THE ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES, 127:293–297, 2000 April
© 2000. The American Astronomical Society. All rights reserved. Printed in U.S.A.

MODELING OF THE EXPANSION OF ULTRA–SHORT-PULSE LASER-PRODUCED PLASMAS IN MAGNETIC FIELDS

TODD DITMIRE
Lawrence Livermore National Laboratory, Livermore, CA 94550; t.ditmire@ic.ac.uk
ALEXANDR RUBENCHIK
Lawrence Livermore National Laboratory, Livermore, CA 94550; rubenchik1@llnl.gov
VLADIMIR V. MIRNOV
Physics Department, Middle East Tech. University, Ankara, 06531, Turkey; vladimir@rorqual.cc.metu.edu.tr
AND
DEFNE UCER
Physics Department, Middle East Tech. University, Ankara, 06531, Turkey; ducer@rorqual.cc.metu.edu.tr
Received 1999 January 19; accepted 1999 December 13

ABSTRACT

The study of hot plasma expansion in a uniform magnetic field is of interest for many astrophysical applications. In order to observe this process in laboratory, an experiment is proposed in which an ultrashort laser pulse produces a high-temperature plasma by irradiation of a jet of atomic clusters. The very high laser light absorption exhibited in such a gas of clusters facilitates the creation of a hot (> 5 keV), dense (10^{19}–10^{20} cm^{-3}) plasma with a sharp boundary. The small scale of the plasma (< 100 μm radius) permits the use of a strong magnetic field ($> 1T$). Pump-probe techniques can then be used to diagnose the density and magnetic field with high spatial and temporal resolution ($< 50fs$). In the present work the expansion rate of the plasma and deceleration caused by the magnetic field are examined analytically. Electrodynamical aspects related to the radiation and transformation of energy are considered as well. The results obtained can be used in treating experimental data, studying magnetic R-T instabilities and other phenomena of astrophysical significance.

Subject headings: magnetic fields — methods: analytical — methods: laboratory — plasmas

1. INTRODUCTION

The problem of sudden expansion of hot plasma into a vacuum in the presence of a uniform external magnetic field has been intensively studied in the mid-1960s in connection with the high-altitude nuclear explosions. It has also been discussed in the analysis of the explosion of a supernova into the intergalactic magnetic field. Both analytical and numerical approaches (Raizer 1963; Kulsrud et al. 1965; O'Neil & Fowler 1966; Golubev 1978) were developed to study radial dynamics and evolution of the initially spherical plasma cloud. Plasma has been considered as a highly conducting matter with zero magnetic field inside. From the point of view of electrodynamics it is similar to the expansion of a superconducting sphere in a uniform external magnetic field.

The property of expanding plasma to push magnetic field out is a sequence of magnetic flux conservation. Even if a magnetic field and, magnetic flux are nonzero initially inside the plasma, then internal field tends to zero fast, providing magnetic flux in plasma to be constant. Plasma is shielded from the penetration of the large external field by means of surface currents circulating inside the thin layer on the plasma boundary. Ponderomotive forces resulting from interaction of these currents with the magnetic field would act on the plasma surface as if there were "magnetic" pressure applied from outside. Although magnetic pressure, $p_{\rm mag} = B^2/8\pi$, is assumed to be small in comparison with the initial plasma pressure $p_{\rm max}$ (on the axis at $t = 0$), after some period of accelerated motion, plasma gets decelerated as a result of this external force acting inward.

An important advantage of short pulse laser–produced plasma is its cylindrical symmetry around the laser-beam direction. Since the external magnetic field is aligned along the same axis, plasma dynamics can be treated in one-dimensional approximation. This approach is certainly valid until plasma starts to be decelerated and R-T instability appears. In this sense cylindrical expansion is better for experimental modeling than the spherical one. In the spherical case the symmetry is broken down because of the uniform magnetic field just from the beginning of expansion when plasma is still involved in acceleration and, therefore, no R-T instability.

