Mon. Not. R. Astron. Soc. **372,** 489–496 (2006) doi:10.1111/j.1365-2966.2006.10893.x

Pulsar braking indices, glitches and energy dissipation in neutron stars

M. Ali Alpar[1] and Altan Baykal[2]★

[1]*Sabancı University, Faculty of Engineering and Natural Sciences, Istanbul, Turkey*
[2]*Middle East Technical University, Physics Department, Ankara, Turkey*

Accepted 2006 July 31. Received 2006 July 31; in original form 2006 June 4.

ABSTRACT

Almost all pulsars with anomalous positive $\ddot{\Omega}$ measurements (corresponding to anomalous braking indices in the range $5 < |n| < 100$), including all the pulsars with observed large glitches ($\Delta\Omega/\Omega > 10^{-7}$) as well as post-glitch or interglitch $\ddot{\Omega}$ measurements, obey the scaling between $\ddot{\Omega}$ and glitch parameters originally noted in the Vela pulsar. Negative second derivative values can be understood in terms of glitches that were missed or remained unresolved. We discuss the glitch rates and a priori probabilities of positive and negative braking indices according to the model developed for the Vela pulsar. This behaviour supports the universal occurrence of a non-linear dynamical coupling between the neutron star crust and an interior superfluid component. The implied lower limit to dynamical energy dissipation in a neutron star with spindown rate $\dot{\Omega}$ is $\dot{E}_{\rm diss} > 1.7 \times 10^{-6} \dot{E}_{\rm rot}$. Thermal luminosities and surface temperatures due to dynamical energy dissipation are estimated for old neutron stars which are spinning down as rotating magnetic dipoles beyond the pulsar death line.

Key words: pulsars: general.

1 INTRODUCTION

Anomalous second derivatives of the rotation rates of radio pulsars may have interesting implications. Very large positive or negative second derivatives are likely to be artefacts of timing noise. Here, we show that second derivatives corresponding to braking indices n in the interval $5 < |n| < 100$ generally fit well with secular interglitch behaviour according to a model previously applied to the Vela pulsar. Pulsars with large glitches ($\Delta\Omega/\Omega \geqslant 10^{-7}$) and measured anomalous second derivatives of the rotation rate, mostly positive (Shemar & Lyne 1996; Lyne, Shemar & Smith 2000; Wang et al. 2000), as well as pulsars with positive or negative anomalous second derivatives but no observed glitches (Johnston & Galloway 1999) scale with the model. We infer that isolated neutron stars older than Vela have dynamical behaviour similar to the Vela pulsar. This implies relatively large energy dissipation rates that can supply a luminosity to older isolated neutron stars.

The spindown law of a pulsar is usually given in the form $\dot{\Omega} = -k\Omega^n$, where n, the braking index, is 3 if the pulsar spindown is determined purely by electromagnetic radiation torques generated by the rotating magnetic dipole moment of the neutron star. The braking index has been conventionally measured through the relation

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} \tag{1}$$

by measuring $\ddot{\Omega}$, the second derivative of the pulsar rotation frequency. An alternative method, suggested recently by Johnston & Galloway (1999) is based on integrating, rather than differentiating, the spindown law, to obtain

$$n = 1 + \frac{\Omega_1\dot{\Omega}_2 - \Omega_2\dot{\Omega}_1}{\dot{\Omega}_1\dot{\Omega}_2\,(t_2 - t_1)}, \tag{2}$$

where Ω_i and $\dot{\Omega}_i$ are values measured at t_i.

Among the known radio pulsars, only young pulsars have braking indices measured with accuracy. These reported braking indices are all less than 3: for the Crab pulsar $n = 2.509 \pm 0.001$ (Lyne, Pritchard & Smith 1988, 1993); for PSR B 1509−58, $n = 2.837 \pm 0.001$ (Kaspi et al. 1994); for PSR B 0540−69, $n = 2.04 \pm 0.02$ (Manchester & Peterson 1989; Nagase et al. 1990; Gouiffes, Finley & Ögelman 1992); for pulsar J 1119−6127, $n = 2.91 \pm 0.05$ (Camilo et al. 2000); for pulsar J 1846−0258, $n = 2.65 \pm 0.01$ (Livingstone et al. 2006). For the Vela pulsar, a long-term (secular) braking index of 1.4 ± 0.2 was reported (Lyne, Pritchard & Smith 1996). This value was extracted with certain assumptions for connecting fiducial epochs across a timing history dominated by glitches and interglitch response.

For old pulsars with $\nu \sim 1$ Hz and $\dot{\nu} \sim 10^{-15}$ Hz s^{-1}, the expected $\ddot{\nu}$ for $n = 3$ is $\sim 10^{-30}$ Hz s^{-2}. This is difficult to measure because the cumulative effect of the second derivative would contribute one extra cycle count [$(\ddot{\nu}t^3)/6 \sim 1$] only after several centuries. For 19 'old' radio pulsars, observations yielded anomalous braking indices extending from $\sim \pm 4$ all the way to $\pm 10^5$ (Gullahorn & Rankin 1982). Later measurements of braking indices of these pulsars have shown that these anomalous values are artefacts produced by timing noise (Cordes 1980; Cordes & Helfand 1980; Cordes & Downs 1985). Some of the old pulsars' (PSRs 0823+26, 1706−16, 1749−28, 2021+51) time of arrival (ToA) data extending

★E-mail: altan@astroa.physics.metu.edu.tr

© 2006 The Authors. Journal compilation © 2006 RAS

Downloaded from https://academic.oup.com/mnras/article-abstract/372/1/489/993600 by Middle East Technical University Library user on 23 July 2020

over more than three decades were investigated for the stability of the pulse frequency second derivatives $\ddot{\nu}$ (Baykal et al. 1999). These pulsars have shown anomalous values of braking indices of the order of $\sim \pm 10^5$. In the framework of low-resolution noise power spectra estimated from the residuals of pulse frequency and ToA data, it is found (Baykal et al. 1999) that the $\ddot{\nu}$ terms of these sources arise from the red torque noise in pulse frequency derivatives.

For pulsars with moderate ages, $\sim 10^5$ yr, anomalous braking indices have values of the order of $\pm 10^2$. These are not noise artefacts. Rather, such braking indices can be understood as part of the neutron star's secular dynamics. The interglitch recovery of pulsars extending through observation time-spans may yield positive anomalous braking indices, while negative anomalous braking indices can be explained by the occurrence of an unobserved glitch causing a negative step $\Delta\dot{\Omega}$ in the spindown rate (as typically observed with resolved glitches), between the different measurements of $\dot{\Omega}$ (Johnston & Galloway 1999). In this work, we show that all pulsars with anomalous $\ddot{\Omega}$ measurements, including all the pulsars with observed glitches as well as post-glitch or interglitch $\ddot{\Omega}$ values (Shemar & Lyne 1996; Wang et al. 2000), obey the same scaling between $\ddot{\Omega}$ and glitch parameters (Alpar 1998a) as in the models developed for the Vela pulsar glitches (Alpar et al. 1993a; Alpar, Ögelman & Shaham 1993b).

