AN ANALYSIS OF NINTH GRADE MATHEMATICS MODELING ACTIVITY BOOK BASED ON MODEL-ELICITING ACTIVITY DESIGN PRINCIPLES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ESMA NUR ÇELİKDİN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MATHEMATICS EDUCATION IN MATHEMATICS AND SCIENCE EDUCATION

SEPTEMBER 2023

Approval of the thesis:

AN ANALYSIS OF NINTH GRADE MATHEMATICS MODELING ACTIVITY BOOK BASED ON MODEL-ELICITING ACTIVITY DESIGN PRINCIPLES

submitted by **ESMA NUR ÇELİKDİN** in partial fulfillment of the requirements for the degree of **Master of Science** i**n Mathematics Education in Mathematics and Science Education, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** _____

Prof. Dr. Mine Işıksal Bostan
Head of the Department, **Math. and Sci. Edu.** _____

Assoc. Prof. Dr. Bülent Çetinkaya
Supervisor, **Math. and Sci. Edu., METU** _____

Examining Committee Members:

Prof. Dr. Ayhan Kürşat Erbaş
Math. and Sci. Edu., METU _____

Assoc. Prof. Dr. Bülent Çetinkaya
Math. and Sci. Edu., METU _____

Assoc. Prof. Dr. Yasemin Sağlam Kaya
Math. and Sci. Edu., Hacettepe University _____

Date: 11.09.2023

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name Last name: Esma Nur Çelikdin

Signature:

ABSTRACT

AN ANALYSIS OF NINTH-GRADE MATHEMATICAL MODELING ACTIVITY BOOK BASED ON MODEL-ELICITING ACTIVITY DESIGN PRINCIPLES

Çelikdin, Esma Nur
Master of Science, Mathematics Education in Mathematics and Science Education
Supervisor : Assoc. Prof. Dr. Bülent Çetinkaya

September 2023, 141 pages

In recent years, with the development of science and technology, mathematical modeling, which connects mathematics to real life, has increasingly gained importance in the teaching and learning of mathematics. Thus, modeling activities began to be used as a tool for students to learn mathematics. However, to effectively teach and learn mathematics through modeling activities, these activities must have certain characteristics. Lesh et al. (2003) outlined six fundamental principles for designing modeling activities: model construction, reality, model generalization, simple prototype, model externalization, and self-evaluation.

This study aimed to examine the ninth-grade mathematics modeling activity book published by the Ministry of National Education [MoNE] (2020a) concerning the compatibility of modeling activities with Lesh et al.'s (2003). The document analysis method was used to collect data, and the content analysis method was used to analyze data.

Results indicated that the modeling activities were generally compatible with the MEA design principles. Analyses showed that the activities were below the medium level regarding only the model generalization principle. Other principles were above

the medium level, with the model construction, reality, and simple prototype principles close to a high level, while the model externalization and self-evaluation principles were slightly above the medium level. Consequently, the modeling activities in the ninth-grade mathematics modeling activity book generally met the design principles of MEA above the medium level. The results of the study provide a framework for curriculum developers, textbook authors, and teachers in the field of mathematical modeling.

Keywords: Modeling, Mathematical Modeling, Model-Eliciting Activities, Model-Eliciting Activity Design Principles, Models and Modeling Perspective

ÖZ

DOKUZUNCU SINIF MATEMATİK MODELLEME ETKİNLİK KİTABININ MODEL OLUŞTURMA ETKİNLİĞİ TASARLAMA PRENSİPLERİNE GÖRE ANALİZİ

Çelikdin, Esma Nur
Yüksek Lisans, Matematik Eğitimi, Matematik ve Fen Bilimleri Eğitimi
Tez Yöneticisi: Doç. Dr. Bülent Çetinkaya

Eylül 2023, 141 sayfa

Son yıllarda bilim ve teknolojinin gelişmesiyle birlikte gerçek hayat ile matematiği birbirine bağlayan matematiksel modelleme, matematiğin öğretilmesi ve öğrenilmesinde giderek önem kazanmıştır. Böylece modelleme etkinlikleri öğrencilerin matematik öğrenmeleri için bir araç olarak kullanılmaya başlandı. Ancak modelleme etkinlikleri yoluyla matematiği etkili bir şekilde öğretmek ve öğrenmek için bu etkinliklerin belirli özelliklere sahip olması gerekir. Lesh ve diğerleri (2003) modelleme etkinliklerini tasarlamak için altı temel prensibi özetlemiştir: model oluşturma, gerçeklik, model genelleştirme, basit prototip, model dışsallaştırma ve öz değerlendirme.

Bu çalışma, modelleme etkinliklerinin Lesh ve diğerleri (2003) tarafından önerilen tasarım prensipleriyle uyumluluğuna ilişkin Millî Eğitim Bakanlığı'nın [MEB] yayınladığı dokuzuncu sınıf matematik modelleme etkinlik kitabını (2020a) incelemeyi amaçlamıştır. Veri toplamak için doküman analizi ve verileri analiz etmek için içerik analizi yöntemi kullanılmıştır.

Sonuçlar, modelleme etkinliklerinin genel olarak MOE tasarlama prensipleriyle uyumlu olduğunu göstermiştir. Yapılan analizler etkinliklerin yalnızca model genelleştirme prensibi açısından orta düzeyin altında olduğunu ortaya koymaktadır. Diğer prensipler orta düzeyin üzerindedir; model oluşturma, gerçeklik ve basit prototip prensipleri yüksek seviyeye yakınken, model dışsallaştırma ve öz değerlendirme prensipleri ortalamanın biraz üzerindedir. Sonuç olarak, dokuzuncu sınıf matematik modelleme etkinlik kitabındaki modelleme etkinliklerinin çoğunlukla MOE tasarlama prensiplerini orta düzeyin üzerinde sağladığı görülmektedir. Bu çalışmanın sonuçları müfredat geliştiricilerine, kitap yazarlara ve öğretmenlere matematiksel modelleme alanında bir çerçeve sunmaktadır.

Anahtar Kelimeler: Modelleme, Matematiksel Modelleme, Model Oluşturma Etkinlikleri, Model Oluşturma Etkinlikleri Tasarlama Prensipleri, Model ve Modelleme Perspektifi

To My Parents

ACKNOWLEDGMENTS

First of all, I would like to express my deepest gratitude to my supervisor, Assoc. Prof. Dr. Bülent Çetinkaya for his guidance, support, motivation, encouragement, time, effort, motivation, and understanding throughout my thesis study.

I would also like to thank dear committee members Prof. Dr. Ayhan Kürşat Erbaş and Assoc. Prof. Dr. Yasemin Sağlam Kaya for their time, valuable feedback, and suggestions. Thanks to these, I had the opportunity to do my work better and improve my thesis.

I would like to express my deepest and greatest thanks to my family members, who are my father, Ahmet; my mother, Hanife; my brother Nursun and his wife, Buse and my lovely nephew İnci Nur for their lifelong support, continuous encouragement, unconditional love, and moral in any condition.

I am also grateful to my sweet cat, Lila Balım, who was always by my side while writing the thesis, sometimes lying on my computer and asking me to pay attention to her instead of the thesis.

I would like to thank all my friends for their valuable friendship, motivation, support, and encouragement. During this process, they provided full support and always believed and encouraged me to finish.

Last but not least, my deepest gratitude goes to my boyfriend, Alperen İnci. His love and support gave me strength and motivation to achieve this goal. I sincerely thank him for always being there for me whenever I need him.

TABLE OF CONTENTS

ABSTRACT v

ÖZ vii

ACKNOWLEDGMENTS x

TABLE OF CONTENTS xi

LIST OF TABLES xiv

LIST OF FIGURES xvi

LIST OF ABBREVIATIONS xvii

CHAPTERS

1 INTRODUCTION 1

1.1 The Rationale for the Study 5

1.2 Purpose of the Study and Research Question 6

1.3 Significance of the Study 7

1.4 Definitions of Terms 9

2 LITERATURE REVIEW 11

2.1 Theoretical Framework 11

2.1.1 Model and Modeling 11

2.1.2 Modeling in Mathematics Education 13

2.1.2.1 Modeling in Mathematics Textbooks 16

2.1.3 Different Mathematical Modeling Approaches and Perspectives 18

2.1.4 Model-Eliciting Activities in MMP 25

2.1.5 Model-Eliciting Activities Design Principles 26

2.2 Previous Research Studies on the Analysis of Mathematical Modeling Activities in Teaching Materials and Textbooks 37
2.2.1 Previous Research Studies in Turkey 37
2.2.2 Previous Research Studies in Other Countries 42
2.3 Summary of the Literature Review 45
3 METHODOLOGY 47
3.1 Design of the Study 47
3.2 Data Set 47
3.3 Data Analysis 52
3.4 Sample Coding from the Study 58
3.5 Validity and Reliability 64
4 RESULTS 67
4.1 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Design Principles of MEA 67
4.1.1 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Model Construction Principle 73
4.1.2 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Reality Principle 78
4.1.3 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Model Generalization Principle 83
4.1.4 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Simple Prototype Principle 87
4.1.5 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Model Externalization Principle 92
4.1.6 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Self-Evaluation Principle 96

5 DISCUSSION, CONCLUSION AND IMPLICATIONS 103

5.1 Limitations and Recommendations..... 106

REFERENCES 111

APPENDICES

A. Criteria for Compatibility with the MEA Design Principles 125

B. Mathematical Modeling Activities in the Ninth-Grade Mathematics Modeling Activity Book 127

LIST OF TABLES

TABLES

Table 1. The Distribution of the Modeling Activities According to Ninth Grade Mathematics Topics 48
Table 2. Sub-criterion of MEA design principles 54
Table 3. Scoring criteria for the level of compatibility with MEA design principles 58
Table 4. Evaluation of the activity 13a in terms of the criteria of MEA design principles 61
Table 5. General Evaluation of Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Design Principles of MEA 67
Table 6. The Compatibility Level of Activities in the 9th Grade Mathematics Modeling Activity Book with the MEA Design Principles and their Distribution According to Units 69
Table 7. The Compatibility Level of Activities in the 9th Grade Mathematics Modeling Activity Book with the MEA Design Principles According to the Units 71
Table 8. The Compatibility Level of Activities in the 9th Grade Mathematics Modeling Activity Book with the Model Construction Principle 73
Table 9. Evaluation of Activity 13a in terms of the Model Construction Principle 76
Table 10. Evaluation of Activity 38 in terms of the Model Construction Principle 77
Table 11. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Reality Principle 79
Table 12. Evaluation of Activity 13a in terms of the Reality Principle 82
Table 13. Evaluation of Activity 12 in terms of the Reality Principle 83
Table 14. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Model Generalization Principle 84
Table 15. Evaluation of Activity 15 in terms of the Model Generalization Principle 86

Table 16. Evaluation of Activity 11 in terms of the Model Generalization Principle 87
Table 17. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Simple Prototype Principle 87
Table 18. Evaluation of Activity 13a in terms of the Simple Prototype Principle . 90
Table 19. Evaluation of Activity 11 in terms of the Simple Prototype Principle ... 91
Table 20. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Model Externalization Principle 92
Table 21. Evaluation of Activity 12 in terms of the Model Externalization Principle 95
Table 22. Evaluation of Activity 38 in terms of the Model Externalization Principle 95
Table 23. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Self-Evaluation Principle 96
Table 24. Evaluation of Activity 15 in terms of Self-Evaluation Principle 99
Table 25. Evaluation of Activity 38 in terms of Self-Evaluation Principle 100
Table 26. General Evaluation of Compatibility Level of Activities with all Sub-criteria of MEA Design Principles 100
Table 27. Criteria for compatibility with the design principles of MEA in the native language (Doruk, 2019) 125

LIST OF FIGURES

FIGURES

Figure 1. Book Cover of Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a) 49
Figure 2. Table of Contents of the Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a, p. 1) 50
Figure 3. Sample Solution Approaches of the Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a, p. 2) 50
Figure 4. Sample Modeling Activity from the Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a, p. 3) 51
Figure 5. Sample Solution Approach from the Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a, p. 66) 52
Figure 6. Activity 13a from the textbook (MoNE, 2020a, p.17) 59
Figure 7. The English Version of the Activity 13a (MoNE, 2020a) 60
Figure 8. Activity 11 from the textbook (MoNE, 2020a, p. 15) 127
Figure 9. Activity 12 from the textbook (MoNE, 2020a, p. 16) 131
Figure 10. Activity 15 from the textbook (MoNE, 2020a, p. 20) 133
Figure 11. Activity 15 from the textbook (cont'd) (MoNE, 2020a, p. 21) 134
Figure 12. Activity 38 from the textbook (MoNE, 2020a, p. 51) 137
Figure 13. Activity 38 from the textbook (cont'd) (MoNE, 2020a, p. 52) 138

LIST OF ABBREVIATIONS

ABBREVIATIONS

ATD: Anthropological Theory of Didactics

CCSSI: Common Core State Standards Initiative

CCSSM: Common Core State Standards for Mathematics

DFE: Department for Education

MCP: The Model Construction Principle

MEA: Model Eliciting Activities

MEB: Milli Eğitim Bakanlığı

MEP: The Model Externalization Principle

MGP: The Model Generalization Principle

MMP: Models and Modeling Perspective

MOE: Model Oluşturma Etkinlikleri

MoNE: Ministry of National Education

NBE: National Board of Education

NCTM: National Council of Teachers of Mathematics

RME: Realistic Mathematics Education

RP: The Reality Principle

SEP: The Self-Evaluation Principle

SPP: The Simple Prototype Principle

CHAPTER 1

INTRODUCTION

The rapid advancements and progress in science and technology have directly impacted people's lives and resulted in shifts in countries' expectations of their citizens. The emphasis now lies not only on possessing knowledge but also on the ability to apply that knowledge effectively to solve problems. This shift is considered vital for keeping pace with the constantly evolving world. Consequently, a country's education policy and teaching approach play a crucial role in training its citizens to acquire such competence.

Adjustments to the curriculum were also made to accommodate the changes in science and technology. As a result of these educational developments, significant innovations have emerged in mathematics teaching. These innovations aim to foster students' mathematical skills, enhance problem-solving abilities, and improve analytical thinking. Within this context, mathematical modeling plays a crucial role in helping individuals acquire mathematical proficiency by tackling challenges encountered during problem-solving. Given the growing importance of innovations in mathematics teaching and their relevance to individuals' lives, there has been a parallel increase in studies focusing on mathematical modeling.

Teaching mathematics, which is directly relevant to everyday life, solely as a collection of abstract concepts and a set of memorized rules in schools, negatively impacts students' interest in and performance within the subject (Baki, 2014). Consequently, students tend to approach mathematical problems mechanically, producing solutions without fully comprehending the underlying process. However, in mathematics education, one of the fundamental goals is to improve students to use their mathematical knowledge and skills while solving real-life problems (Blum & Leiβ, 2007). Mathematical modeling holds an essential role in helping students to

perceive mathematics as important and necessary by allowing them to apply it to address real life situations through modeling (MoNE, 2005). With the help of mathematical modeling, students not only acquire a deeper understanding of mathematical concepts but also gain insights into how this knowledge can be used effectively in their daily lives. Therefore, it is important to incorporate real-life problems into mathematics lessons, providing students with opportunities to gain experience in tackling such issues (Kaiser, 2005).

The Common Core State Standards for Mathematics [CCSSM] (2010) states that modeling with mathematics is one of the standards. According to the National Council of Teachers of Mathematics [NCTM] (2000), modeling is at the heart of mathematics, and problem-solving is integral to mathematics learning. Also, in the mathematics curriculum in Turkey, the general aims are to help students solve real-life problems and relate mathematics and other disciplines (Ministry of National Education [MoNE], 2013a). In this sense, mathematical modeling activities can help the development of students' problem-solving and analytical thinking abilities (Lesh & Doerr, 2003a) and enable students to internalize the fundamental mathematical principles underlying real-life problems (NCTM, 2000). Mathematical modeling activities not only promote students to construct mathematical models to solve the real life complex problems, but they also enable researchers to examine students' mathematical thinking (NCTM, 2000).

Since teaching mathematics through modeling gains more importance for students' mathematical development, conducting research on modeling in this field has a crucial role in mathematics education. Also, in order for the model to have mathematical significance, it should be focused on the basic structural features of modeling activities (Lesh et al., 2003). In Turkey, the literature has revealed that modeling activities are rarely used in school mathematics lessons despite the revisions made to the mathematics curriculum (Tekin Dede & Bukova Güzel, 2013a). One of the most important reasons for this is that the modeling activities in textbooks are quantitatively sufficient but qualitatively insufficient (Urhan & Dost,

2017). Therefore, when examining modeling activities, investigating the basic features of the modeling activities is important.

The "Mathematical Models and Modeling Perspective (MMP)," proposed by Lesh and Doerr (2003a), is a theoretical approach in the field of mathematics education that offers comprehensive explanations for learning, teaching, and problem-solving. MMP provides explanations in mathematics education concerning (i) the nature of reality, (ii) the nature of mathematical knowledge, (iii) the nature of knowledge development in students, (iv) how this knowledge development can be facilitated, (v) the role of context and its relationship to generalization in knowledge development, (vi) the nature of problem-solving, and (vii) the nature of teaching and teacher development (Lesh & Doerr, 2003a, 2003b).

MMP presents a different perspective on problem-solving in mathematics education. The traditional problem-solving process involves providing students with specific information and expecting them to reach a predetermined outcome. It often involves problems isolated from real life or with artificial constructs, requiring students to follow particular solution strategies and steps (Erbaş, et al., 2014; Lesh & Zawojewski, 2007). However, according to MMP, problem-solving should entail non-routine situations involving complex real-life contexts and be open to different thinking approaches and solution methods (Zawojewski & Lesh, 2003). According to MMP, the primary aim is to assist students in developing conceptual models that enable them to interpret and solve real problem situations they encounter in mathematics education. Therefore, problem-solving activities in the classroom should provide students with environments conducive to creating and refining conceptual models. MMP uses the term "model-eliciting activities (MEA)" to refer to problems that engage students in authentic problem-solving experiences.

MEAs are problem-solving activities that employ conceptual tools, which can be shared, adapted, and reused to construct, estimate, explain, and control mathematically significant systems (Lesh & Doerr, 2003a). Numerous studies have demonstrated that MEAs offer significant benefits, enabling students to develop

problem-solving skills, analytical thinking, analysis, and conceptual understanding of fundamental mathematical concepts within real-life problems (Chamberlin & Moon, 2005; Lesh & Zawojewski, 2007). By working on MEAs, students naturally explore mathematical concepts linked to real-life problems and develop models (Erbaş et al., 2014). The successful utilization of MEAs in classes demonstrates their suitability for inclusion in the mathematics curriculum.

Lesh et al. (2003) proposed six design principles for creating productive MEAs: (i) the model construction principle, (ii) the reality principle, (iii) the model generalization principle, (iv) the simple prototype principle, (v) the model externalization principle, and (vi) the self-evaluation principle.

The model construction principle requires developing a model to solve the problem (Chamberlin & Moon, 2005). This principle indicates that the modeling activity task ensures students clearly recognize in which situations they need to construct, or modify a model (Lesh et al., 2000; Lesh & Caylor, 2007).

The reality principle requires that students can make sense of the problem situation using their personal knowledge and experience (Lesh et al., 2000). This principle indicates that the problem situations should be designed using real or realistic data and reflect students' real lives (Lesh et al., 2000; Lesh & Caylor, 2007).

The model generalization principle indicates that the model should not only be useful and applicable to a particular situation but also be used for similar situations, shareable, reusable, or modifiable with other people for different purposes (Chamberlin & Moon, 2005; Lesh et al., 2000; Lesh & Caylor, 2007).

The simple prototype principle indicates that the model should be as simple as possible and memorable for students even after a long time when they encounter structurally similar situations (Chan, 2008; Lesh et al., 2000; Lesh & Caylor, 2007). It also should be mathematically meaningful, effective, and easily interpretable by others (Chamberlin & Moon, 2005).

The principle of model externalization principle states that students should be able to express their thinking processes during the modeling task clearly, and these processes need to be documented in their responses (Chamberlin & Moon, 2005; Lesh et al., 2000; Zawojewski, Lesh & English, 2003).

The self-evaluation principle requires that the purpose of the problem situation should be clear. According to this principle, students can assess their solution processes without any teacher support and approval and select the most appropriate and suitable solution from the alternative solutions by making their own decisions (Chamberlin & Moon, 2005; Lesh et al., 2000; Tekin Dede & Bukova Güzel, 2013b).

1.1 The Rationale for the Study

In recent years, many research studies related to mathematical modeling indicated that more emphasis should be given to modeling activities for all educational levels in school mathematics (Department for Education [DFE], 1997; National Board of Education [NBE], 2011, 2013; NCTM, 1989, 2000). Over the past few decades, research on modeling has gained popularity in Turkey (Koç, 2020).

For effective utilization of modeling in mathematics classrooms, it is crucial for students to be at the center and actively construct knowledge. Hence, teachers are expected to plan well-structured activities before class time. Also, activities, including resources like textbooks and educational materials, should be organized in such a way as to ensure the active participation of students and to be appropriate for their levels (MoNE, 2013a). Therefore, materials and textbooks are important for the effective implementation of modeling.

The research studies at the national level show that the textbook analyses conducted in the mathematical modeling context mainly examined how the activities in textbooks at the elementary school level meet the MEA design principles. The activities in the 5th-8th-grade mathematics textbooks (Çavuş-Erdem, Doğan, Gürbüz, & Şahin, 2017) and mathematics practice course books (Doruk, 2019; Saka

& Alkan, 2022) were analyzed in terms of model-eliciting activity design principles. Only one study examined the tasks at the high school level (9^{th} grade) (Urhan & Dost, 2017). Also, Koç (2020) stated that there is a gap in modeling research in Turkey, particularly the limited number of studies that focused on modeling at the elementary and high school levels. Moreover, in line with the focus on mathematical modeling, it is crucial to assess the extent to which the mathematical modeling activities in the books reflect the modeling process (Saka & Alkan, 2022). Therefore, studying teaching and learning materials at the high school level is important.

The mathematics textbooks or mathematics practice course books examined in the aforementioned studies did not claim to be modeling textbooks. Also, the literature has stated that although the textbooks claim to be modeling books, they do not fully fulfill the promise of mathematical modeling (Meyer, 2015). Thus, analyzing a mathematics textbook that claims to contain modeling tasks would be more valuable for this study.

1.2 Purpose of the Study and Research Question

The purpose of this study is to analyze the mathematical modeling activities in the ninth-grade mathematics modeling activity book (MoNE, 2020a) in terms of their compliance with the model-eliciting activities design principles according to Lesh and others (2003).

For this purpose, the following research question and sub-questions guided the study:

- To what extent do the modeling activities in the ninth-grade Mathematics Modeling Activity Book (MoNE, 2020a) comply with the MEA design principles?
    - To what extent do the modeling activities in the ninth-grade Mathematics Modeling Activity Book (MoNE, 2020a) comply with the principle of model construction?

- To what extent do the modeling activities in the ninth-grade Mathematics Modeling Activity Book (MoNE, 2020a) comply with the principle of reality?
- To what extent do the modeling activities in the ninth-grade Mathematics Modeling Activity Book (MoNE, 2020a) comply with the principle of model generalization?
- To what extent do the modeling activities in the ninth-grade Mathematics Modeling Activity Book (MoNE, 2020a) comply with the principle of simple prototype?
- To what extent do the modeling activities in the ninth-grade Mathematics Modeling Activity Book (MoNE, 2020a) comply with the principle of model-externalization?
- To what extent do the modeling activities in the ninth-grade Mathematics Modeling Activity Book (MoNE, 2020a) comply with the principle of self-evaluation?

1.3 Significance of the Study

Mathematical modeling is crucial in enhancing the students' comprehension of the world around them. It contributes to develop students' understanding of being responsible citizens and active participants in societal advancements (Blum, 2011). It also supports the students' mathematical learning by fostering motivation, comprehension, and retention while promoting the development of various mathematical competencies (e.g., creativity, critical thinking, problem-solving) and fostering positive attitudes toward mathematics. Furthermore, it contributes to creating a comprehensive and adequate perception of mathematics as a whole (Blum, 2011; Blum & Niss, 1991). Also, through modeling, mathematics becomes more meaningful and gains more importance for students, aligning with the secondary schools' core objectives of mathematics teaching (Niss, 1996).

In addition to numerous studies that highlight the pivotal role of mathematical modeling in mathematics education, the Turkish mathematics curriculum also emphasizes solving real-life problems and relating mathematics to real life and other disciplines (MoNE, 2013a). To help students develop their modeling competencies, among other factors, the modeling tasks used in teaching mathematics are of critical importance. Also, books serve as a bridge for the presentation of the curriculum. They contain modeling activities that play a crucial role in transferring mathematical modeling to the learning environment (Saka & Alkan, 2022). That is, it is important for students to do mathematical modeling, learn mathematics through modeling tasks, and develop mathematical thinking skills. This can only be done with real mathematical modeling tasks. Thus, modeling tasks in textbooks or teaching materials should be analyzed and evaluated in this regard.

Blum and Borromeo-Ferri (2009) indicated that a modeling activity should be designed to reflect the process of modeling and developed in a way that shows some critical features. In this context, design principles for modeling tasks have an important role in developing these tasks (Doerr & English, 2003). These principles (Lesh et al., 2003) are one of the tools that analyze the features of the modeling tasks for engaging students in meaningful mathematics learning.

In Turkey, there is only one ninth-grade mathematics textbook consisting of a collection of modeling tasks titled Mathematics Modeling Activity Book (MoNE, 2020a). However, no study investigated if and to what extent this textbook is in compliance with the design principles for modeling activities. Thus, assessing the extent to which the modeling activities in the ninth-grade Mathematics Modeling Activity Book adhere to the design principles of MEA has significance.

This study holds significance for various stakeholders, including the MoNE, researchers, teachers, textbook authors, and curriculum designers. The results of this study will provide valuable guidance for secondary school mathematics curriculum developers and textbook authors to make necessary modifications to instructional materials and resources containing mathematical modeling. Based on the study's

findings, curriculum designers can incorporate a greater number of model-eliciting activities aligned with the design principles of MEA into the teaching materials to enhance students' modeling experiences and boost their mathematics achievements. Also, teachers can make modifications to the modeling tasks in the textbooks or design new tasks to integrate the model-eliciting tasks into their classes better and improve students' modeling skills. Furthermore, this study has the potential to attract national and international researchers' attention to evaluating the quality of modeling activities published for teaching and research purposes.

1.4 Definitions of Terms

In this study, model, modeling, mathematical model, mathematical modeling, model-eliciting activities, models and modeling perspective, and model-eliciting activities design principles are important terms. These terms can be defined as follows:

Models, *as conceptual systems, are composed of elements, operations, relations, and interaction rules, which are expressed through systems of external notation. They are used for the construction, description, and explanation of behaviors exhibited by other systems (Lesh & Doerr, 2003a).*

Modeling *is the process of selecting and applying suitable mathematics, statistics, and mathematical methods to examine real-world situations to enhance comprehension and the decision-making process (Common Core State Standards Initiative [CCSSI], 2010).*

Mathematical model *is a description of a real-world situation by applying different mathematical techniques to make inferences about the situation. Mathematical models concentrate on structural features and functional principles of real-life situations or objects rather than their physical properties. Mathematical models encompass a variety of representations, operations, and relationships to make sense*

of real-life situations instead of relying on a singular approach (Lehrer & Schauble, 2003; Lesh & Doerr, 2003a; NCTM, 2000).

