study presented.

Furthermore, in this study, the originality of students' responses was analyzed based on the rarity of their responses, which is frequently noted in the literature (Baykal, 2009; Kaufman, 2016; Kim et al., 2004; Leikin, 2009; Shamay-Tsoory et al., 2011). Measuring originality remains a significant challenge in the literature (Said-Metwaly et al., 2017). One of the criticisms of rarity or frequency-based scoring is that it is more open to random differences in small samples and that this scoring method is sensitive to sample size (Dumas & Dunbar, 2014; Silvia et al., 2008). However, Beketayev and Runco (2016) argued that the sample dependency of traditional originality scoring is not a limitation, instead, this can provide a

significant advantage due to its context sensitivity. For this reason, in addition to this scoring, expert opinions were also utilized in the present study. In light of these discussions reported in the literature, the originality scoring in this study should be interpreted with caution. Due to the qualitative nature of the present study, no absolute generalization was made from this scoring, and originality scores were used only to make sense of the distribution and rarity of the responses of the participants in the same context. Although the method used in the evaluation and interpretation of originality scores in this study is considered appropriate because it is aligned with the qualitative nature of the study and consistent with the rarity-based definition and assessment of originality frequently used in the literature, testing the functionality of the method used in the evaluation of originality scores in this study with different samples may be a suggestion for further research.

Additionally, during the data collection and data analysis, the researcher noticed some gender-related differences. Firstly, while collecting the data, differences were observed in the interview durations with male and female students. More specifically, the researcher conducted two interview sessions, each lasting approximately two to two and a half hours with female students. However, the male student was able to solve all the problems in one session, which lasted two hours. Furthermore, although the interview time with the male student was shorter, he received the highest originality score in the figural far generalization task. Besides, although all gifted students utilized the counting from a drawing strategy in the figural near generalization task, the purposes for which girls and boys used this strategy differed. To put it more clearly, girls first used this strategy to reach the answer, while the male student used this strategy to check his answer or verify his conjecture. Nevertheless, since mathematical creativity and gender relationships were not the focus of the present study, those insights were not presented and discussed. Further studies can be conducted to provide more comprehensive insights into whether and how the mathematical creativity of gifted male and female students differs across alternative types of realistic problems and the possible reasons behind these differences.

Lastly, the present study uncovered gifted students' strategies and mathematical creativity while solving realistic nonlinear generalization tasks. Future studies can be carried out to expand the findings of the present study by examining gifted students' strategies and mathematical creativity in other content areas beyond generalization tasks. Furthermore, since this study specifically focused on gifted students' strategies and mathematical creativity, future studies can be conducted to compare the strategies and mathematical creativity of nongifted students with the same tasks to shed light on the hallmark of giftedness.

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APPENDICES

A. APPROVAL OF THE METU HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

Konu: Değerlendirme Sonucu 29 KASIM 2023

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof. Dr. Mine Işıksal Bostan

Danışmanlığını yürüttüğünüz Zeynep TUNCER'in ***"Üstün Yetenekli 7, 8 ve 9. Sınıf Öğrencilerinin Gerçekçi Cebir Problemlerindeki Genelleme Stratejilerindeki Matematiksel Yaratıcılıklarının İncelenmesi"*** başlıklı araştırmanız İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek 0532-ODTUİAEK-2023 protokol numarası ile onaylanmıştır

