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Expectancy of large pulsar glitches: a comparison of models with the observed glitch sample

M. Ali Alpar and Altan Baykal
Physics Department, Middle East Technical University, Ankara 06531, Turkey

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ABSTRACT

We study the expectancy of large glitches ($\Delta\Omega/\Omega > 10^{-7}$) from a sample of 430 pulsars other than the Vela pulsar. The pulsars in this sample have exhibited 19 large glitches. We assume that all pulsars experience such glitches, with rates that depend on the pulsar's rotation rate Ω and spin-down rate $\dot{\Omega}$, and on the glitch model. We use the core-quake model, and two variants of the superfluid vortex unpinning model, and choose model parameters such that the observed number of glitches is matched by the theoretically expected number. Among the models investigated, one of the superfluid vortex unpinning models gives good agreement with the observed distribution of glitches and with the parameter values deduced for the Vela pulsar glitches.

Key words: MHD – stars: interiors – pulsars: general – stars: rotation.

1 INTRODUCTION

Pulsar glitches are sudden frequency changes, $\Delta\Omega/\Omega \sim 10^{-9}$–$10^{-6}$, accompanied by jumps in the spin-down rate, $\Delta\dot{\Omega}/\dot{\Omega} \sim 10^{-3}$–$10^{-2}$. Initially, glitches were observed in the Crab and the Vela pulsars. These two fast, young pulsars have each exhibited glitches at intervals of a few years since their discovery (Radhakrishnan & Manchester 1969; Reichley & Downs 1969; Lohsen 1981; Downs 1981; McCulloch, Hamilton & Royle 1981; Hamilton, McCulloch & Royle 1982; Klekociuk, McCulloch & Hamilton 1985; Cordes, Downs & Krause-Polstorff 1988; Hamilton et al. 1989; McCulloch et al. 1990; Flanagan 1989, 1990, 1991). Recent observations show that pulsar glitches are experienced by pulsars of all ages, but more frequently in younger pulsars. Models for these events invoke either quakes in a solid component of the neutron star (Ruderman 1969; Baym & Pines 1971; Pines & Shaham 1972; Pines, Shaham & Ruderman 1972) or sudden unpinning of pinned vortex lines in the neutron star superfluid (Anderson & Itoh 1975; Alpar 1977; Pines et al. 1980; Anderson et al. 1982; Alpar et al. 1981a,b, 1984a,b; Alpar, Cheng & Pines 1989; Alpar et al. 1993). In order to distinguish the quake and unpinning models, Alpar & Ho (1983) investigated the expectancy of large glitches by using 267 pulsars with the three large glitches that had been observed at that time (all the large glitches observed other than those from the Vela pulsar). They found that the statistics available at the time did not favour either model over the other. In the last decade, the number of known pulsars and the total time devoted to pulsar observations have increased significantly. 19 large glitches have been observed, excluding the nine glitches from the Vela pulsar, for which there are detailed model fits to post-glitch data (Alpar et al. 1993). In this paper we re-examine the expectancy of large glitches ($\Delta\Omega/\Omega > 10^{-7}$) from an extended sample of 430 pulsars. We determine average glitch parameters by setting the model expectancies equal to the observed number of large glitches. Our findings show that the vortex unpinning model gives the best agreement with the observed distribution of glitches. For this model, the mean glitch parameter $\langle \delta\Omega/\Omega \rangle$ for the sample agrees with that for the Vela pulsar.

In Section 2 we describe the data base. In Section 3 we present the calculation of glitch expectancies by using Poisson statistics. In Section 4 we derive model parameters for two variants of the superfluid vortex unpinning model and the core-quake model and in Section 5 we do likewise for the core-quake model. The results are discussed in Sections 6 and 7.

2 TIME-SPANS

In the sample, we have included all pulsars for which values of $\dot{\Omega}$ are known and for which time-spans of the observations were available from the published literature or from observers (for references see Table 2). Where there are gaps between the time intervals during which the pulsar was monitored, we check to see if the timing fits in each session were accurate enough that a potential glitch occurring in the

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Table 1. Observed large glitches.

Pulsar	$\Delta\Omega/\Omega$ (10^{-6})	$\Delta\dot{\Omega}/\dot{\Omega}$ (10^{-3})	Reference
0355+54	4.4	100	Lyne (1987)
0833-45	2.35	13	Radhakrishnan and Manchester (1969)
	2.05	18	Reichley and Downs (1971)
	1.99	11	Manchester, Goss and Hamilton (1976)
	3.06	18	Manchester et al. (1983)
	1.14	9	McCulloch et al. (1983)
	2.05	20	Cordes, Downs and Krause-Polstorff (1988)
	1.3	15	McCulloch et al.(1987)
	1.81	160	Flanagan (1989)
	2.7	-	Flanagan (1991)
1325-43	0.12	-	Newton, Manchester and Cooke (1981)
1338-62	1.5	-	Kaspi et al. (1992)
	0.99	-	Kaspi et al. (1992)
1535-56	2.79	1.1	Johnston et al. (1994)
1641-45	0.19	1.6	Manchester et al. (1978)
	0.8	0.5	Flanagan (1993)
1706-44	2.06	4.0	Johnston et al. (1994)
1727-33	3.03	3.5	Johnston et al. (1994)
1737-30	0.42	2.8	McKenna and Lyne (1990)
	0.6	2.0	McKenna and Lyne (1990)
	0.7	1.3	Lyne, McKenna and Shemar (1994)
1758-23	0.2	-	Kaspi et al. (1993)
	0.23	-	Kaspi et al. (1993)
	0.34	-	Kaspi et al. (1993)
1800-21	4.1	8.2	Lyne, McKenna and Shemar (1994)
1823-13	2.7	-	Lyne, McKenna and Shemar (1994)
1830-08	1.9	1.1	Lyne, McKenna and Shemar (1994)
2224+65	1.7	-	Backus, Taylor and Damashek (1982)