Mathematical modeling *is the process of applying mathematics to real-world problems by using mathematical terms to understand them (Cheng, 2001; Niss, Blum & Galbraith, 2007; Pollak, 2016).*

Model-eliciting activities (MEAs) *are challenging, non-routine, and open-ended problems in various real-world situations that can be solved with the help of the interaction between learners' formal and informal mathematical knowledge and requires the learner to develop a model that describes these situations (Wessels, 2014).*

Models and Modeling Perspective (MMP) *is a mathematics education approach that emphasizes using mathematical modeling as a central tool for learning to construct, analyze, and interpret new mathematical models to solve real-world problems. MMP is a problem-solving perspective in which the problems are thought-revealing problems known as model-eliciting activities instead of traditional textbook problems (Lesh & Zawojewski, 2007).*

Model-eliciting activities design principles *are a set of principles to design activities that align with this criterion. These principles serve as a guide for creating productive and effective model-eliciting activities. There are six MEA design principles: model construction, reality, model generalization, simple prototype, model externalization, and self-evaluation principles (Lesh et al., 2000).*

CHAPTER 2

LITERATURE REVIEW

The literature review part includes the theoretical background of model and modeling, modeling in mathematics education, modeling in mathematics education, different mathematical modeling approaches and perspectives, MEAs in MMP, MEAs design principles, which includes Galbraith's (2007) and Lesh and others' (2003) frameworks, and previous research studies on MEAs in Turkey and other countries.

2.1 Theoretical Framework

The theoretical framework includes the development of model and modeling concepts, modeling in mathematics education, specifically in mathematics textbooks, different mathematical modeling approaches and perspectives, and model-eliciting activities in MMP, Galbraith's and Lesh et al.'s modeling activities design principles.

2.1.1 Model and Modeling

Although the concepts of model and modeling are often used as the same concepts, they have different meanings (Kaiser, Blomhøj, & Sriraman, 2006; Niss, et al., 2007). When expressed in a general sense, modeling describes creating a model of a situation, while the model is the final product revealed at the end of the process (Sriraman, 2006).

From a mathematical point of view, a model is any picture, object or drawing representing a concept or that a relationship can be imposed on that concept (Van de

Walle, 2012). Also, Lesh and Doerr (2003a) described a model as a conceptual system containing elements, operations, relationships, and rules that regulate interactions. These models are represented using external symbols and are utilized to construct, explain, describe, and predict the other systems' behaviors (Lesh & Doerr, 2003a). According to Lehrer and Schauble (2003), a model is a form of representation that contains a collection of specific properties and a kind of analogy between an unknown or unfamiliar system and a system that is already known or familiar, and the objects and relations. The models allow people to understand, interpret, and make sense of real-world scenarios.

Modeling is a process of developing an abstract, physical, or symbolic model of a situation (Sriraman, 2006). Modeling is an iterative, creative process that involves making mathematical sense of a problem situation (Moore, Doerr, Glancy & Ntow, 2015). Modeling consists of the application, recognition, and interpretation of basic given models, utilization of various representational models, development and manipulation of complex models, and reflecting on, generalizing, and explaining the outcomes of modeling (Steen, Turner & Burkhardt, 2007).

Mathematical models are conceptual tools used by mathematicians to interpret various situations in a mathematical language that focuses on structural properties and functional principles of the systems in the real world (Lesh & Doerr, 2003a; Lehrer & Schauble, 2003, 2007). The processes of interpretation, construction, explanation, and description consist of schemes to organize, systematize, or mathematize the situations (Lesh & Doerr, 2003a, 2003b).

Mathematical modeling is a process of constructing and developing new models by predetermined models and conceptual systems to relate mathematics and real life (Lesh & Doerr, 2003a). According to Blum and Borromeo Ferri (2009), mathematical modeling is a two-way translation process between the real world and mathematics. Also, mathematical modeling is not only expressing real-life situations mathematically by using existing models but also relating phenomena within the

situation by reinterpreting them with mathematical representations and concepts (Gravemeijer & Stephan, 2002).

In modeling problems, the problem information provided could be incomplete, ill-defined, or ambiguous. Also, the extra data could be, or less information could be given. When confronted with such information, children may tend to make unjustified assumptions or impose inappropriate limitations on the solutions they need to create (English, 2006).

According to the literature, three categories of modeling tasks are described in mathematics education. These are sense-making by meaningful tasks (Freudenthal, 1973; Verschaffel, Greer & DeCorte, 2000), model-eliciting activities by challenging tasks (thought-revealing problems) (Lesh & Doerr, 2003a), and authentic tasks (Kaiser & Schwarz, 2010; Palm, 2007). In this present study, we focus on model-eliciting activities (MEA) because MEA is an effective and significant method for helping learners enhance their problem-solving, analytical thinking skills, and conceptual understanding of fundamental mathematical concepts within real life situations (Chamberlin & Moon, 2005; Lesh & Zawojewski, 2007; Moore et al., 2015).

2.1.2 Modeling in Mathematics Education

According to CCSSI (2010), one of the common core state standards for mathematics is “modeling with mathematics” (p. 7). Based on this standard, students who are proficient in mathematics have the ability to use their mathematical knowledge to solve problems that arise in daily life, society, and the workplace. Also, Principles and Standards for School Mathematics (NCTM, 2000) states that one of the standards is “using mathematical models to represent and understand quantitative relationships” (p. 39). NCTM (2000) highlights that mathematical modeling is among the most potent tools in mathematics education. Hence, it is crucial for

students at all educational levels to have chances to engage in mathematical modeling across a wide range of phenomena adapted to their individual skill levels.

Mathematical modeling is essential for students for several reasons. Mathematical modeling enhances students' problem-solving skills by allowing them to apply mathematical concepts to real-world situations (Lesh et al., 2000). It encourages critical thinking as students need to make decisions about which mathematical tools and techniques are most appropriate for a given problem (Lesh & Doerr, 2003a). Also, mathematical modeling helps students see the interdisciplinary applications of mathematics, which is crucial in today's interconnected world (National Research Council, 2013). It makes mathematics more relevant by showing students how mathematics is used to address real-world problems (NCTM, 2000). Mathematical modeling teaches students how to formulate problems, an essential skill in research and problem-solving (Blum & Leiß, 2007). It prepares students for higher education by introducing them to advanced mathematical concepts and techniques (Lesh & Doerr, 2003a).

English (2006) presents six compelling justifications for incorporating the significance of mathematical modeling into any reformed school curriculum, which can be viewed in light of the roles that mathematical modeling plays in the mathematics classroom:

1. The use of authentic problem situations provides not only real-world situations but also creates real-world solutions.
2. Model exploration and application offer students opportunities to construct, reinforce, and refine their conceptual understanding.
3. Multiple interpretations and approaches allow for critical examination, interpretation, and re-interpretation of problem information.
4. Social development is fostered when students share responsibility for problem-solving.
5. Encouraging multifaceted end products enables students to adopt various modes of representation, demonstrating representational fluency.

6. Engaging in important mathematical processes fosters optimal mathematical development.

Modeling holds significant importance for students as it facilitates deeper comprehension of the world, enhances mathematics learning, fosters the development of diverse mathematical competencies and positive disposition towards mathematics, and contributes to a comprehensive understanding of mathematics (Blum, 2011). Therefore, modeling is so important in mathematics education.

There are only a few modeling examples in mathematics lessons. The main reason is that modeling is difficult for both teachers and students. Because of the cognitive demands of modeling tasks involving the reciprocal translation between mathematics and real life, it makes relevant mathematical concepts and real-world knowledge necessary for students. When students attempt MEAs for the first time, they encounter difficulties. Some of the challenges including a lack of comprehension of the problem and the struggle to construct a well-developed and adequate model. Also, modeling requires other mathematical competencies, such as the competencies of problem-solving, mathematical thinking, and reasoning (Blomhøj & Jensen, 2007; Blum, 2011; Niss, 2003). For these reasons, modeling is so difficult for students.

On the other hand, MEAs need real-world knowledge, and teaching becomes more open and more unpredictable. Teachers encountered difficulties such as students' reluctance to collaborate, persistent need for feedback, and preference for quick solutions without thorough deliberation (Moore et al., 2015; Zawojewski et al., 2003). When incorporating MEAs, the main focus is not on directing students toward a specific solution but rather on cultivating a classroom environment where students are encouraged to share, discuss, and revise their ideas freely. Therefore, creating such a classroom environment can be difficult for teachers. Furthermore, teachers must acquire all the modeling competencies expected from their students (Blum, 2011). Mathematical modeling competencies are the ability to translate a given real world situation into mathematics and to interpret and validate the problems' solution, and develop mathematical models (Kaiser, 2007; Niss et al., 2007).

In addition to the difficulties for teachers and students, there are other factors that prevent the implementation of modeling activities in mathematics lessons. These factors are as follows (Urhan & Dost, 2016):

- The inadequacy of both quantitative and qualitative aspects of mathematical modeling activities in mathematics textbooks and other resources,
- The difficulty of attracting students' attention because of the simplicity of the modeling activities present in textbooks and other resources,
- The low motivation of students towards modeling activities due to their lengthy and complex scenario-based nature,
- The density of the curriculum as the most significant obstacle to the incorporation of modeling activities in mathematics instruction,
- The discrepancy between the question structures in university entrance exam questions and mathematical modeling activities,
- The students' fundamental mathematical deficiencies and lack of readiness levels.

The inadequacy and simplicity of modeling activities in mathematics textbooks and their lack of attractiveness for students are among the reasons that prevent the implementation of modeling activities in mathematics education. Therefore, it is important to examine the place of modeling in mathematics textbooks.

2.1.2.1 Modeling in Mathematics Textbooks

"The mathematics textbook plays a pivotal role in facilitating mathematics instruction and learning." The mathematics textbook plays a pivotal role in facilitating teaching and learning in mathematics lessons (Zwaneveld, Perrenet, Overveld & Borghuis, 2017). Based on the MoNE Course Books and Educational Tools Regulations, the Education Committee undergoes approval a course book before being employed in both formal and non-formal educational institutions in Turkey (MoNE, 2015). These course books are developed by service units, the

Ministry itself, or private publishing companies, adhering to the criteria specified in the Regulations (Urhan & Dost, 2017).

In case of any curriculum modifications, the respective service unit or publishing company is responsible for incorporating these changes into the relevant course books. Therefore, considering the revised curricula in Turkey (MoNE, 2005, 2011, 2013a), it is reasonable to expect that modifications in line with the constructivist approach mentioned above would be included in the course books, incorporating modeling problems and model-eliciting activities.

Research studies conducted in Turkey have shown that MEAs are rarely utilized in school mathematics lessons (Tekin Dede & Bukova Güzel, 2013a). Textbooks are one of the reasons for this. Urhan and Dost (2016) stated that this is due to the fact that the textbooks are insufficient quantitatively and qualitatively with respect to model-eliciting activities. Instead, mathematics course books should incorporate problems that are not overly straightforward, may initially appear ambiguous and complex, and involve mathematical real-world situations (Pollak, 1979).

Modeling activities are reflected in curricula in different ways in different countries. In some countries, modeling activities are included in mathematics textbooks, while in others, there is also another modeling book in addition to the textbooks. For example, mathematics practice course books and ninth-grade mathematics modeling books are separate books specific to modeling in Turkey. Also, COMAP's educational philosophy is centered around mathematical modeling. Their aim is to use mathematical tools to explore real life problems. This is an example of another modeling book.

In the curricula prepared with the focus on problem-solving or Realistic Mathematics Education (RME), the contents of the textbooks are structured in this sense (Zwaneveld et al., 2007). RME is an educational approach that has five main characteristics: (i) using students' experiences in everyday lives, (ii) visualizing reality into models, (iii) using students' activities, (iv) conducting discussion and question-answer for students to realize the mathematics, and (v) connecting concepts

and topics in mathematics education. Some curricula (e.g., Dutch and Indonesian curricula) are prepared using this approach as the important principles for teaching and learning mathematics, and textbooks include mathematical modeling activities that can be achieved by presenting student outcomes and materials close to everyday life (Revina & Leung, 2018).

The textbooks containing mathematical modeling questions are mostly incompatible with modeling processes (Meyer, 2015; Zwaneveld et al., 2017). The literature indicates that the practice of mathematical skills takes precedence over incorporating conceptual development with mathematical modeling in textbooks (Zwaneveld et al., 2017). While modeling textbooks provide an opportunity to perform operations and interpret the results, they offer students very limited modeling opportunities, such as identifying variables, formulating models, and validating the conclusions (Meyer, 2015).

2.1.3 Different Mathematical Modeling Approaches and Perspectives

In mathematics education, modeling is generally categorized based on its purpose (Galbraith, 2012; Gravemeijer, 2002; Julie & Mudaly, 2007; Kaiser, 1995; Niss et al., 2007). According to Kaiser (1995), the modeling perspectives diversified due to the difference in purposes related to modeling and implementation. Kaiser (1995) has divided the modeling goals into four categories:

i. **Pedagogical goals:** Enhancing students' skills to gain a deeper and better comprehension of fundamental aspects of the world.
ii. **Psychological goals:** Promoting and improving students' motivation and attitude towards mathematics and the teaching of mathematics.
iii. **Subject-related goals:** Organizing learning processes and introducing and exemplifying new methods and concepts related to mathematics.

iv. **Science-related goals:** Providing a realistic portrayal of mathematics as a scientific discipline and offering an understanding of how mathematical and extra-mathematical ideas intersect in the historical mathematical progress.

Also, Julie and Mudaly (2007) characterized modeling in two different ways. Modeling is expressed as a vehicle if it is used to develop a mathematical understanding and as a content in which modeling is the learning goal if it is used to improve modeling skills. Like Julie and Mudaly (2007), Niss et al. (2007) categorized the modeling goals as follows (p. 5):

i. "Modeling may be a vehicle for facilitating and supporting students' mathematical learning as a subject (i.e., modeling for the learning of mathematics)"
    a. "Illustrating to students the practical application of mathematics in diverse contexts beyond the mathematics classroom, thereby contributing to a more comprehensive understanding of the nature and role of mathematics."
    b. "Helping students to understand and interpret mathematical activities" and
    c. "Fostering students' motivation to participate in mathematics studies and helping to improve their mathematical attitudes."
ii. "Learning mathematics to develop competency in applying mathematics and building mathematical models for extra-mathematical scenarios (i.e., learning mathematics for modeling)"
    a. "Empowering students with the ability to apply mathematics beyond its own domain" and
    b. "Enabling the use of extra-mathematics with the help of mathematical models and modeling."

That is, generally, the purposes of mathematical modeling are categorized in two main ways (Erbaş et al., 2014):

i. Modeling as the purpose of teaching mathematics: The purpose of teaching mathematics is to provide students with mathematical modeling competency, viewed as a fundamental skill for solving real-world problems in mathematics and various other fields. In this approach, the modeling process begins with providing mathematical concepts and modeling, regarded as pre-existing objects, and then continues adapting these concepts to real-world situations (Gravemeijer, 2002; Niss et al., 2007). Concerning this purpose, it focuses on conceptualizing, developing, and measuring the modeling competencies.

ii. Modeling as a means to teach mathematics: Modeling is viewed as a vehicle to support students in constructing and developing their foundational mathematical knowledge and models.

As a result, there is a duality related to mathematical modeling purposes in the literature. These different purposes play an essential role in developing different perspectives on mathematical modeling (Abassian, Safi, Bush & Bostic, 2019; Nish et al., 2007). Various approaches exist regarding the integration of modeling into mathematics education. As a result of this diversity, different modeling perspectives have developed within the framework of different goals and theoretical orientations in connection with mathematical modeling teaching and learning (Kaiser, Blum, Borromeo Ferri, & Stillman, 2011).

Educators have not made a common and single decision on this issue (Abassian et al., 2019; Galbraith, 2012; Kaiser et al., 2011; Kaiser & Sriraman, 2006). However, according to the literature, the leading perspective of modeling approaches is the classification system of Kaiser and Sriraman (2006) (Erbaş et al., 2014). Within this context, in research about mathematical modeling education, mathematical modeling perspectives are classified into five different categories: (1) realistic modeling or applied modeling, (2) educational modeling, (3) models and modeling perspective or contextual modeling, (4) socio-critical modeling, and (5) epistemological modeling (Kaiser & Sriraman, 2006).

The realistic modeling or applied modeling perspective focuses on solving real-world problems, understanding real life, and developing students' competencies related to mathematical modeling (Blomhøj, 2009; Kaiser & Sriraman, 2006; Kaiser, Sriraman, Blomhøj & Garcia, 2007). According to this view of modeling, the tasks that students work on should be realistic, messy, and authentic, requiring modeling cycle use. Instead of focusing on mathematical content development, this perspective highlights the importance of solving real-world problems. Also, realistic modeling emphasizes interdisciplinary approaches (Blomhøj, 2009).

The educational modeling perspective focuses on integrating models and modeling into mathematics teaching (Blomhøj, 2009). The main objectives of this perspective are to develop students' mathematical modeling competencies and to learn and understand mathematics (Blomhøj, 2009; Kaiser & Sriraman, 2006). That is, this perspective consists of both modeling purposes as mentioned above, which are "modeling as a vehicle" and "modeling as a content" (Julie & Mudaly, 2007; Niss et al., 2007) at the same time. This situation leads the tasks to be simplified more than the authentic tasks given in the previous perspective (Kaiser & Sriraman, 2006). From the educational modeling perspective, the connection between mathematics and the real world has an essential role in the process of mathematics teaching and learning. This perspective motivates and supports students' needs to learn mathematics and describe real-life scenarios.

Also, it emphasizes mathematical modeling activities in different mathematics curricula, problems related to the application of modeling in schools, and the assessment of students' modeling activities. The educational modeling perspective has been divided into didactical and conceptual modeling. While didactical modeling aims to organize the learning processes, conceptual modeling aims to introduce and develop the concepts (Kaiser & Sriraman, 2006).

The contextual modeling perspective focuses on developing a deep understanding of mathematics in the context of modeling (Kaiser & Sriraman, 2006; Lesh & Doerr, 2000). This perspective is called in different ways: "contextual perspective" or

"model-eliciting perspective" (Kaiser & Sriraman, 2006), and "models and modeling perspective (MMP)" (Lesh & Doerr, 2003a). This perspective is referred to as MMP since it describes the mathematical modeling framework proposed by Lesh and others (Lesh et al., 2000). In this framework, there are six guiding principles for designing model-eliciting activities: (i) the model construction principle, (ii) the reality principle, (iii) the model generalization principle, (iv) the simple prototype principle, (v) the model externalization principle, and (vi) the self-evaluation principle (Lesh & Doer, 2003a; Lesh et al., 2000). MEAs are designed to develop learners' mathematical reasoning, thinking, and understanding and teach mathematics and mathematical concepts (Doerr & Lesh, 2011; Kaiser & Sriraman, 2006). Unlike other perspectives, the contextual modeling perspective defines the mathematical model as a conceptual tool for a mathematical system that expresses a certain real-world scenario, and in this perspective, mathematical modeling is not considered a specific competence (Blomhøj, 2009; Lesh, Doerr, Carmona, & Hjalmarson, 2003).

The socio-critical modeling perspective focuses on the social role of mathematics and pedagogical goals such as a critical understanding of the real-world (Blomhøj, 2009; Kaiser & Sriraman, 2006). This perspective is also defined as "modeling as critics" (Barbosa, 2006) to use mathematical concepts to make decisions about society. It includes both internal and external goals of mathematics: to show the power of mathematics and its role as a tool in decision-making, to make societal decisions, and to develop a critical understanding of the process of mathematical modeling (Barbosa, 2006; Kaiser et al., 2007). In this way, this viewpoint of modeling encourages students to develop their skills in mathematical modeling to foster independent and autonomous citizens in society with the help of the teaching of mathematical modeling in mathematics education (Blomhøj, 2009). According to this perspective, a mathematical model serves as a representation of a real-world situation (Barbosa, 2007). Socio-critical modeling perspective differs from the perspectives of educational and realistic modeling since it emphasizes students' critical application of mathematics to make sense of the world instead of

mathematics-related competencies (Ferri, 2013). It also differs from the contextual modeling perspective because it focuses on students' ability to be critical modelers, not on understanding mathematics. Here, mathematical concepts and modeling competencies are considered as means to criticize mathematical models (Barbosa, 2006).

The epistemological modeling perspective focuses on developing comprehensive theories related to the teaching and learning of mathematics (Blomhøj, 2009). This perspective aims to develop mathematical understanding. Unlike other perspectives, the epistemological perspective does not require a real-world context for modeling. According to the epistemological perspective, any mathematical task can be regarded as a modeling task. In this perspective, the modeling process, which is not limited to the mathematization of non-mathematical problems, starts with the real world but ends with mathematics (Kaiser & Sriraman, 2006).

Besides Kaiser's (2006) and Kaiser and Sriraman's (2006) categories of mathematical modeling, there are three frameworks in the modeling field, namely Realistic Mathematics Education (RME), the Anthropological Theory of Didactics (ATD), and the Models and Modelling Perspective (MMP), that specifically emphasize the use of modeling about the aim of enhancing students' understanding of mathematics in mathematics education. Two of these frameworks, RME and ATD, adopt an epistemological viewpoint toward modeling as a vehicle for facilitating students' mathematical learning (Blomhøj, 2019). These frameworks provide comprehensive guidelines for teaching and learning mathematics as a whole. In the frameworks of RME and ATD, the focus on modeling is embedded within broader theories related to the teaching and learning of mathematics. On the other hand, the third framework, MMP, takes a contextual problem-solving approach to modeling (Kaiser & Sriraman, 2006).

Within Realistic Mathematics Education (RME), modeling extends beyond translating real-life problems into mathematical form, embracing modeling as a tool for teaching mathematics (Erbaş et al., 2014). The learning process at RME is viewed

as a dynamic reinvention of mathematical knowledge by the learners during the mathematization process (Freudenthal, 1983). In RME, models are described as mathematical representations of problem situations and the elements comprising students' conceptual systems (Cobb, 2002). The modeling process begins with students mathematizing their real-life experiences in various contexts (i.e., "from real life to the world of symbols" or "model of"), called horizontal mathematization (Freudenthal, 1991). Through this process, students establish a foundation for grasping the theoretical meaning behind their concepts and methods and create more abstract models called vertical mathematization (i.e., "stay in the world of symbols" or "model for") (Doorman & Gravemeijer, 2009). That is, modeling is the process of moving from a "model of" to a "model for," which is considered emergent modeling (Gravemeijer & Doorman, 1999). The emergent modeling perspective developed by Gravemeijer, and his colleagues focuses on the learning process through RME (Kaiser et al., 2007).

The other framework, ATD, provides a general and strong theory for comprehending and designing the teaching of mathematics. The fundamental assumption is the development of mathematical knowledge in order to contribute to the answer of particular types of questions. For this reason, in ATD, mathematics teaching should be defined and utilized as its starting point to teach how to generate such questions for mathematics knowledge in the curriculum. The ATD approach highlights the central role of mathematical modeling in the process of learning mathematics. Instead of viewing modeling competency as a goal, it emphasizes modeling as a didactical means for learning mathematics (Blomhøj, 2019).

MMP is a comprehensive approach to teaching and learning specific mathematical concepts, methods, and problem-solving within various situations and contexts (Lesh & Doerr, 2003a; 2003b). Here, modeling is regarded as a vehicle for both supporting students' mathematical learning and developing students' mathematical modeling and problem-solving competencies. It also focuses on designing modeling activities that allow students to activate precisely defined aspects of these concepts in modeling activities, referred to as MEA. They derive inspiration from everyday real-

life scenarios, meaningful contexts created during instruction, or authentic applications in various fields or professions.

All three frameworks share a fundamental belief that stems from the idea that students' understanding of mathematical concepts can be solidified and deeply rooted through modeling activities (Blomhøj, 2019). By working with modeling and applications across various contexts, students have the opportunity to not only apply their conceptual understanding but also challenge and enhance it.

2.1.4 Model-Eliciting Activities in MMP

The models and modeling perspective (MMP) represent an extensive theoretical framework for understanding learning and teaching mathematics and mathematical problem-solving (Lesh & Doerr, 2003a). Within the context of MMP, models refer to explicit constructions developed by students to represent, describe, and explain significant real-life mathematical situations they come across (Lesh & Doerr, 2003a).

Modeling problems necessitate children comprehending the situation, enabling them to mathematize it independently in a meaningful manner (English, 2006). This process involves a cyclical approach of interpreting the problem information, identifying relevant quantities, exploring operations that lead to new quantities, and generating meaningful representations (Lesh and Doerr, 2003a).

This perspective was built upon the principles of constructivist and socio-cultural theories. According to this viewpoint, individuals actively use their internal conceptual systems, known as mental models, to organize, interpret, and comprehend events, experiences, or problems (Moore et al., 2015). They are encouraged to construct their models, aligning with the fundamental principles of constructivism (Lesh & Lehrer, 2003). To effectively tackle complex problem-solving situations, it is essential for these models to be externalized through representational media, such as symbols and figures.

As mentioned above, MEAs are challenging, non-routine, and open-ended problems in various real-world contexts, designed to be approached by learners at different skill levels. Learners engage with these problems by combining their informal and formal mathematical knowledge (Wessels, 2014). These MEAs offer learners the chance to create models and enhance their creative applied mathematical abilities (Chamberlin & Moon 2005). MEA is an effective method for aiding students in the development and enhancement of crucial mathematical concepts within real-world scenarios (Moore et al., 2015).

MEAs have been specially designed for integration within the MMP. MEAs present students with real-world problems that require them to grasp mathematical concepts intuitively and construct relevant models within a relatively short timeframe (Carlson, Larsen, & Lesh, 2003; Doerr & Lesh, 2011). To promote collaborative learning and foster interaction, MEAs are carried out in teams of three to four students. The objective is for students to create models to be shared and reused among their peers, thus emphasizing the social dimension of learning within the MMP (Zawojewski et al., 2003).

The development and use of MEAs serve two primary purposes (English, 2003; Lesh et al., 2000). Firstly, learners are given opportunities to solidify their existing mathematical knowledge, create mathematical models to solve complex tasks and acquire new insights by modeling complex mathematical problems. Secondly, teachers are afforded the opportunity to observe and study learners' mathematical thinking and determine students' progress (Lesh & Doerr, 2003a; Moore et al., 2015; Wessels, 2014).

2.1.5 Model-Eliciting Activities Design Principles

In the literature, a modeling activity typically possesses the following main features: (i) being open-ended, (ii) being grounded in real-world scenarios, (iii) creating mathematical models, (iv) developing students' modeling competencies, (v) making

assumptions consistent with real-world scenarios, (vi) creating diverse solutions according to the assumptions, and (vii) offering solutions that can be generalized (Blum & Niss, 1991; English, 2003; Kula Unver et al., 2018; Lesh & Doerr, 2003a).

Modeling activities can be designed according to different design perspectives. Here, two different frameworks that are used to design modeling activities are presented: Galbraith's (2007) and Lesh et al.'s (2003) six modeling design principles.

Galbraith's (2007) six design principles provide a guideline for transforming ideas into modeling problems (Galbraith, Stillman & Brown, 2010). These principles are (i) relevance and motivation, (ii) accessibility, (iii) feasibility of approach, (iv) feasibility of outcome, (v) validity, and (vi) didactical flexibility. As documented in Galbraith (2007, p. 55), the first principle requires the modeling activity to have a genuine connection with the students' real world. The context of modeling activities should be a part of students' daily experiences. According to the second principle, modeling activity should provide students an opportunity to identify a mathematically tractable questions extracting from the general problem statement. The third principle states that students can develop solutions to the modeling activities by (a) using mathematics that is available to them, (b) making the necessary assumptions, and (c) collecting the required data. According to the fourth principle, the feasibility of outcome principle, the solution of a basic problem in mathematics should be possible for students with the help of interpretation. The fifth principle, the validity principle, indicates that solutions for the modeling activity can be assessed in terms of their mathematical accuracy and their relevance to the contextual setting. According to the sixth principle, the didactical flexibility principle, the problem should be structured into a series of questions that can still preserve the integrity of the actual situation.