The prototypical Vela pulsar glitches occur at intervals of about 2 yr. Models developed for the Vela pulsar glitches indicate that interglitch intervals scale with $|\dot{\Omega}|^{-1}$. This is borne out by the statistics of large $\Delta\Omega/\Omega > 10^{-7}$ glitches (Alpar & Baykal 1994). Scaling with the spindown rates, the glitch intervals of pulsars at the ages of 10^5–10^6 yr are of the order of $\sim 10^2$ yr.

In Section 2, we review the observations of anomalous braking indices, their errors and methods of deciding if the nominal second derivatives are artefacts of the noise process. In Section 3, we review the interglitch timing behaviour of the Vela pulsar and the simple explanation for this standard behaviour in terms of the model of non-linear vortex creep dynamics in the neutron star superfluid. In Section 4, we show that pulsars with reliable anomalous $\ddot{\Omega}$ measurements can be consistently explained within the same model, with one model parameter whose values are similar, to the order of magnitude, to those obtained in detailed fits to the Vela pulsar timing data. In Section 5, we extend this analysis to pulsars with glitches of size $\Delta\Omega/\Omega > 10^{-7}$, comparable to the Vela pulsar glitches and with reliable anomalous $\ddot{\Omega}$ measurements. This seemingly universal dynamics is characterized by a lag in rotation rate between the observed crust and some interior component of the neutron star, the crust superfluid in current models. The identification of the universal dynamical behaviour leads us to derive a lower limit on the lag, and a corresponding lower limit on the rate of dynamical energy dissipation. In Section 5, we explore the implications of the lower bound on the energy dissipation rate. Estimates of minimum thermal luminosities and surface blackbody temperatures for isolated neutron stars of various ages are presented under the dipole spindown law.

2 OBSERVATIONS OF ANOMALOUS BRAKING INDICES

Pulse arrival time measurements display irregularities in the rotation rate known as 'timing noise'. The timing noise could be due to a noisy component of the secular torque involving fluctuations in the magnetosphere of the neutron star (Cheng 1987a,b, 1989). Alternatively, timing noise could arise from internal torques coupling different components of the neutron star, for example the decoupling and recoupling of the crust superfluid (Alpar, Nandkumar & Pines 1986; Jones 1990). Timing noise for pulsars has been studied for the last three decades (Boynton et al. 1972; Groth 1975; Cordes 1980; Cordes & Helfand 1980; Cordes & Downs 1985; D'Alessandro et al. 1995; Deshpande et al. 1996). Boynton et al. (1972) proposed that the timing noise in the ToA of pulses might arise from 'random walk' processes which are rth order ($r = 1, 2, 3$) time integrals of a 'white noise' time series (that is, a time series of unresolved delta functions). The random walks in phase ϕ, pulse frequency ν and pulse frequency derivative $\dot{\nu}$ are called 'phase noise', 'frequency noise' and 'slowing down noise', respectively (Cordes 1980).

The crosstalk between the timing noise and secular slowing down is very important. Many of the old pulsars with spindown age $\tau = P/2\dot{P}$ greater than about 10^6 yr have shown anomalous trends in their secular frequency second derivative ($\ddot{\nu}$) (Cordes & Downs 1985). These trends make it impossible to recover the braking law $\dot{\nu} \sim \nu^n$ of the pulsar (for pure magnetic dipole radiation $n = 3$). Nominal values of $\ddot{\nu}$ from timing fits gave anomalous braking indices ranging from -10^5 to 10^5 in various pulsars. Recent observations of some young/middle-aged pulsars with glitches also showed anomalous positive braking indices of the order of $\sim$20–200 (Shemar & Lyne 1996; Lyne et al. 2000; Wang et al. 2000). Interglitch recovery between successive glitches can effect the pulsar's dynamical parameters such as $\dot{\nu}$ and $\ddot{\nu}$. For the glitching pulsars, the high values of the second derivative of the rotation rate, $\ddot{\nu}$, and associated braking indices of the order of 20–200 are characteristic of interglitch recovery (Alpar 1998b), which extends from one glitch to the next one, as studied in detail between the glitches of the Vela pulsar (Alpar et al. 1993b). For all middle-aged pulsars, the expected intervals between glitch events are of the order of a few hundred years (Alpar & Baykal 1994). Thus a pulsar is most likely to be observed during the interglitch recovery phase. A sample of pulsars without observed glitches (Johnston & Galloway 1999) displays mostly positive, along with some negative braking indices.

Baykal et al. (1999) have investigated the time series of pulsars on the longest available time-scales by combining observations of 24 pulsars (Downs & Reichley 1983) with later observations (Siegman, Manchester & Durdin 1993; Arzoumanian, Nice & Taylor 1994) containing available timing data for time-spans of the order of 30 yr for several pulsars. Some of these pulsars were eliminated as candidates for secular timing behaviour, since their frequency time series is not consistent with secular quadratic trends (constant $\ddot{\nu}$). Equivalently, polynomial fits to the ToA of these pulsars require higher order polynomials rather than a cubic polynomial. For these pulsars, the time series is dominated by complicated noise processes rather than interglitch recovery. For four pulsars, PSRs 0823+26, 1706−16, 1749−28 and 2021+51, the time series called for a more careful analysis to determine if there is a secular second derivative. While there are significant quadratic trends in frequency histories (cubics in ToA), these trends arise from the cumulative effect of noise. Baykal et al. (1999) estimated the noise strengths for these four pulsars from the residuals of ToA data. In order to see whether the noise strengths are stable or not and to see whether the quadratic trends in pulse frequency and cubic trends in ToA absorb the noise, they estimated alternative sets of noise strengths by removing quadratic polynomials from the pulse frequency data for the longest time-span of data and cubic polynomials from the ToA data for the shorter intervals. They found that for each source these two power spectra are consistent with each other in terms of average noise strength S_r and slope of the power spectra. This suggested

Downloaded from https://academic.oup.com/mnras/article-abstract/372/1/489/993600 by Middle East Technical University Library user on 23 July 2020

that their original noise estimates were robust (consistent with each other in terms of the noise strength parameter, S_r) and were not dominated by either of the two particular polynomial trends. If there were a secular polynomial trend in the data, one would expect that particular polynomial trend to produce a significantly better fit, i.e. a significantly lower, and different, power spectrum of the residuals, compared to the other polynomial models. All pulsars investigated by Baykal et al. (1999) are old pulsars, with characteristic ages $P/2\dot{P} > 10^7$ yr.