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan

Prof.Dr. İ. Semih AKÇOMAK
Üye

Doç. Dr. Ali Emre Turgut
Üye

Doç. Dr. Şerife SEVİNÇ
Üye

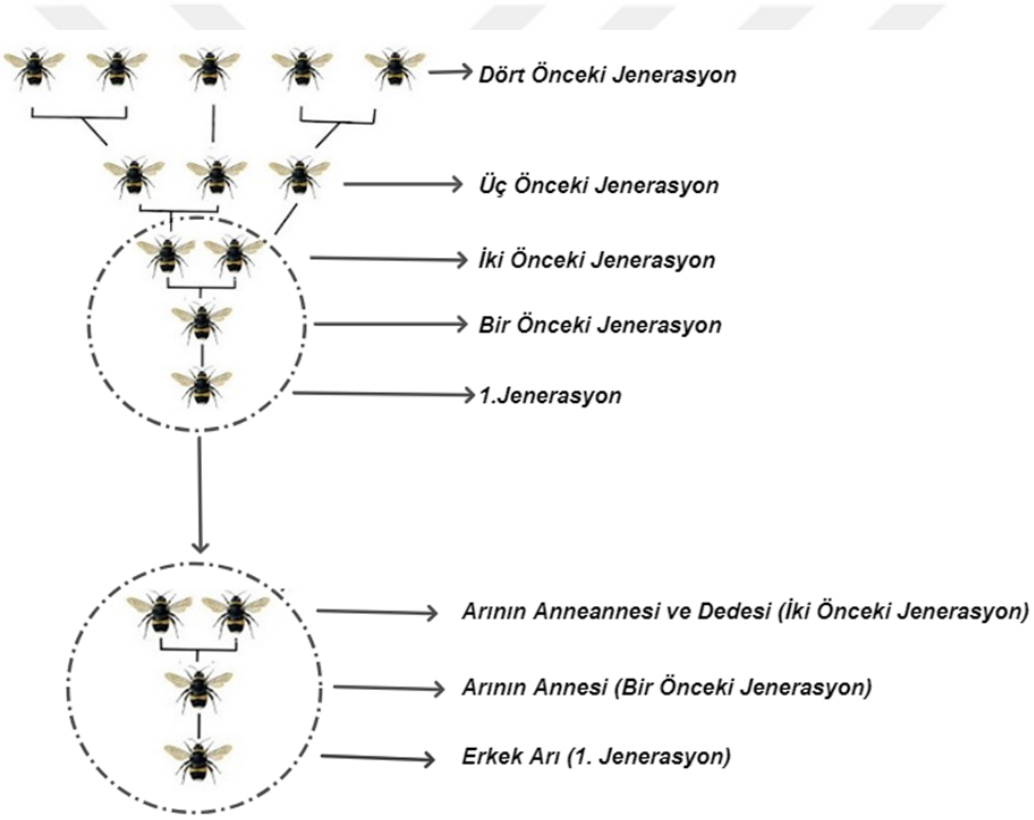
Doç.Dr. Murat Perit ÇAKIR
Üye

Dr. Öğretim Üyesi Süreyya ÖZCAN KABASAKAL
Üye

Dr. Öğretim Üyesi Müge GÜNDÜZ
Üye

B. PROBLEM SHEET

1. Tüm arıların iki ebeveyninin olmadığını biliyor muydunuz? Arıların ilginç bir üreme sistemi ve buna bağlı olarak farklı soyağaçları vardır. Kraliçe arılar genetik çeşitliliği sağlamakla görevli olan dişi arılardır. Kraliçe arıların kovana bıraktığı döllenmemiş yumurtalardan erkek arılar, döllenmiş yumurtadan ise dişi arılar oluşur. Yani erkek arıların yalnızca bir ebeveyni (annesi) varken dişi arıların iki ebeveyni (annesi ve babası) vardır.



Yukarıda bir arının soyağacı verilmiştir. Buna göre:

a) Verilen soyağacında dişi arıları “D”, erkek arıları “E” ile gösteriniz.



b) Yukarıda bir arının görseli verilmiştir. Bu arıyı 1. Jenerasyon kabul ederseniz, 6 önceki jenerasyonda kaç tane arı vardır? Olabildiğince fazla yöntemle cevap veriniz.

c) Her bir jenerasyonda kaç tane arı gözlemlediniz? Herhangi bir jenerasyondaki (n. jenerasyondaki) arı sayısını matematiksel olarak nasıl ifade edersiniz? Olabildiğince fazla yöntem ile cevap veriniz.