Appearance of R-T instability is likely to be in the form of growing flutelike perturbations localized on the plasma boundary. These perturbations are described by the fluid equations linearized around the nonuniform and unsteady radial plasma flow. Within the scope of this paper we will not consider the instability by itself, but focus attention on the structure and scalings valid for the radial plasma flow. For this purpose ideal MHD equations will be used together with the Maxwell equations needed for energy balance analysis.

2. PLASMA AND ELECTROMAGNETIC ENERGY BALANCE

Significant attention has been paid (Raizer 1963) to the question of what fraction of energy is emitted and lost in the form of electromagnetic pulse propagating outward of the expanding plasma. Two statements of this problem are of practical interest. The model of infinite space with uniform magnetic field extending to infinity is more adequate to the astrophysical applications. The case when magnetic field is created inside external solenoid is more relevant to the laboratory conditions. This solenoid can be treated as a continuous super-conducting cylinder which is coaxial with

the plasma and placed at a distance $R \gg a$ from the axis. Azimuthal current circulating around the surface of this cylinder generates uniform field $\boldsymbol{B}^{(0)}$ in the solenoid and the system is supposed to be disconnected from the external energy source.

We consider now the case of the infinite space. When hot cylindrical plasma of the small initial radius a_0 is created at $t = 0$ and starts expanding, external magnetic field $B^{(0)}$ is perturbed by the electromagnetic pulse, $B^{(1)}(r, t)$, $E(r, t)$, propagating outward with the speed of light. The tail of this pulse coincides with the moving plasma boundary $r = a(t)$ while the leading edge is at $r = a_0 + ct$. Ahead of the leading edge, the magnetic field is not perturbed and equals $B^{(0)}$ while the electric field is zero. If plasma radial velocity is small, $v/c \ll 1$, the amplitudes of perturbations $B^{(1)}(r, t)$ and $E(r, t)$ are small as well [of the order of $v\boldsymbol{B}^{(0)}/c$].

The energy radiated to infinity is measured as a Poynting vector integrated over time and over the unitlength distant cylinder $r = R_c$ (control cylinder) enclosing the plasma cylinder [$a(t) \ll R_c$]. It equals the decrease of the magnetic energy, $B^2/2\pi$, inside the control surface R_c (the contribution of the electric energy is small as v^2/c^2) and decrease of plasma energy measured as a sum of kinetic and thermal plasma energies.

In order to illustrate the above statements, we consider a simple case of planar geometry and constant velocity of expansion. It assumes that plasma slab of zero width ($a_0 = 0$) is created at $t = 0$ at midplane $x = 0$ and expands to the right and to the left with the velocities $\pm v$. At $t < 0$ everywhere in space there exists uniform magnetic field $B^{(0)}$ which is aligned along z-axis. Surface currents $\mp I$ are caused by the plasma expansion and appear at $t = 0$. They are oriented along y-axis and localized on the plasma boundaries at $x = \pm vt$, respectively. The problem with initial conditions is considered implying that perturbations $B^{(1)}(x, t)$, $E(x, t)$ are zero at $t < 0$. The same conditions are required to be satisfied at $|x| > ct$.

Introducing vector potential $A(x, t)(B^{(1)} = \partial A/\partial x$, $E = -\partial A/c\partial t)$ yields a wave equation with the source term in the right-hand side, describing the structure of the surface currents during plasma expansion:

$$\frac{\partial^2 A}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = \frac{4\pi}{c} I_{\text{exp}}(x, t) , \tag{1}$$

$$I_{\text{exp}}(x, t) = I_0[\delta(x + vt) - \delta(x - vt)] . \tag{2}$$

With the help of a Fourier transformation on x, the following differential equation for the time-dependent function $A_k(t)$ is derived:

$$\frac{d^2 A_k}{dt^2} + k^2 c^2 A_k = 4icI_0 \sin(kvt) . \tag{3}$$

Taking the surface current I_0 constant of time, the solution to this equation with zero initial conditions, $A_k(0) = 0$, $dA_k/dt(0) = 0$ takes the form

$$A_k(t) = \frac{4iI_0}{k^2(c^2 - v^2)} [c \sin(kvt) - v \sin(kct)] . \tag{4}$$

Making an inverse Fourier transform for $B^{(1)}(x, t)$, $E(x, t)$ gives the expressions for total magnetic $B = B^{(0)} + B^{(1)}$ and electric fields during the expansion.