gap would show up above the accumulated noise on extrapolating from the observing sessions on either side of the gap. We use the criterion

$$\delta P + \delta\dot{P}\Delta t \ll \Delta P, \tag{1}$$

where δP and $\delta\dot{P}$ are the total errors quoted by the observers for the timing fits on two sides of the gap, of length Δt; $\Delta P/P = \Delta\Omega/\Omega$ is the magnitude of glitches. Post-glitch observations in the Vela pulsar have shown that the $\Delta\Omega$ relaxation is not complete in a few years (Alpar et al. 1993). In older pulsars, relaxation times are expected to be longer (Alpar et al. 1989). We therefore assume that a large glitch does not relax quickly and that it will be detected if the above criterion is satisfied. For glitches with $\Delta\Omega/\Omega < 10^{-7}$, many of the gaps cannot be included in the time-base. Therefore we concentrate on large glitches and include the gaps in the time-spans satisfying the above criterion, during which a glitch would be caught. In Table 1 we list the observed large glitches. Table 2 gives the time-spans of observations for 430 pulsars.

3 LARGE GLITCH EXPECTANCY

If the ith pulsar in the sample glitches at intervals $t_{g,i}$ and timing observations on this pulsar have covered a total time-span t_i, in the sense that a glitch occurring within that time-span would be noticed, the probability that n_i glitches are caught from that pulsar is given by Poisson statistics:

$$P(n_i) = \exp(-\beta_i)\beta_i^{n_i}/n_i!, \tag{2}$$

where $\beta_i = t_i/t_{g,i}$. For a sample of N pulsars, the probability of observing a total number $n = \sum n_i$ glitches is

$$P(n) = \exp(-\beta)\beta^n/n!, \tag{3}$$

where $\beta = \sum\beta_i$.

The expected number of glitches from the sample is then

$$\langle n\rangle = \beta. \tag{4}$$

In order to calculate glitch expectancy for a given model for glitches, the time interval between the glitches $(t_{g,i})$ is calculated for that model. In the following sections, we will express the interglitch time-scales $(t_{g,i})$ in terms of superfluid vortex unpinning and core-quake models, and estimate the model parameters by using the observed number of glitches. For each model, $t_{g,i}$ depends on the rotation rate Ω_i and spin-down rate $\dot{\Omega}_i$ of the pulsar. These are taken from Taylor, Manchester & Lyne (1993).

4 REVIEW OF THE SUPERFLUID VORTEX UNPINNING MODEL

In superfluid vortex unpinning theory, the glitch results from the sudden unpinning of vortex lines, whose subsequent motion transfers angular momentum to the crust (Anderson & Itoh 1975; Alpar 1977; Pines et al. 1980; Anderson et al. 1982; Alpar et al. 1981a,b; Pines & Alpar 1985). In this theory, the glitches, of size $\Delta\Omega/\Omega \sim 10^{-6}$ and with a frequency of ~ 2–3 yr, that are observed in the Vela pulsar are attributed to sudden unpinning and outward motion of large numbers ($\sim 10^{11}$) of vortex lines in the crustal neutron superfluid, which are there pinned to the lattice of neutron-rich nuclei. The unpinning occurs when the difference between the rotational frequencies of the normal matter and

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Table 2. Time-spans of glitch-monitoring observations.