Another framework that was developed to assess the modeling nature of the tasks, and that guides turning ideas into modeling activities is Lesh et al.'s (2003) design principles for MEAs. Lesh et al. (2000) stated that a MEA must ensure modeling design principles. The researchers developed and tested these principles with the help

of hundreds of teachers, parents, and community leaders who worked with the researchers in a 15-week multi-stage teaching experiment (Lesh et al., 2000). There are six modeling design principles, namely, the principles of model construction, reality, model generalization, simple prototype, model externalization, and self-evaluation. The following paragraphs give an account of these principles.

The model construction principle indicates that the solution to the problem requires creating a model (Chamberlin & Moon, 2005). A model can be defined as a system including elements, relationships between these elements, and operations. These explain how the elements interact and contain patterns and rules that apply to the operations and relationships (Chamberlin & Moon, 2005; Lesh et al., 2000). Here, the modeling process begins when confronted with the need to construct such a model (Lesh et al., 2000).

To qualify as a model, a system must be employed to explain, make sense of, or predict another system. Furthermore, for the model to hold mathematical significance, it must concentrate on the basic structural features of the described system (Lesh et al., 2000). A situation cannot be considered model eliciting unless it constructs a model for a student. Therefore, this principle is necessary to determine the situation that creates the need for a particular type of model and its underlying concepts (Lesh & Caylor, 2007).

While traditional textbooks include problems that need to make sense of symbolically expressed situations, MEAs require symbolic representations of given situations, called mathematizing (Lesh et al., 2000). As a product of mathematizing, a model is constructed. A variety of concrete, graphic, symbolic, or language-based representation systems may be necessary to describe the connections, operations, and patterns depicted in the underlying model.

To determine whether a model eliciting activity satisfies the model construction principle, the following questions should be considered:

- “Does the task put students in a situation that makes them realize the need to develop a specific type of model to interpret the givens, goals, desired outcomes, and possible solution processes in a complex problem-solving situation?” (Lesh et al., 2000, p. 17)
- “Does the task lead students to get only an answer formulated by others for a question?” (Lesh et al., 2000, p. 17)
- “Does the task include the construction, description, explanation, manipulation, prediction, or control of a system that holds structural significance?” (Lesh et al., 2003, p. 43)
- “How can activities create the need for students to construct, develop, revise, modify, refine, or extend a mathematically significant model and underlying conceptual systems?” (Lesh & Caylor, 2007, p. 181)

Models are essential in cases where predictions are required based on underlying patterns, to justify or explain decisions, to evaluate or assess alternative conclusions, explanations, or interpretations proposed by others, and to describe explicitly requested constructions or explanations (Lesh et al., 2000).

The model construction principle becomes essential when arithmetic methods alone are insufficient for problem solution. In such cases, the solution can only be obtained by developing mathematical models (English, 2006). The principle emphasizes that the purpose of MEAs is not only to get a solution but also to develop the appropriate tools and structures that can be evaluated, explained, and predicted, which leads to the desired outcome (Deniz, 2014; Saka & Alkan, 2022).

The reality principle (RP) is also known as the personal meaningfulness principle since it emphasizes the significance of students attempting to understand a situation based on their own knowledge and experience (Lesh et al., 2000). It requires that a problem presented in the MEA should be a problem that students are likely to encounter in their daily lives, and the problem situation should reflect students’ real life (Lesh et al., 2000; Lesh & Caylor, 2007). This principle states that activities should be designed in accordance with real or realistic data (Deniz, 2014; Saka &

Alkan, 2022; Urhan & Dost, 2017). According to the reality principle for task design, the questions asked to students are real questions instead of mathematics class questions (Zawojewski et al., 2003). This enables students to comprehend the situation through their knowledge and experience. Furthermore, students may realize that an actual need causes the problem, and the model they have created will help solve a real problem (Chamberlin & Moon, 2005; Deniz, 2014).

The reality principle requires that during the solution process, students' ideas are taken seriously, and they are not forced to follow the way the teacher or author considers the right approach for the problem (Doruk, 2016; Lesh et al., 2003).

The realistic nature of the problem situation in the activity allows students to find more creative solutions and understand the problem more easily because they will be familiar with it (Chamberlin & Moon, 2005).

To determine whether a model eliciting activity is in accordance with the reality principle, the following questions can be asked (Lesh et al., 2003, p. 43):

- "Could this scenario happen in real-life situations?"
- "Will students be encouraged to understand the situation by their own personal knowledge and experiences?"
- "Do students' ideas be taken seriously, or are they expected to align with the teacher's notion of the correct way on approaching about the problem situation?"

It is crucial to consider that the interpretation of reality can vary significantly between adults and children when examining the reality of the activity. Furthermore, one child's reality may differ from another's (Doruk, 2019). Therefore, a problem is not expected to be meaningful for all students, and no situation is expected to be problematic for all students (Lesh & Caylor, 2007).

The model-eliciting activities in teaching mathematics can keep students' interest alive, attract their attention, and increase their motivation by establishing connections with their daily lives (Koç, 2020). In addition, solving model-eliciting

activities can increase the group's motivation and contribute to their perseverance by attracting the interest and attention of the individuals in the group in the process of comprehending the problem situation (Zawojewski et al., 2003).

The model generalization principle (MGP) is also known as the construct shareability and reusability principle (Lesh et al., 2000). This principle tells that the elicited model should not only be useful and applicable to a particular situation, but it should also be used for similar situations, shareable with other people (Chamberlin & Moon, 2005; Doruk, 2016; Lesh et al., 2000; Lesh & Caylor, 2007).

To determine whether a model eliciting activity provides the model generalization principle, answers are sought to the following questions (Lesh et al., 2000, p. 32):

- "Is the developed model applicable only to the specific situation in the problem and useful only for the person who created it?"
- "Does the problem situation make students recognize the need for the model to be reusable, shareable, transportable, and easily modifiable for different situations?"

The modeling activities that meet this principle require students to ensure that they develop models that are generalizable instead of developing personal models or producing single-purpose ways of thinking (Lesh et al., 2000; Lesh & Doerr, 2003a).

According to this principle, for the solution to be successful, the created model can also be reused and generalized to other situations that require a similar model (Chamberlin & Moon, 2005). These types of problems prompt students to provide descriptions, explanations, or prediction procedures that others can apply in different contexts, highlighting more potent applications of mathematics compared to problems that do not require any form of generalization (Lesh et al., 2000). According to Lesh and Caylor (2007), this principle provides that the model should not only be useful for a particular situation but also be shareable with other people and reusable or generalizable for different purposes.

The simple prototype principle (SPP) is also known as the effective prototype principle (Lesh et al., 2000). This principle indicates that the developed model should be as simple as possible and mathematically meaningful (Lesh et al., 2000; Lesh & Caylor, 2007). This principle requires that the model created by students can be remembered by the students even after a long time. That is, it should be memorable for students when they encounter structurally similar situations (Chan, 2008; Lesh & Caylor, 2007; Lesh et al., 2000; Tekin Dede & Bukova Güzel, 2013b). Also, the model constructed should be easily interpretable by others (Chamberlin & Moon, 2005).

To determine whether a model eliciting activity is in accordance with the simple prototype principle, the following questions can be asked:

- "Does the solution offer a useful prototype or metaphor for interpreting various other structurally similar situations?" (Lesh et al., 2003, p. 43)
- "Is the situation simplified as much as possible while still requiring a significant model?" (Lesh & Caylor, 2007, p. 184)
- "Long after the problem has been solved, will students think back on it when encountering other structurally similar situations?" (Lesh et al., 2000, pp. 33)

Based on this principle, Doruk (2019) states that since the purpose of MEAs is to focus on mathematical concepts, the solution to the problem should not include complex procedures at a level that will overshadow the students' main goal. Modeling activities that are effective and thought-provoking should be structurally meaningful and important, but they do not need to be operationally complex. If not, students may not remember the model they created before when they encounter similar situations after a long time because they will not be able to realize the basic conceptual structure due to procedural complexities.

The model externalization principle (MEP) is also known as the model documentation principle (Lesh et al., 2003). This principle requires external documentation of mathematical models of the solution process (Zawojewski et al., 2003). The principle of model externalization emphasizes that students should be

able to express their thinking processes during the work on MEAs. Also, these processes should be documented in their responses (Chamberlin & Moon, 2005; Lesh et al., 2000; Tekin Dede & Bukova Güzel, 2013b). This principle helps teachers in assessing their students' thinking regarding mathematical operations, relationships and patterns, and final answers or models during the problem-solving process. It is also beneficial for students as it facilitates their visualizations and enables them to reflect on their thoughts (Chamberlin & Moon, 2005; Lesh et al., 2000; Lesh et al., 2003).

To determine whether a model eliciting activity is in accordance with the model externalization principle, the following questions can be asked:

- "Will the response necessitate students to explicitly reveal their thought process concerning the provided information, goals, and possible solution ways?" (Lesh & Caylor, 2007, p. 183; Lesh et al., 2003, p. 43)
- "Will it offer an "audit trail" that can be examined to understand the types of systems (mathematical objects, relations, operations, patterns, and regularities) that students were thinking with and about?" (Lesh et al., 2000, p. 30)

In model eliciting activities prepared in accordance with this principle, students are expected to create a written document that will clearly reveal their own thoughts about the problem situation and possible solutions during the activity (Doruk, 2019).

The required problem-solving situations must go beyond simply prompting thoughts; that is, they should also reveal the thought processes involved. This principle is the rationale behind using the term "thought-revealing activities" to describe these activities (Lesh et al., 2000).

Since students have the opportunity to evaluate themselves, this principle is directly related to the principle of self-evaluation in terms of the need for students to evaluate their solutions (Chamberlin & Moon, 2005; Lesh & Caylor, 2007; Tekin Dede & Bukova Güzel, 2013b).

The self-evaluation principle (SEP) is also known as the self-assessment principle (Lesh et al., 2000). It refers to the students' self-evaluation of their solution process. The self-evaluation principle requires that the purpose of the problem is clear and suitable for the students' levels (Chamberlin & Moon, 2005; Lesh et al., 2000; Urhan & Dost, 2017).

According to this principle, students should be able to assess the appropriateness and usefulness of their solution approaches independently, without the need for teacher support or approval (Chamberlin & Moon, 2005; Lesh et al., 2000; Tekin Dede & Bukova Güzel, 2013b). Also, this principle allows students to control the accuracy of their own interpretations and conclusions in the activity and to decide for themselves the model they have created that needs to be improved or revised (Doruk, 2019; Lesh et al., 2000).

Since students in a group may have different solutions and ideas in MEAs, this principle contains information that enables students to choose the most appropriate and useful solution from alternative possibilities and eliminate other ideas to achieve the given purpose (Lesh et al., 2000).

Furthermore, during MEA implementation, teachers are expected to employ teaching strategies that do not provide information about the correctness of students' solution approaches. Instead, they should facilitate self-evaluation by allowing students to assess their own work (Doerr & English, 2006). According to Chamberlin and Moon (2005), the self-evaluation principle is consistent with the development of creativity since it is necessary for people engaged in creative work to be capable of self-assessment. To determine whether a model eliciting activity is in accordance with the self-evaluation principle, the following questions can be asked:

- "Does the problem statement strongly suggest suitable criteria for evaluating the usefulness of alternative solutions?" (Lesh et al., 2000, pp. 27-28)
- "Will students be able to evaluate the strengths and weaknesses of alternative solutions?" (Lesh et al., 2003, p. 43)

- "Will students be able to determine when their responses are good enough?" (Lesh et al., 2003, p. 43)
- "Is the purpose clear (what, when, why, where, and for whom)?" (Lesh et al., 2000, p. 28)
- "Will students likely have confidence that their ideas will be taken seriously?" (Lesh & Caylor, 2007, p. 182)
- "Will students believe the responses they are expected to give should align with the teacher's or author's notion of the only "correct" approach to think about the problem situation?" (Lesh & Caylor, 2007, p. 182)
- "Are students able to assess when their responses require improvement, enhancement, refinement, or extension for a specific purpose?" (Lesh et al., 2000, p. 28)

As a result, Lesh and others' (2003) six design principles emphasize the key characteristics of a good model-eliciting activity. They aim to increase the level of revealing students' thoughts and to enhance the effectiveness and quality of the MEAs. Otherwise, the purpose of these principles is not to determine whether any activity is a model-eliciting activity (Lesh et al., 2000; Lesh & Doerr, 2003a). The design principles serve as a guide for the creative progression of new crafting MEA problems. MEAs designed following these six principles possess distinct educational and learning characteristics (Chamberlin & Moon, 2005).

Both Galbraith's (2007) and Lesh et al.'s (2003) six modeling design principles are frameworks for designing mathematical modeling activities and problems. A close examination of these two frameworks shows that they have some similarities and differences. Regarding the similarities between the principles, Lesh et al.'s (2003) reality principle corresponds to Galbraith's (2007) first principle, namely relevance and motivation. Both principles recognize the importance of connecting mathematical problems to real-world contexts. This emphasis helps make mathematical modeling more relevant and engaging for students. Moreover, the self-evaluation principle (Lesh et al., 2003) corresponds to Galbraith's (2007) validity

principle. Both principles involve checking the appropriateness in context and mathematical accuracy of students' responses through evaluation procedures.

On the other hand, there are some differences between these principles. According to Galbraith (2007), the solution of mathematics for a basic problem and the formulation of a solution process can be possible and feasible for students. Also, the problem should be structured into sequential sub-questions to preserve the real situation's integrity. These principles are not stated in this way in the principles of Lesh et al. (2003). On the other hand, Galbraith's (2007) accessibility principle emphasizes the importance of being able to derive specific questions from a general problem statement. In contrast, Lesh et al.'s (2003) model construction principle focuses on developing mathematical models to represent real-world situations. Moreover, in contrast to Galbraith's principles, Lesh et al. (2003) emphasize the generalization of the model and the reusability of the elicited model in similar situations. Also, unlike Galbraith's principles, Lesh et al. (2003) give the importance of expressing students' thinking processes during MEA and documenting their responses. In addition to this, the developed model should be mathematically meaningful and as simple as possible, and students should remember the model even after a long time (Lesh et al., 2003).

In summary, both modeling design principles aim to enhance the design of mathematical modeling tasks by using different focuses and terminology. When these principles are examined, it is concluded that the principles of Lesh and others (2003) provide a broader and more comprehensive framework for designing modeling activities. Indeed, the relevant literature shows that it is also widely used in developing or assessing modeling tasks, or conducting research studies that focus on issues around implementing mathematical modeling tasks. Therefore, in this study, Lesh et al.'s (2003) design principles of MEA are used to determine the compatibility of activities with the MEA design principles.

2.2 Previous Research Studies on the Analysis of Mathematical Modeling Activities in Teaching Materials and Textbooks

This part gives previous research studies related to the analysis of mathematical modeling studies in teaching materials and textbooks in Turkey and other countries.

2.2.1 Previous Research Studies in Turkey

Doruk (2019) carried out a research study aimed at assessing the compatibility level of problem-solving activities in the 5th-grade mathematics applications course teaching material with the principles necessary for designing effective MEAs. This study used the document analysis method to collect the data. A researcher and an expert in the field of mathematics education examined 36 problem-solving activities in the teaching material separately. Based on the relevant literature, the researchers created a form that lists in detail the criteria to be provided for each principle by using the content analysis to understand the compliance of activity with the MEA design principles. The qualitative data obtained using this form consisting of 19 criteria were quantified, and descriptive analyses were performed. As a result of these analyses, they found that the level of providing the MEA design principles was above the intermediate level in general. It is concluded that the activities provide the principles of effective prototype and reality at the highest level, while the principles of model generalization and model construction have the lowest conformity level. Also, the model documentation and self-assessment principles were higher than the medium level in terms of the compatibility of the MEA design principles.

The activities in the secondary school mathematics practices course books (6th, 7th, and 8th grades) were taught as part of the mathematics applications course since 2017 in Turkey. The published by the MoNE were examined by Saka and Alkan (2022) based on the MEA design principles. By using the document analysis method, the data were analyzed according to the MEA design principles. The results showed that the self-evaluation principle was the lowest level of activities in the 6th and 7th-

grade course books, while it was slightly above the intermediate level in the 8th-grade course book activities. For all grade levels, the activities in the books demonstrated more than moderately suitable for the model generalization principle, and they were prepared in compliance with the principles of reality and effective prototype. Regarding the model documentation principle, the activities in the 6th and 8th-grade course books were above the intermediate level, while those in the 7th-grade course book were found to be below the intermediate level. To conclude, it was determined that the activities in the course books for all three grade levels generally met the design principles of MEA above the medium level.

The activities in the 9th-grade mathematics course book were analyzed by Urhan and Dost (2017) according to the MEA design principles defined by Lesh and Doerr (2003a), except for the effective prototype principle. The data source of the research was the questions in the sections "Mathematics Workshop" and "Project" in the ninth-grade mathematics course book. This book, prepared by MoNE (2013b), comprises six units, including sets, equations and inequalities, functions, congruency and similarity of triangles, right triangles and trigonometry, area of triangles and vectors, and data, counting, and probability. While document analysis was used to collect data, content analysis was used to analyze the data. Based on the results, it was determined that 16 out of the 73 activities in the textbook are model-eliciting activities. The results showed that the "Equations and Inequalities" unit contains the most model-eliciting activities; on the other hand, none of the activities in the "Congruency and Similarity of Triangles" and "Right Angled Triangle and Trigonometry; Area of Triangles and Vectors" units could be defined as model-eliciting activities. Also, it was found that none of the model-eliciting activities in the textbook complied with all of the MEA design principles; only 5 out of 16 activities partially complied with the self-evaluation and model externalization principles, and the rest fully adhered to the principles. The research findings indicated that although the ninth-grade mathematics textbook is quantitatively rich in terms of principle-based model-eliciting activities, it is qualitatively insufficient.

Çavuş-Erdem et al. (2017) conducted a research study to investigate how much mathematical modeling is included in secondary school mathematics textbooks and to what extent the concept of modeling reflects mathematical modeling. The data source of the study was all grade levels (5, 6, 7, and 8) mathematics textbooks in the 2016-2017 academic year in Turkey. They were analyzed by document review and content analysis method in terms of the learning area, the grade level, the section where the model is used, and the type of model used. In this context, the code list related to the model type before the analysis was determined as a concrete model, equality/inequality, graphics, and other models in the research. According to the results, modeling is considered as "modeling mathematics" rather than as a means of mathematical modeling in middle school mathematics textbooks. In addition, it has been determined that the concept of modeling is used intensively in some subjects, while it is not used at all in some topics. In addition, it has been determined that the concept of modeling is used a lot in some subjects in mathematics books. In contrast, it is not used at all in some subjects and class levels, and the models used are limited only to concrete and visual structures. Therefore, modeling in mathematics textbooks aims to model mathematical concepts into concrete and visual forms instead of mathematizing the problem. Based on these results, the researchers suggested that the modeling approach in the textbooks should be revised to represent the purpose of mathematical modeling by considering the focus on mathematical modeling in the mathematics education program.

Tekin Dede and Bukova Güzel (2013b) conducted a case study to analyze the design process of an MEA called Obesity Problem, created by four secondary school mathematics teachers in accordance with the framework of MEA design principles. The study's data sources were the activity and transcriptions of the recorded videotape. The content analysis method was used to analyze the video recordings, while the document analysis method was used to analyze the activity. The researchers examined the extent to which the activity aligns with the MEA design principles without using specific tools. They determined whether the task was completely appropriate, partially appropriate, inappropriate, and unidentifiable with

the MEA design principles. Since this problem was not implemented in schools, the effective prototype principle was not included in the analyses. The findings indicated that the activity was completely appropriate for the principles of model construction, reality, model externalization, and model generalization. However, it was partially appropriate for the principle of self-evaluation. Also, the simple prototype principle could not be identified. As a result, the teachers, who started the design process from real-life situations, created a generalizable and appropriate model for secondary school students.

Elçi (2020) conducted a case study to explore how well the model eliciting activities developed by secondary school teacher candidates align with MEA design principles proposed by Lesh et al. (2000). Thirty-eight senior university students taking the Mathematical Modeling course participated in this study in eight groups. The participants were selected by using a convenience sampling method. The data of this study were eight model-eliciting activities and solution reports created by the groups. The document analysis was used to analyze the activities and their analysis designed by groups based on the design principles of MEAs, except for the effective prototype principle. The compatibility of the activities with each principle was examined by coding them as "totally appropriate," "partially appropriate," and "inappropriate." The findings indicated that the designed MEAs complied with the reality principle at the highest level. In contrast, they complied with the construct shareability and reusability principle at the lowest level. As a result, it has been suggested that candidates for secondary school mathematics teachers should gain more experience by practicing more model-eliciting activities and implementing the model-eliciting activities in the classroom.

Şen (2020) investigated the perspectives of primary education prospective mathematics teachers regarding the design of mathematical modeling problems and their experiences throughout the design process. The research study was conducted as a part of the Mathematical Modeling in Primary Education course, and forty-nine prospective mathematics teachers participated in this study. Qualitative research design was used to collect data, and content analysis method was used to analyze

data. The data set of the study included mathematical modeling activities and their reports created by the prospective mathematics teachers. The mathematical modeling activities were evaluated based on the mathematical modeling activity design principles, including the principles of reality, model construction, self-assessment, model documentation, model shareability and reusability, and effective prototype. The findings revealed that prospective teachers encountered difficulties in designing mathematical modeling problems that provided all the specified principles. All teachers designed their mathematical modeling problems in connection with the daily life scenarios. It was determined that 63% of the prospective teachers' designed mathematical modeling problems satisfied the principles of reality, model construction, self-assessment, model documentation, and model shareability and reusability. However, these problems did not fulfill the criterion of being effective prototypes. Also, according to the prospective teachers, the process of designing the mathematical modeling problems required a significant amount of time and they had difficulties in constructing mathematical models and selecting appropriate subjects.

Kula Unver et al. (2018) conducted a research study to examine the factors uncovered during the posing mathematical modeling activities by preservice mathematics teachers. Twenty-seven preservice secondary school mathematics teachers who took the Mathematical Modeling Course participated in the study by forming their groups. Qualitative research method was used to collect the data and content analysis method was used to analyze the data. The instruments of the study were mathematical modeling problems created by the participants, and the reflection papers about the processes of designing and solving the problems. Data analysis was performed by two researchers by evaluating the data and identifying the codes individually. The study's findings indicate that participants took into account several factors when formulating and solving modeling problems, including the development of a mathematical model, relevance to real-life situations, being understandable and interesting, integration of real-world data, and the inclusion of visual elements. Their approach to solutions primarily focused on utilizing

mathematical expressions, ensuring alignment with the modeling process, and being relevance in real-life scenarios.

Bukova Guzel (2011) constructed a case study to analyze the preservice mathematics teachers' approaches to create and solve the mathematical modeling problems, the suitability of the problems and their solutions for modeling. The participants were thirty-five preservice secondary school mathematics teachers within six groups who attended the Mathematical Modeling Course. Data sources of this study were modeling problems prepared by teacher candidates, their group presentations and transcript of video recordings. Data were analyzed by using qualitative analysis methods in terms of the mathematical modeling processes, which are (i) understanding the task, (ii) simplifying the task, (iii) mathematizing, (iv) working mathematically using individual mathematical competencies, (v) interpreting and (vi) validating. Each processes were coded as "complete performance", "insufficient performance", "incorrect displayed" or "no approach displayed". As a result of the study, participants created modeling problems that met the several criteria, including relevance to real-life situations, adaptability these situations to mathematical concepts, understanding and simplifying the problem, alignment with the curriculum, appropriateness of students' levels of mathematical knowledge, and drawing attention. On the other hand, they faced challenges in the interpretation and validation phases of the modeling process.

2.2.2 Previous Research Studies in Other Countries

Berget (2022) investigated Norwegian upper secondary school mathematics textbook modeling tasks and national examination tasks in terms of modeling cycle that could consist of different perspectives on mathematical modeling. This study focused on identifying which steps of the modeling cycle are required to solve modeling tasks in textbooks and national examinations and examining the differences and similarities between the intended, enacted, and assessed curriculum. The steps of the modified modeling cycle from Blum and Leiβ (2007) were

constructing, simplifying/structuring, mathematizing, working mathematically, interpreting, validating, and exposing. This study used the content analysis method to analyze 152 tasks from the written national examination in 2014-2018 and 514 tasks from the three different textbooks. By using a modeling cycle to examine tasks, it is possible to gain an understanding of how the various sub-competencies involved in modeling are emphasized in both the textbooks and the national exam. According to the results, the assessed curriculum (national exam tasks) and enacted curriculum (textbook tasks) were related to the intended curriculum. Furthermore, there were differences between the initial points of the modeling process in the intended curriculum and the tasks presented in textbooks and exams.

Jung (2015) discussed the strategies employed by two middle school teachers and their students during a semester as they engaged in modeling activities. Jung and these two teachers create modeling lessons together using MEAs three times in eleven weeks. Throughout the modeling activity, students presented the mathematical strategies they found by working in groups, and the teachers analyzed students' work. The data sources of this study were audiotaped interviews and discussions with the two eighth-grade teachers, along with their students' written reports. As a result of this study, they presented that the strategies were categorized under six principles of the MEAs to ensure that the models comply with the design principles as follows:

- The reality principle is related to whether students understand the task based on their own real-life experiences.
- The effective prototype principle is about guiding students through inquiry to develop a mathematically meaningful model.
- The model generalizability principle is related to the students' creation of a generalizable model.
- The model documentation principle is related to students documenting their modeling processes.

- The model construction principle is related to the process of developing students' models.
- The self-evaluation principle is related to students applying to peer review to assess their own responses.

Meyer (2015) examined how well textbooks fulfill the promise of one specific standard of CCSSI (2010), which is mathematical modeling. He analyzed 83 tasks in two textbooks, which are a geometry textbook and an algebra 1 textbook published by McGraw-Hill according to the Common Core State Standards for Mathematics [CCSSM] high school modeling standard describes five different actions to complete modeling tasks, namely, identifying essential variables in a situation, formulating models from those variables, performing operations using those models, interpreting the results of those operations, and validating the conclusions of those results (CCSSI, 2010, p. 72). As a result of this study, while textbooks emphasize the modeling actions of performing operations and interpreting results, they do not emphasize the modeling actions of identifying variables, formulating models, and validating conclusions. To conclude, these two textbooks provide specific opportunities for both students and teachers to engage in modeling activities, while other modeling opportunities remain unavailable for student learning and teacher instruction.

Zwaneveld et al. (2017) evaluated the extent to which the tasks in the two most frequently used Dutch upper secondary school mathematics textbooks, "Number and Space" and "Modern Mathematics," comply with the criteria for genuine mathematical modeling with respect to modeling purposes which are explanation, prediction, compression, inspiration, communication, unification, abstraction, analysis, verification, exploration, decision, optimization, specification, realization and steering and control (Van Overveld & Borghuis, 2013), and characteristic modeling activities are conceptualizing, mathematizing, solving, interpreting, verifying, validating, iterating, communicating, and reflecting (Perrenet & Zwaneveld, 2012). Results showed that many of the modeling tasks did not meet

these criteria. The findings revealed that nearly all of the problem-solving tasks involved context-based task-solving. Also, conceptualizing, mathematizing, and interpreting occurred in almost 30% of the tasks, while the remaining five characteristics of modeling activities were either rare or fully absent. In addition, the results related to the presence of modeling purposes showed that while the analysis and explanation purposes were included the most, optimization, decision, and prediction followed this. Also, approximately 60% of the tasks did not have a modeling purpose.

2.3 Summary of the Literature Review

The literature review provides a comprehensive overview of various aspects of mathematical modeling in mathematics education. The fundamental concepts related to this present study, including model, modeling, modeling approaches and perspectives, specifically focusing on MEAs within the MMP framework are presented in detail. Furthermore, the importance of modeling in mathematics education and the role of mathematics textbooks in teaching and learning mathematics are introduced.