$$B(x, t) = \begin{cases} B^{(0)} , & x > ct , \\ B^{(0)}c/(c - v) , & vt < x < ct , \\ 0 , & 0 < x < ct , \end{cases} \tag{5}$$

$$E(x, t) = \begin{cases} 0 , & x > ct , \\ B^{(0)}v/(c - v) , & vt < x < ct , \\ 0 , & 0 < x < vt . \end{cases} \tag{6}$$

The value of the surface current, $I_0 = cB^{(0)}(1 + v/c)/4\pi$, is chosen to provide total magnetic and electric fields to be zero in plasma. Graphs of $B(x, t)$ and $E(x, t)$ are given in Figure 1*a*. They are plotted for $x \geq 0$ only because $B(x, t)$ and $E(x, t)$ are even and odd functions of x, respectively. Although plasma velocities are supposed to be nonrelativistic, exact expressions are derived to be used in the case of periodic motion below. Boundary condition on the moving interface, $E = vB/c$, is satisfied automatically. Surface current I_0 is expressed in terms of magnetic field on the plasma boundary with standard equation, $I_0 = cB(1 - v^2/c^2)/4\pi$, modified for the case of relativistic velocities. Since the problem is symmetric with respect to midplane $x = 0$, we will further consider the motion of the right interface only.

If one assumes that expanding plasma boundary reaches a maximum displacement $a_{\max} = v\tau$ at time $t = \tau$ and is then stopped at this position, the trailing edge of solutions (eqs. [5] and [6]) loses contact with the plasma boundary and profile shown in Figure 1*a* starts, as whole, propagating outward with the speed of light. There appears the gap $a_{\max} < x < a_{\max} + c(t - \tau)$ between plasma boundary at rest and escaping pulse where magnetic and electric field are, respectively, $B = B^{(0)}$, $E = 0$. Integrating a Poynting vector over the distant control surface and over the time during which the pulse localized between $a_{\max} + c(t - \tau) < x < ct$ passes through the surface yields significant fraction of energy, $a_{\max} B^{(0)2}/4\pi(1 - v/c)$, to be carried to the infinity by the outgoing electromagnetic field. Approximately half of this energy is gained from the plasma due to the drag on the external magnetic field, while the other half is gained from the magnetic energy due to the growing hole in initially uniform magnetic configuration. Similar results are valid in the case of expansion of highly conducting cylindrical plasma.

An important feature of the above energy balance is the quasi-reversible character of the energy flux in the case of periodic plasma motion. Let us consider a situation when after expansion and deceleration plasma boundary reaches turning point $x = a_{\max}$ at time $t = \tau$ and then starts moving back to the origin. Due to the change of the direction of motion flux of electromagnetic energy changes the sign as well, and almost all energy returns back to the control volume except a small fraction proportional to v/c.

In order to consider the whole periodic cycle analytically we will assume that plasma boundary moves forth and back with the constant velocity $\pm v$. The change of sign from plus to minus happens instantaneously at $t = \tau$, implying infinitively large deceleration imposed at this instant.

Electromagnetic fields $B^{(1)}(x, t)$, $E(x, t)$ at $t > \tau$ are described by equation (1) with the source term modified accordingly to the plasma compression when boundaries are at the points $x = \pm[a_{\max} - v(t - \tau)]$:

$$I_{\text{com}}(x, t) = I_1[\delta(x + 2v\tau - vt) - \delta(x - 2v\tau + vt)] . \tag{7}$$