PSR	t(days)	Ref.	PSR	t(days)	Ref.	PSR	t(days)	Ref.
0011+47	3130	(D)A	0656+14	880	(D)	1039-55	553	N
0031-07	4319	DH	0727-18	3881	ABS	1044-57	555	N
0037+56	880	(D)	0736-40	4109	DM	1046-58	900	J
0045+33	880	(D)	0740-28	4810	MS	1054-62	3810	SN
0052+51	880	(D)	0743-58	404	N	1055-52	542	N
0053+47	880	(D)	0751+32	746	B	1056-78	556	N
0059+65	880	(D)	0756-15	1037	AB	1056-57	540	N
0105+65	744	B	0808-47	3808	BS	1105-59	542	N
0105+68	880	(D)	0809+47	2777	H	1107-56	900	J
0114+58	880	(D)	0818-13	2174	H	1110-65	542	N
0136+57	746	B	0818-41	3591	NS	1110-69	540	N
0138+59	1842	B	0820+02	3504	T	1112-60	900	J
0144+59	880	(D)	0823+26	4705	DGH	1112+50	3100	B
0148-06	3870	ABS	0826-34	661	N	1114-41	546	N
0149-16	3767	ABS	0834+06	2556	GH	1118-79	554	N
0153+39	880	(D)	0835-41	3807	NS	1119-54	541	N
0154+61	557	B	0839-53	539	N	1125-60	900	J
0203-40	3779	NS	0840-48	540	N	1131-62	900	J
0226+70	880	(D)	0841+80	880	(D)	1133+16	4815	DGH
0254-53	3808	MNS	0844-35	538	N	1133+55	3810	NS
0301+19	1834	GH	0853-33	730	A	1143-60	542	N
0320+39	746	B	0855-61	540	N	1154-62	3779	NS
0329+54	4865	DH	0901-63	540	N	1159-58	542	N
0331+45	880	(D)	0903-42	540	N	1221-63	542	N
0339+53	880	(D)	0904-74	542	N	1222-63	449	N
0353+52	880	(D)	0905-51	3809	NS	1236-68	538	N
0355+54	4465	L	0906-17	1037	AB	1237+25	4526	DGH
0402+61	745	B	0909-71	553	N	1237-41	444	N
0403-76	3779	NS	0917+63	880	(D)	1240-64	4758	MS
0410+69	880	(D)	0919+06	1080	ABN	1254-10	880	(D)
0447-12	3881	ABS	0922-52	542	N	1256-67	443	N
0450+55	3105	B(D)	0923-58	541	N	1257+12	486	(JM)
0450-18	5960	BS	0932-52	540	N	1302-64	541	N
0456-69	2920	C	0940-55	542	N	1303-66	900	J
0458+46	3105	B(D)	0940+16	1055	L	1309-53	540	N
0523+11	3881	ABS	0941-56	3591	NS	1309-12	880	(D)
0525+21	4596	DGH	0942-13	1037	AB	1309-55	540	N
0531+21	1245	L	0943+10	1398	G	1316-60	900	J
0538-75	3779	NS	0950-38	538	N	1317-53	3808	NS
0540+23	1368	GH	0950+08	4140	DGH	1322-66	541	N
0540-69	545	(G)	0953-52	555	N	1322+83	880	(D)
0559-05	1037	AB	0957-47	538	N	1323-63	541	N
0559-57	539	N	0959-54	3675	NS	1323-627	900	J
0609+37	880	(D)	1001-47	555	N	1323-58	541	N
0611+22	1584	GH	1011-58	900	J	1323-62	539	N
0621-04	1037	AB	1014-53	553	N	1325-43	484	N
0626+24	781	B	1015-56	553	N	1325-49	452	N
0628-28	4384	DM	1016-16	880	(D)	1334-61	900	J
0643+80	880	(D)	1036-58	900	J	1336-64	542	N
0655+64	2377	(J)	1039-19	730	A	1338-62	810	K
1352-51	541	N	1630-44	900	J	1740-03	730	A
1356-60	542	N	1630-59	542	N	1740-13	880	(D)
1358-63	3809	NS	1633+24	880	(D)	1742-30	820	S
1409-62	900	J	1634-45	900	J	1745-12	1037	AB
1417-54	539	N	1635-45	900	J	1745-56	539	N
1424-55	3808	NS	1636-47	900	J	1746-30	900	J
1426-66	3809	NS	1641-45	4802	MS	1747-31	900	J
1436-63	541	N	1641-68	539	N	1747-46	3806	NS
1449-64	1485	MN	1642-03	4555	DH	1749-28	5521	DS
1451-68	2018	M	1643-43	900	J	1750-24	1095	L
1454-51	534	N	1647-528	539	N	1753+52	880	(D)
1503-51	483	N	1647-52	539	N	1753-24	1095	L
1503-66	532	N	1648-42	841	S	1758-23	2263	L
1504-43	542	N	1648-17	1037	AB	1758-29	900	J
1507-44	539	N	1650-38	900	J	1758-03	880	(D)
1508+55	1245	H	1657-13	880	(D)	1800-27	900	J
1508-57	900	J	1657-45	900	J	1800-21	2236	L
1510-48	533	N	1658-37	900	J	1802+03	880	(D)
1518-58	900	J	1659-60	539	N	1804-27	730	A
1523-55	539	N	1700-18	880	(D)	1804-08	1079	ABN
1524-39	537	N	1701-75	531	N	1805-20	1095	L
1530+27	880	(D)	1702-19	1245	L	1806-21	1095	L
1530-53	3776	BS	1703-40	900	J	1806-53	538	N
1530-539	900	J	1706-44	900	J	1809-173	1095	L
1535-56	900	J	1706-16	6610	DHS	1809-176	1095	L
1540-06	1037	AB	1707-53	530	N	1810+02	880	(D)
1541-52	541	N	1709-15	880	(D)	1811+40	652	B
1541+09	1668	G	1714-34	900	J	1813-17	1095	L
1550-54	538	N	1715-40	900	J	1813-36	637	N
1552-31	730	A	1717-29	843	S	1815-14	1095	L
1552-23	730	A	1717-16	880	(D)	1817-13	1095	L
1555-55	539	N	1718-35	900	J	1817-18	900	J
1556-44	2031	M	1718-02	1037	AB	1818-04	4385	DH
1556-57	3809	NS	1718-32	820	S	1819-22	1245	L
1557-50	2031	M	1718-36	900	J	1820-14	1095	L
1558-50	539	N	1719-37	539	N	1820-31	539	A
1600-27	730	A	1726-00	880	(D)	1821-19	3809	NS
1600-49	541	N	1727-33	900	J	1821+05	746	B
1601-52	539	N	1727-47	1162	M	1821-11	1095	L
1604-00	4109	DGH	1729-41	532	N	1822-14	1095	L
1607-52	900	J	1730-37	900	J	1822+00	880	(D)
1607-13	880	(D)	1732-02	880	(D)	1822-09	3588	NS
1609-47	540	N	1734-35	900	J	1823-11	1095	L
1610-50	900	J	1735-32	1095	L	1823-13	2579	L
1611-55	900	J	1736-31	1095	L	1824-10	1095	L
1612+07	745	B	1737-30	2579	L	1826-17	843	S
1620-42	540	N	1737+13	1037	AB	1828-10	1095	L
1620-09	1245	L	1737-39	538	N	1828-60	538	N
1627-47	900	J	1738-08	1037	AB	1829-08	915	L
1629-50	900	J	1740-31	900	J	1830-08	2500	L