The literature presents several previous research studies related to the analysis of mathematical modeling activities in teaching materials and textbooks, both in Turkey and other countries. These studies revealed both similarities and differences. In Turkey, most of the research studies (Çavuş-Erdem et al., 2017; Doruk, 2019; Saka & Alkan, 2022; Urhan & Dost, 2017) have primarily focused on assessing the compatibility of modeling activities in mathematics teaching materials and textbooks with mathematical modeling design principles, mostly Lesh et al.'s (2000, 2003) design principles of MEAs. Also, some of the research studies (Elçi, 2020; Kula Unver et al., 2018; Şen, 2020; Tekin Dede & Bukova Güzel, 2013b) have examined the modeling problems prepared by preservice teachers. Research studies in other countries (Berget, 2022; Jung, 2015; Meyer, 2015; Zwaneveld et al., 2017) have explored various aspects of mathematical modeling tasks and activities in textbooks

and national examinations in terms of different perspectives, such as the modeling cycle, modeling actions and standards, modeling purposes, and characteristics of modeling activities. In studies conducted in other countries, university-level mathematics textbooks are generally examined, while in Turkey, mostly elementary school-level textbooks have been analyzed. These studies collectively provided insights into the state of mathematical modeling within educational materials and textbooks.

CHAPTER 3

METHODOLOGY

The purpose of this study is to analyze the mathematical modeling activities in the ninth-grade mathematics modeling activity book (MoNE, 2020a) based on their compliance with the model-eliciting activities principles according to Lesh et al. (2003). In the methodology chapter, the design of the study, data set, data analysis, sample coding from the study, validity, and reliability are explained.

3.1 Design of the Study

In this study, the document analysis method is used to collect, review, and analyze data. Document analysis is a qualitative research method that requires interpretation and examination of data to comprehend the information in the documents and the underlying concepts to provide empirical knowledge (Corbin & Strauss, 2008; Rapley, 2007). Also, document analysis is a systematic procedure for reviewing or evaluating printed and electronic documents (Bowen, 2009).

The data of the study is analyzed by the content analysis method. Content analysis is a qualitative research method in which the systematic set of methods for the rigorous analysis, investigation, and verification of the written data contents (Fraenkel, Wallen & Hyun, 2011). To analyze the modeling activities in the textbook, the design principles for model eliciting defined by Lesh and others (2003) are used.

3.2 Data Set

The data set of the study is the ninth-grade mathematics modeling activity book (MoNE, 2020a). MoNE (2020b) stated that the resources containing activities based

on the area, life, and basic skills of the courses found in the course schedules of ninth grades have been made available as of November 20, 2020, to provide resources to students in order to minimize the learning losses of students during the COVID-19 pandemic. One of the books prepared for this purpose is the ninth-grade mathematics modeling activity book (MoNE, 2020a).

The ninth-grade mathematics modeling activity book (MoNE, 2020a) covers the units of the ninth-grade textbook except sets: logic, equations and inequalities, triangles, and data. As a 127-page supplementary book, it consists of 47 modeling activities, sample solution approaches to these activities. This activity book (MoNE, 2020a) does not provide particular information about which ninth-grade topic the modeling activities are related to, and no topic/subject index is provided in the book. The distribution of the modeling activities according to topics has been determined by the researcher (see Table 1).

Table 1. The Distribution of the Modeling Activities According to Ninth Grade Mathematics Topics

Unit Name	**The number of activities**
Logic	1, 2
Sets	-
Equations and Inequalities	3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13a, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44
Triangles	13b, 42, 43, 45
Data	46, 47

As can be seen from the Table 1, there are two activities from logic, none from sets, 40 from equations and inequalities, 4 from triangles, and 2 from data.

At the beginning of the book, the first two pages show the table of contents and the page numbers of the sample solution approaches. After that, 47 modeling activities

are presented, followed by sample solution approaches for these activities. Below, some sample pages from the book are presented:

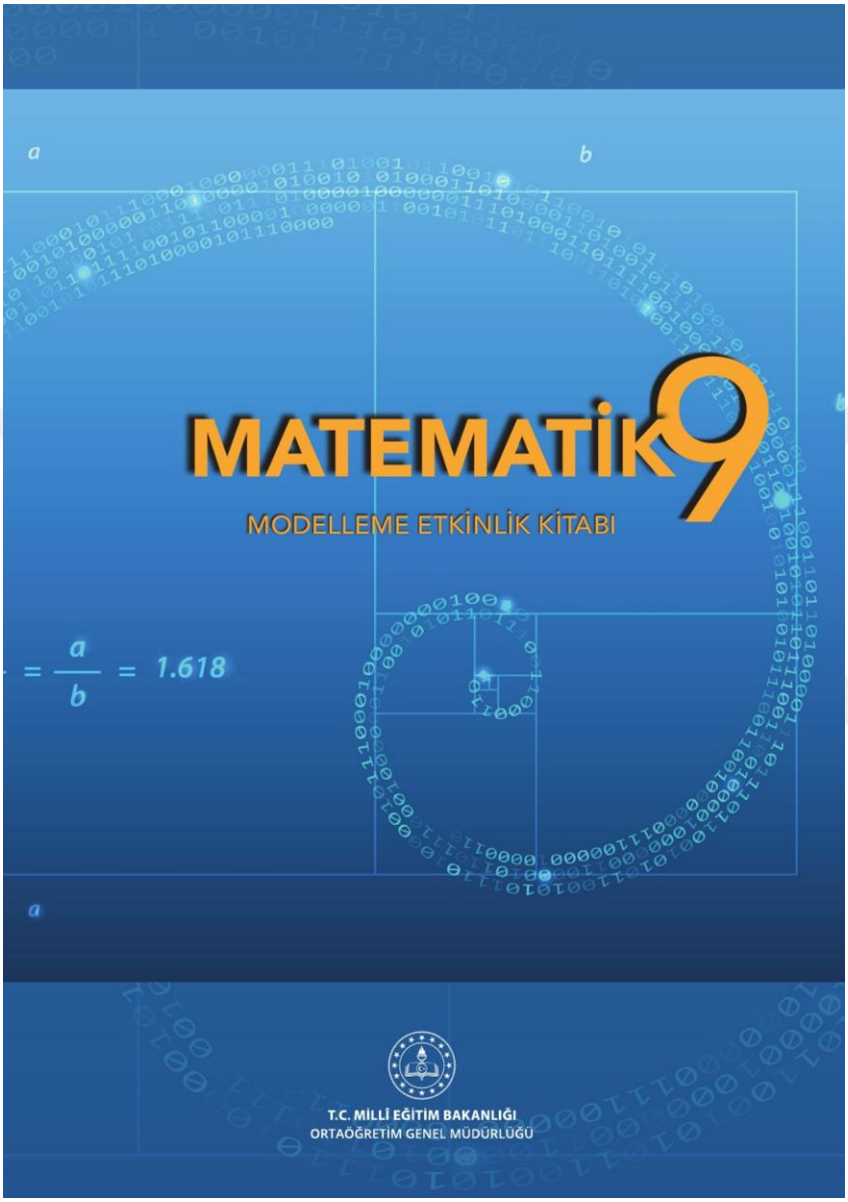


Figure 1. Book Cover of Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a)



Figure 2. Table of Contents of the Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a, p. 1)



Figure 3. Sample Solution Approaches of the Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a, p. 2)

İKRAMİYE KİMİN HAKKI?



Züccaciye mağazası sahibi olan Selma Hanım'ın mağazasında Kemal, Selim ve Burcu adlı üç çalışanı vardır.

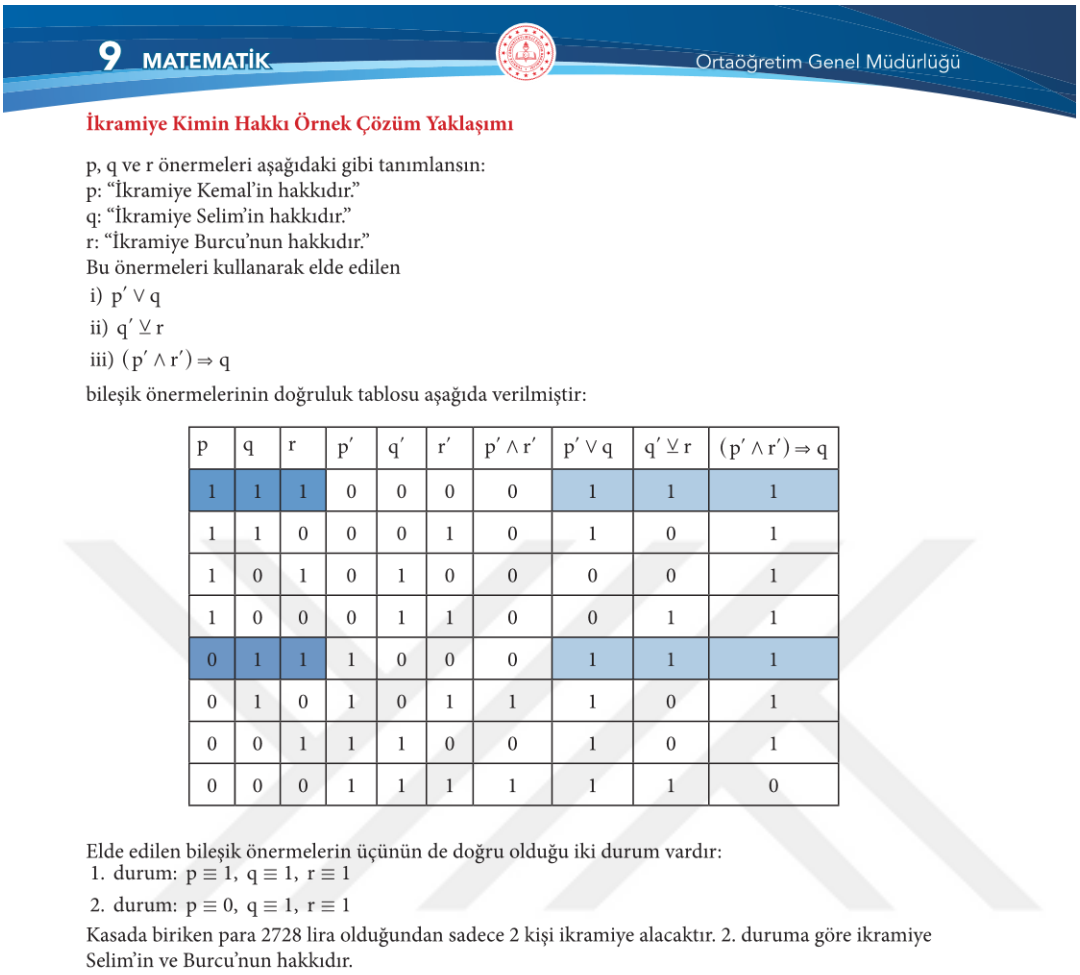
Selma Hanım bir günde en az 1000 Türk lirası satış yapan çalışanlarına ikramiye vereceğini söylemiştir. Günün sonunda kasada 2728 Türk lirası para birikmiştir ancak kimin ne kadarlık satış yaptığı bilinmemektedir.

Aşağıda verilen ifadelerin doğru olduğu kabul edilsin.

- İkramiye Kemal'in hakkı değildir veya Selim'in hakkıdır.
- İkramiye Selim'in hakkı değildir ya da Burcu'nun hakkıdır.
- İkramiye hem Kemal'in hem de Burcu'nun hakkı değil ise Selim'in hakkıdır.

İkramiyeyi hak eden kişinin kim ya da kimler olduğunu bulamayan Selma Hanım'a bu konuda yardımcı olacak bir modelleme yapınız.

Figure 4. Sample Modeling Activity from the Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a, p. 3)



9 MATEMATİK Ortaöğretim Genel Müdürlüğü

İkramiye Kimin Hakkı Örnek Çözüm Yaklaşımı

p, q ve r önermeleri aşağıdaki gibi tanımlansın:
p: "İkramiye Kemal'in hakkıdır."
q: "İkramiye Selim'in hakkıdır."
r: "İkramiye Burcu'nun hakkıdır."
Bu önermeleri kullanarak elde edilen

i) $p' \vee q$

ii) $q' \veebar r$

iii) $(p' \wedge r') \Rightarrow q$

bileşik önermelerinin doğruluk tablosu aşağıda verilmiştir:

p	q	r	p′	q′	r′	p′ ∧ r′	p′ ∨ q	q′ ⊻ r	(p′ ∧ r′) ⇒ q
1	1	1	0	0	0	0	1	1	1
1	1	0	0	0	1	0	1	0	1
1	0	1	0	1	0	0	0	0	1
1	0	0	0	1	1	0	0	1	1
0	1	1	1	0	0	0	1	1	1
0	1	0	1	0	1	1	1	0	1
0	0	1	1	1	0	0	1	0	1
0	0	0	1	1	1	1	1	1	0

Elde edilen bileşik önermelerin üçünün de doğru olduğu iki durum vardır:
1. durum: $p \equiv 1,\ q \equiv 1,\ r \equiv 1$
2. durum: $p \equiv 0,\ q \equiv 1,\ r \equiv 1$

Kasada biriken para 2728 lira olduğundan sadece 2 kişi ikramiye alacaktır. 2. duruma göre ikramiye Selim'in ve Burcu'nun hakkıdır.

Figure 5. Sample Solution Approach from the Ninth Grade Mathematics Modeling Activity Book (MoNE, 2020a, p. 66)

3.3 Data Analysis

In this study, modeling activities are analyzed considering the following design principles of model eliciting activities: model construction, reality, model generalization, simple prototype, model externalization, and self-evaluation principles proposed by Lesh et al. (2003). To determine whether a model eliciting activity comply with the design principles, a form, developed by Doruk (2019) was adapted and used. Doruk (2019) developed this form with the help of the relevant literature (e.g., Lesh et al., 2000; Lesh et al., 2003). The original form is in Turkish

and is given in Appendix A. It has been adapted from Turkish to English. The form contains 19 criteria in total. There are five criteria for the model construction principle, four for the reality principle, two for the model generalization principle, three for the simple prototype principle, two for the model externalization principle, and three for the self-evaluation principle. Also, the researcher analyzes modeling activities by categorizing them according to the units of ninth grade mathematics textbook. To do this, the researcher solves the modeling activities at first, and then decides which modeling activity belongs to which unit by examining the solution approaches.

In the form (Doruk, 2019), some of the criteria did not allow for a clear and complete analysis. To better understand and clarify these criteria, Doruk has been contacted via e-mail. B. K. Doruk stated that "the criteria were developed in Turkish format by examining studies in the literature, primarily referencing the work of Lesh and others (2000; 2003). Therefore, the source of these items is originally from English texts, which may not be entirely clear in our culture" (personal communication, May 16, 2023). Therefore, in this study, the criteria that are not clearly understood have been clarified with the help of relevant literature. For example, an ambiguous criterion "the framework of the task is clear," which is one of the criteria related to the self-evaluation principle, has been clarified as "the purpose is clear (what, when, why, where, and for whom)" according to Lesh et al. (2000, p. 28). See Table 2 for the form adapted from Doruk (2019).

Although some of the studies conducted in this field did not examine the activities in terms of the effective prototype principle (Elçi, 2020; Tekin Dede & Bukova Güzel, 2013b; Urhan & Dost, 2017) by indicating that a certain amount of time must elapse after the completion of the activity for the evaluation of this principle, the evaluation criterion form proposed by Doruk (2019) is used in this study included the effective prototype principle criteria by stating that this principle can be evaluated by experts even if the activity is not applied.

Table 2. Sub-criterion of MEA design principles

Principle	Sub-criterion number	Sub-criterion
The Model Construction Principle (MCP)	MCP1	The task puts students in a situation where they recognize the need to develop a model for interpreting the givens, goals, and possible solution processes in a complex problem-solving situation.
	MCP2	The task does not lead students to produce only an answer to a question that was formulated by others.
	MCP3	The task creates the need for students to revise, correct, explain, and modify the developed model.
	MCP4	The task explicitly states that students need to develop a model for solving the problem.
	MCP5	The task supports developing the most appropriate symbolic descriptions of meaningful situations instead of making meaning out of symbolically stated questions.
The Reality Principle (RP)	RP1	The problem situation included in the task could really happen in a real-life situation.
	RP2	The problem situation included in the task has a structure that can be perceived as realistic for the students.
	RP3	Students can make sense of the situation based on extensions of their own personal knowledge and experience.
	RP4	During the solution process, students' ideas are taken seriously, and they are not forced to follow the way that the author considers the right approach for the problem.

Table 2. (cont'd)

Principle	Sub-criterion number	Sub-criterion
The Model Generalization Principle (MGP)	MGP1	The task allows the developed model to be useful not only to the person who developed it and applicable only to the particular situation presented in the problem but also to be used in other similar situations.
	MGP2	The task leads students to think about creating models that can be shareable, transportable, easily modifiable, and reusable.
The Simple Prototype Principle (SPP)	SPP1	The problem situation is kept away from complexity in a way that allows the student to produce a logical response and develop a prototype.
	SPP2	The solution provides a useful prototype, or metaphor, for interpreting other structurally similar situations.
	SPP3	The solution to the problem does not include complex computational procedures at a level that would prevent noticing conceptual relationships that are essential for dealing with the intended structure of the task.
The Model Externalization Principle (MEP)	MEP1	The response to the problem is in a way that reveals explicitly how students are thinking during the modeling process.

Table 2. (cont'd)

Principle	Sub-criterion number	Sub-criterion
The Model Externalization Principle (MEP)	MEP2	The problem requires students to create a document that clearly reveals their own thoughts and solution approaches regarding the problem situation during the solution process.
The Self-Evaluation Principle (SEP)	SEP1	Students can check the accuracy of their interpretations and conclusions while performing the task and decide whether any improvements or corrections are necessary.
	SEP2	The problem statement strongly suggests appropriate criteria for assessing the usefulness of alternative solutions.
	SEP3	The purpose is clear (what, when, why, where, and for whom).

Note. Adapted from Doruk (2019)

Using the adapted form developed by Doruk (2019), the compatibility of each modeling activity is coded as "completely compatible," "partially compatible," or "incompatible." When coding activities according to principles, for each principle, 2 points are given if it is completely compatible, 1 point if it is partially compatible, and 0 if it is not compatible at all.

There are a total of 47 activities in the ninth-grade mathematics modeling activity book. Since the activity numbered 13 had two different sub-questions, this activity is divided into 13a and 13b. That is, the number of activities to be examined is 48.

The researcher conducts the coding procedure. At first, coding is done together with an associate professor who is an expert in the field of modeling. Thanks to this, it enables the researcher to understand the criteria of the principles better and learn

how to analyze the activities according to these criteria. In this data analysis process, three activities (7, 13a, and 21) are used. After that, another researcher, who holds a master's degree in mathematics education and writes a thesis about mathematical modeling, is asked to code according to the coding form to provide inter-rater reliability. At this stage, nine activities (2, 5, 22, 27, 32, 34, 36, 37, 38) are used. Later, the researchers compare the coding, and the different codes are reviewed again, and a compromise is reached on the conflicting coding results.

In the process of analyzing the modeling activities, each one of the activities are solved by the researcher, and sample solution approaches in the book are also examined. Examining the solution approaches supports data analysis in terms of determining if and how the activity meet the criteria for each MEA design principles. For example, while deciding on the suitability of the activity to the model generalization principle, sample solution approaches show whether it is a generalizable or reusable model. Also, for the simple prototype principle, these solution approaches allow us to determine whether the solutions provides a useful prototype or metaphor for interpreting other structurally similar situations.

The data obtained as a result of coding are qualitative. To interpret the compatibility levels of the modeling activities to the MEA design principles by descriptive statistical methods, it is necessary to convert qualitative data into quantitative data with the help of some calculations. To do this, the total score obtained from the criteria of each principle is calculated and divided into the highest total score that can be obtained from the criteria of that principle. Finally, the sum of the scores obtained for each principle is divided by the highest score that can be obtained from all criteria. In this way, a score, which is between 0 and 1 is obtained that shows the compatibility level of a modeling activity for each principle. Through this calculation, the qualitative findings obtained through the examination of each activity are digitized (Doruk, 2019). This final score gives the general compatibility of the modeling activities for the design principles of MEA. Thanks to this, the compatibility of the activities to the MEA design principles can be analyzed by descriptive statistical methods with the help of MS Excel. If a score approaching 1

indicates that the modeling activity has a high level of compatibility with the design principles; otherwise, the score approaching 0 shows that the level of compatibility is decreasing.

According to the literature (Doruk, 2019), the following scoring system is used when identifying the compliance of the activities with the MEA design principles:

Table 3. Scoring criteria for the level of compatibility with MEA design principles

The range of score	The level of compatibility with MEA design principles
0 - 0.25	Low level
0.25 - 0.50	Moderate
0.50 - 0.75	Above average
0.75 - 1	High level

Note. 0.50 is accepted as medium level

At the end of data analysis, the modeling activities are interpreted according to this scoring criteria to determine the level of compatibility with MEA design principles. Analyses are presented on the basis of units and as a whole book.

3.4 Sample Coding from the Study

To show the sample coding from the study, Activity 13a, titled "Youth of our Neighborhood's Vefa Social Support Group" is given below to be analyzed.

MAHALLEMİZİN VEFA SOSYAL DESTEK GRUBU GENÇLERİ



Koronavirüs (Covid-19) salgınıyla mücadelede ülkemiz pek çok tedbir almış ve almaya da devam etmektedir. Bu tedbirlerden biri de Sağlık Bakanlığı ve Koronavirüs Bilim Kurulu kararları doğrultusunda; İçişleri Bakanlığımızca ülkemiz genelinde öncelikli risk grubunda bulunan 65 yaş ve üstü ile kronik hastalığı bulunan yaşlılarımızın sokağa çıkmalarının yasaklanması uygulamasıdır.

Kronik hastalığı bulunanlara ve yaşlılarımıza virüs bulaşmasını önlemenin yolu, vatandaşlarımızın gönüllü olarak kendilerine karantina uygulamalarıdır.

Geçmişi geleceğe taşıyan, geçmişin canlı şahitleri olan yaşlılarımızı özenle korumalıyız. 65 yaş ve üstü olan büyüklerimizden maddi durumu iyi olmayan, kronik rahatsızlığı bulunan, evlerinde ihtiyaçlarını karşılayacak yakınları bulunmayanların sokağa çıkmadan temel ihtiyaçlarının karşılanması amacıyla ülkemiz genelinde Vefa Sosyal Destek Grupları kurulmuştur.

112, 155 ve 156'ya bu vatandaşlarımızdan gelen çağrı ve talepleri yerine getirmek için Vefa İletişim Merkezlerimizde görevli polis, jandarma, belediye, AFAD, Kızılay ve sivil toplum kuruluşlarımızın ekipleri yoğun bir gayret göstermektedir.

Vefa Sosyal Destek Grubu çalışmalarına destek vermek amacıyla mahallelerinde ikamet eden 65 yaş üstü olan Hasan amcayı, Tuna 2, Tolga 4, Tülay 5 günde bir ziyaret ederek ihtiyaçlarını karşılamaktadır. İkinci kez birlikte 7 Eylül Pazartesi günü ziyaretlerini gerçekleştirdikten sonra Tolga rahatsızlanarak ziyaretlerine ara vermek zorunda kalır. Tuna ve Tolga'nın ziyaretlerinin çakıştığı günlerde tek ziyaret olmak üzere Tolga rahatsızlanmasaydı üçüncü kez birlikte ziyaret yapacakları güne kadar Tuna, Tolga'nın yerine ziyaretleri gerçekleştirir.

Hasan amcanın evi Tuna, Tolga ve Tülay'ın evlerini ikişer ikişer birleştiren doğrusal yolların oluşturduğu üçgenin kenarortaylarının kesişme noktasındadır.

1. Tuna, Tolga ve Tülay birlikte üçüncü ziyaretlerini yaptıktan sonra görevlerini Emre, Eray ve Erdem'e devrediyor.

Ziyaretlerini sırasıyla 3, 5 ve 6 gün aralıklarla gerçekleştiren Emre, Eray ve Erdem'e Hasan amcayı ziyaret edecekleri ilk günlerini bulmada yardımcı olunuz.

Figure 6. Activity 13a from the textbook (MoNE, 2020a, p.17)

The English version of the activity is as follows:

Our country has taken and continues to take many measures in the fight against the coronavirus (Covid-19) pandemic. One of these measures is the implementation of banning elderly people aged 65 and older, who are in the priority risk group, as well as individuals with chronic diseases, from going out on the streets by the Ministry of Internal Affairs in accordance with the decisions of the Ministry of Health and the Coronavirus Science Board throughout the country.

The way to prevent transmission of the virus to those with chronic diseases and to our elderly is for our citizens to apply quarantine to themselves voluntarily.

We must carefully protect our elders, who carry the past into the future and serve as living witnesses of the past. To meet the basic needs of our elders who are 65 years old and older, especially those who have financial difficulties, chronic diseases, and do not have relatives to meet their needs at home, Social Support Groups called "Vefa" have been established throughout our country to meet their basic needs without going outside.

The teams of the police, gendarmerie, municipality, AFAD (Disaster and Emergency Management Authority), Turkish Red Crescent (Kızılay), and non-governmental organizations working in our Vefa Communication Centers are making intensive efforts to fulfill the calls and requests received from these citizens through 112, 155 and 156.

To support the activities of the Vefa Social Support Group, Hasan, an elderly individual over the age of 65 residing in their neighborhood, is visited by Tuna every two days, Tolga every four days, and Tülay every five days to meet their needs. After their second visit together on Monday, September 7th, Tolga falls ill and has to temporarily take a break from his visits. On the days when Tuna and Tolga's visits coincide, Tuna takes over the visits instead of Tolga, if Tolga had not been ill until they were supposed to visit together for the third time.

Hasan's house is located at the intersection of the medians of the triangle formed by linear paths that connect the houses of Tuna, Tolga, and Tülay in pairs.

1) After Tuna, Tolga, and Tulay make their third visit together, they transfer their duties to Emre, Eray, and Erdem.

Help Emre, Eray, and Erdem, who visit Uncle Hasan at intervals of 3, 5, and 6 days respectively, to find their first day to visit him.

Figure 7. The English Version of the Activity 13a (MoNE, 2020a)

Table 4 is given below to demonstrate the evaluation of Activity 13a in terms of the MEA design principles criteria.

Table 4. Evaluation of the activity 13a in terms of the criteria of MEA design principles

Sub-criterion*	Point**	Explanation
MCP1	0	This question can be solved mathematically using the least common multiple without the need for a model.
MCP2	1	The task does not ask students to produce only an answer to this question formulated by others. However, it has only one correct result; that is, everyone who solves it will reach this same result.
MCP3	0	The task is incompatible according to the third criterion of the model construction principle since it does not put students in a situation where they will need to review, correct, explain and modify the developed model.
MCP4	0	The task does not explicitly state that students should develop a model to solve the problem because there is no statement in the question about the necessity of creating a model either. Therefore, it is incompatible with this criterion.
MCP5	0	The question does not support developing symbolic descriptions of the problem situation.

Table 4. (cont'd)

Sub-criterion*	Point**	Explanation
RP1	1	During the coronavirus pandemic, social support centers have been opened to help the elderly and disabled people, but for children who do not experience this period, there may not be a situation that they can encounter in real life.
RP2	0	The problem situation does not have a structure that can be perceived as completely realistic for students.
RP3	1	Students who have experienced the coronavirus pandemic can make sense of the situation based on extensions of their own personal knowledge and experience.
RP4	1	Students do not have to follow the author's solution approach during the solution process, but since the correct result is clear and single, students' ideas do not change this result.
MGP1	1	It can enable the use of the greatest common divisor (GCD) and least common multiple (LCM) for similar periodic condition problems, but it is not applicable or useful for every similar condition.
MGP2	0	The question does not lead students to create a shareable, transportable, easily modifiable, and reusable model.
SPP1	0	The problem situation is not suitable for generating a logical response and developing a prototype.

