INVESTIGATING THE PRESERVICE PRIMARY SCHOOL TEACHERS' MATHEMATICAL KNOWLEDGE IN TEACHING OF DIVISION OPERATION

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ABSTRACT

INVESTIGATING THE PRESERVICE PRIMARY SCHOOL TEACHERS' MATHEMATICAL KNOWLEDGE IN TEACHING OF DIVISION OPERATION

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The aim of the study is to investigate preservice primary school teachers' mathematical knowledge in teaching regarding division operation according to the Knowledge Quartet Model. The qualitative research, specifically the case study approach, was used as a research design. The study was conducted with two preservice primary school teachers studying at a foundation university in Ankara, and they were selected through convenient sampling. The data were collected via readiness tests, observations, lesson plans, and semi-structured interviews. The data were analyzed through the Knowledge Quartet Model. The results demonstrated that preservice primary school teachers' definitions of division operation were ill-defined before the teaching practice in a real classroom setting; however, after they prepared lesson plans and had their teaching practice, there was an improvement in their definitions of division operation. Furthermore, it was observed that preservice teachers relied on and used standard algorithms very often before and during teaching implementation. Additionally, their usage of examples, representations, demonstrations, and materials was limited and lacked diversity while planning and

implementing the lesson. It was also seen that they had difficulties while handling unexpected moments during teaching.

Keywords: Preservice Primary School Teachers, Mathematical Knowledge in Teaching, Knowledge Quartet Model, Division Operation

ÖZ

SINIF ÖĞRETMENİ ADAYLARININ BÖLME İŞLEMİNE YÖNELİK ÖĞRETİMSEL MATEMATİK BİLGİLERİNİN İNCELENMESİ

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Bu çalışmanın amacı sınıf öğretmeni adaylarının bölme işlemine yönelik öğretimsel matematik bilgilerini Dörtlü Bilgi Modeli'ne göre incelemektir. Araştırma deseni olarak nitel araştırma seçilmiş ve olay incelemesi yöntemi kullanılmıştır. Çalışma, Ankara'da vakıf üniversitesinde okuyan iki sınıf öğretmeni adayıyla yapılmıştır ve sınıf öğretmeni adayları uygun örnekleme yöntemiyle seçilmiştir. Çalışmanın verileri hazırbulunuşluk testi, gözlemler, ders planları ve yarı yapılandırılmış görüşmelerle toplanmış olup, Dörtlü Bilgi Modeli ile analiz edilmiştir. Çalışmanın bulguları sınıf öğretmeni adaylarının bölme işlemini gerçek bir sınıf ortamında öğretim yapmalarında önce yeterince tanımlayamazken ders planlarını hazırlayıp ve öğretimlerini yaptıktan sonra bölme işlemini tanımlamalarında gelişme olduğunu göstermiştir. Aynı zamanda öğretmen adayları standart algoritmaları öğretimden önce ve öğretim sırasınca sıklıkla kullanmışlardır. Buna ek olarak, öğretmen adaylarının örnek, temsil, gösterim ve materyal kullanımlarının kısıtlı ve çeşitlilik göstermemektedir. Aynı zamanda sınıf öğretmeni adaylarının öğretim esnasında beklenmedik durumlarından üstesinde gelemedikleri görülmüştür.

Anahtar Kelimeler: Sınıf Öğretmeni Adayları, Öğretimsel Matematik Bilgisi, Dörtlü Bilgi Modeli, Bölme İşlemi

To my beloved family

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CHAPTER 1

INTRODUCTION

This chapter includes the statement of the research problem, the definition of important key terms, the significance of the study, and limitations accordingly.

According to the Turkish Language Association, a teacher is a person whose profession is teaching knowledge (TLA, n.d.). Since teachers are responsible for conveying knowledge, including their existing knowledge, its quality becomes important for the quality of education. Many studies state that teacher knowledge is important in effective teaching and instruction (Ball et al., 2008; Hill et al., 2008; Petrou & Goulding, 2011; Walshaw, 2012). National Council of Teachers of Mathematics (NCTM) (2000) also stated that "effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies" (p.17). That is, teachers should know and understand what to teach in mathematics, know the students and their characteristics and know how to convey their knowledge about mathematics to their students.

Additionally, the importance of teacher knowledge is often emphasized by the studies that focus on teachers' role in the efficiency of teaching. These studies indicate that teacher knowledge is a determinative factor in instruction (Ball, 1990; Crespo & Nicol, 2006; Fennema & Franke, 1992; Hill et al., 2004; Tirosh & Graeber, 1991; Tirosh, 2000) and consequently, it benefits the student's understanding and achievement (Ball, 1990; Montes et al., 2018). Teachers who do not fully understand a subject may lack the skills and expertise needed to teach students on that subject (Ball et al., 2008), and also, the deficiency in their knowledge may result in possible misconceptions for the students (Even 1990). On the other hand, when teachers have well-founded knowledge, they can easily teach a concept more meaningfully (Even, 1990). As McLaughlin and Talbert (1993) argue that

> Teachers with only superficial knowledge of their subject matter will have little flexibility in their pedagogical choices….by contrast, teachers who have mastered the rich interconnections and multiple forms of knowledge found in a subject area will have the substantive control of the subject needed to develop the kinds of activities and strategies involved in teaching for understanding (p. 3).

In brief, it could be deduced that limited subject matter knowledge of teachers has a negative effect on their pedagogical behaviors, while broad subject matter knowledge can positively affect their pedagogical behaviors and teaching.