In the technique developed by Johnston & Galloway (1999), the braking index is obtained from ν and $\dot{\nu}$ values. Errors of braking indices depend on the errors of ν and $\dot{\nu}$. Johnston and Galloway applied their methods to 20 pulsars. They found that the braking indices of old pulsars are insignificant because of large error bars. However, pulsars with middle ages have yielded significant braking indices. Due to the sparseness of timing data, power spectrum techniques cannot be applied to these pulsars. All 'middle aged' ($10^5 < \tau < 10^7$ yr) and young pulsars have large spindown rates compared to the spindown rates of old pulsars. Observations of anomalous braking indices suggested that the old pulsars' braking indices are artefacts of timing noise. For the young and middle-aged pulsars, timing noise does not have a strong effect on $\ddot{\nu}$ values. In this work, we take the young and middle-aged pulsars' braking indices to be real and older pulsars' braking indices to be artefacts of timing noise. This is in agreement with the result of Johnston & Galloway (1999), on the basis of the data from 20 pulsars, and with the results of Baykal et al. (1999) for four old pulsars.

3 THE MODEL FOR GLITCHES AND INTERGLITCH DYNAMICS

Extensive timing observations on the Vela pulsar now cover a period of about 35 yr and encompass 14 glitches with post-glitch relaxation and interglitch timing behaviour. A detailed empirical model interprets the glitches and post-glitch–interglitch response in terms of angular momentum exchange between a 'pinned crust superfluid' and the observed crust of the pulsar (Alpar et al. 1984a,b; Alpar, Cheng & Pines 1989; Alpar et al. 1993b). The time t_g between glitches scales as $|\dot{\Omega}|^{-1}$ in this model. The hypothesis that all pulsars experience glitches similar to the Vela pulsar glitches, at rates proportional to the $|\dot{\Omega}|$ of the individual pulsars, is borne out by the statistics of Vela type ($\Delta\Omega/\Omega > 10^{-7}$) glitches from the entire pulsar sample (Alpar & Baykal 1994). The observations of glitches and interglitch measurements of $\ddot{\Omega}$ (Shemar & Lyne 1996; Johnston & Galloway 1999; Lyne et al. 2000; Wang et al. 2000) provide us with many pulsars actually observed in behaviour like the Vela pulsar prototype. Our first task is to demonstrate this similarity in dynamical behaviour. We start with a summary of the model developed for the Vela pulsar. The basic features will be brought forth in a description involving the observed neutron star crust and one interior component, and independent of the microscopic details of the coupling between the two components.

In the absence of evidence that the pulsar electromagnetic torque changes at a glitch, and with the established impossibility of explaining the large ($\Delta\Omega/\Omega > 10^{-7}$) and frequent (intervals $\sim$2 yr) Vela pulsar glitches with starquakes, the glitch is modelled as a sudden angular momentum exchange between the neutron star crust and an interior component,

$$I_c \Delta\Omega_c = I_s \delta\Omega = (I_A/2 + I_B)\,\delta\Omega. \quad (3)$$

Here, $\Delta\Omega_c$ is the observed increase of the crust's rotation rate at the glitch. I_c is the effective moment of inertia of the crust, including all components of the star dynamically coupled to the crust on time-scales shorter than the resolution of the glitch event. The observations imply that I_c includes practically the entire moment of inertia of the star, and the theory of the dynamical coupling mechanisms of the neutron star core (Alpar, Langer & Sauls 1984) provides an understanding of this by furnishing crust-core coupling times shorter than the resolution of glitch observations. The coupling mechanism relies on the simultaneous presence of superfluid neutrons and superconducting protons in the core of the neutron star. Recent arguments that the protons in the core of the neutron star are either normal or in the type I superconductor phase would kill this coupling mechanism for all or some regions of the core neutron superfluid, which carries almost the entire moment of inertia. This would then require another mechanism of short time-scale coupling of the core superfluid neutrons to the effective crust to explain the empirical fact that almost the entire moment of inertia of the neutron star seems to couple to the observed crust rotation on time-scales less than a minute. However, the argument for the absence of type II proton superconductivity is not strictly valid because it rests on the premises that (i) the observed long-term modulation in timing and pulse shapes of the pulsar PSR B 1828−11 is due to precession of the neutron star and not due to some surface or magnetospheric excursion of the magnetic field pattern and (ii) such precession of the observed period and amplitude cannot take place in the presence of pinning. Of these premises, (i) is not necessarily the case and (ii) is not valid because at finite temperature pinning does not give an absolute constraint on precession (Alpar 2005). In the following, we replace I_c with I, the total moment of inertia of the star.

In current models, the sudden transfer of angular momentum is associated with a superfluid in the inner crust of the neutron star, where the rotational dynamics of the superfluid is constrained by the existence of pinning forces exerted by the crust lattice on the superfluid's vortex lines. $\delta\Omega$ describes the decrease in the rotation rate of the pinned superfluid at the glitch. I_A and I_B are the parts of the superfluid's effective moment of inertia I_s associated with different dynamical behaviour. The vortex lines are the discrete carriers of the superfluid's angular momentum. Vortex lines under pinning forces respond to the driving external pulsar torque, as this torque makes the normal crust lattice spindown.

There are two modes of this response. Some vortices will remain pinned until critical conditions matching the maximum available pinning force are reached. Then they will unpin catastrophically and move rapidly in the radially outward direction, thereby transferring angular momentum to the crust only in glitches. The element of the superfluid through which unpinned vortices move rapidly in a glitch, and there is no vortex flow otherwise, has moment of inertia I_B and contributes angular momentum $I_B\delta\Omega$ to the glitch in rotation frequency, as indicated in equation (3). It does not spindown continuously between glitches, rather it spins down only by discrete steps of the angular momentum transfer at glitches, analogous to a capacitor which does not transmit electric current except in discharges. Since it does not contribute to spindown between glitches, it does not contribute to the glitch-induced sudden change in spindown rate.

In other parts of the superfluid, vortices are not pinned all the time, but unpin and repin, at thermally supported rates. I_A is the moment of inertia of those parts of the superfluid that allow a continuous vortex flow, in analogy with the current in a resistive circuit element. In the presence of finite energy barriers, there will always be a continuous current of vortices, in addition to the discrete discharges that we call glitches. This continuous current of vortices moving radially outwards through the inner crust 'vortex creep' makes the superfluid spindown continuously in response to the driving spindown torque

Downloaded from https://academic.oup.com/mnras/article-abstract/372/1/489/993600 by Middle East Technical University Library user on 23 July 2020

on the pulsar. At finite temperature, the motion of these vortices against the pinning energy barriers is made possible by thermal activation. A different possibility, operating even at $T = 0$, is quantum tunnelling. It can easily be shown that if vortices unpinned in a glitch are unpinned at a uniform density throughout the creep regions of moment of inertia I_A, then the angular momentum transfer from these regions to the normal crust is $I_A \delta\Omega/2$, as in the right-hand side of equation (3) (Alpar et al. 1984a,b).

The continuous spindown between glitches is governed by

$$I_c \dot{\Omega}_c = N_{ext} + N_{int} = N_{ext} - I_A \dot{\Omega}_s, \tag{4}$$

where N_{ext} is the external torque on the neutron star and N_{int} is the internal torque coupling the superfluid to the 'effective crust' with moment of inertia $I_c \cong I$.