2. Ülkemizde ilk olarak 2020 yılında görülen bulaşıcı COVID19 Koronavirüs hastalığı tüm dünyayı etkilemiştir. Tedbirlerin önceden alınabilmesinin önem arz ettiği o dönemde, COVID19 vakalarının doğru bir şekilde tahmin edilebilmesi için Manav Bhatnagar isimli bir araştırmacı COVID19 virüsünün yayılımını gösteren bir matematiksel model geliştirmiştir. Bhatnagar kapanma gibi tedbirlerin virüsün yayılımına etkisini gözlemleyebilmek için ilk olarak hiçbir önlem alınmadığı durumda COVID19 un toplumsal yayılımını gösteren modeli şu senaryodan yola çıkarak oluşturmuştur:

*Koronavirüs pozitif bir hastanın ülkeye geldiğini ve başka insanlarla temas ettiğini varsayalım. Koronavirüs bulaşıcı bir hastalık olduğu için başka insanlara da bulaşır. Hasta kişiyi A ile temsil edelim. Bir birimlik zamanda A kişisi **a** sayıda başka kişiye hastalığı bulaştırabilir. Artık hasta olan kişilerin hepsi aktiftir ve hasta olan kişilerin her biri de **a** sayıda başka kişiye hastalığı bulaştırabilir ve bu böyle devam eder.*

Önemli Not: Bu modelde virüsün aktif olma süresi hesaba katılmamıştır. Hasta olan bir kişinin hep enfekte olduğu yani sürekli olarak hastalığı bulaştırmaya devam ettiği varsayılmıştır.

Bhatnagar toplam hasta kişi sayısını matematiksel olarak nasıl ifade etmiş olabilir? Olabildiğince fazla yöntem ile cevap vermeye çalışınız.

C. INFORMED CONSENT FORM

ARAŞTIRMAYA GÖNÜLLÜ KATILIM FORMU

Bu çalışma Orta Doğu Teknik Üniversitesi Matematik eğitimi programı yüksek lisans öğrencisi Zeynep Tuncer ve öğretim üyesi Prof. Dr. Mine Işıksal Bostan tarafından yürütülmektedir. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir?

Çalışmanın amacı, 7, 8 ve 9.sınıf üstün yetenekli öğrencilerin gerçekçi cebir problemlerindeki genelleme stratejilerindeki matematiksel yaratıcılıklarının incelenmesidir. Bu çalışmanın bulguları üstün yetenekli öğrencilerinin matematiksel yaratıcılıklarının geliştirilmesine yönelik farklılaştırılmış matematik eğitimi çalışmalarına ışık tutma potansiyeline sahiptir.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz?

Araştırma öğrenci için uygun olan herhangi bir mekânda ve araştırmacı ve öğrenci için uygun olan bir zamanda yüz yüze gerçekleştirilecektir. Bu çalışmaya gönüllü olarak katılmayı kabul ederseniz sizden iki adet gerçekçi cebir problemini çözmeniz ve araştırmacının sizin çözüm sürecini anlayabilmesi adına soracağı görüşme sorularına yanıt vermeniz beklenmektedir. Araştırmacının görüşme sırasında tüm cevapları not alması zor olacağı için, görüşme sırasında ses kaydı alınacaktır. Ses kayıtları yalnızca araştırmacının erişebileceği kilitli bir dosyada tutulacak, hiçbir şekilde kimseyle paylaşılmayacaktır.

Sizden Topladığımız Bilgileri Nasıl Kullanacağız?

Araştırmaya katılanlardan toplanan veriler tamamen gizli tutulacak, veriler ve kimlik bilgileri herhangi bir şekilde eşleştirilmeyecektir. Katılımcıların isimleri bağımsız bir listede toplanacaktır. Ayrıca toplanan verilere sadece araştırmacılar ulaşabilecektir. Bu araştırmanın sonuçları bilimsel ve profesyonel yayınlarda veya eğitim amaçlı kullanılabilir, fakat katılımcıların kimliği gizli tutulacaktır.

Katılımınızla ilgili bilmeniz gerekenler:

Çalışma, genel olarak kişisel rahatsızlık verecek sorular içermemektedir. Ancak, katılım sırasında sorulardan ya da herhangi başka bir nedenden ötürü kendinizi rahatsız hissederseniz cevaplama işini yarıda bırakıp çıkmakta serbestsiniz. Böyle bir durumda araştırmacıya, görüşmeyi sonlandırmak istediğinizi söylemek yeterli olacaktır.