As a result of the apparent connection between teachers' knowledge and students' learning, researchers naturally gravitated to investigate the teachers' knowledge (Ball et al., 2008; Ma, 1999; Monk, 1994; Rowan et al., 1997; Tchoshanov et al., 2008; Wilson et al., 1987). Moreover, Ponte (2012) emphasized that it was important to comprehend the structure of the teacher's knowledge due to its impact on teaching. Accordingly, while academicians tried to understand this structure, they developed several different models and frameworks to identify and make sense of teacher knowledge, such as the Shulman Model (Shulman, 1986), Mathematical Knowledge for Teaching (Ball et al., 2008), Mathematics Teachers' Specialized Knowledge (Carrillo-Yanez et al.,2018) and Knowledge Quartet (Rowland et al., 2003).

1.1 Problem Statement

While the need to investigate teacher knowledge is evident, many researchers studied teachers' knowledge about particular mathematics subjects such as addition, subtraction, or multiplication (Ball, 1990; Carpenter, 1988; Leavitt, 2008; Rowland, 2007; Turner, 2007; Turner & Rowland, 2008), fraction and its operations (Ball, 1990; Borko et al., 1992; Isiksal-Bostan & Cakiroglu, 2008; Leavitt, 2008; Tekin-Sitrava, 2019), probability (Maher & Muir, 2014), algebra (Girit-Yildiz & Akyuz, 2019; Li, 2007), measurement (Dogan Coskun et al., 2021; Dogan Coskun & Isiksal-

Bostan, 2018; Dogan Coskun & Isiksal-Bostan, 2022; Livy et al., 2012) and problem posing (Tekin-Sitrava, 2019). Most of these studies showed that teachers' knowledge on these subjects is limited.

Division operation is one of the four fundamental operations, and understanding and being able to do this operation is important for a meaningful understanding of other concepts, such as fractions, decimals, rational numbers and so on. (Ball, 1990; McCallum & Schmitt, 2011; Yorulmaz, 2018). However, considering the four fundamental operations, division operation is the most challenging concept to understand for both students and teachers (Carpenter et al., 1988; National Research Council, 2001). There are many reasons why this concept is the most difficult to understand, such as the poor connection between conceptual and procedural knowledge (English & Halford,1995), intuitive thinking of division as equal sharing (Fischbein et al., 1985), or the semantic structure of the division operation (Anghileri, 1989; Kouba, 1989). Since division operation is vital for understanding the following mathematics topics, researchers are encouraged to identify teacher's knowledge of division operation, and most of them found that teachers' knowledge regarding division operation is limited (Baki, 2013; Ball, 1990; Dogan Coskun & Cimen, 2019; Roche & Clark, 2013; Simon, 1993; Tekin-Sitrava et al., 2020).

Furthermore, these limited number of studies were conducted in hypothetical settings where teachers' knowledge was examined through questionnaires or test items. Nevertheless, in recent years, examining teachers' knowledge in natural settings such as the classroom gains important. According to Connelly and colleagues (1997), "the most important area is what teachers know and how their knowing is expressed in teaching" (p.665). Since today's preservice teachers are the future teachers, examining their knowledge of division operation in hypothetical and classroom settings becomes crucial. Thus, the current study aims to examine preservice primary school teachers' knowledge of the division operation. The following title lists the research question related to the aim.

1.2 Research Questions

As the importance of teacher knowledge is evident, and the literature review points to the need for examining the teacher knowledge regarding the division operation, this study aims to investigate pre-service primary school teachers' mathematical knowledge in teaching regarding the division operation through the framework of the Knowledge Quartet Model. Regarding this purpose, the research question and sub-questions of this study are phrased as follows:

1) What is the nature of preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Knowledge Quartet Model?
    a) What are the evidences of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Foundation dimension of the KQ Model?
    b) What are the evidences of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Transformation dimension of the KQ Model?
    c) What are the evidences of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Connection dimension of the KQ Model?
    d) What are the evidences of the preservice primary school teachers' mathematical knowledge in teaching regarding division operation according to the Contingency dimension of the KQ Model?

1.3 Definition of Important Key Terms

Below, definitions of some important key terms that are used throughout the study were given, such as division operation, teacher knowledge, Knowledge Quartet, foundation dimension, transformation dimension, connection dimension, contingency dimension, and preservice teachers.

Division Operation: In the current study, the division is explained as "at its foundation, the division has to do with forming groups" (Ball, 1990, p. 452). Fundamentally, division operation has two meanings: partition or partitive (fair sharing) and measurement quotative (repeated subtraction) (Fischbein et al., 1985; Reys et al., 2014; Van de Walle, 2013). The operation will be explained in the subsequent chapter in detail.

Mathematical Knowledge in Teaching: It refers to teacher knowledge related to mathematics, which was used to facilitate students' learning (Rowland & Ruthven, 2011). In the current study, this term refers to preservice teachers' knowledge related to mathematics regarding division operation to support students' learning.

Knowledge Quartet: Knowledge Quartet is an "empirically-based conceptual framework for lesson review discussions with a focus on the mathematics content of the lesson, and the role of the trainee's mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK)" (Rowland, 2013, p.18). It includes four dimensions: Foundation, Transformation, Connection and Contingency.

> *Foundation Dimension*: In the Knowledge Quartet Model, it refers to teachers' "… knowledge, understanding and ready recourse to what was learned at school and at college/university, including initial teacher education in preparation (intentionally or otherwise) for their role in the classroom" (Rowland, 2013, p. 22-23). It also includes teachers' beliefs and theoretical background regarding mathematics (Rowland et al., 2005). In the current study, this dimension was related to preservice teachers' theoretical knowledge and understanding of division operation while studying to be primary school teachers.
>
> *Transformation Dimension*: In the Knowledge Quartet Model, this dimension shows how teachers transform their existing knowledge so that their students understand (Rowland et al., 2005). In the current study, this dimension refers to how preservice teachers transmit their existing knowledge about division

operation to their students through their lesson plans and teaching implementations.