In a cylindrically symmetric situation, the spindown rate of the superfluid is proportional to the mean vortex velocity in the radial direction, which in turn is determined by the lag $\omega = \Omega - \Omega_c$ between the superfluid and crust rotation rates:

$$\dot{\Omega}_s = -\frac{2\Omega_o}{r} V_r(\omega). \tag{5}$$

As the glitch imposes a sudden change in ω, it will offset the superfluid spindown, and therefore the observed spindown rate of the crust, according to equation (4). The glitch is followed by transient relaxation processes in which the crust rotation frequency and spindown rate relax promptly as an exponential function of time (Alpar et al. 1984a,b). It is the long-term interglitch relaxation of the spindown rate, after the transients are over, that determines the interglitch behaviour of the observed crust spindown rate. Labelling the moment of inertia associated with long-term offset in spindown rate with I_A, from equation (4) we have

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} = \frac{I_A}{I}. \tag{6}$$

We refer the reader to earlier papers (Alpar et al. 1984a,b, 1989) for details. The contribution of the regions I_A to the glitch in the rotation frequency is $I_A \delta\Omega/(2I)$. Together the contributions of the 'resistive' (continuous vortex current) regions A and the 'capacitive' vortex trap (accumulation) regions B give equation (3).

The long-term offset $\Delta\dot{\Omega}/\dot{\Omega}$ is observed to relax as a linear function of time:

$$\frac{\Delta\dot{\Omega}(t)}{\dot{\Omega}} = \frac{I_A}{I}\left(1 - \frac{t}{t_g}\right). \tag{7}$$

The constants in this description of the observed long-term $\Delta\dot{\Omega}(t)$ are labelled following the model for Vela (Alpar et al. 1984b). The time between glitches t_g is the time it takes the spindown rate $\dot{\Omega} = N_{ext}/I$ determined by the external torque to replenish the glitch-induced offset $\delta\Omega$ in ω:

$$t_g = \delta\Omega / |\dot{\Omega}| \tag{8}$$

and

$$\frac{\Delta\Omega}{\Omega} = \left(\beta + \frac{1}{2}\right)\left(\frac{\Delta\dot{\Omega}}{\dot{\Omega}}\right)\frac{\delta\Omega}{\Omega}, \tag{9}$$

where $\beta = I_B/I_A$. Using equations (6)–(9), the long-term second derivative of Ω to be observed between glitches is

$$\ddot{\Omega} = \frac{I_A}{I}\frac{\dot{\Omega}^2}{\delta\Omega} = (\beta + 1/2)(\Delta\dot{\Omega}/\dot{\Omega})_{-3}{}^2/(\Delta\Omega/\Omega)_{-6}(\dot{\Omega}^2/\Omega). \tag{10}$$

This is equivalent to the positive 'anomalous' braking index

$$n = (\beta + 1/2)(\Delta\dot{\Omega}/\dot{\Omega})_{-3}{}^2/(\Delta\Omega/\Omega)_{-6}. \tag{11}$$

The time to the next glitch can be expressed as

$$t_g = 2 \times 10^{-3}(\Delta\Omega/\Omega)_{-6}/[(\beta + 1/2)(\Delta\dot{\Omega}/\dot{\Omega})_{-3}]\tau_{sd}, \tag{12}$$

where $\tau_{sd} = \Omega/(2|\dot{\Omega}|)$ is the characteristic dipole spindown time.

We will show, in the next section, that the 'anomalous' braking index behaviour of older pulsars is consistent with this model, indicating that all pulsars older than Vela experience glitches with $\Delta\Omega/\Omega > 10^{-7}$ and the universal interglitch behaviour described by equations (10) and (12). The hypothesis that all pulsars conform to this glitch behaviour model developed for the Vela pulsar was first applied to Geminga (Alpar et al. 1993b). Its universal application and implications for energy dissipation were introduced by Alpar (1998a,b).

The significance of identifying this universal behaviour is that it implies a lower bound to the lag ω between crust and superfluid: $\omega > \delta\Omega$ since the superfluid's loss of rotation rate at glitches should not overshoot the lag $\omega = \Omega_s - \Omega_c$. This lower bound in turn leads to a lower bound in the energy dissipation rate.

4 ANOMALOUS BRAKING INDICES, GLITCHES AND INTERGLITCH BEHAVIOUR

Braking indices were measured, at various degrees of accuracy as the data permitted, from eight (excluding the Crab and Vela pulsars) out of 18 glitching pulsars studied by Lyne et al. (2000), and from nine (excluding the Vela pulsar) out of 11 glitching southern pulsars studied by Wang et al. (2000). Some of these pulsars are common to both surveys. We exclude the Crab and Vela pulsars in the present work because detailed post-glitch and interglitch data and fits exist for these pulsars; indeed the long-term interglitch behaviour of the Vela pulsar provides the prototype dynamical behaviour that we are searching for in pulsars older than the Vela pulsar. For three pulsars common to both surveys, PSRs J 1341−6220, J 1709−4428 and J 1801−2304, Wang et al. (2000) quote $\ddot{\Omega}$ measurements, while Lyne et al. (2000) quote upper limits to $\ddot{\Omega}$ for two of these pulsars. Thus there are now published $\ddot{\Omega}$ measurements for 14 out of 23 glitching pulsars excluding the Crab and Vela pulsars. We have tabulated 10 of these according to the significance of error bars.

In addition, Johnston & Galloway (1999) have obtained braking indices for 20 pulsars to demonstrate the method they proposed, applying equation (2) to rotation frequency and spindown rate measurements at two different epochs. These pulsars were not known glitching pulsars, and they were not observed to glitch during these observations. Anomalous braking indices were found for all 20 pulsars, with negative values in six pulsars and positive values in the rest. Of the data in the Johnston and Galloway sample, we will take into consideration those data sets for which the quoted errors in the braking index are less than the quoted value, so that there is no ambiguity in the sign of the braking index. With these criteria, we study 18 pulsars, five with negative and 13 with positive braking indices. From two of these pulsars, Johnston and Galloway reported two distinct data sets. Thus our sample contains 20 determinations of the braking index from 18 pulsars. Johnston & Galloway (1999) have interpreted the positive anomalous braking indices as due to interglitch recovery, without evoking a specific model. They interpreted the negative braking indices as reflecting an unresolved glitch during their observation time-spans. All glitches result in long-term decrease of the spindown rate, i.e. a negative step, an increase in the absolute value, of the rate of spindown. Since the pulsars were not monitored continuously, a glitch occurring between two timing observations would lead to a negative $\ddot{\Omega}$ inference, equivalent to a negative braking index.