Table 4. (cont'd)

Sub-criterion*	Point**	Explanation
SPP2	0	The solution does not provide a useful prototype for interpreting other structurally similar situations.
SPP3	2	In solving the problem, it is expected that three of the students will find the day they visited together using LCM and then determine the days of new visitors according to the result. This does not require complex calculation procedures.
MEP1	1	Since the students are not asked to explain their thoughts or prepare a report on the question, it does not reveal how they think during the modeling process. However, it may be understandable by their solutions.
MEP2	0	In the question, the requirement for students to create a document is not clearly stated.
SEP1	1	While students can check the accuracy of their results themselves, there is no situation in which they can decide whether it is necessary to correct or improve them.
SEP2	0	Since the problem statement is not open to assumptions and different ways of solution, it does not propose criteria suitable for alternative solutions.
SEP3	2	The purpose of the question is clear.

*: MCP: "Model Construction Principle", RP: "Reality Principle", MGP: "Model Generalization Principle", SPP: "Simple Prototype Principle", MEP: "Model Externalization Principle", SEP: "Self-Evaluation Principle"
**: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"

3.5 Validity and Reliability

In qualitative research studies, validity and reliability are essential considerations to ensure the findings' credibility and trustworthiness (Fraenkel et al., 2011). While they are often associated with quantitative research, they are also relevant in qualitative research.

Validity is considered the appropriateness, usefulness, correctness, and meaningfulness of the particular inferences researchers derive from the data they gather, while reliability is considered the consistency of the scores or inferences over time, location, and circumstances (Fraenkel et al., 2011). Also, reliability is considered as the degree of consistency in the obtained scores, individuals across multiple administrations of an instrument, and across different sets of items (Fraenkel et al., 2011).

For qualitative studies, reliability corresponds to inter-rater reliability (Lincoln & Guba, 1985). Inter-rater reliability refers to the degree of agreement among different researchers who analyze the same data. Researchers can enhance inter-rater reliability by providing clear guidelines and definitions for coding and interpretation.

In the evaluation of the model-eliciting activities in the ninth-grade mathematics modeling activity book, the activities of two researchers working in the field of mathematics education are scored separately in terms of coding criteria of MEA design principles described in Table 2. Later, the scores are examined, and it is determined that the scoring values of the two researchers are largely compatible. Internal reliability is ensured by reviewing the criteria that differed in scoring and reaching a consensus on scoring.

There are a total of 48 activities in the ninth-grade mathematics modeling activity book. Nine of them (activity numbers: 2, 5, 22, 27, 32, 34, 36, 37, 38) are examined together with another researcher. That is, 18.75% of the whole activities are coded by two researchers. Since all activities have 19 different criteria, each coder makes 171 codes. Then, they come together to compare the coding results. Among 171

coding, 19 contradictions are found. That is, the agreement level is 88.9% by using inter-rater reliability formula (Miles & Huberman, 1994). To eliminate the contradictions and reach an agreement, the two researchers come together again and review the coding that is different. As a result of the discussion, they reached a consensus on the different coding, and the level of agreement reached 100%. Thus, inter-rater reliability is provided.

CHAPTER 4

RESULTS

The study aims to analyze the mathematical modeling activities in the ninth-grade mathematics modeling activity book (MoNE, 2020a) according to their compliance with the design principles of model-eliciting activities proposed by Lesh et al. (2003).

In this chapter, the findings obtained as a result of the analyses are presented. Evaluation of compatibility of the activities in the ninth-grade mathematics modeling activity book with the MEA design principles, which are model construction, reality, model generalization, simple prototype, model externalization, and self-evaluation is given separately.

4.1 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Design Principles of MEA

The mathematics modeling activity book, consisting of 48 activities, is examined according to the design principles of model-eliciting activities. As a result of these analyses, the descriptive statistical inferences of the findings obtained are presented below in Table 5:

Table 5. General Evaluation of Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Design Principles of MEA

MEA Design Principle	Mean	Min	Max
The model construction principle	0.70	0.1	1
The reality principle	0.74	0.38	1
The model generalization principle	0.22	0	1

Table 5. (cont'd)

MEA Design Principle	Mean	Min	Max
The simple prototype principle	0.74	0.33	1
The model externalization principle	0.51	0	1
The self-evaluation principle	0.52	0.33	1

Table 5 shows the means of the compatibility levels obtained by considering all the principles for the conformity of the modeling activities included in the ninth-grade mathematics modeling activity book. Also, the minimum and maximum values on the Table 5 show the lowest and highest values that modeling activities receive according to the MEA design principles for each principle.

As a result of the analyses, general mean is calculated by dividing the scores obtained from the sub-criteria of all MEA design principles by the total score that can be obtained from all criteria in the form. This value indicates the general compliance of modeling activities with MEA design principles. It is found that the general mean of compatibility of the activities in the ninth-grade mathematics modeling activity book is 0.57. This shows that the ninth-grade mathematics modeling activity book provides the design principles of MEA above the medium level which is 0.5.

As can be seen in Table 5, the principles that are determined to be at the highest level are the principles of reality and simple prototype with a mean of 0.74. In contrast, the principle at the lowest level is determined as the model generalization principle with a mean of 0.22. The analysis of activities according to the design principles shows that except for the model generalization principle, the mean scores for each principle are above the medium level. The mean score for the principle of model generalization is at a low level.

The compatibility level of 48 activities with the design principles of MEA in the ninth-grade mathematics modeling activity book and the distribution of these activities according to units is demonstrated in Table 6 below.

Table 6. The Compatibility Level of Activities in the 9th Grade Mathematics Modeling Activity Book with the MEA Design Principles and their Distribution According to Units

Unit name	Activity number	MCP	RP	MGP	SPP	MEP	SEP	Mean
Logic	**1**	0.6	0.63	0.25	0.83	0.5	0.5	**0.58**
	2	0.5	0.75	0.5	1	1	0.5	**0.68**
Equations and Inequalities	**3**	0.8	1	0	0.67	0.5	0.5	**0.66**
	4	0.8	1	0	0.5	0.5	0.5	**0.63**
	5	0.4	0.88	0.25	0.83	0.25	0.5	**0.55**
	6	0.9	0.75	0	0.83	0.5	0.5	**0.66**
	7	0.9	1	0.75	0.83	0.25	0.5	**0.76**
	8	0.5	1	0	0.83	0.5	0.67	**0.63**
	9	0.5	0.88	0	0.83	0.25	0.5	**0.55**
	10	1	0.88	0.25	0.83	0.5	0.5	**0.74**
	11	0.9	0.38	1	1	0.5	0.5	**0.71**
	12	0.3	1	0.75	0.67	0	0.5	**0.55**
	13a	0.1	0.38	0.25	0.33	0.25	0.5	**0.29**
	14	0.9	0.88	0.5	0.83	0.25	0.5	**0.71**
	15	0.5	0.75	0	0.33	0.25	0.33	**0.42**
	16	0.6	0.75	0	0.67	0.25	0.5	**0.53**
	17	0.5	0.88	0.5	0.83	0.25	0.5	**0.61**
	18	0.9	0.88	0.25	0.83	0.5	0.5	**0.71**
	19	0.4	0.75	0	0.83	0.25	0.5	**0.5**
	20	0.5	0.88	0	0.83	0.75	0.5	**0.61**
	21	0.8	0.63	0.75	0.83	1	0.5	**0.74**
	22	0.6	0.5	0.25	1	1	0.5	**0.63**
	23	0.9	0.75	0.25	0.67	0.5	0.5	**0.66**

MCP: Model Construction Principle, RP: Reality Principle, MGP: Model Generalization Principle, SPP: Simple Prototype Principle, MEP: Model Externalization Principle, SEP: Self-Evaluation Principle

Table 6. (cont'd)

Unit name	Activity number	MCP	RP	MGP	SPP	MEP	SEP	Mean
Equations and Inequalities	**24**	0.5	0.75	0	0.83	0.25	0.5	**0.53**
	25	0.5	1	0.25	0.83	0.25	0.5	**0.61**
	26	0.4	0.5	0	0.83	0.25	0.5	**0.45**
	27	0.4	0.38	0	0.83	0.25	0.5	**0.42**
	28	1	0.75	0	0.67	1	0.67	**0.74**
	29	0.9	0.75	0.5	0.83	0.75	0.5	**0.74**
	30	0.9	0.38	0	0.67	0.5	0.5	**0.55**
	31	0.5	0.75	0	0.83	0.5	0.5	**0.55**
	32	0.8	0.63	0	0.5	0.25	0.5	**0.53**
	33	0.7	0.63	0	0.67	0.25	0.5	**0.53**
	34	0.9	0.75	0.25	0.67	1	0.67	**0.74**
	35	0.9	0.75	0	0.67	1	0.5	**0.68**
	36	1	0.88	0.5	0.5	1	0.83	**0.82**
	37	0.5	0.75	0	0.33	1	0.5	**0.53**
	38	1	0.88	0.25	0.5	1	1	**0.82**
	39	0.9	0.88	0	0.67	0.5	0.5	**0.66**
	40	0.5	0.75	0	0.67	0.25	0.5	**0.5**
	41	0.9	0.75	0	0.67	0.5	0.5	**0.63**
	44	0.7	0.88	0.25	0.83	0.5	0.5	**0.66**
Triangles	**13b**	0.7	0.63	0.25	0.83	0.5	0.5	**0.61**
	42	0.8	0.38	0.75	0.83	0.75	0.5	**0.66**
	43	0.8	0.38	0.25	0.83	0.5	0.5	**0.58**
	45	1	0.63	0.25	0.83	0.75	0.5	**0.71**
Data	**46**	0.8	0.88	0.25	0.83	0.25	0.5	**0.66**
	47	0.6	1	0.25	0.83	0.25	0.5	**0.63**

MCP: Model Construction Principle, RP: Reality Principle, MGP: Model Generalization Principle, SPP: Simple Prototype Principle, MEP: Model Externalization Principle, SEP: Self-Evaluation Principle

Table 6 shows that four out of 48 activities (13a, 15, 26, and 27) are at moderate levels (between 0.25 and 0.50) in terms of meeting the design principles of MEA. Also, only three out of 48 activities (7, 36, and 38) meet the design principles of MEA at a high level (above 0.75). The remaining 41 activities take values between 0.50 and 0.75, which is above average.

According to Table 6, the compatibility level of 24 (50%) of the activities for MCP, 34 (70.8%) of the activities for RP, 5 (10.4%) of the activities for MGP, 29 (60.4%) of the activities for SPP, 13 (27.1%) of the activities for MEP, and 2 (4.2%) of the activities for SEP are at high level. If the activities at the moderate level of compatibility are added to these values, the proportion of the activities above the average compatibility for MCP, RP, MGP, SPP, MEP, and SEP become 87.5%, 87.5%, 20.8%, 93.7%, 60.4%, and 98%, respectively. Furthermore, the compatibility level is at a low level for one of the activities in the MCP and MEP dimensions and 22 (45.8%) activities in the MGP dimension. The data analysis also shows that activities 36 and 38 meet the design principles of MEA at the highest level, whereas activity 13a meets the principles at the lowest level.

Table 6 shows that the least number of modeling activities belongs to the Units of Logic and Data, and the most are from the Unit of Equations and Inequalities according to the units in the ninth-grade mathematics curriculum. There are 2 activities from the Units of Logic and Data,, 4 from the Units of Triangles and 40 from the Units of Inequalities. There are no activities related to the Units of Sets in the modeling activity book. Accordingly, modeling activities related to logic and data are 4.17% of the activities in the book, equations and inequalities are 83.33% and triangles are 8.33%.

Table 7. The Compatibility Level of Activities in the 9th Grade Mathematics Modeling Activity Book with the MEA Design Principles According to the Units

Unit name	MCP	RP	MGP	SPP	MEP	SEP	Mean
Logic	0.55	0.69	0.38	0.92	0.75	0.5	0.63

Table 7. (cont'd)

Unit number	MCP	RP	MGP	SPP	MEP	SEP	Mean
Equations and Inequalities	0.69	0.76	0.19	0.72	0.5	0.53	0.62
Triangles	0.83	0.51	0.38	0.83	0.63	0.5	0.64
Data	0.7	0.94	0.25	0.83	0.25	0.5	0.64

Table 7 shows that all units are above average in terms of the compatibility of activities to the principles of model construction, reality, simple prototype and self-evaluation. On the other hand, the compliance of all units with the model generalization principle is below average.

The compatibility of the model construction principle is highest for the Unit of Triangles while it is at the lowest level for the Unit of Logic. From the point of view of the reality principle, the Unit of Data has the highest compatibility, while the Unit of Triangles has the lowest. The compliance of the activities with the model generalization principle is low in general and it is higher in the Units of Logic and Triangles than they are in the Unit of Equations and Inequalities. According to the simple prototype principle, the Unit of Logic has the highest level of compatibility, while The Unit of Equations and Inequalities has the lowest level. In terms of the model externalization principle, there is a big difference between the highest and the lowest level of compatibility. The Unit of Logic is at a high level with 0.75, while the Data is at a low level with 0.25. The compatibility levels show similarities in the self-evaluation principle. The highest and the lowest values are very close to each other. The Unit of Equations and Inequalities is the highest with 0.53, while the other subjects are at the same level with 0.50.

The following sections present values that show the extent to which the activities in the modeling activity book are compatible with each of the design principles based on each sub-criterion.

4.1.1 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Model Construction Principle

The compatibility level of 48 activities in the ninth-grade mathematics modeling activity book according to the model construction principle is given in Table 8 below.

Table 8. The Compatibility Level of Activities in the 9th Grade Mathematics Modeling Activity Book with the Model Construction Principle

Activity number	MCP1	MCP2	MCP3	MCP4	MCP5	Mean
1	2	0	0	2	2	0.6
2	1	1	2	0	1	0.5
3	2	2	1	2	1	0.8
4	2	2	1	2	1	0.8
5	1	1	0	0	2	0.4
6	2	2	1	2	2	0.9
7	2	2	1	2	2	0.9
8	1	2	1	0	1	0.5
9	1	2	1	0	1	0.5
10	2	2	2	2	2	1
11	2	2	1	2	2	0.9
12	0	2	0	0	1	0.3
13a	0	1	0	0	0	0.1
13b	1	2	1	1	2	0.7
14	2	2	1	2	2	0.9
15	1	2	0	0	2	0.5
16	1	2	1	0	2	0.6
17	0	2	1	0	2	0.5
18	2	2	1	2	2	0.9
19	0	2	0	0	2	0.4

Table 8. (cont'd)

Activity number	MCP1	MCP2	MCP3	MCP4	MCP5	Mean
20	0	2	1	0	2	0.5
21	2	2	1	1	2	0.8
22	1	1	1	1	2	0.6
23	2	2	1	2	2	0.9
24	0	2	1	0	2	0.5
25	1	2	0	0	2	0.5
26	0	2	0	0	2	0.4
27	0	1	1	0	2	0.4
28	2	2	2	2	2	1
29	2	2	1	2	2	0.9
30	2	2	1	2	2	0.9
31	0	2	1	0	2	0.5
32	1	2	2	1	2	0.8
33	2	2	0	1	2	0.7
34	2	2	2	1	2	0.9
35	2	2	1	2	2	0.9
36	2	2	2	2	2	1
37	1	1	1	0	2	0.5
38	2	2	2	2	2	1
39	2	2	1	2	2	0.9
40	0	2	1	0	2	0.5
41	2	2	1	2	2	0.9
42	2	2	1	1	2	0.8
43	2	2	1	1	2	0.8
44	2	1	0	2	2	0.7

Table 8. (cont'd)

Activity number	MCP1	MCP2	MCP3	MCP4	MCP5	Mean
45	2	2	2	2	2	1
46	2	2	1	1	2	0.8
47	1	2	1	0	2	0.6
Mean	0.66	0.90	0.48	0.51	0.91	0.70

MCP: Model Construction Principle

Table 8 shows that the criteria of model construction principle other than the third criterion related to creating the need for students to review, correct, explain, and change the developed model are above the medium level. Based on the scoring criteria in Table 3, the second and fifth criteria are at a high level, the first and fourth, related to the need to develop a model, are above the medium level, whereas the third is moderate.

While 26 of 48 activities (54.2%) are completely compatible, 12 (25%) are partially compatible, and 10 (20.8%) are incompatible with the first criterion (MCP1). It is observed that tasks usually put students in a situation where they will realize the need to develop models.

While 40 of 48 activities (83.3%) are completely compatible, 7 (14.6%) are partially compatible, and 1 (2.1%) are incompatible with the second criterion (MCP2). That is, since the answers to the questions are mostly specific and everyone will reach the same result, the tasks ask students to answer a question formulated by others.

While 8 of 48 activities (16.7%) are completely compatible, 30 (62.5%) are partially compatible, and 10 (20.8%) are incompatible with the third criterion (MCP3). It can be said that the task usually does not create a need for students to review, correct, explain, and change the created model.

While 20 of 48 activities (41.7%) are completely compatible, 9 (18.8%) are partially compatible, and 19 (39.5%) are incompatible with the fourth criterion (MCP4). This criterion is very close to the average, which means in some questions, students

clearly are asked to develop a model to solve the problem, while in others, they do not.

While 41 of 48 activities (85.4%) are completely compatible, 6 (14.6%) are partially compatible, and 1 (2.1%) are incompatible with the fifth criterion (MCP5), which indicates that the tasks expect students to develop the most appropriate symbolic descriptions of meaningful situations instead of making sense out of symbolically expressed questions.

According to Table 8, the 10th, 28th, 36th, 38th, and 45th activities meet the design principles of MEA at the highest level, while activity 13a meets the design principles of MEA at the lowest level. Considering the model construction principle, activity 38, one of the activities with the highest score, and activity 13a, which has the lowest score, are examined in terms of the criteria related to the relevant principle.

The analysis of activity 13a (in Appendix B), which has the lowest score in the dimension of the model construction principle, is given in the following table:

Table 9. Evaluation of Activity 13a in terms of the Model Construction Principle

Sub-criterion**	**Point***	**Explanation**
MCP1	0	This question can be solved mathematically using the least common multiple without the need for a model. That is, this activity directs students to find results rather than putting them in a situation where they will recognize the need to develop a model.
MCP2	1	The task does not ask students to produce only an answer to this question formulated by others. However, it has only one correct result; that is, everyone who solves it will reach this same result.

Table 9. (cont'd)

Sub-criterion**	Point*	Explanation
MCP3	0	The task is incompatible according to the third criterion of the model construction principle since it does not put students in a situation where they will need to review, correct, explain, and modify the developed model.
MCP4	0	The task does not explicitly state that students should develop a model to solve the problem because there is no statement in the question about the necessity of creating a model either. Therefore, it is incompatible with this criterion.
MCP5	0	The question does not support developing symbolic descriptions of the problem situation.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: MCP: Model Construction Principle

The analysis of activity 38 (in Appendix B), which has the highest score in the dimension of the model construction principle, is given in the following table:

Table 10. Evaluation of Activity 38 in terms of the Model Construction Principle

Sub-criterion**	Point*	Explanation
MCP1	2	The task asks students to notice that they need to develop a model to be able to interpret the given numerical values contained in the tables and to solve the complex problem situation.
MCP2	2	The answers to the questions in the task are clear, but they do not lead the students to a solution that has been formulated by others. Students can also produce different answers to the question.

Table 10. (cont'd)

Sub-criterion**	Point*	Explanation
MCP3	2	Since it is indicated that "Explain in detail…" in the question, this task creates the need for students to explain the developed model.
MCP4	2	Since the task includes an expression that states, "Create a mathematical model that explains…", it explicitly requests students to develop a model.
MCP5	2	By making sense of the values given in the tables in the questions, the task supports students to reach the desired results with symbolic representations created by themselves. That is, it does not lead to make meaning out of symbolically stated questions.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: MCP: Model Construction Principle

4.1.2 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Reality Principle

The compatibility level of 48 activities in the ninth-grade mathematics modeling activity book according to the reality principle is given in Table 11 below.

Table 11. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Reality Principle

Activity number	**RP1**	**RP2**	**RP3**	**RP4**	**Mean**
1	2	1	1	1	0.63
2	2	2	1	1	0.75
3	2	2	2	2	1
4	2	2	2	2	1
5	2	2	2	1	0.88
6	2	1	1	2	0.75
7	2	2	2	2	1
8	2	2	2	2	1
9	2	1	2	2	0.88
10	2	1	2	2	0.88
11	0	0	1	2	0.38
12	2	2	2	2	1
13a	1	0	1	1	0.38
13b	1	1	1	2	0.63
14	2	1	2	2	0.88
15	2	1	1	2	0.75
16	2	1	1	2	0.75
17	2	1	2	2	0.88
18	2	1	2	2	0.88
19	2	1	1	2	0.75
20	2	1	2	2	0.88
21	2	0	1	2	0.63
22	2	1	1	0	0.5
23	2	1	1	2	0.75
24	2	1	1	2	0.75
25	2	2	2	2	1

Table 11. (cont'd)

Activity number	RP1	RP2	RP3	RP4	Mean
26	1	0	1	2	0.5
27	1	1	1	0	0.38
28	2	1	1	2	0.75
29	2	1	1	2	0.75
30	1	0	0	2	0.38
31	2	1	1	2	0.75
32	1	1	1	2	0.63
33	2	0	1	2	0.63
34	2	1	1	2	0.75
35	2	1	1	2	0.75
36	2	2	1	2	0.88
37	2	2	1	1	0.75
38	2	2	1	2	0.88
39	2	2	1	2	0.88
40	2	1	1	2	0.75
41	2	1	1	2	0.75
42	1	0	0	2	0.38
43	1	0	0	2	0.38
44	2	2	1	2	0.88
45	2	1	0	2	0.63
46	2	2	1	2	0.88
47	2	2	2	2	1
Mean	0.89	0.57	0.60	0.90	0.74

RP: Reality Principle

Table 11 shows that all criteria are above the medium level. As can be seen, the first criterion (RP1) related to a real-life problem situation, and the fourth criterion (RP4) is about the students being free in the solution approach are at a high level, and the

second criterion (RP2) is about realistic problem structure and the third criterion (RP3) related to the appropriateness of the questions to the students' knowledge and experiences are above the medium level based on the scoring criteria in Table 3.

The analysis of data shows that 39 of 48 activities (81.3%) are completely compatible, 8 (16.7%) are partially compatible, and 1 (2.1%) is incompatible with the first criterion of RP. That is, problem situations in activities can be expected to really happen in a real-life situation.

While 15 of 48 activities (31.3%) are completely compatible, 25 (52.1%) are partially compatible, and 8 (16.7%) are incompatible with the second criterion of RP. As can be understood from these values, more than half of the activities are partially compatible with this criterion. This indicates that some problem situations involved in the tasks can be perceived realistically by the students, while others cannot.

While 14 of 48 activities (29.2%) are completely compatible, 30 (62.5%) are partially compatible, and 4 (8.3%) are incompatible with the third criterion of RP. It can be observed that students can make sense of the situation based on their own personal knowledge and experience in a way that is above average.

Not all activities that could happen in real life seem realistic to students. At the same time, it can also be interpreted by looking at the scores in Table 11, where most of the situations that are realistic for students can be made sense of with their own experience and knowledge.

While 41 of 48 activities (85.4%) are completely compatible, 5 (10.4%) are partially compatible, and 2 (4.2%) are incompatible with the fourth criterion of RP. It can be concluded that students' ideas are mostly taken seriously during the solution process.

According to Table 11, the activities that provide the design principles of MEA at the highest level are activities 3, 4, 7, 8, 12, 25, and 47, while the activities that provide the design principles of MEA at the lowest level are activities 11, 13a, 27, 30, 42 and 43.

Considering the reality principle, Activity 12, which is one of the activities with the highest score, and Activity 13a, which is one of the activities with the lowest score, are examined in terms of the criteria related to the relevant principle.

The analysis of Activity 13a (in Appendix B), which has the lowest score in the dimension of the reality principle, is given in the following table:

Table 12. Evaluation of Activity 13a in terms of the Reality Principle

Sub-criterion	Point*	Explanation
RP1	1	During the coronavirus pandemic, social support centers have been opened to help the elderly and disabled people, but for children who do not experience this period, there may not be a situation that they can encounter in real life.
RP2	0	The problem situation does not have a structure that can be perceived as entirely realistic for students.
RP3	1	Students who have experienced the coronavirus pandemic can make sense of the situation based on extensions of their own personal knowledge and experience while those who have not experienced this period may not be able to understand it.
RP4	1	Students do not have to follow the author's solution approach during the solution process, but since the correct result is clear and single, students' ideas do not change this result.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: RP: Reality Principle

The analysis of activity 12 (in Appendix B), which has the highest score in the dimension of the reality principle, is given in the following table:

Table 13. Evaluation of Activity 12 in terms of the Reality Principle

Sub-criterion**	Point*	Explanation
RP1	2	This activity is completely compatible with this criterion for high school students who will usually be preparing for the university exam since they often take essay exams and take private lessons or courses in different lessons that they have difficulty understanding in their daily lives.
RP2	2	The problem situation is in a structure that can be perceived realistically because it is a situation that students may often encounter in their daily lives.
RP3	2	Students can make sense of the situation based on their own personal knowledge and experiences.
RP4	2	Students are not forced to follow the way the author considers the right solution. On the contrary, students are expected to explain the answers they have found with their own ideas.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: RP: Reality Principle

4.1.3 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Model Generalization Principle

The compatibility level of 48 activities in the ninth-grade mathematics modeling activity book according to the model generalization principle is given in Table 14 below.

Table 14. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Model Generalization Principle

Activity number	MGP1	MGP2	Mean
1	1	0	0.25
2	1	1	0.5
3	0	0	0
4	0	0	0
5	1	0	0.25
6	0	0	0
7	2	1	0.75
8	0	0	0
9	0	0	0
10	0	1	0.25
11	2	2	1
12	2	1	0.75
13a	1	0	0.25
13b	1	0	0.25
14	1	1	0.5
15	0	0	0
16	0	0	0
17	1	1	0.5
18	1	0	0.25
19	0	0	0
20	0	0	0
21	2	1	0.75
22	1	0	0.25
23	1	0	0.25
24	0	0	0
25	1	0	0.25
27	0	0	0

Table 14. (cont'd)

Activity number	MGP1	MGP2	Mean
28	0	0	0
29	1	1	0.5
30	0	0	0
31	0	0	0
32	0	0	0
33	0	0	0
34	0	1	0.25
35	0	0	0
36	1	1	0.5
37	0	0	0
38	1	0	0.25
39	0	0	0
40	0	0	0
41	0	0	0
42	2	1	0.75
43	1	0	0.25
44	1	0	0.25
45	1	0	0.25
46	1	0	0.25
47	1	0	0.25
Mean	0.30	0.14	0.22

MGP: Model Generalization Principle

Table 14 represents that both criteria of the model generalization principle are below the medium level. Based on the scoring criteria in Table 3, the first criterion (MGP1) related to the usability of the developed model in other similar situations is considered moderate, and the second criterion (MGP2) about the reusability and shareability of the created models is considered low level. The data in the Table 14 shows that only five activities are completely compatible with MGP1, while half of

the activities do not comply with it. This indicates that most of the models developed at the end of the modeling process are useful only for the person developing it and valid only for the specific situation presented in the problem. Table 14 shows that the case for MGP2 is even worse. While 1 of 48 activities (2.1%) are completely compatible, 11 (22.9%) are partially compatible, and 36 (75%) are incompatible with MGP2. Only one activity completely complies with this criterion, while 36 (75%) of activities are incompatible with the criterion. This indicates that almost none of the tasks lead students to think about developing shareable and reusable models.

Considering the model generalization principle, Activity 11, which has the highest score, and Activity 15, which is one of the activities with the lowest score, are examined in terms of the criteria related to the relevant principle.