*Connection Dimensio*n: In the Knowledge Quartet Model, it is related to the teachers' decisions about lessons and lesson plans to make the lesson coherent, such as the sequencing of the lessons, anticipating possible difficulties and making relationships between lessons and concepts (Rowland et al., 2005). In the present study, it refers to preservice teachers' decisions about their lesson plans and teaching implementation of division operation to make their lessons coherent so that their students can learn division operation meaningfully.

Contingency Knowledge: In the Knowledge Quartet Model, it specifies all unexpected situations that occur in lessons and how teachers handle these unexpected moments and situations (Rowland et al., 2005). In the current study, it refers to preservice teachers' unexpected moments and their deviations from their plans and how they deal with these unexpected moments and manage their deviations.

Preservice Teachers: Preservice teachers are university students who study in education programs and have a practicum in the subject and education they will teach in a school environment (MoNE, 2012). In the current study, preservice teachers are considered senior students who have completed all their necessary mathematics method courses and have been taking Teaching Practice Course II.

1.4 Significance of the Study

Teachers' knowledge becomes essential for teacher education programs and teacher educators since they raise the teachers of the future. According to Grossman and Richert (1988), "…special attention needs to be paid to if, where, and how that knowledge base should be addressed in the pre-service curriculum" (p. 61). However, at its heart, teacher education programs provide primarily theoretical

knowledge regarding teaching (Ben-Peretz, 1990; Bullough & Gitlin, 2001; Korthagen et al., 2006). Yet, as Connelly and colleagues said, “Teacher knowledge research is part of a revolution in how educators think about classroom practice” (1997, p. 666). That is, teacher knowledge should be examined both theoretically and practically (Adler & Ball, 2009; Connelly et al., 1997) since teacher knowledge cannot be dissociated from practical knowledge (National Research Council [NRC], 2001; Stylianides & Ball, 2004) and teacher knowledge affects practical knowledge as well (Connelly et al., 1997; Lederman et al., 1994). Therefore, identifying teacher knowledge both theoretically and practically and recognizing the deficiencies in their knowledge gain importance. Hence, this study aims to investigate preservice teachers’ knowledge of division operation not just theoretically but also practically in real classroom settings during their practicum. In this way, this study could give valuable information to teachers, teacher educators and policymakers on teachers’ classroom practices regarding division operation. Additionally, the current study's findings, which aim to examine preservice teachers’ knowledge, might help teacher educators and policymakers to understand preservice teachers’ knowledge and reveal the points that need to be focused on during teacher education both theoretically and practically.

In addition, the current study needs attention since it contributes to the Knowledge Quartet literature in Turkey's context. Even though there are studies conducted in Turkey, examining preservice teachers’ knowledge of division operation in the Turkish context might contribute to the literature and researchers who want to investigate preservice teachers’ knowledge of specific topics and concepts. Thus, the present study would give worthy feedback on the validity and adaptability of the Knowledge Quartet Model regarding division operation in the Turkish context.

CHAPTER 2

LITERATURE REVIEW

The purpose of the study is to analyze pre-service primary school teachers' mathematical knowledge in teaching regarding division operation through the framework of the Knowledge Quartet Model. Therefore, this chapter includes the teacher knowledge models, related studies, the subject of division operation, and studies related to division operation. The chapter ends with a summary of the literature review.

2.1 Teacher Knowledge Models

Though the teaching process has been investigated for many years, there has yet to be a consensus about what teaching is and what it brings. According to Ball and Forzani, "Teaching is one of the most common and also one of the most complicated— human activities" (2010, p.43). Despite the complex structure of teaching, it is proven that the quality and specifications of teacher knowledge, one of the main study areas of teaching, are essential for student success and enhancing teaching. The relationship between teacher knowledge and student success (Monk, 1994; Rowan et al., 1997; Wilson et al., 2002; Tchoshanov et al.,2008) and the relationship between teacher knowledge and teaching (Ball et al., 2008; Ma, 1999; Shulman, 1986) have been emphasized by many studies. Since teacher knowledge has such importance and effect, it directed researchers to investigate what kind of elements the teacher knowledge must have (Shulman, 1986; Ball et al., 2008; Carrillo-Yañez et al., 2018) and consequently has brought out several teacher

knowledge models such as Shulman Model (Shulman, 1986), Mathematical Knowledge for Teaching (Ball et al., 2008), Mathematics Teachers Specialized Knowledge (Carrillo-Yañez et al.,2018) and Knowledge Quartet (Rowland et al., 2003). In the following titles, these models and frameworks are mentioned.

2.1.1 Shulman Model

According to Shulman (1986), teacher knowledge about the content that would be taught was "the missing paradigm," and it did not have as much importance as classroom management, behaviors towards students, and developing assignments and lesson plans. However, questions about the origins of teachers' explanations, representations, and methods to overcome misconceptions went unanswered. While Shulman put emphasis on the teacher's knowledge of the content, he also stressed that it was meaningless without blending it with pedagogical understanding. Therefore, in order to answer these unanswered questions, Shulman identified teacher knowledge in three levels, such as "Subject Matter Knowledge (SMK)," "Pedagogical Content Knowledge (PCK)," and "Curricular Knowledge" (1986, p. 9-10). Shulman (1986) explained the SMK as "the amount and organization of knowledge per se in the mind of the teacher" (p. 9). Followingly, PCK, which was first defined by Shulman, is described as knowledge of " the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others" (1986, p.9). Lastly, curricular knowledge refers to knowledge of objectives of a certain grade level and approprite instructional activites in line with this grade level. In addition to that, it includes knowing students' prior and future learning as well as other lesson subjects in the same grade level (Shulman, 1986). After Shulman determined what kind of knowledge types a teacher must have, this model inspired other researchers to develop a more content-specific teacher knowledge model since Shulman's model was based primarily on general educational sciences without being specific to one

subject. Ball et al. (2008) developed one of these models focusing on mathematics and mathematics teachers, as mentioned in the following title.