Downloaded from https://academic.oup.com/mnras/article-abstract/372/1/489/993600 by Middle East Technical University Library user on 23 July 2020

5 BRAKING INDICES OF PULSARS NOT OBSERVED TO GLITCH

We start our analysis with the braking indices measured by Johnston & Galloway (1999) from pulsars that were not observed to glitch, proceeding to the glitching pulsars in the next section. All glitches bring about a sudden negative change $\Delta\dot{\Omega}$ in $\dot{\Omega}$, that is, a fractional increase $\Delta\dot{\Omega}/\dot{\Omega}$ by 10^{-3}–10^{-2} in the spindown rate. If the unresolved glitch happens in a time-span of length t_i, the offset $\Delta\dot{\Omega}$ in the spindown rate will mimic a negative second derivative of the rotation rate, $\ddot{\Omega} = \Delta\dot{\Omega}/t_i$. Let us first elaborate on the statistical analysis of the negative braking index pulsars as those suffering an unobserved glitch during a gap within the time-span of the observations, following the analysis of Johnston & Galloway (1999) and using, as these authors did, the statistical glitch parameters of Alpar & Baykal (1994). The probability that pulsar i has one glitch during the time-span t_i of the observations is given by the Poisson distribution

$$P(1;\lambda_i) = \lambda_i \ \exp(-\lambda_i), \tag{13}$$

where the parameter λ_i is given by

$$\lambda_i = \frac{t_i}{t_{\mathrm{g},i}} \tag{14}$$

and $t_{\mathrm{g},i}$ is the time between glitches for pulsar i. To derive $t_{\mathrm{g},i}$ with equation (8), one needs to know the decrease $\delta\Omega_i$ in superfluid rotation rate at the previous glitch. In this sample of pulsars from which glitches have not been observed, we estimate the value of $\delta\Omega_i$ by making two alternative hypotheses about the constancy of average glitch parameters among pulsars older than the Vela pulsar and equating the parameters to their average values for the Vela pulsar glitches. Under the first hypothesis, $\delta\Omega$ is assumed to be constant for all pulsar glitches, and is set equal to $\langle\delta\Omega\rangle_{\mathrm{Vela}}$, the average value inferred for the Vela pulsar glitches:

$$\delta\Omega_i{}^{(1)} = \langle\delta\Omega\rangle_{\mathrm{Vela}} \tag{15}$$

$$\lambda_i{}^{(1)} = \frac{t_i|\dot{\Omega}|_i}{\langle\delta\Omega\rangle_{\mathrm{Vela}}}. \tag{16}$$

Under the second hypothesis, $\delta\Omega/\Omega$ is assumed to be constant for all glitches of pulsars older than the Vela pulsar. Johnston & Galloway (1999) adopted this hypothesis, taking the value estimated by Alpar & Baykal (1994) from glitch statistics, which agrees with the range of values of $\delta\Omega/\Omega$ inferred for the Vela pulsar glitches:

$$\langle\delta\Omega/\Omega\rangle_i^{(2)} = 1.74\times10^{-4} \tag{17}$$

$$\lambda_i^{(2)} = 5.75\times10^3\frac{t_i|\dot{\Omega}|_i}{\Omega_i} = 2.87\times10^{-3}\frac{t_i}{\tau_{i,6}}. \tag{18}$$

Here t_i is in years and $\tau_{i,6}$ is the dipole spindown age of pulsar i in units of 10^6 yr. Table 1 gives the values of $\lambda_i^{(1)}$ and $\lambda_i^{(2)}$. The corresponding probabilities $P(1;\lambda_i)$ for a (unobserved) glitch to fall within the observation time-span devoted to pulsar i, or, equivalently, pulsar i mimicking a negative second derivative, are quite low for either hypothesis, while the probabilities $P(0;\lambda_i) \cong 1$ for no glitch occurring within the observation time-span of pulsar i, or, equivalently, a positive anomalous braking index being measured for pulsar i. The probability that five out of the 18 pulsars' 20 data sets sampled have had unresolved glitches within the observation time-spans, so that they have negative anomalous second derivatives, is given by

$$P\left(5;\lambda^{(j)}\right) = \left(\lambda^{(j)}\right)^5 \ \exp\left(-\lambda^{(j)}\right)/5!, \tag{19}$$

where

$$\lambda^{(j)} = \sum_{i=1}^{20}\lambda_i{}^{(j)}, \tag{20}$$

for the hypotheses $j = 1, 2$. The index in this runs over all data sets, since two of the eight pulsars have two independent data sets each in the sample of Johnston & Galloway (1999). We find that

$$\lambda^{(1)} = 1.33 \tag{21}$$

$$P\left(5;\lambda^{(1)}\right) = 0.0092 \tag{22}$$

$$\lambda^{(2)} = 3.11 \tag{23}$$

$$P\left(5;\lambda^{(2)}\right) = 0.11. \tag{24}$$

This means that hypothesis 2 is likely to be true since it gives a total expected number of glitches falling within observation time-spans to be 3.11 against the number 5 implied by this interpretation of negative braking indices, as Johnston & Galloway (1999) noted. With hypothesis 1, the expected number of glitches is $\lambda^{(1)} = 1.33$ and 5 glitches within observation time-spans has a lower $P(5;\lambda^{(1)}) = 0.0092$ probability so this hypothesis is not favoured. The same conclusion was reached by Alpar & Baykal (1994) on the basis of statistics of large pulsar glitches: with hypothesis 1, that $\delta\Omega$ is roughly constant in all pulsars older than Vela, the statistics implied $\langle\delta\Omega\rangle = 0.0188$, which does not agree with $\langle\delta\Omega\rangle_{\mathrm{Vela}} = 0.0094$.

In Table 1, the fractional changes in the spindown rate in the five unobserved glitches are given, as inferred from the negative braking indices, by Johnston & Galloway (1999) according to

$$\left(\frac{\Delta\dot{\Omega}_i}{\dot{\Omega}_i}\right)_{\mathrm{missed}} = \frac{\ddot{\Omega}_i t_i}{\dot{\Omega}_i} = \frac{n_i\dot{\nu}_i t_i}{\nu_i}. \tag{25}$$

These values, $\Delta\dot{\Omega}_i/\dot{\Omega}_i \sim 10^{-4}$–$10^{-3}$, are typical for glitching pulsars, all measured values of $\Delta\dot{\Omega}/\dot{\Omega}$ for the Crab and Vela pulsars' large or small glitches are in the 10^{-4}–10^{-3} range. Using these estimated values, and equation (9), we can also estimate $\Delta\Omega/\Omega$ for the missed glitches. We assume that β has similar values, $\beta \sim 0(1)$, in all glitching pulsars. Thus, taking $\beta + 1/2 = 1$,

$$\left(\frac{\Delta\Omega}{\Omega}\right)_{\mathrm{missed}} = \left(\frac{\Delta\dot{\Omega}}{\dot{\Omega}}\right)_{\mathrm{missed}}\left\langle\frac{\delta\Omega}{\Omega}\right\rangle = 1.74\times10^{-4}\left(\frac{\Delta\dot{\Omega}}{\dot{\Omega}}\right)_{\mathrm{missed}}. \tag{26}$$

We tabulate in Table 1 the estimated sizes of the missed glitches $(\Delta\Omega/\Omega)_{\max} \sim (0.2\text{–}7)10^{-7}$, for the five pulsars with negative braking indices. Finally, we can check if glitches of the estimated magnitudes would have been missed in Johnston and Galloway's observations. The minimum glitch magnitude that can be detected through a mismatch of timing fits before and after the glitch is

$$\left(\frac{\Delta\Omega_i}{\Omega_i}\right)_{\mathrm{detectable}} = \frac{\dot{\nu}_i t_i}{\nu_i}, \tag{27}$$

which is of the order of 10^{-6}–10^{-5} for the data sets on these five pulsars. Thus, the interpretation that these negative braking indices indeed reflect undetected glitches is consistent with standard glitch models.