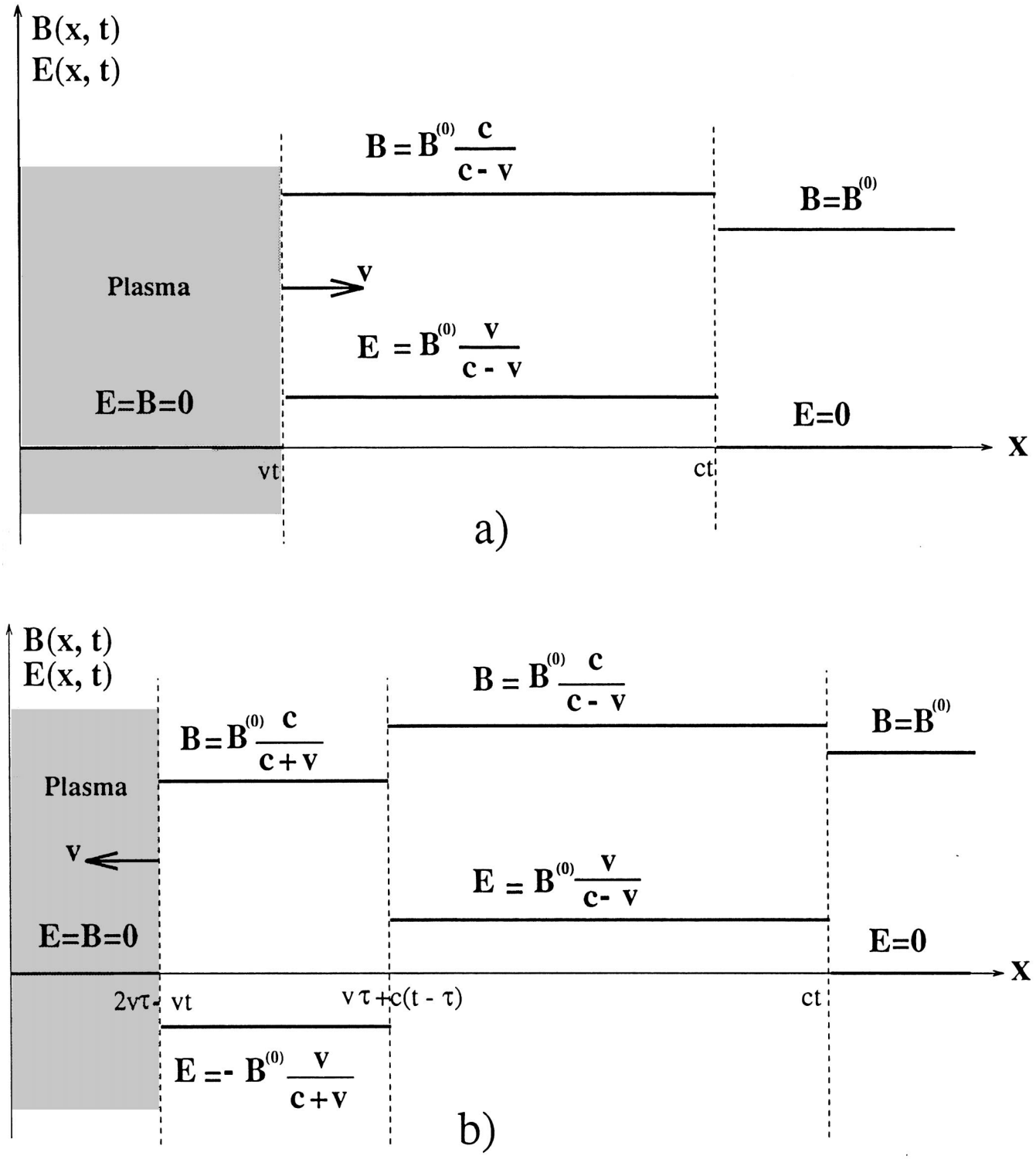


FIG. 1.—Graphs of functions $B(x, t)$, $E(x, t)$ vs. $x \geq 0$. Since the problem is symmetric with respect to midplane $x = 0$, only right part of the plasma slab is shown: (*a*) period of plasma expansion; (*b*) period of plasma compression.