2.1.2 Mathematical Knowledge for Teaching (MKT)

The other framework is Mathematical Knowledge for Teaching (MKT) by Ball et al. (2008). This framework was created using Shulman's framework, taking into account its purposes, methods, and some unresolved issues. Also, unlike the Shulman Model, this model focuses on mathematics and is specific to mathematics teaching. According to Ball et al. (2008), MKT indicates the knowledge of mathematics needed for teachers to teach mathematics at all grade levels.

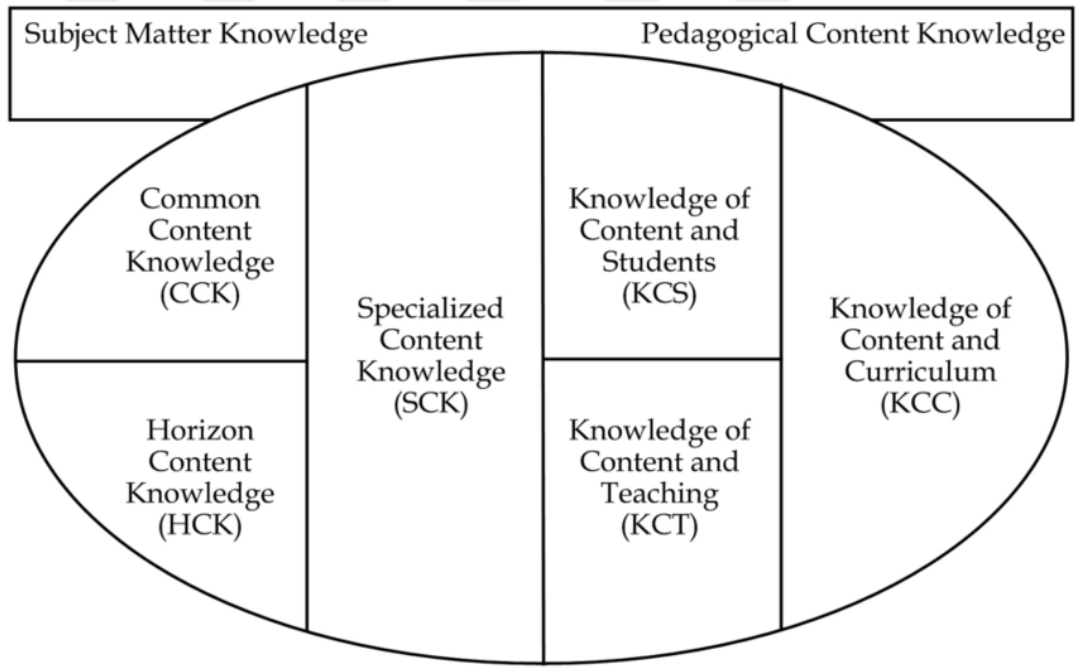


Figure 2.1 Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008)

Subject Matter Knowledge of this framework includes three categories as Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Content Knowledge (HCK). According to Ball (2008), mathematical knowledge that people from different backgrounds could have refers to CCK. Specifically, it is the "'pure mathematics,' i.e., knowledge concerning mathematical theories, concepts,

definitions, results, rules, proofs, and symbols used in different areas of mathematics" (Koponen et al., 2017, p. 35). The second category SCK covers the type of knowledge that only a mathematics teacher can have and is needed for instructional aims. For example, while dividing with fractions, knowledge of flipping the second fraction is considered as CCK, and knowledge of the reason for this procedure is considered as SCK. Lastly, as for HCK, Ball et al. (2008) described it as

> An awareness of how mathematical topics are related over the span of mathematics included in the curriculum. First-grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later. (p. 403)

In brief, HCK is the mathematical knowledge of the relationship between mathematical concepts and how these concepts progressed within and between grade levels (Ball et al., 2008).

PCK forms the second dimension of the framework, and it includes three categories: Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), and Knowledge of Content and Curriculum (KCC). KCT refers to the relationship between teachers' teaching and their knowledge, i.e., their ability to teach a certain topic (Ball et al., 2008). KCS refers to knowing possible difficulties students could have, and it includes knowing students in terms of their developmental levels, prior knowledge and needs for a certain mathematics topic. Lastly, the KCC indicates the curricular knowledge, curriculum's objectives and purposes. However, Ball's contemporaries argue that this model had some limitations, such as difficulties in differentiating the types of knowledge from each other, especially SCK from CCK (Silverman & Thompson, 2008; Carrillo-Yañez et al., 2018). Therefore, considering these shortcomings, a group of academicians developed a new framework called Mathematics Teacher Specialized Knowledge

with well-defined dimensions specific to mathematics teachers (Carrillo-Yañez et al., 2018).

2.1.3 Mathematics Teachers' Specialized Knowledge (MTSK)

The other framework specific to teacher knowledge is the Mathematics Teachers' Specialized Knowledge (MTSK) developed by Carrillo-Yañez et al. (2018). This framework is developed through other existing frameworks' shortcomings. While some frameworks deal with general teacher knowledge without focusing on a specific subject, the other frameworks focus on teaching without acknowledging the teacher's academic background and knowledge. As a result, considering these limitations, Carrillo-Yañez and his colleagues developed a framework with more clear-cut dimensions and, consequently, more specific to mathematics teachers. Figure 2.2 illustrates the model, its domains, and subdomains.