The pulsars having positive braking indices reported by Johnston & Galloway (1999) must have been observed during interglitch relaxation. None of these pulsars has experienced a glitch during the observation time-spans t_i. The values of $\lambda_i^{(1)}$ and $\lambda_i^{(2)}$

© 2006 The Authors. Journal compilation © 2006 RAS, MNRAS **372,** 489–496

Downloaded from https://academic.oup.com/mnras/article-abstract/372/1/489/993600 by Middle East Technical University Library user on 23 July 2020

Table 1. Pulsars with positive or negative braking indices.[a]

PSR B	t (d)	Ω (rad Hz)	$\dot{\Omega}_{17}$ (rad Hz s^{-1})	λ^1 ($\times 10^{-2}$)	λ^2 ($\times 10^{-2}$)	$\frac{\Delta\Omega}{\Omega}$ ($\times 10^{-3}$)	$\frac{\Delta\Omega}{\Omega}_{\rm m}$ ($\times 10^{-7}$)	$\frac{\Delta\dot{\Omega}}{\dot{\Omega}}_{\rm m} = \frac{\Omega t_i}{\Omega}$ ($\times 10^{-5}$)	n
0114+58	2271.1	61.9	−35.3	7.37	6.43	0.11	0.19	1.1	-9.6 ± 1.5
0136+57	4492.0	23.1	−9.1	3.74	8.75	1.2	2.1	1.5	-81 ± 4.7
0154+61	4336.5	2.7	−2.1	0.86	17.11				28 ± 14
0540+23	5543.5	25.5	−16.0	8.16	17.28				11.1 ± 8.6
	5990.5			8.82	18.67				11.81 ± 0.12
0611+22	5541.5	18.8	−33.4	16.98	48.84				20.1 ± 1.1
0656+14	2163.3	16.3	−23.3	4.63	15.34				14.7 ± 1.4
0740−28	4245.2	37.7	−38.0	14.83	21.25				17.7 ± 1.4
	5827.2			20.35	29.17				25.6 ± 0.8
0919+06	4521.7	14.6	−4.6	1.93	7.16				28.9 ± 4.1
1221−63	6661.3	29.0	−6.6	4.06	7.56				18.7 ± 12.3
1719−37	4824.0	26.6	−12.2	5.42	11.02	3.5	6.1	1.9	-183 ± 10
1742−30	1581.0	17.1	−5.0	0.72	2.26	0.52	0.91	0.39	-132 ± 5
1829−08	1541.0	9.7	−9.5	1.34	7.50				2.5 ± 0.9
1907+10	5842.5	22.1	−2.1	1.10	2.70				24 ± 17
1915+13	6080.5	32.3	−11.9	6.67	11.16				36.08 ± 0.48
2000+32	1381.0	9.0	−13.6	1.72	10.30	4.1	7.1	1.8	-226 ± 4.5
2002+31	6076.5	3.0	−1.0	0.58	10.64				23.3 ± 1.0
2148+52	2307.2	18.9	−5.7	1.21	3.45				49.6 ± 3.5
2334+61	2347.1	12.7	−48.8	10.52	44.78				8.6 ± 0.13

[a]Johnston & Galloway (1999).

in Table 1 show that the probabilities $P(0; \lambda_i^{(j)})$ for no glitch occurring within the observation time-span of pulsar i are close to 1 under either hypothesis. The positive interglitch $\ddot{\Omega}$ values of these pulsars are related to the parameters of the previous glitch through equation (10). Using this equation, we obtain the range of β values corresponding to the range of positive braking indices, $n = 2.5$–50 quoted by Johnston and Galloway. Thus we expect $\beta = 2.5$–50, if $\Delta\Omega/\Omega = 10^{-6}$, $\Delta\dot{\Omega}/\dot{\Omega} = 10^{-3}$, while $\beta = 0.25$–5 is obtained if $\Delta\Omega/\Omega = 10^{-7}$ and $\Delta\dot{\Omega}/\dot{\Omega} = 10^{-3}$.

6 PULSARS WITH ANOMALOUS BRAKING INDICES AND OBSERVED GLITCHES

In this section, we discuss the pulsars which have been observed to glitch, and for which observations of anomalous braking indices, which are not noise artefacts, exist. So far, samples of such pulsars have been reported by Lyne et al. (2000) and Wang et al. (2000).

Many of these pulsars have exhibited multiple glitches, of varying magnitudes, from $\Delta\Omega/\Omega \sim 10^{-9}$ to $\Delta\Omega/\Omega \sim 10^{-6}$. Reported $\ddot{\Omega}$ measurements are both negative and positive. Quoted errors in $\ddot{\Omega}$ are typically very large, especially among the negative $\ddot{\Omega}$ values reported. There is only one instance of a negative $\ddot{\Omega}$ with low error, among the glitching pulsars reported by Wang et al. (2000), $\ddot{\Omega} = -1.2318 \pm 0.019 \times 10^{-25}$ rad Hz s^{-1} in one particular epoch of observations for PSR J 1614−5047. The epoch of this measurement does not coincide with the only data set containing a glitch from this pulsar. We select from the data reported by Wang et al. (2000) and Lyne et al. (2000) all those glitches with $\Delta\Omega/\Omega \geqslant 10^{-7}$. Among the 10 large glitches, with $\ddot{\Omega}$ measurements at or immediately following the glitch, eight glitches have positive $\ddot{\Omega}$ measurements. The two large glitches with subsequent negative second derivative measurements are from PSR J 1105−6107, with $\ddot{\Omega} = -3.078 \pm 0.314 \times 10^{-26}$ rad Hz s^{-2}, and PSR J 1801−2451, with $\ddot{\Omega} = -8.796 \pm 3.769 \times 10^{-26}$ rad Hz s^{-2}. As has been observed from the Vela pulsar, in post-glitch relaxation after a large glitch, smaller glitches, with $\Delta\Omega/\Omega \sim 10^{-9}$ can sometimes occur. There is a possibility that the post-glitch data set following these two glitches contains unresolved small glitches, $\Delta\Omega/\Omega \leqslant 10^{-9}$, which determines the second derivative, and makes comparison with the model impossible. We therefore include only the eight large glitches with measured positive post-glitch frequency second derivatives. Observed values of $\Delta\Omega/\Omega$, $\Delta\dot{\Omega}/\dot{\Omega}$ and $\ddot{\Omega}$ are given in Table 2. We evaluate these quantities in terms of the 'standard' interglitch response model given in equations (6)–(12). The extracted values of β, $\delta\Omega$, $\delta\Omega/\Omega$ and $t_{\rm g}$ are also given in Table 2. The values of β derived here are comparable to β values inferred from model fits to the interglitch relaxation of the Vela pulsar with an exception for PSR 1709−4428. The $\delta\Omega$ values vary between 0.057×10^{-2} and 1.48×10^{-2}, while $\delta\Omega/\Omega$ variation is less limited; the $\delta\Omega/\Omega$ values are similar to the values inferred for the sample of negative braking index pulsars (Table 1), and also to $\langle\delta\Omega/\Omega\rangle \cong 1.74 \times 10^{-4}$ inferred earlier from statistics.