Nonzero initial conditions at $t = \tau$ are determined by the functions $A_k(\tau)$ and $dA_k/dt(\tau)$ given by equation (4) from the previous interval $0 < t < \tau$. Solution at later times $t > \tau$ turns out to be a superposition of two different parts. The first part reproduces the structure of the previous electromagnetic pulse. It is localized between $a_{\max} + c(t - \tau) < x < ct$ and looks as if the profiles (eq. [5]), (eq. [6]) at $t = \tau$ were carried to the right with the speed of light. The second part represents a new field which is localized in the gap $2v\tau - vt < x < v\tau + c(t - \tau)$ between the plasma boundary and the escaping old pulse. This solution is obtained from equations (5) and (6) by changing the sign of v. The same

result is valid for the new surface current $\boldsymbol{I}_1 - cB^{(0)}(1 - v/c)/4\pi$. The picture of $B^{(1)}(x, t)$, $E(x, t)$ distributions at $t > \tau$ is illustrated in Figure 1*b*.

With the help of solutions obtained, the total flux of escaping electromagnetic energy measured as a Poynting vector through the distant control surface integrated over the period of plasma expansion and compression is given by

$$W_{\text{loss}} = \frac{a_{\max} B^{(0)2}}{8\pi} \frac{4(v/c)}{1 - v^2/c^2} . \tag{8}$$

Since initial and final configurations are the same, there is no variation of magnetic energy between these two states and plasma energy loss is equal to W_{loss}. It is confirmed independently by calculating the mechanical energy performed by plasma on the external magnetic pressure.

There is an issue how much value of W_{loss} is sensitive to the choice of $a(t)$. In order to analyze this question more realistic parabolic profile of $a(t)$ was treated:

$$a(t) = v_0 t - \frac{v_0^2}{4a_{\max}} t^2 . \tag{9}$$

It corresponds to the initial radial velocity v_0 and constant deceleration $v_0^2/a_{\max}$. This approximation better matches time variation of plasma speed analyzed below in § 3. Calculating net energy loss in the case of the parabolic profile (eq. [9]), yields

$$W_{\text{loss}} = \frac{B_0^2 c^2 a_{\max}}{2\pi v_0^2} \left[\ln\left(\frac{1 + v_0/c}{1 - v_0/c}\right) - 2v_0/c \right] = \frac{B_0^2 a_{\max}}{\pi} \left(\frac{v_0}{3c} + \frac{v_0^3}{5c^3} + \dots \right) . \tag{10}$$

Comparing equations (8) and (10) in nonrelativistic limit shows that both expressions are proportional to the first power of v/c. In order to provide the same $a_{\max}$ during the same expansion time τ, initial plasma velocity v_0 in equation (9) must be $v_0 = 2v$. Taking this into account gives approximately the same net energy loss in the first and second cases ($W_{\text{loss}}^{\text{unif}} = (\frac{3}{4}) W_{\text{loss}}^{\text{parab}}$). The choice of a smoother parabolic profile does not practically effect on the net energy loss, because it is determined by the integral of energy flux over time where details of $a(t)$ and $\dot{a}(t)$ are not important.

3. DYNAMICS OF PLASMA EXPANSION

In the previous section the motion of the plasma boundary was approximated as the motion with constant velocity ($\pm v$). In this section a quantitative analysis of plasma dynamics is developed on the basis of one-dimensional cylindrical radial model. Within the scope of this analysis the initial stage of plasma acceleration, later stage of deceleration and the process of stopping at the point of maximum expansion are considered.

According to the previous results, the irreversible part of energy loss is small and, therefore, can be neglected. Reversible part of energy, which is transferred from plasma to electromagnetic field is the mechanical work performed by the plasma on the external constant magnetic pressure $B^2/8\pi$. Taking into account this effect and omitting small relativistic corrections, the equation of balance of plasma energy normalized per unit length of plasma cylinder is as follows:

$$2\pi \int_0^{a(t)} \frac{p}{\gamma - 1} r\, dr + 2\pi \int_0^{a(t)} \frac{\rho v^2}{2} r\, dr = W_0 - 2\pi \frac{B^{(0)2}}{8\pi} \int_{a_0}^{a(t)} r\, dr , \tag{11}$$

where W_0 is initial thermal energy, $a(t) \leq a_0$. Initial plasma velocity $v(r, 0)$ and kinetic energy at $t = 0$ are supposed to be zero.