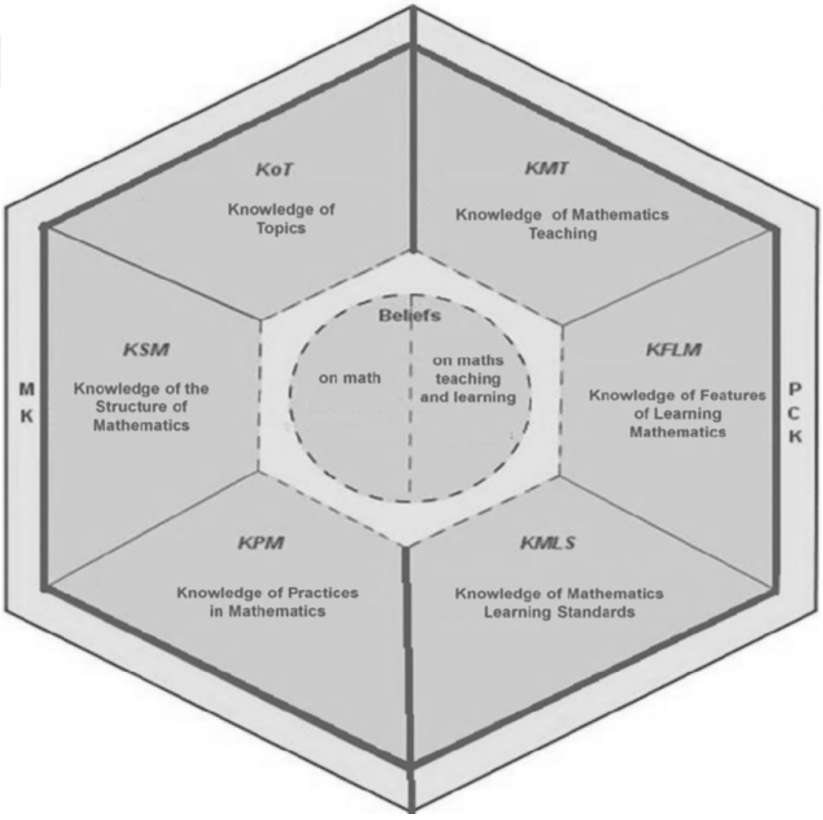


Figure 2.2 Mathematics Teachers Specialized Knowledge (MTSK) (Carrillo-Yañez et al., 2018)

This framework includes two fundamental dimensions: Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK), each consisting of three subdimensions.

According to Carrillo-Yañez et al. (2018), MK is similar to Shulman's Subject Matter Knowledge (1986) and refers to the mathematics teachers' knowledge of mathematics "as a scientific discipline" (p. 240). One subdimension related to MK is Knowledge of Topics (KoT), and it covers the knowledge of "procedures, definitions, properties, representations, applications" of a specific mathematics topic (Carrillo-Yañez et al., 2018, p. 242). Rather than superficial knowledge, it requires knowing the mathematics topic at a more formal and higher level. The following subdimension is Knowledge of the Structure of Mathematics (KSM) and includes the knowledge of teachers about the relationship among the mathematical topics. The last subdimension, which is Knowledge of Practices in Mathematics (KPM), is the knowledge of the teacher "about how mathematics is developed beyond any particular concept (e.g., knowing the meaning of necessary and sufficient conditions)" (Carrillo-Yañez et al., 2018, p. 245).

The other dimension that constitutes the framework is PCK. According to Carrillo-Yañez and his colleagues (2018), PCK is a crucial element for mathematics teachers; however, it can help only if it is supported by mathematical knowledge. According to the model, this dimension consists of three subdimensions. The first subdimension, Knowledge of Features of Learning Mathematics (KFLM), covers the knowledge of learning mathematics, such as theories, conceptions, and misconceptions of the students. The following subdimension is the Knowledge of Mathematics Teaching (KMT), which refers to teaching mathematics, e.g., how to teach mathematics with effective instructional methods, tasks and activities. The last subdimension is the Knowledge of Mathematics Learning Standards (KMLS), which

covers the knowledge of curriculum, e.g., how mathematics is handled in mathematics, the objectives and topics order (Carrillo-Yañez et al., 2018).

Although MTSK provides more distinguishable and well-defined dimensions and is exclusive to mathematics teachers, it is limited for the current study since MTSK is mainly based on in-service teachers (Carrillo-Yañez et al., 2018; Savuran, 2022).

Since the current study aims to investigate Turkish pre-service primary school teachers' knowledge about division operation through their prior knowledge, lesson plans, and teaching implementations in an actual classroom setting, Knowledge Quartet (Rowland et al., 2003) is determined as a theoretical and analytical framework. Concisely, the KQ Model focuses on mathematics teachers, is mainly based on pre-service teachers and is used as a theoretical and analytical framework in different cultural contexts, including Turkey (Rowland, 2013). Additionally, it provides a practice-based and holistic view of pre-service teachers' lesson processes (from foundational knowledge to lesson implementation). It deals with what is happening in the classroom (Rowland, 2013). A detailed explanation of the KQ Model is presented in the following section.

2.1.4 Knowledge Quartet (KQ)

The Knowledge Quartet Model is a theoretical framework developed by Rowland et al. in 2003 to investigate the preservice teachers' mathematical content knowledge. Instead of focusing on classroom management or organization, this 'empirically-based conceptual framework' (Rowland, 2013, p.37) deals with the occurrences in a mathematics lesson regarding the teachers' subject matter knowledge of the topic being taught. Since this framework is used as the theoretical framework for this current study, it is discussed in detail under the following title.

2.1.4.1 Knowledge Quartet as a Theoretical Framework

Tim Rowland and Fay Turner have developed the Knowledge Quartet Model as a framework for observing mathematics lessons and developing mathematics teaching. More specifically, it is a "conceptual framework for lesson reviews with a focus on the mathematics content of the lesson and the role of the trainee's mathematics SMK and PCK" (Rowland & Turner, 2007, p.109). By analyzing a trainee teacher's mathematics lessons, the framework of Knowledge Quartet is based on empirically grounded theory, and it includes four main dimensions, or what they called 'units' and a total of 21 invented codes for all four units (Rowland & Turner, 2007, p. 109-110). These units are listed in Figure 2.3, which illustrates the relationship of units with each other. According to the figure, Foundation is the main knowledge that nurtures the other three knowledge types. While Foundation does so, Transformation and Connection Knowledge reinforce the Contingency Knowledge separately.