Araştırmayla ilgili daha fazla bilgi almak isterseniz:

Çalışma sonunda, bu çalışmayla ilgili sorularınız cevaplanacaktır. Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Çalışma hakkında daha fazla bilgi almak için Matematik Eğitimi öğretim üyelerinden Prof. Dr. Mine Işıksal Bostan (e-posta:) veya araştırma görevlilerinden Zeynep Tuncer (E-posta) ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katılıyorum.
(Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyad Tarih İmza
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D. Parental Consent Form

Veli Onay Formu

Sayın Veliler, Sevgili Anne-Babalar,

Bu çalışma Orta Doğu Teknik Üniversitesi Matematik eğitimi programı yüksek lisans öğrencisi ve araştırma görevlisi Zeynep Tuncer ve öğretim üyesi Prof. Dr. Mine Işıksal Bostan tarafından yürütülmektedir.

Bu çalışmanın amacı nedir? Çalışmanın amacı, 7,8 ve 9.sınıf üstün yetenekli öğrencilerin gerçekçi cebir problemlerindeki genelleme stratejilerindeki matematiksel yaratıcılıklarının incelenmesidir. Bu çalışmanın bulguları üstün yetenekli öğrencilerinin matematiksel yaratıcılıklarının geliştirilmesine yönelik farklılaştırılmış matematik eğitimi çalışmalarına ışık tutma potansiyeline sahiptir.

Çocuğunuzun katılımcı olarak ne yapmasını istiyoruz?: Bu amaç doğrultusunda, çocuğunuzdan iki adet gerçekçi cebir problemini çözmesini ve yaptığımız görüşmede problem çözme sürecini daha detaylı anlayabilmem adına soracağım görüşme sorularını cevaplamasını isteyeceğiz. Öğrencilerden çözümlerini problem kağıdına yazmalarını isteyeceğiz ve görüşmeler sırasında cevapların tamamını not almak zor olacağı için ses kaydı alacağız. Sizden çocuğunuzun katılımcı olmasıyla ilgili izin istediğimiz gibi, çalışmaya başlamadan çocuğunuzdan da sözlü olarak katılımıyla ilgili rızası mutlaka alınacak.

Çocuğunuzdan alınan bilgiler ne amaçla ve nasıl kullanılacak?: Çocuğunuzdan alacağımız cevaplar tamamen gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Elde edilecek bilgiler sadece bilimsel amaçla kullanılacak, çocuğunuzun ya da sizin ismi ve kimlik bilgileriniz, hiçbir şekilde kimseyle paylaşılmayacaktır. Ses kayıtları yalnızca araştırmacının erişebileceği kilitli bir dosyada tutulacak, hiçbir şekilde kimseyle paylaşılmayacaktır.

Çocuğunuz ya da siz çalışmayı yarıda kesmek isterseniz ne yapmalısınız?: Çalışma çocuğunuza psikolojik veya fiziksel zarar verecek hiçbir soru içermemektedir. Yine de katılım sırasında sorulan sorulardan ya da herhangi bir uygulama ile ilgili başka bir nedenden ötürü çocuğunuz kendisini rahatsız hissettiğini belirtirse, ya da kendi belirtmese de araştırmacı çocuğun rahatsız olduğunu öngörürse, çalışmaya sorular tamamlanmadan ve derhal son verilecektir. Şayet siz çocuğunuzun rahatsız olduğunu hissederseniz, böyle bir durumda çalışmadan sorumlu kişiye çocuğunuzun çalışmadan ayrılmasını istediğinizi söylemeniz yeterli olacaktır.

Bu çalışmayla ilgili daha fazla bilgi almak isterseniz: Çalışmaya katılımınızın sonrasında, bu çalışmayla ilgili sorularınız yazılı biçimde cevaplandırılacaktır. Çalışma hakkında daha fazla bilgi almak için Matematik Eğitimi öğretim üyelerinden Prof. Dr. Mine Işıksal Bostan (e-posta:) ve araştırma görevlilerinden Zeynep Tuncer ile (e-posta: ile iletişim kurabilirsiniz.
Bu çalışmaya katılımınız için şimdiden teşekkür ederiz

Yukarıdaki bilgileri okudum ve çocuğumun bu çalışmada yer almasını onaylıyorum (Lütfen alttaki iki seçenekten birini işaretleyiniz.