The analysis of Activity 15 (in Appendix B) with the lowest score in terms of the model generalization principle is given in the following table:

Table 15. Evaluation of Activity 15 in terms of the Model Generalization Principle

Sub-criterion**	**Point***	**Explanation**
MGP1	0	The question asks how many daily PCR tests should be done in order for additional precautions to be taken, in your opinion. Since the task does not lead students to a generalizable model, the developed model is useful only for this particular situation, not other similar situations.
MGP2	0	In the activity, students are expected to solve a specific situation; that is, they are not led to create models that can be modified and reused.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: MGP: Model Generalization Principle

The analysis of the Activity 11 (in Appendix B) with the highest score in terms of the model generalization principle is given in the following table:

Table 16. Evaluation of Activity 11 in terms of the Model Generalization Principle

Sub-criterion**	Point*	Explanation
MGP1	2	In the activity, students are expected to create a mathematical formula by stating, "create a mathematical formula that gives …". Since this formula could be used in other similar situations, the task leads students to develop a generalizable model.
MGP2	2	Since the formula that students are expected to develop is reusable and shareable, the task directs students to construct such models.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: MGP: Model Generalization Principle

4.1.4 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Simple Prototype Principle

The compatibility level of 48 activities in the ninth-grade mathematics modeling activity book according to the simple prototype principle is given in Table 17 below.

Table 17. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Simple Prototype Principle

Activity number	SPP1	SPP2	SPP3	Mean
1	2	1	2	0.83
2	2	2	2	1
3	2	1	1	0.67
4	2	0	1	0.5

Table 17. (cont'd)

Activity number	SPP1	SPP2	SPP3	Mean
5	1	2	2	0.83
6	2	1	2	0.83
7	2	2	1	0.83
8	2	1	2	0.83
9	2	1	2	0.83
10	2	1	2	0.83
11	2	2	2	1
12	2	0	2	0.67
13a	0	0	2	0.33
13b	2	1	2	0.83
14	2	1	2	0.83
15	1	0	1	0.33
16	2	0	2	0.67
17	2	1	2	0.83
18	2	1	2	0.83
19	2	1	2	0.83
20	2	1	2	0.83
21	1	2	2	0.83
22	2	2	2	1
23	2	0	2	0.67
24	2	1	2	0.83
25	2	1	2	0.83
26	2	1	2	0.83
27	2	1	2	0.83
28	2	0	2	0.67
29	2	1	2	0.83
30	2	0	2	0.67

Table 17. (cont'd)

Activity number	SPP1	SPP2	SPP3	Mean
31	2	1	2	0.83
32	1	0	2	0.5
33	2	0	2	0.67
34	2	0	2	0.67
35	2	0	2	0.67
36	2	0	1	0.5
37	1	0	1	0.33
38	1	1	1	0.5
39	2	0	2	0.67
40	2	0	2	0.67
41	2	0	2	0.67
42	2	1	2	0.83
43	2	1	2	0.83
44	2	1	2	0.83
45	2	1	2	0.83
46	2	1	2	0.83
47	2	1	2	0.83
Mean	0.91	0.38	0.92	0.74

SPP: Simple Prototype Principle

Table 17 represents that the first criterion of the simple prototype principle (SPP1) about the complexity of the problem situations, the third criterion (SPP3) is related to whether the solution of the problem involves complex computational procedures are high level, and the second criterion related to the solution that provides a useful prototype is moderate.

While 41 of 48 activities (85.4%) are completely compatible, 6 (12.5%) are partially compatible, and 1 (2.1%) is incompatible with SPP1. It can be concluded that most

of the problem situations are free from complexity in a way that allows the student to generate a logical response and develop a prototype.

While 6 of 48 activities (12.5%) are completely compatible, 25 (52.1%) are partially compatible, and 17 (35.4%) are incompatible with SPP2. That is, most of the solutions do not provide a useful prototype or metaphor for interpreting other structurally similar situations.

Moreover, 41 of 48 activities (85.4%) are completely compatible, and 7 (14.6%) are partially compatible with SPP3, and there are no activities that do not comply with this criterion. It shows that most of the solutions to the modeling activities do not involve complex calculation procedures so as not to prevent seeing the conceptual relationships intended in the question.

According to Table 17, the activities that provide the design principles of MEA at the highest level are activities 2, 11, and 22. In contrast, the activities that provide the design principles of MEA at the lowest level are activities 13a, 15, and 37.

Considering the simple prototype principle, Activity 11, one of the activities with the highest score, and Activity 13a, one of the activities with the lowest score, are examined in terms of the criteria related to the relevant principle.

The analysis of Activity 13a (in Appendix B), which has the lowest score in terms of the simple prototype principle, is given in the following table:

Table 18. Evaluation of Activity 13a in terms of the Simple Prototype Principle

Sub-criterion**	**Point***	**Explanation**
SPP1	0	The problem situation is not very complicated, but it is not suitable for generating a logical response and developing a prototype.

Table 18. (cont'd)

Sub-criterion**	Point*	Explanation
SPP2	0	The solution does not provide a useful prototype for interpreting other structurally similar situations.
SPP3	2	In solving the problem, it is expected that three of the students will find the day they visited together using lcm and then determine the days of new visitors according to the result. This does not require complex calculation procedures.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: SPP: Simple Prototype Principle

The analysis of Activity 11 (in Appendix B) with the highest score in terms of the simple prototype principle is given in the following table:

Table 19. Evaluation of Activity 11 in terms of the Simple Prototype Principle

Sub-criterion**	Point*	Explanation
SPP1	2	The problem situation in this activity is clear and uncomplicated, allowing the student to generate a logical response and develop a prototype.
SPP2	2	In the problem situation, it is asked to develop a mathematical formula as a solution. This formula provides a useful prototype for interpreting other structurally similar situations.

Table 19. (cont'd)

Sub-criterion**	Point*	Explanation
SPP3	2	In this problem, the conceptual relationships expected to be noticed are root numbers and exponential numbers. Here, the solution of the problem does not involve complex calculation procedures at a level that would prevent this.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: SPP: Simple Prototype Principle

4.1.5 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Model Externalization Principle

The compatibility level of 48 activities in the ninth-grade mathematics modeling activity book according to the model externalization principle is given below:

Table 20. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Model Externalization Principle

Activity number	MEP1	MEP2	Mean
1	1	1	0.5
2	2	2	1
3	1	1	0.5
4	1	1	0.5
5	1	0	0.25
6	1	1	0.5
7	1	0	0.25
8	1	1	0.5
9	1	0	0.25
10	1	1	0.5
11	1	1	0.5

Table 20. (cont'd)

Activity number	MEP1	MEP2	Mean
12	0	0	0
13a	1	0	0.25
13b	1	1	0.5
14	1	0	0.25
15	1	0	0.25
16	1	0	0.25
17	1	0	0.25
18	1	1	0.5
19	1	0	0.25
20	1	2	0.75
21	2	2	1
22	2	2	1
23	1	1	0.5
24	1	0	0.25
25	1	0	0.25
26	1	0	0.25
27	1	0	0.25
28	2	2	1
29	1	2	0.75
30	1	1	0.5
31	1	1	0.5
32	1	0	0.25
33	1	0	0.25
34	2	2	1
35	2	2	1
36	2	2	1
37	2	2	1
38	2	2	1

Table 20. (cont'd)

Activity number	MEP1	MEP2	Mean
39	1	1	0.5
40	1	0	0.25
41	1	1	0.5
42	1	2	0.75
43	1	1	0.5
44	1	1	0.5
45	1	2	0.75
46	1	0	0.25
47	1	0	0.25
Mean	**0.58**	**0.44**	**0.51**

MEP: Model Externalization Principle

Table 20 shows that the first criterion of the model externalization principle (MEP1) is related to the fact that the response reveals the thoughts in the modeling process are above the medium level, and the second criterion (MEP2) is related to whether it requires them to create a document revealing their solution approaches is moderate.

Only one of the activities receives zero points from the first criterion (MEP1). Also, 9 of the 48 activities (18.75%) completely comply with this criterion, while 38 (79%) partially comply. These values show that the compatibility of the modeling activities is average in revealing how students think during the modeling process.

On the other hand, the second criterion (MEP2) is below the medium level, which is considered as moderate. While 13 of the 48 activities (27%) are completely compatible and 16 (33.4%) are partially compatible with this principle, 19 (39.6%) are incompatible. These values show that the activities differ substantially in meeting the design principles, meaning that although some activities require students to document their thoughts and solution approaches, many modeling activities do not request any explanations that would elicit students' ways of thinking.

According to Table 20, activity numbers 2, 21, 22, 28, 34, 35, 36, 37, and 38 provide the design principles of MEA at the highest level, while activity number 12 provides the design principles of MEA at the lowest level. Below, the compatibility of activities 12 and 18 in meeting the model externalization principle is documented in terms of the criteria related to the relevant principle.

The analysis of Activity 12 (in Appendix B), which has the lowest score in terms of the model externalization principle, is given in the following table:

Table 21. Evaluation of Activity 12 in terms of the Model Externalization Principle

Sub-criterion**	Point*	Explanation
MEP1	0	Students are asked to find out which course Ahmet attended and how many times. The answer given here does not reveal the students' thoughts in the modeling process.
MEP2	0	The activity does not expect students to create a document that clearly reveals their solutions.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: MEP: Model Externalization Principle

The analysis of the activity 38 (in Appendix B) with the highest score in terms of the model externalization principle is given in the following table:

Table 22. Evaluation of Activity 38 in terms of the Model Externalization Principle

Sub-criterion**	Point*	Explanation
MEP1	2	This activity involves statements like "Explain the model to find which car Ms. Yeliz should…". That is, students are expected to explain the created model in their own words.

Table 22. (cont'd).

Sub-criterion**	Point*	Explanation
MEP2	2	This activity involves statements like "Please give a price offer by explaining in detail on the table." That is, the activity requires students to create a document that clearly reveals their thoughts and solution approaches during the solution process.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: MEP: Model Externalization Principle

4.1.6 Compatibility of the Activities in the 9th Grade Mathematics Modeling Activity Book with the Self-Evaluation Principle

The compatibility level of 48 activities in the ninth-grade mathematics modeling activity book according to the self-evaluation principle is given in Table 23 below.

Table 23. Evaluation of Activities' Compatibility Level of 9th Grade Mathematics Modeling Activity Book with the Self-Evaluation Principle

Activity number	SEP1	SEP2	SEP3	Mean
1	1	0	2	0.5
2	1	0	2	0.5
3	1	0	2	0.5
4	1	0	2	0.5
5	1	0	2	0.5
6	1	0	2	0.5
7	1	0	2	0.5
8	2	0	2	0.67
9	1	0	2	0.5

Table 23. (cont'd)

Activity number	SEP1	SEP2	SEP3	Mean
10	1	0	2	0.5
11	1	0	2	0.5
12	1	0	2	0.5
13a	1	0	2	0.5
13b	1	0	2	0.5
14	1	0	2	0.5
15	1	0	1	0.33
16	1	0	2	0.5
17	1	0	2	0.5
18	1	0	2	0.5
19	1	0	2	0.5
20	1	0	2	0.5
21	1	0	2	0.5
22	1	0	2	0.5
23	1	0	2	0.5
24	1	0	2	0.5
25	1	0	2	0.5
26	1	0	2	0.5
27	1	0	2	0.5
28	1	1	2	0.67
29	1	0	2	0.5
30	1	0	2	0.5
31	1	0	2	0.5
32	1	0	2	0.5
33	1	0	2	0.5
34	1	1	2	0.67
35	1	0	2	0.5

Table 23. (cont'd)

Activity number	SEP1	SEP2	SEP3	Mean
36	2	1	2	0.83
37	1	0	2	0.5
38	2	2	2	1
39	1	0	2	0.5
40	1	0	2	0.5
41	1	0	2	0.5
42	1	0	2	0.5
43	1	0	2	0.5
44	1	0	2	0.5
45	1	0	2	0.5
46	1	0	2	0.5
47	1	0	2	0.5
Mean	0.53	0.05	0.99	0.52

SEP: Self-Evaluation Principle

Table 23 represents that the first criterion of the self-evaluation principle (SEP1) related to the ability of students to control the accuracy of their interpretations and conclusions results themselves is above the medium level based on Table 3. While there are no activities that do not meet the first criterion, 45 (93.8%) of them provide it partially. That is, students can check the accuracy of their results and decide whether there is a need for correction. Moreover, the second criterion (SEP2) related to whether the problem contains criteria for evaluating alternative solutions is at a low level. Only three activities (6.25%) are partially compatible, and one (2.1%) is completely compatible with this criterion. This shows that most of the problem statements in the activities do not include appropriate criteria for evaluating the usefulness of alternative solutions. On the other hand, the third criterion (SEP3) about the clarity of the purpose is at a high level. Except for one, all of the activities

are completely compatible with the third criterion of the self-evaluation principle, clearly stating the purposes of the activities.

According to Table 23, the activity number that provides the design principles of MEA at the highest level is 38, while the activity number that provides the design principles of MEA at the lowest level is 15. The analysis of Activity 15 (in Appendix B) with the lowest score in terms of the self-evaluation principle is given in the following table:

Table 24. Evaluation of Activity 15 in terms of Self-Evaluation Principle

Sub-criterion**	**Point***	**Explanation**
SEP1	1	Students are asked for the number of daily PCR tests to be performed to take additional measures. In this case, students should determine a range of cases for the R_0 value. These ranges are determined by themselves and may vary according to their assumptions. Therefore, students can check their results based on their assumptions, but they may not be able to decide if correction and improvement are necessary due to the lack of a definitive conclusion.
SEP2	0	The problem statement does not suggest appropriate criteria for assessing the usefulness of alternative solutions.
SEP3	1	The purpose of this problem is not completely clear. In the problem situation, it is asked the number of PCR tests per day that should be reached to take additional, but the range of values that we should take is not explicit.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: SEP: Self-Evaluation Principle

The analysis of the activity 38 (in Appendix B) with the highest score in terms of the self-evaluation principle is given in the following table:

Table 25. Evaluation of Activity 38 in terms of Self-Evaluation Principle

Sub-criterion	Point*	Explanation
SEP1	2	Since students are expected to clearly explain their solutions and create documents (e.g., "Create a mathematical model that explains..."), they can check the accuracy of their results.
SEP2	2	The problem statement suggests appropriate criteria for assessing the usefulness of alternative solutions. The criteria in the question are presented in the form of items and tables.
SEP3	2	The purpose of the task is clear: for whom, what, when, why, and where. There are five sub-questions in the problem. The purpose is stated explicitly in all of the questions.

*: 0: "incompatible", 1: "partially compatible", and 2: "completely compatible"
**: SEP: Self-Evaluation Principle

The following table shows the average of all the sub-criteria of MEA design principles. This table is used in the discussion part.

Table 26. General Evaluation of Compatibility Level of Activities with all Sub-criteria of MEA Design Principles

MEA Design Principle	Sub-criterion	Mean
MCP	MCP1	0.66
	MCP2	0.90
	MCP3	0.48
	MCP4	0.51
	MCP5	0.91
RP	RP1	0.89
	RP2	0.57

Table 26. (cont'd).

MEA Design Principle	Sub-criterion	Mean
RP	RP3	0.60
	RP4	0.90
MGP	MGP1	0.30
	MGP2	0.14
SPP	SPP1	0.91
	SPP2	0.38
	SPP3	0.92
MEP	MEP1	0.58
	MEP2	0.44
SEP	SEP1	0.53
	SEP2	0.05
	SEP3	0.99

MCP: Model Construction Principle, RP: Reality Principle, MGP: Model Generalization Principle, SPP: Simple Prototype Principle, MEP: Model Externalization Principle, SEP: Self-Evaluation Principle

CHAPTER 5

DISCUSSION, CONCLUSION AND IMPLICATIONS

In this present research study, the activities included in the ninth-grade mathematics modeling activity book published by MoNE and taught in Turkey since 2020 are analyzed in terms of their compatibility with the MEA design principles. The results of this study reveal that the activities in the ninth-grade mathematics modeling activity book meet these principles above average.

The results of the study show that in the 9th-grade mathematics modeling activity book, except for the model generalization principle, the compatibility with each design principle is above the medium level. While the compatibility levels for model construction, reality, and simple prototype principles are much higher than the medium level and close to the high level, the compatibility levels for the model externalization and self-evaluation principles are slightly above the medium level. On the other hand, the compatibility with the model generalization principle is at a low level. The findings also indicate that compatibility with reality and simple prototype principles are at the highest level.

The findings of this study show that the general compatibility level of the activities in the 9th-grade mathematics modeling activity book according to the MEA design principles is at above average. That is, the level of compliance in all studies is above average, so in this respect, their results are parallel to each other.

The general average of the activities regarding the model construction principle is 0.70, above the medium level. Saka and Alkan (2022) find that the compliance level of 6th, 7th, and 8th-grade mathematics practices course books with the MEA design principles concerning the model construction principle are 0.46, 0.63, and 0.52, respectively. Doruk (2019) finds the compatibility level as 0.46. That is, the results

are consistent with Saka and Alkan's (2022) results for 7th and 8th grade, while they are inconsistent with the results of Doruk (2019) and Saka and Alkan (2022) for 6th grade since they are below the medium level. Also, according to the results of Urhan and Dost's (2017) study, no activities are incompatible with the model construction principle, and their scores are above the medium level. Therefore, these results are in parallel with this present study.

The findings of the study indicate that the mean compatibility level in terms of the reality principle is 0.74, which is above the medium level. Saka and Alkan (2022) find that the compliance level of 6th, 7th, and 8th-grade mathematics practices course books with the MEA design principles concerning the reality principle are 0.64, 0.62, and 0.65, respectively. According to the results of Doruk's (2019) study, the compatibility level in terms of the reality principle is 0.68. Furthermore, Urhan and Dost (2017) state that all the activities were found to be completely compatible with the principle of reality. That is, the result is aligned with these studies.

Considering the model generalization principle, the activities comply with the MEA design principle 0.22, which is below the medium level. Saka and Alkan (2022) find that the compliance levels of 6th, 7th, and 8th-grade mathematics practices course books with the MEA design principles with respect to the model generalization principle are 0.58, 0.56, and 0.56, respectively. Doruk (2019) finds a level of 0.37 below the medium level. Also, Urhan and Dost (2017) find that all activities in the 9th-grade course book are fully compatible with the model generalization principle. Therefore, while the result is coherent with only Doruk's (2019) study, it is incoherent with the results of Urhan and Dost (2017) and Saka and Alkan (2022) for all grades.

When the activities in the ninth-grade mathematics modeling activity book are analyzed in terms of the simple prototype principle, the mean score of 0.74 is above the medium level. According to Saka and Alkan (2022), the compatibility level of 6th, 7th, and 8th-grade activities in mathematics practices course books with the MEA design principles in terms of the simple prototype are 0.82, 0.72 and 0.73,

respectively. Similarly, Doruk (2019) finds the activities' compatibility level regarding the simple prototype principle to be 0.75. Therefore, since all the results are above the medium level, they are consistent with the previous results (Doruk, 2019; Saka & Alkan, 2022).

Regarding the model externalization principle, the compatibility level of the activities in the ninth-grade mathematics modeling activity book is found to be 0.52, slightly above the medium level. Saka and Alkan (2022) find that the compliance levels of 6th, 7th, and 8th-grade mathematics practices course books with the MEA design principles with respect to the model externalization principle are 0.55, 0.46, and 0.51, respectively. According to the findings of Doruk's (2019) study, the compatibility level of the activities with the principle of model externalization is 0.53. Moreover, Urhan and Dost (2017) also conclude that no activities are completely compatible, and most of the activities are found to be partially compatible with the model externalization principle. That is, the results are consistent with Saka and Alkan's (2022) results for 6^{th} and 8^{th} grade and Doruk's (2019) above the medium level, while they are inconsistent with the results of Saka and Alkan (2022) for 7^{th} grade, and Urhan and Dost (2017) with below the medium level.

The results related to the self-evaluation principle indicate that the mean compatibility level is 0.52, slightly above the medium level. Saka and Alkan (2022) find that the compliance levels of 6th, 7th, and 8th-grade mathematics practices course books with the MEA design principles with respect to the self-evaluation principle are 0.44, 0.33, and 0.51, respectively. According to the results of Doruk's (2019) study, the compatibility level is 0.68. Urhan and Dost (2017) state that all the activities are partially compatible with the self-evaluation principle. In this respect, the results are consistent with the studies of Saka and Alkan (2022) for 8^{th} grade and Doruk (2019) since they are found to be above the medium level, while they contradict the results of Saka and Alkan (2022) for 6^{th} and 7^{th}-grade levels as they are below the medium level.

The studies that examine preservice mathematics teachers' constructions of modeling tasks show that they pay attention to some principles more than others when creating modeling activities. Similar to the findings of the present study, these studies (Bukova Guzel, 2011; Kula Unver et al., 2018; Tekin Dede & Güzel, 2013b) reported that the modeling activities developed by preservice mathematics teachers focused on the principles of reality and model construction in their studies. Moreover, Tekin Dede and Bukova Güzel (2013b) indicated that the modeling activity developed by preservice mathematics teachers partially met the self-evaluation principle, as in this study. On the other hand, while Tekin Dede and Bukova Güzel (2013) found that the modeling activity was fully appropriate to the model generalization principle, the present study showed that this principle has the lowest level of compatibility.

5.1 Limitations and Recommendations

In this study, only the ninth-grade mathematics modeling activity book was analyzed. The MoNE has not prepared a mathematical modeling activity book for other high school grade levels. However, there are skill-based activity books published by the MoNE for each grade level. Skill-based activity books are supplementary resources to improve and enhance students' mathematical skills, including problem-solving, critical thinking, mathematical modeling, computational skills, and understanding mathematical concepts. They contain various exercises, problems, modeling tasks, and activities designed to align with the mathematics curriculum. They cover the relevant mathematical units and learning objectives. Therefore, the activities in these books can be examined by the modeling perspective and principles used in this study or other modeling principles (e.g., Galbraith, 2007).

In this study, the MMP was used to analyze the modeling activities. MMP is contextual modeling perspective. It is also related to the realistic and educational modeling perspectives in terms of their purposes. While the realistic modeling perspective focuses on understanding real life, the educational modeling perspective

focuses on integrating modeling into mathematics teaching and learning to develop students' modeling competencies and understand mathematics. However, MMP is not directly related to the socio-critical modeling perspective, which focuses on the social role of mathematics in using mathematical concepts to make decisions about society. Moreover, RME and ATD frameworks, other categorizations of modeling, provide general and broader theories for designing mathematics education (Kaiser & Sriraman, 2006). Not examining activities from this point of view may be a limitation of this study. Thus, the activities in this book can be examined by these perspectives (socio-critical modeling perspective, ATD, and RME) depending on the purpose of the study.

The activities in this book were not implemented in the ninth-grade classrooms before being analyzed. The solution approaches have been limited to the researcher's own solutions and the solution approaches included in the book. Getting support from experts in this field to understand whether there are different solutions can facilitate better analysis, particularly in the dimension of the simple prototype principle. Furthermore, in supporting the analysis, the activities can be implemented with ninth-grade students for a year, and then the results of this implementation can be used in the analysis of the activities for their compatibility with design principles. It can be expected that a sufficient amount of time passed after the implementation can also help to assess the compatibility with this principle more clearly.

Consequently, this study holds significance for various stakeholders, including the MoNE, researchers, textbook authors, and teachers. The MoNE can utilize the findings of this study to incorporate MEAs into the mathematics curriculum, taking into account the analysis conducted on the six design principles of the MMP. Additionally, this study draws attention to the importance of evaluating the quality of modeling activities before their implementation, which may be of interest to researchers. It sheds light on the researchers for the future study. If a modeling activity book is also prepared for other grade levels, the prepared books can be analyzed by researchers before publication. They can also review mathematics skill-based activity books within this framework.

According to the general evaluation of the compatibility level of the modeling activities in the mathematics modeling activity book, it is observed that the principle with the lowest level of compatibility is the model generalization principle. The level of compliance of modeling activities with this principle should be increased to strengthen the compatibility of the teaching material with the MMP. For this purpose, in accordance with the model generalization principle, arrangements should be made that will allow the model requested from students to be useful and applicable not only to a particular situation but also to be used in other similar situations. Also, the activity should lead students to think about producing models that can be reusable, shareable, modifiable, and transportable. Therefore, textbook authors should review the activities in this context and organize them in such a way that students are asked to create a generalizable model. For example, in activities, students may be asked to create a generalizable model in the problem statement, or questions can be designed to contain incomplete information, be open-ended, and have results that can vary based on assumptions and not have a definite solution.

Considering the findings of this study, principles that are slightly above the medium level are the principles of model externalization and self-evaluation. In this regard, although the modeling activities in the book do not completely align with these principles, they have been prepared by considering them. However, it is still suggested to develop and revise activities that meet these principles at a higher level. Regarding the model externalization principle, the problem situation should require students to create a document in such a way as to reveal their solution processes. Also, the activities should be prepared in a way that enables students to check the accuracy of their conclusions and to suggest appropriate criteria for evaluating the usefulness of alternative solutions in terms of the self-evaluation principle. In this context, it is recommended for future book authors that the activities should be designed in accordance with these principles. In addition, necessary changes can also be made to activities, especially those that remain at a low level in the book according to these principles by authors who want to revise this modeling book.

The mathematics modeling activity book (MoNE, 2020a) has been prepared by the MoNE only at the ninth-grade level. It is recommended to prepare a modeling activity book for other high school grade levels. Taking into account of the results of this study, it is expected to eliminate the deficiencies in the previous book that will be prepared in the new ones and to create modeling activities according to the principles of MEA design.

In the secondary education mathematics course curriculum (MoNE, 2018), the course hours of the subjects logic, sets, equations and inequalities, triangles, and data are 12, 20, 98, 70, and 16, respectively. The weights of the subjects are %6, %9, %46, %32 and 7%, respectively. However, the percentage of modeling activities in the book with respect to the topics are %4.17, %0, %83.33, %8.33, and %4.17, respectively. These values are not compatible with the percentages given in the curriculum. In revising the book or in developing new books for other grade levels, the distribution and weight of topics in the curriculum should be considered.

It is not possible for teachers to implement all of these activities in their classrooms according to the curriculum. Therefore, teachers can decide which activity to implement in which unit according to the results of this study. Selecting the modeling activities with the highest average in terms of meeting the design principles is recommended. For example, based on the findings, teachers may prefer activity 2 for the Unit of Logic, activities 7, 36, and 38 for the Unit of Equations and Inequalities, 45 for the Unit of Triangles, and 46 for the Unit of Data.

The analysis of the results shows that although the activities have the highest averages, their compliance with all criteria may not be above average. For example, although they have high averages in terms of meeting the design principles, activities 38, 45, and 46 are below the average according to the model generalization principle. Based on the results of this study, teachers can make revisions and refinements in activities considering dimensions that are below the average. For instance, teachers can revise the question item or the content of the activity to make the modeling activity more generalizable before applying it in their classrooms and to lead students

to create a model that can also be used in similar situations. They may also ask students to prepare a document in which they can reveal their solution approaches and thoughts.

REFERENCES

Abassian, A., Safi, F., Bush, S., & Bostic, J. (2020). Five different perspectives on mathematical modeling in mathematics education. *Investigations in Mathematics Learning. 12*(1). 53–65. https://doi.org/10.1080/19477503.2019.1595360

Baki, A. (2014). *Matematik tarihi ve felsefesi* (1. Basım). Ankara: Pegem Akademi.

Barbosa, J. C. (2006). Mathematical modelling in classroom: A socio-critical and discursive perspective. *ZDM – Mathematics Education, 38*(3), 293–301. https://doi.org/10.1007/BF02652812

Barbosa, J. C. (2007). Report from the working group modelling and applications-mathematical modelling and parallel discussions. In D. Pitta-Pantazi, & C. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2101–2109). University of Cyprus and ERME.