This equation can be effectively used if profiles of velocity $v(r, t)$, pressure $p(r, t)$, and mass density $\rho(r, t)$ are known functions of $a(t)$. We will take these dependences from the solution of a similar problem which deals with sudden expansion of cylindrical plasma into a vacuum without ambient magnetic field. The simplest class of solutions available in this case are so-called self-similar solutions. They are realized under the specified initial conditions. We will set the initial conditions with a parabolic distribution of pressure and a tablelike mass density, which describe hot and dense initial plasma state with sharp boundary localized at $r = a_0$.

$$p(r, 0) = \begin{cases} 0 , & r \geq a_0 , \\ p_{\max}(1 - r^2/a_0^2) , & r \leq a_0 ; \end{cases}$$

$$\rho(r, 0) = \begin{cases} 0 , & r \geq a_0 , \\ M/\pi a_0^2 = \rho_0 , & r \leq a_0 ; \end{cases} \tag{12}$$

where $M = \text{const}$ is the total mass of plasma per unit length of cylinder.

The self-similar solutions are characterized by a velocity distribution linearly dependent on r (uniform velocity profile):

$$v(r, t) = r\dot{a}(t)/a(t) , \quad r \leq a(t) , \tag{13}$$

where unknown $a(t)$ is the radius of sharp plasma boundary while $\dot{a}(t)$ is the velocity of the boundary. The specification of the tablelike mass density profile:

$$\rho(r, t) = M/\pi a^2(t) , \quad r \leq a(t) , \tag{14}$$

and equation (11) for velocity, automatically satisfies the continuity equation for an arbitrary function $a(t)$. The problem considered is not isentropic, since the density of all elements is the same while the pressures are different. Substitution of ρ and p into the entropy equation gives a following solution for the pressure:

$$p(r, t) = p_{\max}[a_0/a(t)]^2[1 - r^2/a^2(t)] , \quad r \leq a(t). \tag{15}$$

Substituting above expressions into the equation of motion yields a second-order differential equation governing the motion of the plasma boundary $a(t)$.

Equations (13)–(15) are an exact solution in the case of expansion into a vacuum without magnetic field. However, equation (15) does not satisfy the boundary condition, $p[a(t), t] = B^{(0)2}/8\pi$, which is imposed in the case of expansion into an ambient magnetic field. Since magnetic pressure is much smaller than the plasma pressure, $p_{\text{mag}}/p_{\max} \simeq 10^{-5}$, the difference between the exact solution in the magnetic field and free expansion model is small and localized in a narrow area near the surface of the cloud. These deviations are additionally reduced due to integration in the

equation of energy balance. Estimating accuracy of the free expansion model, one should take into account that the long stage of plasma deceleration corresponds to a high expansion ratio, $a(t)/a_0 \gg 1$. Average plasma pressure drops significantly and plays no role in energy balance equation (11) during this stage. In accordance with the above boundary condition local pressure near the plasma edge must be equal to the constant magnetic pressure outside. It causes deviation from tablelike profile equation (14) and accumulation of plasma in this area. This is confirmed independently by the numerical calculations (Golubev 1978). In the limiting case when all plasma is localized near the front, one can expect 4 times increase of the kinetic energy and 2 times longer stage of plasma deceleration as compared with the free expansion model.

Substituting equations (11)–(13) into (9) and integrating over r yields first-order differential equation for $a(t)$, which already satisfies initial condition $\dot{a}(0) = 0$:

$$\dot{x}^2 + (1/x)^{2(\gamma-1)} - 1 + [2p_{\rm mag}(\gamma - 1)/p_{\rm max}](x^2 - 1) = 0. \quad (16)$$

New dimensionless variables are introduced as follows: $x(t) = a(t)/a_0$, $t \to t/t_0$, $t_0 = [M(\gamma - 1)/2\pi p_{\rm max}]^{1/2} = a_0/v_{\rm free}$, where $v_{\rm free} = [2p_{\rm max}/(\gamma - 1)\rho_0]^{1/2}$ is a velocity of plasma expansion, achieved asymptotically at $t \to \infty$ in the case of expansion into a vacuum without magnetic field.