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Table 2 – *continued*

PSR	t(days)	Ref.	PSR	t(days)	Ref.	PSR	t(days)	Ref.
1831-03	1095	L	1911+11	561	G	2021+51	4384	DH
1831-04	582	B	1913+167	595	G	2022+50	880	(D)
1832-06	1095	L	1913+10	1325	AG	2028+22	1323	ABG
1834-10	1037	AB	1914+09	1295	AG	2034+19	880	(D)
1834-10	1095	L	1914+13	1249	AG	2035+36	880	(D)
1834-06	700	L	1915+13	1602	G	2036+53	880	(D)
1838-04	1095	L	1916+14	365	G	2043-04	1037	AB
1839+09	1037	AB	1917+00	1318	G	2044+15	1037	AB
1839-04	1095	L	1918+26	880	(D)	2045+56	880	(D)
1839+56	730	A	1918+19	922	G	2045-16	4769	DHM
1841-05	1095	L	1919+14	1289	AG	2048-72	539	N
1841-04	1095	L	1919+21	2791	GH	2053+21	880	(D)
1842-02	1095	L	1920+21	506	G	2053+36	745	B
1842+14	1037	AB	1922+20	730	A	2106+44	1946	B
1842-04	1095	L	1923+04	1037	AB	2110+27	880	(D)
1844-04	819	S	1924+16	2073	ABG	2111+46	4384	DH
1845-19	539	N	1924+14	556	G	2113+14	1037	AB
1845-01	758	S	1925+22	544	G	2122+13	880	(D)
1848+13	880	(D)	1927+13	549	G	2123-67	538	N
1848+12	880	(D)	1929+10	4568	DGH	2148+63	745	B
1849+00	1095	L	1929+20	1310	AG	2148+52	880	(D)
1851-79	532	N	1930+22	267	G	2151-56	540	N
1851-14	1037	AB	1933+16	4576	DGH	2152-31	539	N
1855+02	1095	L	1933+15	543	G	2154+40	1648	H
1857-26	1245	L	1935+25	880	(D)	2210+29	880	(D)
1859+03	1226	G	1937-26	3040	AS	2217+47	4384	DH
1859+01	880	(D)	1940-12	1037	AB	2224+65	747	B
1859+07	2236	L	1941-17	1037	AB	2227+61	880	(D)
1900+05	1297	AG	1942-00	1610	A(D)	2241+69	880	(D)
1900+01	606	G	1943-29	1245	L	2255+58	1944	BH
1902-01	880	(D)	1944+22	568	G	2303+30	2173	GH
1903+07	1095	L	1944+17	1638	GH	2306+55	745	B
1904+06	1095	L	1943-29	1245	L	2310+42	745	B
1905+39	1037	AB	1946+35	1691	GH	2315+21	649	B
1906+09	500	G	1951+32	584	L	2319+60	1577	H
1907+00	780	G	1952+29	1408	G	2321-61	539	N
1907+02	1298	G	1953+50	746	B	2323+63	651	B
1907+10	993	G	2000+32	1245	L	2324+60	1185	L
1907+03	1037	AB	2000+40	880	(D)	2327-20	3807	NS
1907-03	1037	AB	2002+31	1606	GH	2334+61	880	(D)
1907+12	585	G	2003-08	652	B	2351+61	745	B
1910+20	1116	G	2011+38	880	(D)			
1911+13	1291	AG	2016+28	4437	DGH			
1911-04	4346	DH	2020+28	2045	GH			