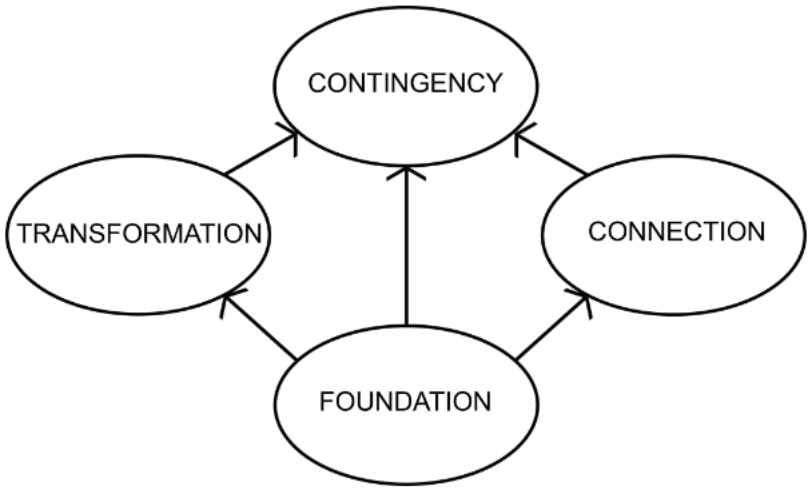


Figure 2.3 The relationships of the four dimensions that comprise KQ (Rowland et al., 2009)

Considering the model, Rowland and his colleagues (2013) used it for the first time to analyze lessons conducted by a preservice elementary teacher in their end-of-teacher training through KQ. The preservice teacher was teaching subtraction to the students who were 5 and 6 years old, and her lesson was analyzed with all the

dimensions. In 2005, Thwaites and her colleagues again analyzed a preservice teacher's lessons; however, this time, the authors used another method along with KQ and interviewed the preservice teacher. In the interview, the preservice teacher watched her videotaped lesson and was asked to reflect on her lesson. In conclusion, it was determined that KQ helped analyze preservice teachers' teacher knowledge with an emphasis on mathematics. Also, preservice teachers' being able to reflect on their lessons would provide a base for them to enhance their knowledge.

As mentioned before, while there are four units in KQ, each has its related codes. Each unit and its related codes are explained in detail in the following titles.

2.1.4.1.1 Foundation Dimension

Foundation is the first dimension that covers preservice teachers' "knowledge, understanding and ready recourse to what was learned at school and at college/university, including initial teacher education in preparation (intentionally or otherwise) for their role in the classroom" (Rowland, 2013, p. 22-23). Specifically, derived from the "comprehension" aspect of Shulman's idea on pedagogical reasoning (1987, p. 14), it includes mathematical content knowledge, theoretical knowledge of how to learn and teach mathematics, and conceptions and beliefs related to mathematics and mathematics learning and this knowledge and beliefs can be acquired or experienced by pre-service teachers throughout their whole academic life (Rowland, 2008; Rowland et al., 2009; Rowland, 2013). Also, as seen from the above figure, this dimension reinforces the other three dimensions regarding emergence and interpretation.

The foundation dimension consists of seven codes out of a total of twenty-one codes. These codes are listed as follows,

- Awareness of purpose
- Identifying pupil errors

- Overt display of subject knowledge
- Theoretical underpinning of pedagogy
- Use of mathematical terminology
- Adherence to textbook
- Concentration on procedures

The code of *awareness of purpose* refers to pre-service teachers' knowing which mathematical subjects to teach and related objectives in a specific grade level. This code also includes being aware of the general aims of mathematics education at a certain school level and grade level. The second code *identifying pupil errors* comprises the teacher's pinpointing possible mathematical errors that an element of the learning environment reveals. These elements could be students, the learning process, or the textbook. In the code of *overt display of subject knowledge*, what is essential is the teachers' conceptual and critical understanding of the subject they will teach. The code of *theoretical underpinning of pedagogy* refers to noticing how the learning process occurs and knowing the circumstances for an effective learning process. Teachers' usage of terminology and mathematical language during instruction or any teaching phase constructs the code of *use of mathematical terminology,* and how teachers benefit from textbooks constructs the code of *adherence to textbook.* The last code in foundation knowledge is *concentration on procedures*, which indicates teachers' usage of algorithms, rules, or any standard instructions while teaching a concept. In brief, unlike other units, this unit and its related codes deal with what kind of knowledge a pre-service teacher brings into play from their personal and academic background before implementing a lesson.

2.1.4.1.2 Transformation Dimension

In this dimension, KQ deals with how teachers plan and execute their lessons, i.e., 'knowledge-in-action' (Rowland et al., 2003). This dimension is mainly based upon

Shulman's idea (1986) and indicates teachers' ability to transfer their foundation knowledge to students regarding students' learning processes and pedagogical circumstances of learning (Rowland, 2003).

The four codes that construct Transformation Knowledge are as follows:

- Choice of examples
- Teacher demonstration
- Choice of representations
- Use of instructional materials

By looking at these three codes, it can be seen that they include purposefully chosen actions during planning or teaching. For example, the code of *choice of examples* refers to teachers using examples they deliberately prefer for a specific aim. Similarly, *choice of representations* includes selecting an appropriate representation for a specific aim. Followingly, using instructional materials refers to utilizing appropriate instructional materials, such as manipulatives, activity sheets, etc., for a specific subject, e.g., using ten base blocks for teaching place value. Lastly, *teacher demonstration* is considered as teachers selecting a demonstration to teach a specific concept or process. Shortly, Transformation Knowledge entails knowing how to deliver Foundation Knowledge to students in an understandable way, such as through examples, demonstrations, and representations.