Berget, I. K. L. (2022). Mathematical modelling in textbook tasks and national examination in Norwegian upper secondary school. *Nordic Studies in Mathematics Education, 27*(1), 51–70.

Blomhøj, M. (2009). Different perspectives in research on the teaching and learning mathematical modelling—Categorising the TSG21 papers. In M. Blomhøj, & S. Carreira (Eds.), *Mathematical applications and modelling in teaching and learning of mathematics (proceedings from topic study group 21 at the 11th international congress on mathematical education, Monterrey, Mexico, July 6–13, 2008)* (pp. 1–17). IMFUFA.

Blomhøj, M. (2019). Towards integration of modelling in secondary mathematics teaching. In G. A. Stillman, & J. P. Brown (Eds.), *Lines of inquiry in mathematical modelling research in education* (pp. 37–52). Springer. https://doi.org/10.1007/978-3-030-14931-4_3

Blomhøj, M., & Jensen, T. H. (2007). What's all the fuss about competencies? In W. Blum, P. L. Galbraith, H. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 45–56). Springer.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15–30). Springer. https://doi.org/10.1007/978-94-007-0910-2_3

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45–58.

Blum, W., & Leiß, D. (2007). How do students and teachers deal with modelling problems. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 222–231). Chichester: Horwood Publishing.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects – State, trends and issues in mathematics instruction. *Educational Studies in Mathematics, 22*(1), 37–68.

Borromeo Ferri, R. (2013). Mathematical modelling in European education. *Journal of Mathematics Education at Teachers College, 4*(2), 18–24.

Bowen, G. A. (2009). Document analysis as a qualitative research method. *Qualitative Research Journal, 9*(2), 27–40.

Bukova Guzel, E. (2011). An Examination of pre-service mathematics teachers' approaches to construct and solve mathematical modelling problems. *Teaching Mathematics and Its Applications, 30*(1), 19-36.

Carlson, M., Larsen, S., & Lesh, R. (2003). Integrating a models and modeling perspective with existing research and practice. In R. A. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on*

mathematics problem solving, learning, and teaching (pp. 465–478). Lawrence Erlbaum Associates.

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics, 109*, 205–236. https://doi.org/10.1007/s10649-021-10104-6

Chamberlin, S. A. & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Journal of Secondary Gifted Education, 17*(1), 37–47.

Chan, E. C. M. (2008). Using model-eliciting activities for primary mathematics classrooms. *The Mathematics Educator, 11*(1), 47–66.

Cheng, A. K. (2001). Teaching mathematical modeling in Singapore schools. *The Mathematics Educator, 6*(1), 63–75.

Cobb, P. (2002). Modeling, symbolizing, and tool use in statistical data analysis. In K. Gravemeijer, R. Lehrer, B. Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 171–196). Kluwer Academic Publishers.

Common Core State Standards Initiative [CCSSI]. (2010). *Common core state standards for mathematics.* https://learning.ccsso.org/common-core-state-standards-initiative

Corbin, J., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (4th ed.). Sage.

Çavuş-Erdem, Z., Doğan, M. F., Gürbüz, R., & Şahin, S. (2017). The reflections of mathematical modeling in teaching tools: Textbook analysis. *Adıyaman University Journal of Educational Sciences, 7*(1), 61–86. https://doi.org/10.17984/adyuebd.309793

Deniz, D. (2014). *Ortaöğretim matematik öğretmenlerinin matematiksel modelleme yöntemine uygun etkinlik oluşturabilme ve uygulayabilme yeterlikleri.* [Doctoral dissertation, Atatürk University].

Department for Education. (1997). *Mathematics in the national curriculum*. DFE Welch Office.

Doerr, H. M., & English, L. D. (2003). A modeling perspective on students' mathematical reasoning about data. *Journal for Research in Mathematics Education, 34*(2), 110–136

Doerr, H. M., & Lesh, R. (2011). Models and modelling perspectives on teaching and learning mathematics in the twenty-first century. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling: ICTMA 14* (Vol. 1, pp. 247–268). Springer.

Doorman, L. M., & Gravemeijer, K. (2009). Emerging modeling: Discrete graphs to support the understanding of change and velocity. *ZDM – Mathematics Education, 41,* 199–211.

Doruk, B. K. (2016). Realistic real world context: Model eliciting activities. *International Journal for Mathematics Teaching and Learning, 17*(2). https://doi.org/10.4256/ijmtl.v17i2.24

Doruk, B. K. (2019). Analysis of fifth grade mathematics practices course teaching material activities based on model-eliciting design principles. *Necatibey Faculty of Education Electronic Journal of Science & Mathematics Education, 13*(2), 879-908.

Elçi, A. N. (2020). The compatibility of model eliciting activities of secondary school teacher candidates with design principles. *Journal of Computer and Education Research, 8*(15), 305–322. https://doi.org/10.18009/jcer.695253

English, L. D. (2003). Reconciling theory, research, and practice: A models and modeling perspective. *Educational Studies in Mathematics, 54*, 225–248. https://doi.org/10.1023/B:EDUC.0000006167.14146.7b

English, L. D. (2006). Mathematical modeling in the primary school: Children's construction of a consumer guide. *Educational Studies in Mathematics. 63*, 303–323. https://doi.org/10.1007/s10649-005-9013-1

Erbaş, A. K., Kertil, M., Çetinkaya, B., Çakıroğlu, E., Alacacı, C., & Baş, S. (2014). Mathematical modeling in mathematics education: Basic concepts and approaches. *Educational Sciences: Theory & Practice, 14*(4), 1607–1627.

Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2011). *How to design and evaluate research in education* (8th ed.). McGraw-Hill Education.

Freudenthal, H. (1973). *Mathematics as an educational task.* Reidel.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Reidel.

Freudenthal, H. (1991). *Revisiting mathematics education*. Kluwer Academic Publishers.

Galbraith, P. (2007). Dreaming a "possible dream": More windmills to conquer. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 44–62). Chichester: Horwood Publishing.

Galbraith, P. (2012). Models of modelling: Genres, purposes or perspectives. *Journal of Mathematical Modeling and Application, 1*(5), 3–16.

Galbraith, P., Stillman, G., & Brown, J. (2010). Turning ideas into modeling problems. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 133–144). Springer. https://doi.org/10.1007/978-1-4419-0561-1_11

Gravemeijer, K. (2002). Preamble: From models to modeling. In K. Gravemeijer, R. Lehrer, B. Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 7–22). Kluwer Academic Publishers.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39*, 111–129.

Gravemeijer, K., & Stephan, M. (2002). Emergent models as an instructional design heuristic. In K. Gravemeijer, R. Lehrer, B. Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 145–169). Kluwer Academic Publishers.

Julie, C., & Mudaly, V. (2007). Mathematical modelling of social issues in school mathematics in South Africa. In W. Blum, P. L. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: 14th ICMI Study* (pp. 503–510). Springer.

Jung, H. (2015). Strategies to support students' mathematical modeling. *Mathematics Teaching in the Middle School, 21*(1), 42–48. https://doi.org/10.5951/mathteacmiddscho.21.1.0042

Kaiser, G. (1995). Results from a comparative empirical study in England and Germany on the learning of mathematics in context. In C. Sloyer, W. Blum, & I. Huntley (Eds.), *Advances and perspectives on the teaching of mathematical modelling and applications* (pp. 83–95). Water Street Mathematics.

Kaiser, G. (2005). Mathematical modelling in school–examples and experiences. In G. Kaiser, & H. W. Henn (Eds.), *Mathematikunterricht im spannungsfeld von evolution und evaluation* (pp. 99–108). Hildesheim: Franzbecker.

Kaiser, G. (2006). Introduction to the working group "Applications and Modelling". In M. Bosch (Ed.), *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1613–1622). Universitat Ramon Llull.

Kaiser, G. (2007). Modelling and modelling competencies in school. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 110–119). Horwood. https://doi.org/10.1533/9780857099419.3.110

Kaiser, G., Blomhøj, M., & Sriraman, B. (2006). Towards a didactical theory for mathematical modelling. *ZDM – Mathematics Education, 38*(2), 82–85. https://doi.org/10.1007/BF02655882

Kaiser, G., Blum, W., Borromeo Ferri, R., & Stillman, G. (2011). Preface. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling: ICTMA 14* (pp. 1–5). Springer.

Kaiser, G., & Schwarz, B. (2010). Authentic modelling problems in mathematics education – Examples and experiences. *Journal für Mathematik-Didaktik, 31*, 51–76.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM – Mathematics Education, 38*(3), 302–310. https://doi.org/10.1007/BF02652813

Kaiser, G., Sriraman, B., Blomhøj, M., & Garcia, F. J. (2007). Report from the working group modelling and applications—Differentiating perspectives and delineating commonalities. In D. Pitta-Pantazi, & G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Research in Mathematics Education* (pp. 2035–2041). University of Cyprus and ERME.

Koç, D. (2020). *An investigation on theses and dissertations on mathematical modeling in Turkey in the last two decades.* [Master's thesis, Middle East Technical University].

Kula Unver, S., Hidiroglu, C. N., Tekin Dede, A., & Bukova Guzel, E. (2018). Factors revealed while posing mathematical modelling problems by Mathematics student teachers. *European Journal of Educational Research, 7*(4), 941-952. doi: 10.12973/eu-jer.7.4.941

Lehrer, R., & Schauble, L. (2003). Origins and evaluation of model-based reasoning in mathematics and science. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 59–70). Lawrence Erlbaum.

Lehrer, R., & Schauble, L. (2007). A developmental approach for supporting the epistemology of modeling. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education* (pp. 153–160). Springer.

Lesh, R. & Caylor, B. (2007). Introduction to special issue: Modeling as application versus modeling as a way to create mathematics. *International Journal of Computers for Mathematical Learning, 12*(3), 173–194.

Lesh, R., Cramer, K., Doerr, H. M., Post, T., & Zawojewski, J. S. (2003). Model development sequences. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 35–58). Lawrence Erlbaum Associates.

Lesh, R., & Doerr, H. M. (2003a). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3–33). Lawrence Erlbaum Associates.

Lesh, R., & Doerr, H. M. (2003b). In what ways does a models and modeling perspective move beyond constructivism. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 519–556). Lawrence Erlbaum Associates.

Lesh, R., Doerr, H. M., Carmona, G., & Hjalmarson, M. (2003). Beyond constructivism. *Mathematical Thinking and Learning, 5*(2&3), 211–233. https://doi.org/10.1080/10986065.2003.9680000

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought revealing activities for students and teachers. In A. Kelly, & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-646). Lawrence Erlbaum.

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical Thinking and Learning, 5*(2&3), 109–129.

Lesh, R., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763–804). Information Age Publishing.

Lincoln, Y. S. & Guba, E. G. (1985). *Naturalistic inquiry*. Sage.

Meyer, D. (2015). Missing the promise of mathematical modeling. *The Mathematics Teacher, 108*(8), 578–583. https://doi.org/10.5951/mathteacher.108.8.0578

Miles, M. B., & Huberman, A. M. (1994). *An expanded source book: Qualitative data analysis.* Thousand Oaks, CA: Sage.

Ministry of National Education [MoNE]. (2005). *Ortaöğretim matematik dersi (9–12 sınıflar) öğretimi programı* [Secondary school mathematics (9-12 classes) curriculum]. Author.

Ministry of National Education [MoNE]. (2011). *Ortaöğretim matematik dersi (9–12 sınıflar) öğretimi programı* [Secondary school mathematics (9-12 classes) curriculum]. Author.

Ministry of National Education [MoNE]. (2013a). *Ortaöğretim matematik dersi (9–12 sınıflar) öğretimi programı* [Secondary school mathematics (9-12 classes) curriculum]. Author.

Ministry of National Education [MoNE]. (2013b). *Ortaöğretim matematik dersi dokuzuncu sınıf ders kitabı* [The ninth grade textbook of secondary school mathematics]. Author.

Ministry of National Education [MoNE]. (2015). *Ders kitapları ve eğitim araçları yönetmeliğinde değişiklik yapılmasına dair yönetmelik. sayı: 26502* [Regulation on the Amendment of the Regulation on Textbooks and Educational Instruments. Issue: 26502].

Ministry of National Education [MoNE]. (2018). *Ortaöğretim matematik dersi (9–12. sınıflar) öğretimi programı* [Secondary school mathematics (9-12 classes) curriculum]. Author.

Ministry of National Education [MoNE]. (2020a). *9. sınıf matematik modelleme etkinlik kitabı*. Author. https://ogmmateryal.eba.gov.tr/panel/upload/etkilesimli/kitap/beceri_temelli/9/matematik-modelleme/index.html

Ministry of National Education [MoNE]. (2020b). *Lise öğrencilerine beceri ve kazanım temelli kaynak desteği.* https://www.meb.gov.tr/lise-ogrencilerine-beceri-ve-kazanim-temelli-kaynak-destegi/haber/21995/tr

Moore, T. J., Doerr, H. M., Glancy, A. W., & Ntow, F. D. (2015). Preserving pelicans with models that make sense. *Mathematics Teaching in the Middle School, 20*(6), 358–364. DOI:10.5951/mathteacmiddscho.20.6.0358.

National Board of Education [NBE]. (2011). *Ortaöğretim matematik dersi (9, 10, 11 ve 12. sınıflar) öğretim programı.* Devlet Kitapları Müdürlüğü.

National Board of Education [NBE]. (2013). *Ortaöğretim matematik dersi (9, 10, 11 ve 12. sınıflar) öğretim programı.* T.C. Millî Eğitim Bakanlığı.

National Council of Teachers of Mathematics [NCTM]. (1989). *Curriculum and evaluation standards for school mathematics.* Author.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics.* Author.

National Council of Teachers of Mathematics [NCTM]. (2000). *Curriculum and Evaluation Standards for School Mathematics*. Author.

National Research Council. (2013). Assessing 21st Century Skills: Summary of a workshop. National Academies Press.

Niss, M. (1996). Goals of mathematics teaching. In A. Bishop et al. (Eds.), *International handbook of mathematical education* (pp. 11–47). Springer.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis, & S. Papastavridis (Eds.), *Proceedings of the Third Mediterranean conference on mathematical education* (pp. 115–124). The Hellenic Mathematical Society.

Niss, M., Blum, W., & Galbraith, P. L. (2007). Introduction. In W. Blum, P. L. Galbraith, H. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 3–32). Springer.

Palm, T. (2007). Features and impact of the authenticity of applied mathematical school tasks. In W. Blum et al. (Eds.), *Modelling and applications in mathematics education* (pp. 201–208). Springer.

Perrenet, J. C., & Zwaneveld, B. (2012). The many faces of the modelling cycle. *Journal of Mathematical Modelling and Application, 1*(6), 3–21.

Pollak, H. (1979). The interaction between mathematics and other school subjects. In UNESCO (Ed.), *New trends in mathematics teaching IV* (pp. 232–248). UNESCO.

Pollak, H. (2016). Foreword. In C. Hirsch, & A. R. McDuffie (Eds.), *Annual perspectives in mathematics education 2016: Mathematical modeling and modeling mathematics* (pp. vii–viii). The National Council of Teachers of Mathematics.

Rapley, T. (2007). *Doing conversation, discourse, and document analysis*. Sage.

Revina, S., & Leung, F. K. S. (2018). Educational borrowing and mathematics curriculum: Realistic mathematics education in the Dutch and Indonesian

primary curriculum. *International Journal on Emerging Mathematics Education*, *2*(1), 1–16. http://dx.doi.org/10.12928/ijeme.v2i1.8025

Saka, E. & Alkan, S. (2022). Examining the secondary school mathematics practices course books in Turkey according to the principles of the model eliciting activity design. *International Journal of Curriculum and Instruction 14*(1), 523–542.

Sriraman, B. (2006). Conceptualizing the model-eliciting perspective of mathematical problem solving. In M. Bosch (Ed.), *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1686–1695). Universitat Ramon Llull.

Steen, L.A., Turner, R., & Burkhardt, H. (2007). Developing mathematical literacy. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 285–294). Springer. https://doi.org/10.1007/978-0-387-29822-1_30

Şen, E. Ö. (2020). Mathematical modeling problems designed by pre-service teachers and their views on the design process. *Journal of Kırşehir Education Faculty, 21*(3), 1530 - 1560. DOI: 10.29299/kefad.853996.

Tekin Dede, A. & Bukova Güzel, E. (2013a). Ortaöğretim matematik öğretmenlerinin model oluşturma etkinlikleri ve matematik derslerinde kullanımlarına ilişkin görüşleri. *Journal of Faculty of Education, 2*(1), 300-322.

Tekin Dede, A. & Bukova Güzel, E. (2013b). Examining the design process of modeling activity of mathematics teachers: Obesity problem. *Elementary Education Online*, *12*(4), 1100–1119.

Urhan, S., & Dost, Ş. (2016). Matematiksel modelleme etkinliklerinin derslerde kullanımı: Öğretmen görüşleri. *Elektronik Sosyal Bilimler Dergisi, 15*(59), 1279–1295. https://doi.org/10.17755/esosder.263231

Urhan, S., & Dost, Ş. (2017). Analysis of ninth grade mathematics course book activities based on model-eliciting principles. *International Journal of Science and Mathematics Education, 16*(5), 985–1002.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2012). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). Pearson.

Van Overveld, K., & Borghuis, T. (2013). *From problems to numbers and back, Lecture Notes to "A Discipline-neutral introduction to mathematical modelling"*. Eindhoven University of Technology.

Verschaffel, L., Greer, B., & DeCorte, E. (2000). *Making sense of word problems*. Swets & Zeitlinger.

Wessels, H. (2014). Levels of mathematical creativity in model-eliciting activities. *Journal of Mathematical Modeling and Application, 1*(9), 22–40.

Zawojewski, J. S., & Lesh, R. (2003). A models and modelling perspective on problem solving. In R. A. Lesh, & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 317–336). Lawrence Erlbaum.

Zawojewski, J. S., Lesh, R., & English, L. (2003). A models and modeling perspective on the role of small group learning activities. In R. A. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 337–358). Lawrence Erlbaum.

Zwaneveld, B., Perrenet, J., van Overveld, K., Borghuis, T. (2017). Mathematical modelling in Dutch textbooks: Is it genuine mathematical modelling?. In G. Stillman, W. Blum, & G. Kaiser (Eds.), *Mathematical modelling and applications. International perspectives on the teaching and learning of mathematical modelling* (pp. 503–514). Springer. https://doi.org/10.1007/978-3-319-62968-1_42

APPENDICES

A. Criteria for Compatibility with the MEA Design Principles

Table 27. Criteria for compatibility with the design principles of MEA in the native language (Doruk, 2019)

1.Aşağıdaki maddeler model oluşturma prensibine uygunluk hakkındadır.	0	1	2
Görev öğrencileri karmaşık bir problem durumunda verilenler, istenenler ve muhtemel çözüm süreçlerini yorumlamak için bir model geliştirme gereksinimini fark ettirecek bir durum içine koyuyor.	◯	◯	◯
Görev öğrencileri bir soru için başkaları tarafından formüle edilmiş olan cevabı elde etmeye yönlendirmiyor.	◯	◯	◯
Görev üretilecek modeli, gözden geçirmeyi, düzeltmeyi, açıklamayı ve değiştirmeyi gerektiriyor.	◯	◯	◯
Görev, öğrencilerden problemin çözümü için bir model geliştirmeleri gerektiğini açıkça ifade ediyor.	◯	◯	◯
Görev, sembolik olarak ifade edilmiş durumları anlamlandırma yerine, o duruma uyan en uygun sembolik gösterimleri geliştirmeyi destekliyor.	◯	◯	◯
2. Aşağıdaki maddeler gerçeklik prensibine uygunluk hakkındadır.			
Görevin içerdiği problem durumu bire bir gerçek yaşamda karşılaşılabilecek yapıdadır.	◯	◯	◯
Görevin içerdiği problem durumu öğrenciler için gerçekçi algılanabilecek bir yapıdadır.	◯	◯	◯
Öğrenciler kendi deneyimleri ve bilgilerine dayanarak problem durumunu anlamlandırabilir.	◯	◯	◯
Çözüm sürecinde öğrencilerin fikirleri ciddiye alınıyor, yazarın problem için doğru yol olarak düşündüğü yola uymaya zorlanmıyor.	◯	◯	◯
3. Aşağıdaki maddeler model genelleştirme prensibine uygunluk hakkındadır.			
Görev oluşturulan modelin sadece onu geliştiren kişi için kullanışlı ve sadece özel bir duruma uygulanabilir olmasına değil, benzer başka durumlarda da kullanabilmesine olanak sağlıyor.	◯	◯	◯
Görev öğrencileri tekrar kullanılabilir, paylaşılabilir, üzerinde değişiklikler yapılabilir modeller üretmek için kafa yormaya yönlendiriyor.	◯	◯	◯
4. Aşağıdaki maddeler etkili ve basit prototip prensibine uygunluk hakkındadır.			

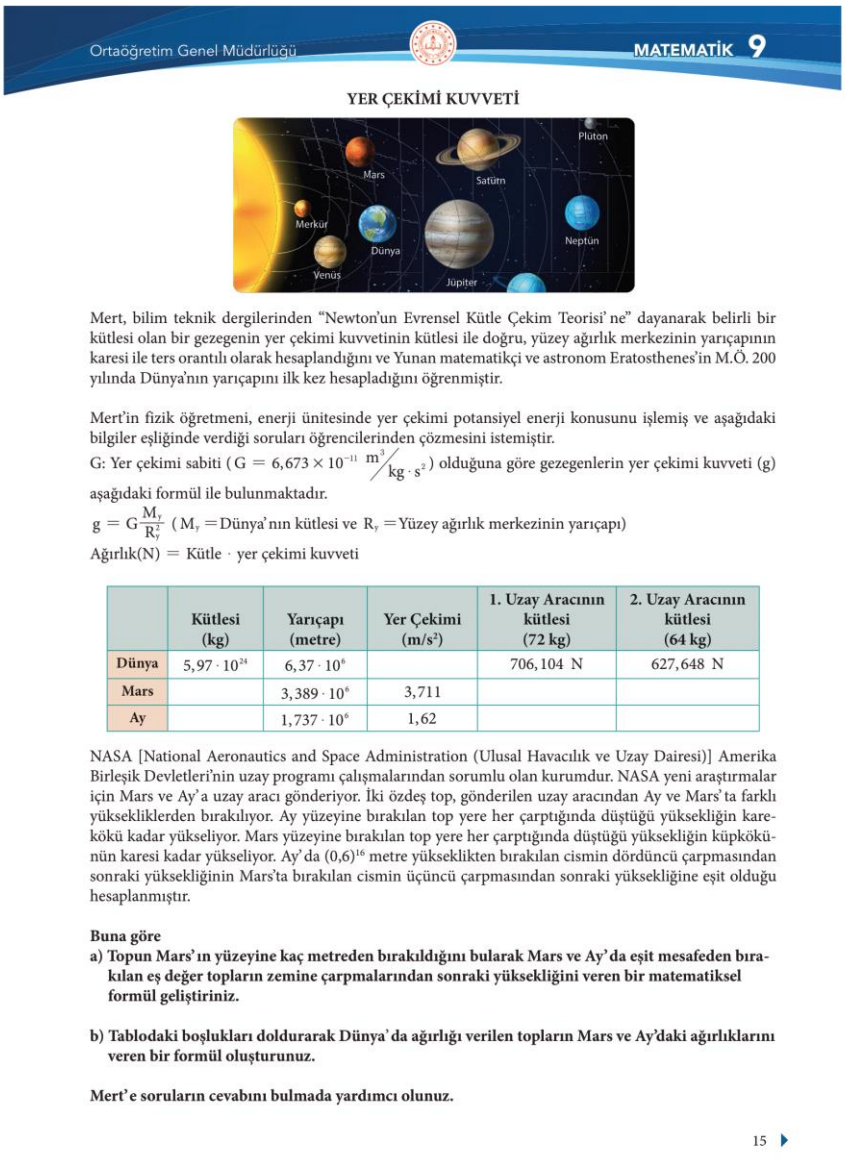
Görevin içerdiği problem durumu öğrencinin mantıklı bir cevap üretebilmesine (prototip geliştirmesine) olanak sağlayacak şekilde karmaşıklıktan uzak.	◯	◯	◯
Görevin içerdiği problemin çözümü yapısal olarak benzer problem durumlarını yorumlamak için kullanışlı bir prototip (örnek model) veya metafor (mecaz) sağlıyor.	◯	◯	◯
Görevin içerdiği problemin çözümü kavramsal ilişkileri fark etmeyi (prototip geliştirmeyi) engelleyecek düzeyde karmaşık hesaplama prosedürlerini içermiyor.	◯	◯	◯
5. Aşağıdaki maddeler model dışsallaştırma (düşüncelerini belgelendirme) prensibine uygunluk hakkındadır.			
Görevin içerdiği probleme verilen yanıt, öğrencilerin modelleme sürecinde neler düşündüklerini açığa çıkaracak şekilde.	◯	◯	◯
Görevin içerdiği problem öğrencilerin çözüm süreci boyunca problem durumuyla ilgili kendi düşünceleri ve çözüm yollarını açıkça ortaya koyan bir belge oluşturmalarını gerektiriyor.	◯	◯	◯
6. Aşağıdaki maddeler öz değerlendirme prensibine uygunluk hakkındadır.			
Öğrenci, görevi yerine getirirken kendi yorumlarının ve ulaştığı sonuçların doğruluğunu kendi kontrol edebilir, geliştirme veya düzeltmeye gerek olup olmadığına karar verebilir.	◯	◯	◯
Görevin içerdiği problem ifadesi güçlü bir şekilde alternatif çözümlerin kullanışlılığını değerlendirmek için uygunluk kriterleri öneriyor.	◯	◯	◯
Görevin çerçevesi nettir.	◯	◯	◯

0: “Sağlamıyor”, 1: “Kısmen sağlıyor”, 2: “Tam olarak sağlıyor”

B. Mathematical Modeling Activities in the Ninth-Grade Mathematics Modeling Activity Book

Activity Number: 11

Activity Name: Gravitational Force



Ortaöğretim Genel Müdürlüğü — MATEMATİK 9

YER ÇEKİMİ KUVVETİ

Mert, bilim teknik dergilerinden "Newton'un Evrensel Kütle Çekim Teorisi' ne" dayanarak belirli bir kütlesi olan bir gezegenin yer çekimi kuvvetinin kütlesi ile doğru, yüzey ağırlık merkezinin yarıçapının karesi ile ters orantılı olarak hesaplandığını ve Yunan matematikçi ve astronom Eratosthenes'in M.Ö. 200 yılında Dünya'nın yarıçapını ilk kez hesapladığını öğrenmiştir.

Mert'in fizik öğretmeni, enerji ünitesinde yer çekimi potansiyel enerji konusunu işlemiş ve aşağıdaki bilgiler eşliğinde verdiği soruları öğrencilerinden çözmesini istemiştir.

G: Yer çekimi sabiti ($G = 6{,}673 \times 10^{-11}\ {}^{m^3}/_{kg \cdot s^2}$) olduğuna göre gezegenlerin yer çekimi kuvveti (g) aşağıdaki formül ile bulunmaktadır.