References: A: Ashworth & Lyne (1981); B: Backus et al. (1982); D: Downs G. S. (1982, private communication); G: Gullahorn & Rankin (1978); H: Helfand et al. (1980); M: Manchester et al. (1983); N: Newton et al. (1981); (D): Dewey et al. (1988); (J): Jones & Lyne (1988); T: Taylor & Dewey (1988); C: McConnell et al. (1991); (G): Gouiffes et al. (1992); (JM): Johnston et al. (1992); Kaspi et al. (1992); D: Siegman, Manchester & Durdin (1993); J: Johnston et al. (1994); L: Lyne (private communication).

the superfluid in a boundary layer with vortex pinning reaches a critical value $\delta\Omega$. This parameter $\delta\Omega$ determines the time between glitches through the relation

$$t_g = \frac{\delta\Omega}{\dot{\Omega}} . \tag{5}$$

We shall explore two hypotheses for our statistics: (i) that $\delta\Omega$ is the same in all neutron stars; and (ii) that $\delta\Omega/\Omega$ is the same in all neutron stars.

When a glitch is observed in detail, and post-glitch timing observations exist, $\delta\Omega$ can be obtained from the observed quantities. The subsequent angular momentum transfer involves a region of the pinning layer with effective moment of inertia I_p. With the use of angular momentum conservation, the increase $\Delta\Omega$ in the rotation rate of the crust can be written as

$$\Delta\Omega = \frac{I_p \delta\Omega}{I} , \tag{6}$$

where I is the total moment of inertia of all the layers of the neutron star that are coupled to the crust on short timescales; I turns out to include effectively the whole star (Alpar, Langer & Sauls 1984c; Alpar & Sauls 1988). The glitch is expected to lead to a temporary uncoupling of the superfluid pinning layers, with moment of inertia of the order of I_p, so that, after the glitch, the external torque acts on a lower moment of inertia (Alpar et al. 1981a,b); the moment of inertia of the decoupled component is then reflected in the

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fractional change of the spin-down rate observed in a glitch:

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}}=\frac{I_{\rm p}}{I}. \tag{7}$$

Actually, detailed post-glitch timing fits of the Vela pulsar show that the effective moments of inertia entering the angular momentum balance, equation (6), and the change in spin-down rate, equation (7), are not the same quantity (Alpar et al. 1993). For the Vela pulsar, we infer that parts of the superfluid, with moment of inertia $I_{\rm B}$, contribute only to angular momentum transfer in glitches, and not to the spin-down rate, while other parts, $I_{\rm A}$, contribute to both, so that for Vela,

$$\Delta\Omega=\left(\frac{1}{2}\,\frac{I_{\rm A}}{I}+\frac{I_{\rm B}}{I}\right)\delta\Omega, \tag{8}$$

and

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}}=\frac{I_{\rm A}}{I}. \tag{9}$$

For the Vela pulsar, a third observational quantity, a large constant second derivative, $\ddot{\Omega}$, between glitches, enables the solution for all three parameters, $I_{\rm A}$, $I_{\rm B}$ and $\delta\Omega$, from the observed quantities, $\Delta\Omega$, $\Delta\dot{\Omega}$ and $\ddot{\Omega}$. The distinction between these effective moments of inertia, however, necessitates post-glitch observations and the existence of observable repeating glitches. For the present discussion of expectancies to apply to a sample where most pulsars exhibit no glitches, some a single glitch, and only a few repeat, we are, of necessity, restricted to using the same, single parameter, $I_{\rm p}$ in both equation (6) and equation (7).

With the assumption that the average value of the critical glitch parameter applies for all pulsars, the interglitch time can be expressed as $t_{{\rm g},i}=\langle\delta\Omega\rangle/\dot{\Omega}_i$. From equations (4) and (5) we obtain

$$\langle n\rangle=\frac{1}{\langle\delta\Omega\rangle}\sum t_i\dot{\Omega}_i. \tag{10}$$

In order to set the expected number, $\langle n\rangle$, to the observed number, 19, we require that $\langle\delta\Omega\rangle=0.0188$ rad s^{-1}. This value is roughly a factor of 2 larger than the average value for the Vela pulsar glitches, for which $\delta\Omega$ is in the range 0.0049–0.013 rad s^{-1}. $\delta\Omega$ reflects the number of vortices unpinned at a glitch, which is not the same for different pulsars. In Fig. 1(a) we plot a histogram for the $t_i\dot{\Omega}_i$ values of the pulsars in our sample. As can be seen from the figure, most of the glitches occur at the higher values of expected rates (because $\beta\sim t_i\Omega_i$).