2.1.4.1.3 Connection Dimension

The third unit, Connection Dimension, refers to "the coherence of the planning or teaching displayed across an episode, lesson or series of lessons" (Rowland, 2003, p.98). In this knowledge type, what is important is to adapt before mentioned purposefully chosen actions for a mathematics lesson in a coherent and well-organized way. The codes in the dimension are:

- Making connections between procedures
- Making connections between concepts
- Anticipation of complexity
- Decisions about sequencing
- Recognition of conceptual appropriateness

Considering the first two codes, *making connections between procedures* and *making connections between concepts*, they are similar in principle; however, while one of them shows teachers' ability to connect different procedures of a specific concept, the other one suggests teachers' ability to connect different concepts (Rowland, 2013). For example, while being able to connect addition to multiplication refers to *making connections between procedures,* being able to connect fractions to decimals refers to *making connections between concepts.* The code of *anticipation of complexity* indicates teachers' recognizing possible restrictions of students while they deal with concepts and activities. Teachers organizing the concepts, activities, or any part of teaching in deliberate order comprises the code of *decisions about sequencing* (Rowland, 2008). The last code of Connection Knowledge is *recognition of conceptual appropriateness,* and it refers to teachers recognizing students' needs and levels while planning a lesson and teaching a concept. Concisely, this unit emphasizes presenting and ordering tasks, procedures, and concepts logically by recognizing the students' needs and levels.

2.1.4.1.4 Contingency Dimension

The last dimension of KQ is the Contingency Dimension, and it is mainly about unexpected events occurring in the teaching process. Even though a mathematics teacher goes to a classroom with lesson plans, prepared tasks and activities, and predetermined objectives, since every class and student is unique, students' behaviors and responses are unplanned and unexpected. Therefore, Contingency

Dimension generally refers to teachers' ability to deal with these unplanned situations. In this dimension, the code is listed below:

- Responding to students' ideas
- Deviation from agenda
- Teacher insight
- Responding to (un)availability of tools and resources

The first code *responding to students' ideas,* refers to how the teacher takes care of students' unexpected ideas. Also, sometimes teachers should make changes because of these unexpected ideas, students, or opportunities. In that case, these changes in the lesson plan refer to code *deviation from agenda,* and it includes whether the teacher could accommodate herself/himself and how he/she does so. Followingly, the code *teacher insight* entails teachers noticing the deficiencies in the lesson plan during the teaching process. Lastly, the code of *responding to (un)availability of tools and resources* refers to utilizing a tool or a resource that is unplanned in the lesson plan but can be integrated into the lesson.

2.1.4.1.5 Summary of Knowledge Quartet

In brief, Rowland's Knowledge Quartet is a framework that enables examining the mathematics lessons while concentrating on teachers, especially pre-service teachers, and their subject matter knowledge and pedagogical content knowledge. All four dimensions focus on a specific part of teaching. Concisely, the focal point of the Foundation Dimension is teachers' knowledge before their teaching, such as what they know, understand and believe as a mathematics teacher. Followingly, the Transformation Dimension emphasizes how teachers carry out their teaching and how they transform their knowledge to teach their students by considering the required circumstances of students' learning. Consecutively, the Connection Dimension targets teachers' deliberate actions and decisions to make the lesson

coherent. Lastly, the Contingency Dimension is about how teachers respond to unexpected and unplanned situations. Totally these four dimensions form the framework, and its focal point is the SMK and PCK of mathematics teachers before and during their teaching. In line with the aim of the current study, which investigates the preservice primary school teachers' mathematical knowledge in teaching in a real classroom setting, the Knowledge Quartet Model is determined as a theoretical and analytical framework of the study. Since the study aims to examine teacher knowledge, the following section covers studies on teacher knowledge by different teacher knowledge models, including Knowledge Quartet.

2.1.5 Studies Related to Teacher Knowledge

Teacher knowledge is a concept that has been studied previously. Thus, it was and has been studied from different aspects. While some studies focused on its effect on teaching, some dealt with particular types of knowledge, such as subject matter or pedagogical content knowledge, in terms of various mathematics topics. For instance, Hill and colleagues (2008) studied whether subject matter knowledge of in-service teachers affects their teaching, and they concluded that there is a relationship between them. In similar, Wagner and colleagues (2007) indicated that insufficient subject matter knowledge of teachers affects their decisions about instructional goals adversely. Besides its effect on teaching, many scholars analyzed primary school and middle school teachers' knowledge considering specific mathematics subjects, such as numbers, measurement, algebra, etc., internationally and nationally. In the following section, these international and national studies were listed.

2.1.5.1 International Studies

Regarding international studies, these studies could be categorized according to their sample type, such as in-service or preservice teachers. Considering teacher knowledge of in-service teachers, researchers studied this concept in different topics

such as numbers and operations, algebra and so on. For example, Carpenter et al. (1988) examined the PCK of 40 in-service primary school teachers regarding whether they could distinguish addition and subtraction problems and whether they knew about students' strategies for these problems through the PCK level of Shulman's model. The result showed that even though teachers could tell the difference between the main classes of addition and subtraction, they could only do it for each problem and could not generate a model to identify the problem types. The reason why they failed to develop a model could be that preservice teachers have inadequate knowledge regarding this issue. In another study done with in-service teachers, Leavitt (1998) researched in-service mathematics teachers CK and PCK in several subjects, such as arithmetic operations with multi-digit numbers, fractions, perimeter, and area. By implementing a questionnaire and interviewing 20 in-service teachers, it was found that teachers' knowledge about division with fractions was limited, and they had difficulties connecting real life with fractions. Followingly, Li (2007) conducted a study to investigate whether there is a relationship between the in-service teachers' knowledge of algebraic equations and their academic background and experience. The findings indicated that the teachers lacked the knowledge to recognize misconceptions and difficulties, and they needed to improve their subject matter knowledge. Also, while their teaching was affected by their experiences, their academic background did not have an effect on it. Concisely, according to these, in-service teachers' knowledge of particular mathematics topics was found to be limited. In addition, there are studies conducted with in-service teachers through the KQ Model. In a study conducted by Rowland and Zazkis (2013), three mathematics teachers' knowledge was examined through the Contingency dimension of the KQ Model in three different cases. They concluded that mathematics teachers' knowledge had an effect on how they handled the contingent moments, especially the problematic contingent moments.

