On a relation between shock profiles and stabilization mechanisms in a hyperbolic-elliptic coupled system

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We discuss L^∞-stability of traveling waves or shock waves to a hyperbolic-elliptic coupled system of

$$u_t + f(u)_x + q_x = 0, \tag{20a}$$

$$-q_{xx} + q + u_x = 0 \tag{20b}$$

for $x \in \mathbb{R}$, where u, q are scalar functions and f is a convex function. This system appears in a gas dynamics with radiation or in a regularized Chapman-Enskog expansion model for scalar conservation laws. If $U(\pm\infty) = u_\pm$ for certain constants $u_\pm$, the propagation speed σ is $(f(u_-)-f(u_+))/(u_- - u_+)$. A sufficient condition for the existence of smooth solutions was given by Schochet and Tadmor [**1**]:

$$1 + 4 \inf_{u\in(u_+,u_-)} f''(u)H(u) \geq 0, \quad H(u) := f(u) - f(u_-) - \sigma(u - u_-) \tag{21}$$

as well as a necessary condition for that:

$$1 + 4f''(u_*)H(u_*) \geq 0, \quad u_* := (f')^{-1}(\sigma). \tag{22}$$

Lattanzio, Mascia and Serre [**2**] showed that the system admits arbitrary large amplitude, possibly discontinuous shock waves. Kawashima and Nishibata [**3**] thoroughly classified the regularity of shock profiles in terms of shock strength $u_- - u_+$ for the Burgers's flux $f(u) = u^2/2$.

We first prove that subcritical shock waves, for which the left hand side of (21) is positive, are L^∞-stable to small perturbations by virtue of radiation. While in a critical case where we assume an even function f satisfies (21) and $1+4f''(u_*)H(u_*) = 0$ in addition to $u_- + u_+ = 0$, we show that u_x blows up if certain types of perturbations are added to traveling waves no matter how small they may be. Finally we see that sufficiently strong shock waves which violate (22) and are therefore discontinuous regain stability owing to the contribution of convection under the presence of discontinuity.

These results could be understood as a rationalization of the dependence of the continuity of traveling waves on the shock strength from the view point of L^∞-stability, making an interesting contrast to the L^1-stability by Serre [**4**], which is applicable to arbitrary strong shock waves.

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Analysis of the Relativistic Burgers models on spacetime geometries

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Several relativistic and non-relativistic Burgers models have recently been derived and generalized on spacetime geometries. LeFloch and his collaborators initiated the derivation of the model on a flat background by identifying a hyperbolic balance law which satisfies the same Lorentz invariance property as the one satisfied by the Euler equations of relativistic compressible fluids [**5**]. The relativistic generalization of this model was then extended to Schwarzschild, FLRW, de Sitter and Schwarzschild-de Sitter geometries [**2**, **3**]. A numerical scheme based on finite volume methodology was developed

by taking into account geometrical effects [**1**, **4**]. In this work we consider these spacetime geometries in order to analyze the derived Burgers models theoretically and numerically. We establish a second-order Godunov-type finite volume scheme for approximation of discontinuous solutions and compare each model of interest by illustration of shock and rarefaction wave propagation.

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Influence of the coastline on Western oceanic boundary layers

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We are interested in finding steady solutions for a model of Western boundary layers in an ocean gyre (e.g. the Gulf Stream). By rescaling the 2D geophysical Navier-Stokes equation, we obtain a Prandtl-like system containing a parameter λ, which is a function that depends on the inverse of the derivative of the Western coastline profile. In our work [**1**], we study existence and uniqueness problems for this equation, and seek solutions which have positive latitudinal (South-to-North) velocity. Numerical simulations in [**2**] suggest that this can be done for λ large, while, when λ is near zero, recirculation appears.

By using a von Mises change of variables, and carefully constructing sub- and super-solutions for the degenerate parabolic PDE thus obtained, we confirm that positive solutions do exist for λ large enough, and we show precise behaviour for these solutions in the interior limit. Indeed, the rate of convergence of the stream function towards its interior value is shown to satisfy algebraic relations that are only possible when $\lambda > 2$. Recalling how λ depends on geometry of the boundary, this means that sharp changes in the coastline can lead λ to dip below the threshold for guaranteeing existence of a solution with no recirculation. To our knowledge, our result is the first mathematical clue that the geometry could be behind Western current separation from the coast.

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