In the above, we have taken the critical quantity $\langle\delta\Omega\rangle$ for unpinning, which is proportional to the number of pinned vortices, to be the same for different pulsars. It is possible, however, that the critical conditions for unpinning may also involve a critical fraction of the vortex line density. This corresponds to a critical value of $\delta\Omega/\Omega$, because the average density of vortex lines is proportional to Ω. Thus another hypothesis is that there is a critical constant value of $\langle\delta\Omega/\Omega\rangle\sim\langle\delta N/N\rangle$, where N is the total number of vortices in the star and δN is the number of vortices unpinning in the glitch, which determines the interval between successive glitches. In this case we can write

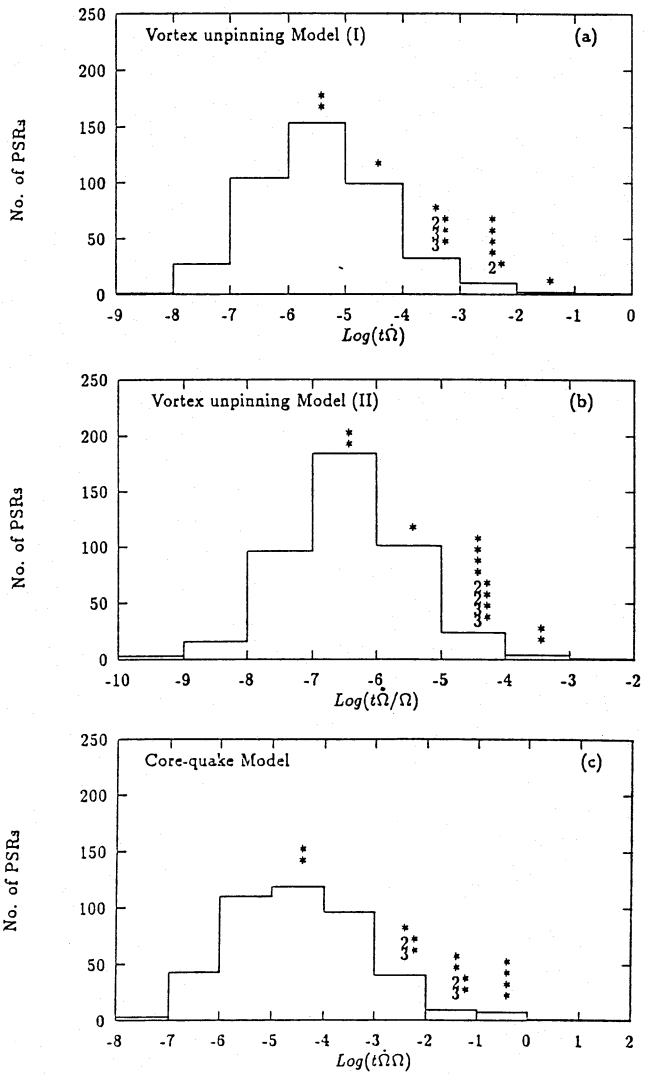


Figure 1. The distribution of the pulsars in logarithmic intervals of (a) $\dot{\Omega}t$; (b) $\dot{\Omega}t/\Omega$; (c) $\Omega\dot{\Omega}t$. For each pulsar, Ω (rad s^{-1}) is the rotation rate, $\dot{\Omega}$ (rad s^{-2}) is the spin-down rate, t(s) is the total time for which the pulsar was effectively monitored for large glitches. The asterisks denote the number of large glitches for each pulsar.

$$\langle n\rangle=\frac{1}{\langle\delta\Omega/\Omega\rangle}\sum t_i\dot{\Omega}_i/\Omega_i. \tag{11}$$

Again, by setting $\langle n\rangle$ equal to the observed number of glitches, 19, we find that the critical average value $\langle\delta\Omega/\Omega\rangle=1.74\times10^{-4}$, which is in close agreement with glitch parameters for the Vela pulsar (for the Vela pulsar $\langle\delta\Omega/\Omega\rangle$ is in the range 0.7–1.9×10^{-4}). This is an indication that $\langle\delta\Omega/\Omega\rangle$, rather than $\langle\delta\Omega\rangle$, may be the critical quantity for glitches arising from vortex unpinning. With its nine post-glitch fits, the Vela pulsar provides a good test case for glitch parameters that are derived statistically. Fig. 1(b) shows a histogram for $t_i\dot{\Omega}_i/\Omega_i$, the rate parameter for the present model. In this case, the accumulation of glitches at the higher expected rates is more pronounced than in the previous variant of the superfluid unpinning model (shown in Fig. 1a).

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5 REVIEW OF THE CORE-QUAKE MODEL

In starquake models, glitches occur as a result of a sudden quake in a solid component of the neutron star (Ruderman 1969; Smoluchowski 1970; Baym & Pines 1971; Pines & Shaham 1972; Shaham & Ruderman 1972). This model assumes that the star has a non-spherical shape as a result of its rotation. Recent models (Ruderman 1991a,b) where the crust quakes are driven by the motion of magnetic flux tubes, rather than by rotation, will not be treated here. The moment of inertia I deviates from that of a spherical star, I_0, with

$$I = I_0(1+\epsilon), \tag{12}$$

where ϵ is the oblateness. The total energy for the solid component of the star can be written as

$$E = \frac{L^2}{2I} + A\epsilon^2 + B(\epsilon_0 - \epsilon)^2, \tag{13}$$

where A is the gravitational energy due to the oblate shape, B is the rigidity of the lattice, and ϵ_0 is a reference oblateness determined by the configuration of the solid component immediately after the last quake. Oblateness (ϵ) will change as the pulsar slows down. The equilibrium of the solid's oblateness can be obtained by minimizing the energy at constant angular momentum:

$$\epsilon = \frac{I_0\Omega^2}{4(A+B)} + \frac{B}{A+B}\epsilon_0. \tag{14}$$

The elastic change of oblateness,

$$\dot{\epsilon} = \frac{I_0\Omega\dot{\Omega}}{2(A+B)}, \tag{15}$$

will cause a quake when $\epsilon - \epsilon_0$ increases to the critical strain of the solid. In the quake, the reduction in the oblateness $\Delta\epsilon$ is related to other parameters by

$$\Delta\epsilon = \frac{\Delta I}{I} = -\frac{\Delta\Omega}{\Omega} \approx \frac{B}{A+B}\Delta\epsilon_0. \tag{16}$$

The time between glitches is

$$t_g = \frac{(\Delta\epsilon - \Delta\epsilon_0)}{\dot{\epsilon}} = \frac{2(A+B)A}{B}\,\frac{\Delta\Omega/\Omega}{I_0\Omega\dot{\Omega}}; \tag{17}$$

with $I_0 \sim 10^{45}$ g cm^2, $A \sim 10^{52}$ erg and $B \sim 10^{48}$ erg for the crust solid, the interval between glitches in the Vela pulsar is much longer than a few years. Vela pulsar glitches occur at intervals of a few years only if the neutron star models have a quaking core solid with $B \sim A \sim 10^{52}$ erg (Pines et al. 1972; Anderson & Palmer 1971; Clark & Chao 1972; Palmer, Tosatti & Anderson 1973). This could be possible with stiff equations of state, and if the neutron stars are massive ($M > 2$ $M_\odot$) (Pandharipande, Pines & Smith 1976). The interglitch interval t_g is then defined as the time to rebuild a certain fraction ϕ of the strain released in the previous glitch:

$$t_g \sim \frac{2(A+B)\phi\Delta\Omega/\Omega}{I_0\Omega\dot{\Omega}}. \tag{18}$$

Then, using equations (4) and (18), one obtains

$$\langle n \rangle = \left\langle \frac{I_0}{2(A+B)\phi\Delta\Omega/\Omega} \right\rangle \sum \Omega_i\dot{\Omega}_i t_i. \tag{19}$$

Again normalizing our sample with the observed glitch rates, we calculate the mean value

$$\left\langle \frac{I_0}{2(A+B)\phi\Delta\Omega/\Omega} \right\rangle = 0.33. \tag{20}$$

For the Vela pulsar glitches, if they are caused by core-quakes, we obtain

$$\left\langle \frac{I_0}{2(A+B)\phi} \right\rangle = \frac{1}{\Omega\dot{\Omega}} \left\langle \frac{\Delta\Omega/\Omega}{t_g} \right\rangle \approx 3.69 \times 10^{-6}\,(\mathrm{rad}^{-2}\,\mathrm{s}^2) \tag{21}$$

where t_g is the time to the next glitch. If we now assume that $I_0/2(A+B)\phi$ is the same in all neutron stars, we find that

$$\left\langle \frac{1}{\Delta\Omega/\Omega} \right\rangle^{-1} \sim 1.1 \times 10^{-5} \left\langle \frac{\Delta\Omega}{\Omega} \right\rangle. \tag{22}$$

Thus even the largest observed glitch is smaller by a factor of 3, and the mean observed glitch size is smaller by a factor of 8, than the statistical estimate that fits the number of events for this model. If the glitch rates are to fit the observations, the sizes of the events in the core-quake model should be significantly larger than the observed glitch sizes from both the Vela pulsar and the sample. For the core-quake model to survive, we have to assume that, although the typical glitch size $\Delta\Omega/\Omega > 1.1 \times 10^{-5}$, so far we have only seen smaller (and more frequent) glitches (19 of them!). Note that, if we normalize the model parameters for $1338-62$, $1641-45$, $1737-30$ and $1758-23$ (the pulsars other than Vela that glitch repeatedly), the discrepancy becomes stronger because $\Delta\Omega/\Omega > 10^{-4}$ is obtained. The distribution of the sample in $\Omega_i\dot{\Omega}_i t_i$ is shown in the histogram in Fig. 1(c).

6 GLITCH PROBABILITIES AND REPETITIVE GLITCHES

By using the mean number of glitches, β_i, for the ith pulsar, we have determined the probabilities of observing n glitches according to the different models. In Table 3 we compare the probabilities of the particular observed distribution of the glitches. Most pulsars that glitch have had a single glitch, but there are two pulsars that have glitched twice, and two that have glitched three times. The Poisson probabilities for the observed number, n_i, of glitches for each pulsar are evaluated using the means, β_i, calculated above for the different models – the superfluid model with $\langle\delta\Omega/\Omega\rangle$ and the core-quake model. The probabilities $\Pi_i P(n_i; \beta_i)$ are, of course, very small for this particular observed realization; however, we keep in mind that the glitching and repeating pulsars all fall in the most likely bins in the histograms, and that there are a large number of equally probable distributions of the 19 observed glitches. It is useful to compare the probabilities of the particular observed distribution in the different models. This comparison for the individual glitching pulsars consistently favours the superfluid vortex unpinning model, as Table 3 shows. The preference is very

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Table 3. Comparison of the glitch probabilities in different models.