Comparing different terms in equation (14) yields conclusion that during short initial period of acceleration, $t_{\rm ac} \simeq (4\text{–}5)t_0 \simeq 3 \times 10^{-11}$ s, plasma boundary is accelerated accordingly to $a(t)/a_0 = (\gamma - 1)(t/t_0)^2/2$, while the ratio a/a_0 increases up to 5–10, and almost all initial thermal energy is transferred into kinetic energy of free radial expansion at constant velocity $v_{\rm free} \simeq 2 \times 10^8$ cm s^{-1}.

The stage of free expansion with $a(t) \simeq v_{\rm free}\, t$, continues until $a(t)$ reaches some value $a_{\rm dec}$ given by $a_{\rm dec}/a_0 = (p_{\rm max}/2p_{\rm mag})^{1/2\gamma} \simeq 30$. This critical radius and time $t_{\rm dec} \simeq a_{\rm dec}/v_{\rm free} \simeq 3 \times 10^{-10}$ s correspond to the beginning of plasma deceleration. Further plasma motion is an expansion with slowing-down velocity. It ends up at the turning point which corresponds to the maximum of expansion, $a_{\rm max}/a_0 = [(p_{\rm max}/2p_{\rm mag})(\gamma - 1)]^{1/2} \simeq 300$. The maximum radius of plasma cloud is limited by the drag on external magnetic field and can be potentially reached if there is no RT instability resulting from plasma deceleration at earlier times.

A characteristic time of plasma motion up to the full stop at the turning point is given by the integral of the equation (14):

$$t_{\rm max} = \int_1^{a_{\rm max}} dx/\{1 - x^{2(1-\gamma)} - [2p_{\rm mag}(\gamma - 1)/p_{\rm max}](x^2 - 1)\}^{-1/2}. \quad (17)$$

Calculating time $t_{\rm max}$ needed for plasma to reach this point, one can simplify the integrand taking into account that the main contribution comes from the vicinity of upper limit of integration. This yields universal expression, $t_{\rm max} = (\pi M/16p_{\rm mag})^{1/2} \simeq 3 \times 10^{-9}$ s. It is worth mentioning that in the case of weak magnetic field, $p_{\rm mag}/p_{\rm max} \ll 1$, this time does not depend on initial plasma radius and temperature. It corresponds to one quarter period of oscillation in parabolic potential formed by the magnetic pressure term in equation (11). This is a specific feature of the cylindrical geometry where large amplitude radial motion is driven by the linear force. It causes turnaround time to be determined by effective Alfven frequency and independent from initial energy.

4. SUMMARY

Preliminary analysis of the cylindrical plasma expansion into ambient magnetic field shows that there are processes of acceleration, retardation and stopping at the point of maximum expansion that are very distinct and separated in space and time. Since even at this point, the plasma radius does snot exceed 3 mm and the length of the initial plasma column is of the order of 3–4 mm, the cylindrical model considered above seems to be quite adequate to the problem.

All scaling laws obtained are the function of one dimensionless parameter, $p_{\rm mag}/p_{\rm max}$, which can be easily varied by means of the choice of the external magnetic field. It allows to test the different regimes of plasma deceleration and, to observe the dynamics of the Rayleigh-Taylor instability in a wide range of external conditions.

The authors would like to acknowledge B. Remington, who initiated this activity and a visit of one of the authors (V. V. M.) to the 2d International Workshop on Laboratory Astrophysics with Intense Lasers (March 19–21, 1998, Tucson, Arizona).

REFERENCES

Golubev, A. I., Solov'ev, A. A., & Terekhin, V. A. 1978, Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, 33
Kulsrud, R. M., Bernstein, I. B., Kruskal, M., Fanucci, J., & Ness, N. 1965, ApJ, 142, 491
O'Neil, T., & Fowler, T. K. 1966, Phys. Fluids, 9, 2219
Raizer, Yu. P. 1963, Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, 19