7 DISCUSSION

We find that in glitching pulsars with measured braking indices, in the current sample, all pulsars exhibit positive second derivatives corresponding to interglitch recovery with model parameters similar to those obtained in detailed fits to interglitch behaviour of the Vela pulsar with the vortex creep model. This extends similar conclusions already reported on the basis of earlier, limited data.

The main uncertainty in comparing these glitching pulsars with the model lies in the interpretation of the observed jumps $\Delta\dot{\Omega}/\dot{\Omega}$ in spindown rate. These glitch observations do not resolve the glitch occurrence time or the time dependence of $\Delta\dot{\Omega}$. Thus the quoted $\Delta\dot{\Omega}/\dot{\Omega}$ values may contain contributions from transients. The

Downloaded from https://academic.oup.com/mnras/article-abstract/372/1/489/993600 by Middle East Technical University Library user on 23 July 2020

Table 2. Observed parameters of glitching pulsars.

PSR	Ω (rad Hz)	$\lvert\dot{\Omega}_{-11}\rvert$ (rad Hz s^{-1})	$\ddot{\Omega}_{-22}$ (rad Hz s^{-2})	$\left(\frac{\Delta\Omega}{\Omega}\right)_{-6}$	$\left(\frac{\Delta\dot{\Omega}}{\dot{\Omega}}\right)_{-3}$	β	t_g (d)	$\delta\Omega_{-2}$	$\left(\frac{\delta\Omega}{\Omega}\right)_{-4}$
1048−583[a]	50.8	3.95	9.22	2.995	3.7	6.07	1834	0.626	1.23
			24.5	0.771	4.62	2.37	865	0.296	0.582
1341−6230[a]	32.5	4.25	11.93	0.99	0.7	42.87	288	0.106	0.326
			16.96	1.636	3.3	4.08	958	0.352	1.082
1614−5047[a]	27.1	5.8	21.99	6.456	9.7	0.72	2950	1.48	5.576
1709−4428[a]	61.32	5.57	10.87	2.012	0.2	1080.16	119	0.057	0.09
1730−3350[b]	45.2	2.76	6.28	3.0	12.	0.27	6154.1	1.467	3.24
1740−3015[b]	10.68	0.79	6.59	0.4	3.	4.51	416.4	0.028	0.26
			3.95	0.6	2.	9.63	936.9	0.063	0.58
1803−2137[b]	47.12	4.77	8.04	4.0	9.2	0.28	6373.1	2.62	5.56
1801−2451[a]	50.31	5.15	25.07	1.998	4.85	3.54	1153	0.51	1.02
1803−2137[a]	47.	4.7	18.00	3.2	10.7	1.16	2894	1.17	2.5

[a]Wang et al. (2000); [b]Lyne et al. (2000).

second derivatives characteristic of interglitch recovery are linked to only the long-term offset in $\Delta\dot{\Omega}/\dot{\Omega}$, after the transients are over. The transients and long-term contributions to $\Delta\dot{\Omega}/\dot{\Omega}$ are comparable in the Vela pulsar. Thus, this uncertainty introduces errors in β estimates by factors of the order of 1.

We have also explored Johnston and Galloway's measurements of positive and negative anomalous braking indices from a sample of pulsars which were not observed to glitch. These authors suggested that negative braking indices are due to the negative $\Delta\dot{\Omega}$ signs of unresolved glitches, while positive braking indices correspond to interglitch recovery. We have applied these suggestions specifically in the context of the phenomenology of Vela pulsar glitches and interglitch recovery. The glitch model parameters are once again in agreement with parameters obtained for the Vela pulsar.

Thus, on the basis of data from all pulsars with measured reliable secular anomalous braking indices, including both glitching pulsars and those without observed glitches, we conclude that pulsars older than the Vela pulsar experience glitches which are similar to the Vela pulsar's glitches. The interval between glitches is

$$t_g = \frac{\delta\Omega}{\Omega}\frac{\Omega}{|\dot{\Omega}|} \cong 2\left\langle\frac{\delta\Omega}{\Omega}\right\rangle\tau_{sd} \cong 3.5\times10^{-4}\tau_{sd}. \quad (28)$$

The last equality is on the basis of the strong indication, both from the analysis of the statistics of all large glitches (Alpar & Baykal 1994) and also from the analysis in this paper of the specific samples of pulsars with anomalous braking indices.

A particularly interesting implication of the universality of glitch behaviour is the provision of a lower limit to the rate of energy dissipation due to vortex creep in neutron stars. As developed first by Alpar et al. (1984b), this energy dissipation rate is

$$\dot{E}_{diss} = I_p\,\omega|\dot{\Omega}|, \quad (29)$$

where $I_p \cong 10^{43}$ gm cm^2 is the moment of inertia of the pinned inner crust superfluid where vortex creep takes place and ω is the lag in the rotation rates between this inner crust superfluid and the observed outer crust. This expression is actually quite model independent. Upper limits on $\dot{E}_{diss}$ are obtained from observations of thermal X-ray emission from PSR B 1929+10 (Alpar et al. 1987; Slowikowska, Kuiper & Hermsen 2005) and PSR B 0950+58 (Becker et al. 2004; Zavlin & Pavlov 2004). The glitch-related decrease in the rotation rate of the superfluid, $\delta\Omega$, provides a lower limit in the energy dissipation rate, since $\delta\Omega < \omega$:

$$\dot{E}_{diss} = I_p\omega|\dot{\Omega}| > I_p\delta\Omega|\dot{\Omega}| \cong \frac{I_p}{I}\left\langle\frac{\delta\Omega}{\Omega}\right\rangle I\Omega\dot{\Omega} \cong 1.7\times10^{-6}\dot{E}_{rot}, \quad (30)$$

taking the moment of inertia ratio $I_p/I = 10^{-2}$ and $\langle\delta\Omega/\Omega\rangle = 1.74\times10^{-4}$.