Evet onaylıyorum___ ***Hayır, onaylamıyorum___***

Velinin adı-soyadı: _____ Bugünün Tarihi:_____

Çocuğun adı soyadı ve doğum tarihi:_____

(Formu doldurup imzaladıktan sonra araştırmacıya ulaştırınız).

E. INTERVIEW PROTOCOL

Birinci Kısım:

1. Hangi okula gidiyorsun? (Özel/Devlet)
2. BİLSEM'de hangi gruptasın (genel yetenek, müzik, resim vs.)? / Hangi derslerdesin?
3. BİLSEM'de hangi dersleri alıyorsun?
4. BİLSEM'e kaçıncı sınıftan beri gidiyorsun?
5. Neler yapıyorsunuz BİLSEM'de?
6. Matematiğinin iyi olduğunu düşünür müsün? Matematik sınavlarından genelde kaç alırsın? En son matematik ortalaman kaç?
7. Matematik problemlerini çözmede kendine güveniyor musun? Kendine güvenini düşük, orta ve yüksek olarak nasıl tanımlarsın? /Nasıl derecelendirirsin?
8. Sınıftaki matematik derslerinde/BİLSEM' bir problemi birden fazla çözebilir misin? Kendini fazla çözüm üretmede ne derece yeterli görüyorsun? Düşük-orta-yüksek? Neden?
9. Öğretmenine farklı alternatif çözüm yöntemleri üretiyor musun / öğretmenine farklı çözüm yöntemleri önerir misin? Kendini fazla ve farklı çözümler üretmede ne derece yeterli görüyorsun? (düşük/orta/ yüksek). Neden?
10. Peki bu çözümlerin sınıfta başka kişiler tarafından nadir üretilen veya üretilmeyen yeni çözümler olduğunu düşünüyor musun? Kendini bu konuda ne derece yeterli görüyorsun (düşük/orta/ yüksek) Neden?
11. Çözümlerinin arkasındaki gerekçeyi açıklayabiliyor musun? Kendini bu konuda ne derece yeterli görüyorsun? (düşük/orta/yüksek) Neden?

İkinci Kısım: Problemlere ve Çözümlere Yönelik Sorular

1. Sorudan ne anladın?

2. Her bir çözüm yöntemini hiç bilmeyen bir arkadaşına anlatır gibi bana anlatır mısın? Çözümlerinin arkasındaki detayları ve nasıl düşündüğünü merak ediyorum. Hangi detaylara/ayrıntılara dikkat ettin? Neleri göz önünde bulundurdun?
3. Şu anda aklına gelen bunların dışında eklemek **istediğin yeni ve farklı** çözüm yolu var mı?

F. PERMISSION TO USE THE HONEYBEE FAMILY TREE PROBLEM (EMAIL CONFIRMATION)

Re: Permission Request

Sian Zelbo göndericisinden 2023-09-28 15:03 tarihinde
Ayrıntılar

Hello!

Thank you for your email. It's fine to use anything on the site. It's under a creative Commons license which just means you can take it and use it with attribution.
If you're adapting it, I would just give the source and say that it was adapted from a problem on the website.

good luck!

Sian

> On Sep 28, 2023, at 7:39 AM, > wrote:
>
> Dear Dr Zelbo,
> I am Zeynep Tuncer, a graduate student and research assistant at Middle East Technical University, Ankara, Turkey. I hope this email finds you well. I would like to say that the problems you publish on the 1001 Math Problems blog are inspiring. I am studying for a master's degree in mathematics education and planning to work on the mathematical creativity of gifted middle school students. Could I use the "Honeybee Family Tree" problem shared on the 1001 math problems in part of my master's thesis by adapting/modifying it according to the purpose of my study? I would appreciate receiving your permission. If you need any additional information, please contact me at Thank you for your consideration.
>
> Respectfully yours,
> Zeynep Tuncer