$g = G\frac{M_y}{R_y^2}$ (M_y = Dünya'nın kütlesi ve R_y = Yüzey ağırlık merkezinin yarıçapı)

Ağırlık(N) = Kütle · yer çekimi kuvveti

	Kütlesi (kg)	Yarıçapı (metre)	Yer Çekimi (m/s^2)	1. Uzay Aracının kütlesi (72 kg)	2. Uzay Aracının kütlesi (64 kg)
Dünya	$5{,}97 \cdot 10^{24}$	$6{,}37 \cdot 10^{6}$		706,104 N	627,648 N
Mars		$3{,}389 \cdot 10^{6}$	3,711		
Ay		$1{,}737 \cdot 10^{6}$	1,62		

NASA [National Aeronautics and Space Administration (Ulusal Havacılık ve Uzay Dairesi)] Amerika Birleşik Devletleri'nin uzay programı çalışmalarından sorumlu olan kurumdur. NASA yeni araştırmalar için Mars ve Ay'a uzay aracı gönderiyor. İki özdeş top, gönderilen uzay aracından Ay ve Mars'ta farklı yüksekliklerden bırakılıyor. Ay yüzeyine bırakılan top yere her çarptığında düştüğü yüksekliğin karekökü kadar yükseliyor. Mars yüzeyine bırakılan top yere her çarptığında düştüğü yüksekliğin küpkökünün karesi kadar yükseliyor. Ay'da $(0{,}6)^{16}$ metre yükseklikten bırakılan cismin dördüncü çarpmasından sonraki yüksekliğinin Mars'ta bırakılan cismin üçüncü çarpmasından sonraki yüksekliğine eşit olduğu hesaplanmıştır.

Buna göre

a) Topun Mars'ın yüzeyine kaç metreden bırakıldığını bularak Mars ve Ay'da eşit mesafeden bırakılan eş değer topların zemine çarpmalarından sonraki yüksekliğini veren bir matematiksel formül geliştiriniz.

b) Tablodaki boşlukları doldurarak Dünya'da ağırlığı verilen topların Mars ve Ay'daki ağırlıklarını veren bir formül oluşturunuz.

Mert'e soruların cevabını bulmada yardımcı olunuz.

15

Figure 8. Activity 11 from the textbook (MoNE, 2020a, p. 15)

The English version of the Activity 11:

Mert learned that based on Newton's Universal Law of Gravitation, a science technical journal, the gravitational force of a planet with a certain mass is calculated by directly proportional to its mass and inversely proportional to the square of the radius of its center of gravity. He also discovered that the Greek mathematician and astronomer Eratosthenes first calculated the radius of the Earth in 200 BC.

Mert's physics teacher taught the topic of gravitational potential energy in the energy unit and asked his/her students to solve the following questions based on the given information.

Since G is the gravitational constant ($G = (6{,}673 \times 10^{-11})\, {}^{m^3}/_{kg \cdot s^2}$), the gravitational force (g) of planets is found by the following formula.

$$g = G\frac{M_y}{R_y^2} M_y R_y$$

(M_y = the mass of the earth, and R_y = the radius of the surface center of gravity)

Weight (N) = mass · gravitational force

	Mass (kg)	Radius (m)	Gravitation (m/s^2)	The mass of the first spacecraft (72 kg)	The mass of the second spacecraft (64 kg)
Earth	$5{,}97 \cdot 10^{24}$	$6{,}37 \cdot 10^{6}$		706,104 N	627, 648 N
Mars		$3{,}389 \cdot 10^{6}$	3,711		
Moon		$1{,}737 \cdot 10^{6}$	1,62		

NASA [National Aeronautics and Space Administration] is the institution responsible for the work of the United States space program. NASA sends spacecraft to Mars and the Moon for new research. Two identical balls are dropped from different heights on the Moon and Mars from the spacecraft. Whenever the ball hits the ground on the Moon, it rises as high as the square root of the height it fell. Whenever the

ball hits the ground on Mars, it rises as high as the square of the cube root of the height it fell. It has been calculated that the height of an object left on the Moon from a height of $(0{,}6)^{16}$ meters after the fourth hit equal the height of an object left on Mars after the third hit.

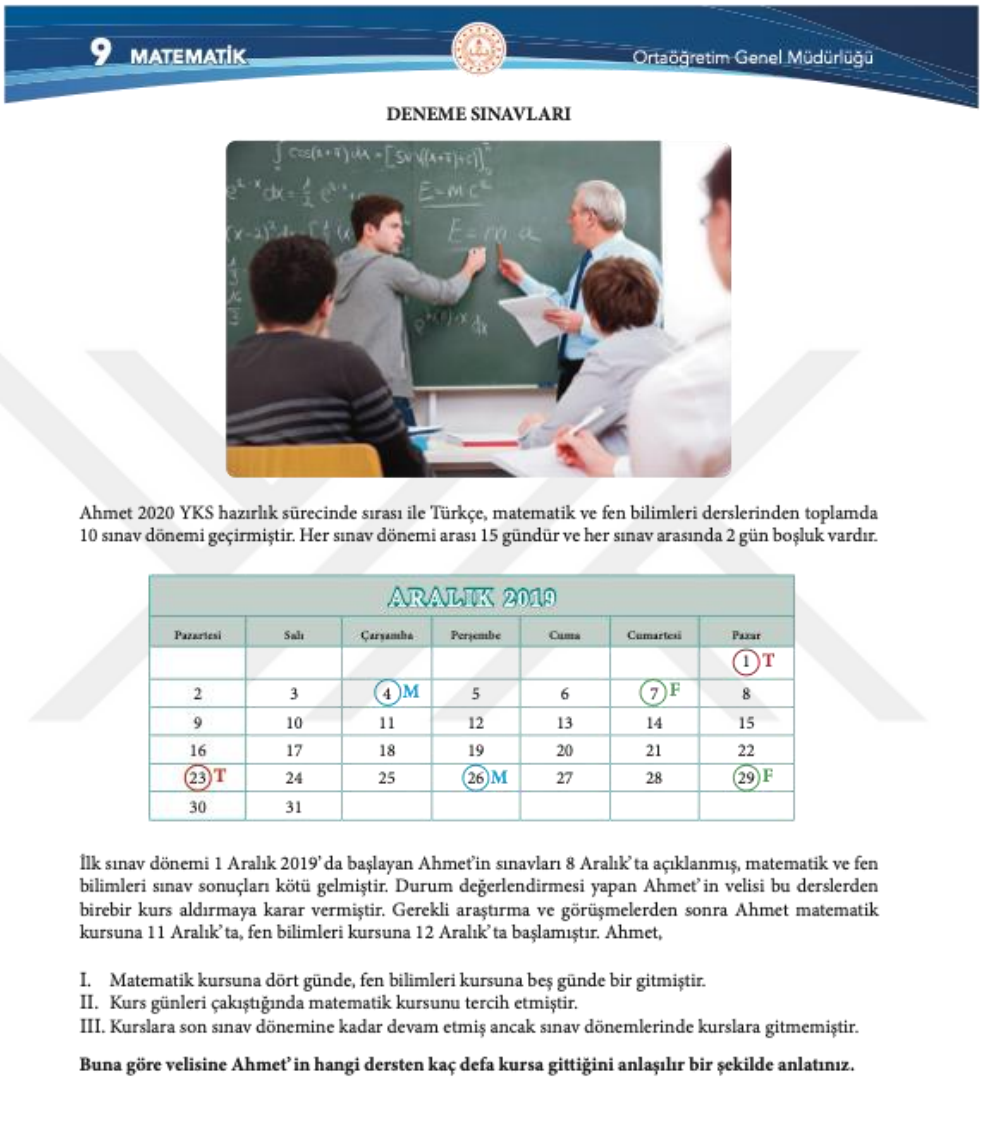
Accordingly,

a) By finding out how many meters the ball was left on the surface of Mars, develop a mathematical formula that gives the height of equivalent balls left at equal distances on Mars and the Moon after they hit the ground.
b) By filling in the blanks in the table, create a formula that gives the weights of the balls given weight in the Earth on Mars, and the Moon.

Help Mert to find the answers to the questions.

Activity Number: 12

Activity Name: Practice Exams



9 MATEMATİK — Ortaöğretim Genel Müdürlüğü

DENEME SINAVLARI

Ahmet 2020 YKS hazırlık sürecinde sırası ile Türkçe, matematik ve fen bilimleri derslerinden toplamda 10 sınav dönemi geçirmiştir. Her sınav dönemi arası 15 gündür ve her sınav arasında 2 gün boşluk vardır.

ARALIK 2019

Pazartesi	Salı	Çarşamba	Perşembe	Cuma	Cumartesi	Pazar
						(1) T
2	3	(4) M	5	6	(7) F	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
(23) T	24	25	(26) M	27	28	(29) F
30	31					

İlk sınav dönemi 1 Aralık 2019'da başlayan Ahmet'in sınavları 8 Aralık'ta açıklanmış, matematik ve fen bilimleri sınav sonuçları kötü gelmiştir. Durum değerlendirmesi yapan Ahmet'in velisi bu derslerden birebir kurs aldırmaya karar vermiştir. Gerekli araştırma ve görüşmelerden sonra Ahmet matematik kursuna 11 Aralık'ta, fen bilimleri kursuna 12 Aralık'ta başlamıştır. Ahmet,

I. Matematik kursuna dört günde, fen bilimleri kursuna beş günde bir gitmiştir.
II. Kurs günleri çakıştığında matematik kursunu tercih etmiştir.
III. Kurslara son sınav dönemine kadar devam etmiş ancak sınav dönemlerinde kurslara gitmemiştir.

Buna göre velisine Ahmet'in hangi dersten kaç defa kursa gittiğini anlaşılır bir şekilde anlatınız.

16

Figure 9. Activity 12 from the textbook (MoNE, 2020a, p. 16)

The English version of Activity 12:

Ahmet has passed ten exam periods in Turkish, mathematics, and science courses, respectively, during the preparation process for the 2020 Higher Education Entrance Exam (YKS). A 15-day break follows each exam period, and there is a gap of 2 days between each exam period.

DECEMBER 2019						
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
						1 (T)
2	3	4 (M)	5	6	7 (S)	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23 (T)	24	25	26 (M)	27	28	29 (S)
30	31					

T: Turkish, M: Mathematics, S: Science

Ahmet's first exam period started on December 1, 2019. His exams were announced on December 8, and he received low points in the mathematics and science exams. His parents, who evaluated the situation, decided to enroll him in a one-to-one course from these lessons. After conducting the necessary research and interviews, Ahmet started his mathematics course on December 11 and his science course on December 12. Ahmet,

I. He attended the mathematics course every four days and the science course every five days.
II. When the course days coincide with the same day, he preferred the mathematics course.
III. He continued the courses until the last exam period but did not attend the courses during the exam periods.

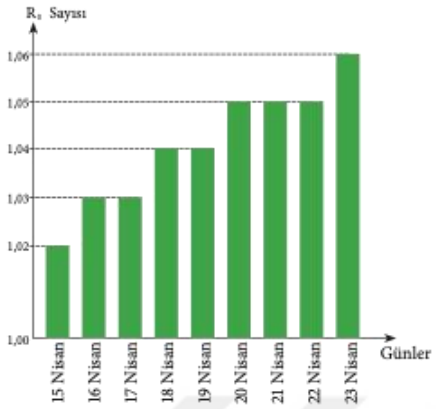
Accordingly, explain clearly to his parent which course Ahmet attended and how many times.

Activity Number: 15

Activity Name: Coronavirus



Figure 10. Activity 15 from the textbook (MoNE, 2020a, p. 20)



Yapılan test sayısı arttıkça tespit edilen yeni Covid-19 vaka sayısı da artmaktadır. R_0 değeri 1,09 değerine ulaştığı zaman hastalığın yayılmasını önlemek için ilave tedbirler alınacaktır.

İlave tedbirlerin alınabilmesi için sizce yapılacak günlük PCR test sayısı kaç olmalıdır?

NOT: R_0 değerinin belirlenmesinde bulaşma yolu, kuluçka süresi, temas riski, yoğunluğu ve süresi gibi birçok faktör etkilidir. Örnek çözümde bu faktörler hesaba katılmamaktadır.

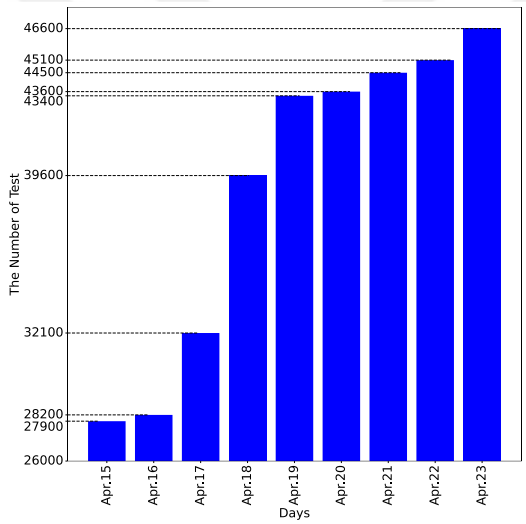
Figure 11. Activity 15 from the textbook (cont'd) (MoNE, 2020a, p. 21)

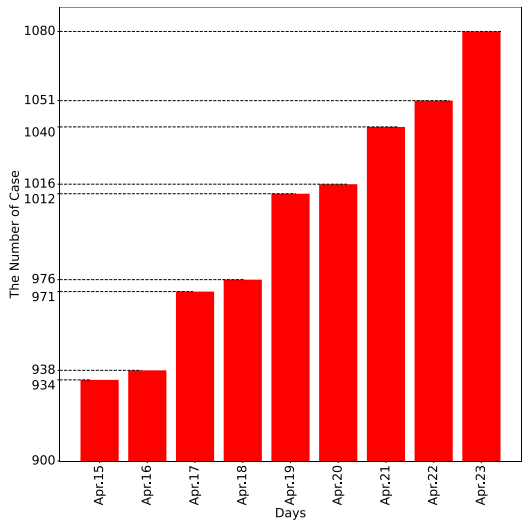
The English version of the Activity 15:

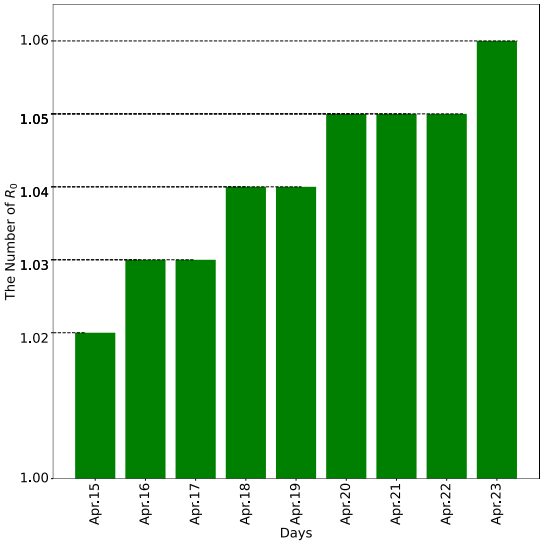
Due to the COVID-19 pandemic, which had its impact worldwide in 2020, a certain number of people are being tested by the “PCR” test every day to identify those who have contracted this disease and to take measures to prevent the spread of the disease.

Daily R_0 values are formed according to the number of new cases. These values indicate how many people a person with Covid-19 can transmit the disease to.

The following graphs provide the number of tests conducted, the number of new cases, and the value of the transmission coefficient (R_0) of this disease according to the days:







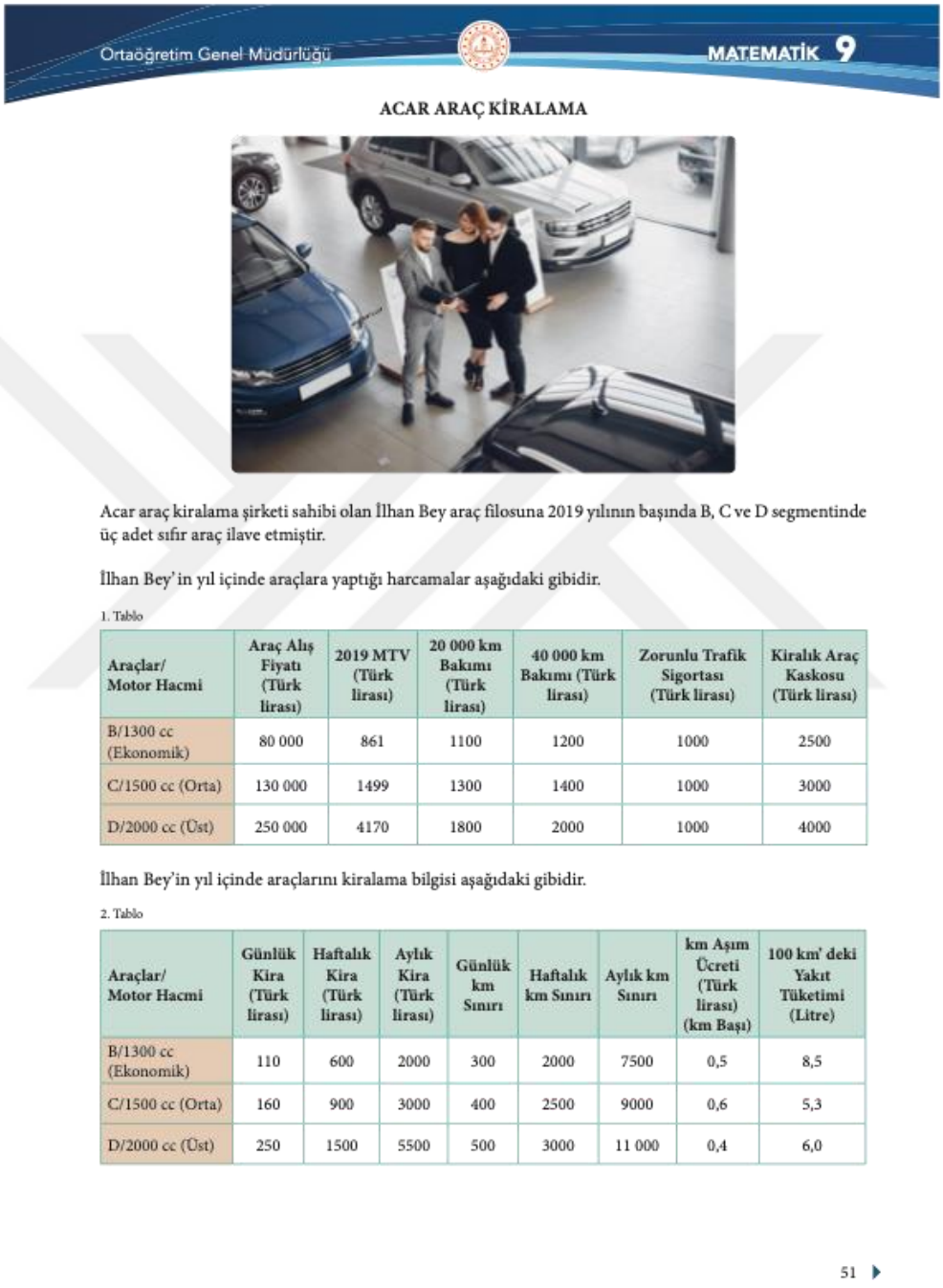
As the number of tests conducted increases, the number of newly detected Covid-19 cases also increases. When the R_0 value reaches 1.09, additional measures will be taken to prevent the spread of the disease.

In your opinion, how many daily PCR tests should be done in order for additional measures to be taken?

NOTE: The determination of the R_0 value is influenced by various factors such as the mode of transmission, incubation period, contact risk, intensity, and duration. These factors are not taken into account in the sample solution.

Activity Number: 38

Activity Name: Acar Car Rental



Ortaöğretim Genel Müdürlüğü

MATEMATİK 9

ACAR ARAÇ KİRALAMA

Acar araç kiralama şirketi sahibi olan İlhan Bey araç filosuna 2019 yılının başında B, C ve D segmentinde üç adet sıfır araç ilave etmiştir.

İlhan Bey'in yıl içinde araçlara yaptığı harcamalar aşağıdaki gibidir.

1. Tablo

Araçlar/ Motor Hacmi	Araç Alış Fiyatı (Türk lirası)	2019 MTV (Türk lirası)	20 000 km Bakımı (Türk lirası)	40 000 km Bakımı (Türk lirası)	Zorunlu Trafik Sigortası (Türk lirası)	Kiralık Araç Kaskosu (Türk lirası)
B/1300 cc (Ekonomik)	80 000	861	1100	1200	1000	2500
C/1500 cc (Orta)	130 000	1499	1300	1400	1000	3000
D/2000 cc (Üst)	250 000	4170	1800	2000	1000	4000

İlhan Bey'in yıl içinde araçlarını kiralama bilgisi aşağıdaki gibidir.

2. Tablo

Araçlar/ Motor Hacmi	Günlük Kira (Türk lirası)	Haftalık Kira (Türk lirası)	Aylık Kira (Türk lirası)	Günlük km Sınırı	Haftalık km Sınırı	Aylık km Sınırı	km Aşım Ücreti (Türk lirası) (km Başı)	100 km' deki Yakıt Tüketimi (Litre)
B/1300 cc (Ekonomik)	110	600	2000	300	2000	7500	0,5	8,5
C/1500 cc (Orta)	160	900	3000	400	2500	9000	0,6	5,3
D/2000 cc (Üst)	250	1500	5500	500	3000	11 000	0,4	6,0

51

Figure 12. Activity 38 from the textbook (MoNE, 2020a, p. 51)

Kiralama en az 1 günlüktür (24 Saat). Daha kısa süreli kiralamalarda günlük ücret uygulanır. Gecikme halinde 4 saate kadar her saat için günlük ücretin 1/4 ü, gecikmenin 4 saati geçtiği durumlarda ise tam gün ücreti alınır.

- 2. Tablo' daki kilometre sınırları şirketin İzmir, Antalya, Bodrum, Dalaman, Fethiye, Ortaca ve Marmaris'teki tüm ofisleri için geçerlidir. Türkiye genelindeki diğer ofislerde araçların günlük km sınırı bulunmamaktadır.
- Aracı kiralayan kişi, kiralama döneminde oluşabilecek her türlü ek masrafı (trafik cezası, OGS/HGS kaçak geçiş veya her türlü maddi zarar) ödeyeceğini bu sözleşmeyi imzalayarak kabul etmiş sayılır.
- Benzinin litresi ortalama 6,5 Türk lirasıdır.

1. Yeliz Hanım, tatile geldiği Marmaris'te kiralık araç kullanarak 3 günlük kültür turu planlıyor. 1600 km lik kültür turu için aracı zamanında ve hasarsız olarak teslim ettiği takdirde Yeliz Hanım'ın hangi aracı tercih etmesi gerektiğini nedenleri ile açıklayan bir matematiksel model oluşturunuz.

2. 2. Tablo' daki bilgileri kullanarak 40 günlük gezi programı için Yeliz Hanım'ın 17 000 Türk lirası olan bütçesi ile (C segmenti) orta sınıf bir araç kiralayarak en fazla kaç km yol gidebileceğini bulup ayrıntılı olarak açıklayınız. (Bu hesaplamada bir ayın 30 gün üzerinden ücretlendirildiğini, aracın zamanında ve hasarsız olarak teslim edildiğini hesaba katınız.)

3. Araçların yıl içerisindeki kiralanma durumu aşağıdaki tabloda verilmiştir.

3. Tablo

Araç Sınıfı/Motor Hacmi	Gün	Hafta	Ay	Aracın Yaptığı Toplam Kilometre
B/1300 cc (Ekonomik)	90	9	3	62 000
C/1500 cc (Orta)	110	8	1	82 300
D/2000 cc (Üst)	60	3	1	40 000

Şirket sahibi İlhan Bey'in yıl sonundaki kazancı nasıl olmuştur? Sizce yeni yılda hangi araçlardan alırsa hem sermayesini korur hem de kazancı yüksek olur?

4. Dokuz günlük araç kiralamak isteyen Sibel Hanım, toplamda 5000 km yol gitmeyi planlıyor. Sizden hangi aracı seçmesi gerektiğine dair öneri istiyor. Önerdiğiniz araç için 2. Tablo'daki verileri kullanarak tablo şeklinde ayrıntılarıyla açıklayarak fiyat teklifi vermenizi istiyor. (Aracın zamanında ve hasarsız olarak teslim edildiğini hesaba katarak teklif veriniz.)

5. Acar araç kiralama şirketine gelen iki aileden biri ekonomik, diğeri orta sınıf araç kiralıyor. Beraber çıktıkları sekiz günlük tatilde 3600 km yol gidiyorlar. Sizce bu ailelerden hangisi daha fazla ücret ödemiştir? Cevabınızı tablo üzerinde ayrıntılarıyla açıklayınız. (Araçların zamanında ve hasarsız olarak teslim edildiğini hesaba katınız.)

52

Figure 13. Activity 38 from the textbook (cont'd) (MoNE, 2020a, p. 52)

The English version of the Activity 38:

Mr. Ilhan, the owner of Acar Car Rental Company, added three brand new vehicles in the B, C, and D segments to the vehicle fleet at the beginning of 2019.

Mr. Ilhan's expenditures on vehicles throughout the year are as follows.

Table 1.

Vehicles/Engine volume	Vehicle purchase price (TL)	2019 MTV (TL)	20 000 km maintenance (TL)	40 000 km maintenance (TL)	Compulsory traffic insurance (TL)	Collision insurance of rental car (TL)
B/1300 cc (economic)	80 000	861	1100	1200	1000	2500
C/1500 cc (medium)	130 000	1499	1300	1400	1000	3000
D/2000 cc (top)	250 000	4170	1800	2000	1000	4000

Mr. Ilhan's information about renting vehicles throughout the year is as follows.

Table 2.

Vehicles/Engine volume	Daily rent (TL)	Weekly rent (TL)	Monthly rent (TL)	Daily km limit	Weekly km limit	Monthly km limit	km exceeding fee (TL)	Fuel consumption per 100 km
B/1300 cc (economic)	110	600	2000	300	2000	7500	0,5	8,5
C/1500 cc (medium)	160	900	3000	400	2500	9000	0,6	5,3
D/2000 cc (top)	250	1500	5500	500	3000	11000	0,4	6,0

The minimum rental duration is one day (24 hours). For rentals shorter than a day, a daily cost is applied. In case of delay, for each hour up to 4 hours, 1/4 of the daily cost is charged, and if the delay exceeds 4 hours, the total daily cost is charged.

- The kilometer limits in Table 2 are valid for all the company's offices in İzmir, Antalya, Bodrum, Dalaman, Fethiye, Ortaca, and Marmaris. There is no daily km limit for vehicles in other offices throughout Turkey.
- By signing this contract, the person renting the vehicle is deemed to have accepted that they will pay any additional expenses (traffic fine, OGS/HGS illegal passage, or any kind of financial damage) that may occur during the rental period.
- The average liter of gasoline is 6.5 Turkish liras.

1. Ms. Yeliz, who is on vacation in Marmaris, plans a 3-day cultural tour using a rental car. Create a mathematical model that explains which car Ms. Yeliz should choose and the reasons behind it, provided that she returns the vehicle on time and without any damage, for a 1600 km cultural tour.

2. Using the information in Table 2, find and explain in detail how many kilometers Ms. Yeliz can travel at most by renting a medium-class vehicle (C segment) with a budget of 17 000 Turkish liras for a 40-day trip program. (In this calculation, consider that one month is charged based on 30 days and assume that the car is returned on time and without any damage.

3. The rental status of the vehicles throughout the year is given in the table below.

Table 3.

Vehicle grade / Engine volume	Day	Week	Month	The total kilometer taken by the vehicle
B/1300 cc (economic)	90	9	3	62 000
C/1500 cc (medium)	110	8	1	82 300
D/2000 cc (top)	60	3	1	40 000

How was the company owner Mr. Ilhan's earnings at the end of the year? Which vehicles do you think he would buy in the new year that can both protect his capital and provide high profit?

4. Ms. Sibel, who wants to rent a car for nine days, plans to travel 5000 km. She wants a recommendation from you about which vehicle she should choose. She also requests a detailed price offer in tabular form using the information in Table 2, explaining the details. (Please make an offer taking into account that the vehicle was returned on time and without any damage.)

5. One of the two families that come to Acar Car Rental Company is renting an economical vehicle, and the other is renting a middle-class vehicle. They are traveling 3600 km on an eight-day vacation they are going on together. Which of these families do you think has paid more? Explain your answer in detail in the table. (Please consider that the vehicles are returned on time and without any damage.)