Neutron stars older than a few 10^6 yr will have cooled to luminosities below $\sim10^{31}$ erg s^{-1}. The neutron star is then kept reheated by energy dissipation. Thus $\dot{E}_{diss}$ is actually a lower limit to the thermal luminosity of an old neutron star. The corresponding lower limit to the surface blackbody temperature of the neutron star is

$$T_s \geqslant 2.2\times10^{-4}\dot{E}_{rot}^{1/4}R_6^{-1/2}. \quad (31)$$

For a radio pulsar spinning down as a pure dipole, extrapolating with the parameters of the Vela pulsar,

$$\dot{E}_{rot} = 8.6\times10^{32}I_{45}t_6^{-2}, \quad (32)$$

where t_6 is the age in 10^6 yr. Thus the lower limit becomes

$$L_{th} \cong \dot{E}_{diss} \geqslant 1.5\times10^{27}I_{45}t_6^{-2}, \quad (33)$$

$$T_s \geqslant 3.8\times10^4 I_{45}^{1/4}R_6^{-1/2}t_6^{-1/2}. \quad (34)$$

Unfortunately, this limit on the blackbody temperature is in the *UV* band at an age of 10^7 yr. If the actual energy dissipation rate is close to the upper limits applied by the PSRs B 1929+10 and B 0950+58, we have

$$L_{th} \leqslant 1.2\times10^{30}(I_p\omega)_{43}t_6^{-3/2}, \quad (35)$$

$$T_s \leqslant 2.0\times10^5(I_p\omega)_{43}^{1/4}R_6^{-1/2}t_6^{-3/8}. \quad (36)$$

If a neutron star is spinning down under a more constant torque like for instance a propeller torque from a fallback disc, and if such spindown extends beyond the few 10^6 yr of the initial cooling era, the luminosity and surface temperature sustained by energy dissipation, according to equations (32) and (33), might be observable.

ACKNOWLEDGMENTS

We thank the anonymous referee for his/her comments that led to clarification of several points. MAA thanks the Turkish Academy of Sciences, and we thank the Sabancı University Astrophysics and Space Forum for research support.

© 2006 The Authors. Journal compilation © 2006 RAS, MNRAS **372,** 489–496

Downloaded from https://academic.oup.com/mnras/article-abstract/372/1/489/993600 by Middle East Technical University Library user on 23 July 2020

REFERENCES

Alpar M. A., 1998a, in Buccheri R., van Paradijs J., Alpar M. A., eds, Proc. NATO ASI, The Many Faces of Neutron Stars. Kluwer, Dordrecht, p. 59

Alpar M. A., 1998b, Adv. Space Res., 21, 159

Alpar M. A., 2005, in Baykal A., Yerli S. K., Inam S. C., Grebenev S., eds, Proc. NATO ASI, The Electromagnetic Spectrum of Neutron Stars, p. 33

Alpar M. A., Baykal A., 1994, MNRAS, 269, 849

Alpar M. A., Anderson P. W., Pines D., Shaham J., 1984a, ApJ, 276, 325

Alpar M. A., Anderson P. W., Pines D., Shaham J., 1984b, ApJ, 278, 791

Alpar M. A., Langer S. A., Sauls J. A., 1984, ApJ, 282, 533

Alpar M. A., Nandkumar R., Pines D., 1986, ApJ, 311, 197

Alpar M. A., Brinkman W., Ögelman H., Kızıloglu Ü., Pines D., 1987, A&A, 177, 101

Alpar M. A., Cheng K. S., Pines D., 1989, ApJ, 346, 823

Alpar M. A., Chau H. F., Cheng K. S., Pines D., 1993a, ApJ, 409, 345

Alpar M. A., Ögelman H., Shaham J., 1993b, A&A, 273, L35

Arzoumanian Z., Nice D. J., Taylor J. H., 1994, ApJ, 422, 671

Baykal A., Alpar M. A., Boynton P. E., Deeter J. E., 1999, MNRAS, 306, 207

Becker W., Weisskopf M. C., Tennant A. F., Jessner A., Dyks J., Harding A. K., Shuang N. Z., 2004, ApJ, 615, 908

Boynton P. E., Groth E. J., Hutchingson D. P., Nanos G. P., Partridge R. B., Wilkinson D. T., 1972, ApJ, 175, 217

Camilo F., Kaspi V. M., Lyne A. G., Manchester R. N., Bell J. F., D'Amico N., McKay N. P. F., Crawford F., 2000, ApJ, 541, 367

Cheng K. S., 1987a, ApJ, 321, 799

Cheng K. S., 1987b, ApJ, 321, 803

Cheng K. S., 1989, in Ögelman H., van den Heuvel E. P. J., eds, NATO ASI Ser. Vol. 262, Timing Neutron Stars. Kluwer, Dordrecht, p. 503

Cordes J. M., 1980, ApJ, 237, 216

Cordes J. M., Downs G. S., 1985, ApJS, 59, 343

Cordes J. M., Helfand D. J., 1980, ApJ, 239, 640

D'Alessandro F., McCulloch P. M., Hamilton P. A., Deshpande A. A., 1995, MNRAS, 277, 1033

Deshpande A. A., D'Alessandro F., McCulloch P. M., 1996, JA&A, 17, 7

Downs G. S., Reichley P. E., 1983, ApJS, 53, 169

Gouiffes C., Finley J. P., Ögelman H., 1992, ApJ, 394, 581

Groth E. J., 1975, ApJS, 29, 443

Gullahorn G. E., Rankin J. M., 1982, ApJ, 260, 520

Johnston S., Galloway D., 1999, MNRAS, 306, L50

Jones P. B., 1990, MNRAS, 246, 364

Kaspi V. M., Manchester R., Siegman B., Johnston S., Lyne A. G., 1994, ApJ, 422, L83

Livingstone M. A., Kaspi V. M., Gotthelf E. V., Kuiper L. 2006, ApJ, 647, 1286

Lyne A. G., Pritchard R. S., Smith F. G., 1988, MNRAS, 223, 667

Lyne A. G., Pritchard R. S., Smith F. G., 1993, MNRAS, 265, 1003

Lyne A. G., Pritchard R. S., Smith F. G., Camilo F., 1996, Nat, 381, 497

Lyne A. G., Shemar S. L., Smith F. G., 2000, MNRAS, 315, 534

Manchester R. N., Peterson B. A., 1989, ApJ, 342, L23

Nagase F., Deeter J., Lewis W., Dotani T., Makino F., Mitsuda K., 1990, ApJ, 351, L13

Shemar S. L., Lyne A. G., 1996, MNRAS, 282, 677

Siegman B. C., Manchester R. N., Durdin J. M., 1993, MNRAS, 262, 449

Slowikowska A., Kuiper L., Hermsen W., 2005, A&A 434, 1097

Wang N., Manchester R. N., Pace R. T., Bailes M., Kaspi V. M., Stappers B. W., Lyne A. G., 2000, MNRAS, 317, 843

Zavlin V. E., Pavlov G. G., 2004, ApJ 616, 452

This paper has been typeset from a TEX/LATEX file prepared by the author.

Downloaded from https://academic.oup.com/mnras/article-abstract/372/1/489/993600 by Middle East Technical University Library user on 23 July 2020