Pulsar	number of observed glitches n_i	$(P(n_i))^u/(P(n_i))^q$
0355+54	1	9
1325-43	1	126
1338-62	2	$3\ 10^6$
1535-56	1	26
1641-45	2	$6\ 10^3$
1706-44	1	3
1727-33	1	7
1737-30	3	$8\ 10^6$
1758-23	3	$1\ 10^6$
1800-21	1	5
1823-13	1	2
1830-08	1	3
2224+65	1	21

Notes: $(P(n_i))^u$ denotes the probability of the observed number n of glitches occurring in the observational time-span devoted to each glitching pulsar according to the vortex unpinning model. $(P(n_i))^q$ is the probability according to the core-quake model.

pronounced in the sources that are repetitively glitching. The superfluid vortex unpinning model with $\langle\delta\Omega\rangle$ fixed also does significantly better than the core-quake model, but not as well as the fixed $\langle\delta\Omega/\Omega\rangle$ variant used for the table. In the superfluid vortex unpinning model, $\delta\Omega=\dot{\Omega}t_g$ is the critical quantity for determining successive glitches. If a pulsar is followed through a succession of glitches, this quantity can be determined for that pulsar. When there are two observed glitches, only one t_g gives $\delta\Omega$; for three glitches there are two $\delta\Omega$ values. In Table 4, we present the $\delta\Omega/\Omega$ parameters of successively glitching pulsars. For $1641-45$, $1737-30$ and $1758-23$ this parameter is considerably (roughly a factor of $1/10$) less than the average value obtained from our sample. By solving equations (8) and (9) for these pulsars' glitches, we see that the variations in glitch repetition times and in $\delta\Omega$ are linked with differences in I_A/I and I_B/I. This implies that, for some sources, $\delta\Omega/\Omega$ may be less than the pulsar sample mean and we are seeing more glitches from these particular sources. The fact that the parameters $\delta\Omega/\Omega$, I_A/I and I_B/I are variable within the observed range, and not strictly constant for all pulsars, does not invalidate the vortex unpinning model. The variability of $\delta\Omega$ is present even in a single pulsar, as the Vela pulsar glitches demonstrate. In the nine Vela pulsar glitches, $\delta\Omega$ varies from glitch to glitch within a factor of 2.6 (Alpar et al. 1993). If all pulsars had $\langle\delta\Omega/\Omega\rangle\sim0.12\times10^{-4}$, the value inferred for $1641-45$, we should see $(1.74/0.12)\times19\sim275$ glitches instead of the 19 from our sample.

7 CONCLUSIONS

A re-examination of the expectancy of large glitches with the present data base gives a statistical preference for superfluid vortex unpinning models, unlike the previous work by Alpar & Ho (1983), which was inconclusive on the basis of the then existing database. By using Poisson statistics and the observed rates of large glitches in the sample, we have found the glitch parameters $\langle\delta\Omega\rangle$ and $\langle\delta\Omega/\Omega\rangle$ for the superfluid vortex unpinning model and the sizes of glitches $\langle\Delta\Omega/\Omega\rangle$ for the core-quake model, as implied by the numbers of observed glitches. In terms of model parameters, the vortex unpinning model with constant fractional vortex density

Table 4. $\delta\Omega/\Omega$ parameters from pulsars with repeated glitches.

Pulsar	number of glitches	$(\delta\Omega/\Omega)_{-4}$
1338-62	2	0.75
1641-45	2	0.12
1737-30	3	0.45
		0.35
1758-23	3	0.19
		0.089

appears to be the most consistent model for the statistics of our extended sample, as the mean $\langle\delta\Omega/\Omega\rangle$ required by the statistics agrees with the $\langle\delta\Omega/\Omega\rangle$ inferred for the Vela pulsar's post-glitch timing analysis. For the core-quake model, expected glitch sizes have been found to be a factor of 8 larger than the average observed glitch magnitudes.

We have also discussed the individual expectancies of the observed glitches. We have found that the superfluid vortex unpinning model gives a higher relative probability than the core-quake model for the observed realization of the glitches. In the repetitive glitches, this relative probability indicates that the core-quake models cannot accumulate the energy released in the last glitch in the observed time-interval to the next glitch. This has been known since the observation of the Vela pulsar's second glitch. Here we have shown that, even if we artificially fix the core-quake model parameters to accommodate the Vela pulsar glitches (or one of the other repeatedly glitching pulsars, for which we know t_g and can use equation 21 to set core-quake parameters), the resulting event sizes are larger than the observed sizes. Connected with this, the glitch intervals are longer (equation 18), and the probabilities of having observed glitches from the particular pulsars that have glitched are significantly lower than in the case of the superfluid models. This contrast in probabilities according to the different models naturally becomes sharper for the repeatedly glitching pulsars.

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