Apart from the studies done with in-service teachers, there were several studies focused on preservice teachers. In 2014, Maher and Muir studied preservice teachers' detection of students' misconceptions about probability. The data was

collected by interviews focusing on four probability questions from seven pre-service teachers. Data were analyzed regarding three dimensions: knowledge of content, learning, and teaching. According to the results, regarding their knowledge of content, preservice teachers had limited conceptual and procedural understanding of probability. Considering their knowledge of teaching, they also demonstrated insufficient teaching strategies. The authors advocate that this lack of understanding and knowledge possibly affected preservice teachers' teaching of probability and also their recognition of students' misconceptions. As a suggestion to teacher educators, it was concluded that preservice teachers should participate more in activities about probability since authors believed that CK was in a close relationship with PCK.

Regarding not only students' misconceptions but also teachers' misconceptions, Livy et al. (2012) conducted a study to investigate what pre-service teachers know about area and perimeter, their misconceptions about these concepts, and how their CK affected their PCK. The data was collected by interviews with 24 preservice teachers from two different universities and by test questions with 222 preservice teachers from one university. Results showed that although preservice teachers could solve problems related to area and perimeter, they did not explain these concepts and distinguish them. As well as these results, the preservice teachers had limited use of mathematical terms and tended to rely on formula rather than conceptually explain it. Additionally, even though these preservice teachers were in their final year, they had the misconceptions that primary school students had. The authors suggested that enabling preservice teachers to notice their understanding, errors, and misconceptions, identify students' errors, and encourage them to participate in professional discussions regarding these points would be better for preservice teachers to improve their CK and PCK. In brief, these studies conducted with preservice teachers showed that they had their own misconceptions and difficulties detecting misconceptions. Also, these two studies emphasized the relationship between CK and PCK and suggested the importance of improving these types of knowledge.

Looking at the literature related to preservice teachers' knowledge, there were also a number of studies focusing on algorithms. For example, Borko et al. (1992) conducted a study with preservice teachers. They analyzed a case of a preservice teacher where the preservice teacher failed to justify the algorithm of fraction division. While the author analyzed the case, they considered what the preservice teacher knew and how this subject was handled in the method courses. Data was collected through mainly semi-structured interviews and observations. The findings demonstrated that she failed to justify this because the pure mathematics classes she took did not emphasize the meanings of algorithms.

Similarly, Ball (1990) conducted a study with 19 preservice mathematics teachers to investigate their SMK before their teacher training considering the division with fractions. The results revealed that the preservice teachers had a limited understanding of division with fractions. Even though they knew the algorithm without meanings, it was concluded that what was essential for effective teaching was being aware of the nature of processes and relationships (Ball, 1990). According to the studies, it could be concluded that preservice teachers had difficulty in explaining what division of fraction meant and why the algorithm worked.

Additionally, while the studies on teacher knowledge used different models and frameworks, such as Shulman's Model or MKT, many studies used the KQ model since the model was focused mainly on preservice teachers. Turner and Rowland (2008) conducted one of these studies, which was also a part of Turner's longitudinal study. It was aimed to analyze a preservice primary school teacher's mathematical content knowledge using KQ through a lesson where the preservice teacher was teaching multiplication. When analyzing the lesson in detail, the preservice teacher introduced the algorithm as a different method without relating it to the first method she used. Doing so showed that she also lacked connection knowledge.

Additionally, it was found that having more than one way to do multiplication could be evidence of her foundational knowledge. However, lacking deliberate order between examples indicated her lack of transformational knowledge. Lastly, the

preservice teacher could not utilize the opportunity of a student's giving the wrong answer; this was also evidence of her contingency knowledge. Likewise, another study that originated from the aforementioned longitudinal study analyzed the preservice teachers' knowledge through KQ. In this separate publication, Turner (2007) investigated the representations of two preservice teachers who were participants in the more extensive study and were teaching addition and subtraction. Findings showed no relationship between teachers' goals and their choice of representations. Additionally, one of the preservice teachers preferred to use a hundred chart; however, her usage of this and the procedures she showed caused students to understand these procedures as a rule and led them to understand the topic without conceptual understanding. Nevertheless, that was expected for preservice teachers, and it was seen that enabling them to reflect on their teaching was necessary so that they would contemplate their choice of representations. Also, some other research that supported the effect of teachers' knowledge on students' understanding was done by Rowland (2007). In this research, Rowland analyzed the foundational knowledge of two teachers through their lessons. One of these teachers was a preservice teacher teaching subtraction, and the other was an in-service teacher teaching graphs and functions. The author advocated that the students' difficulty learning these concepts was not related to teachers but to the concept itself. According to the author, these elementary concepts should be analyzed thoroughly by the teacher and made attainable by the students since these concepts became automatic for the teacher. This led teachers to be unaware of the complexity of these concepts. Studies showed that, through KQ, researchers could investigate preservice teachers' teachings in real classroom settings. It also revealed that preservice teachers had various difficulties and limitations in their teachings regarding different units of KQ.

Followingly there are several studies that examine preservice teachers' knowledge related to their practicum or small-scale teaching practice in method courses. For example, Nicol and Crespo (2006) investigated four preservice teachers and how they used textbooks in their practicum. As a result, the authors found that preservice

teachers used textbooks in different ways and had hardships when they used textbooks. However, practicing teaching encouraged preservice teachers to create and be flexible while using textbooks during practicum. Another study that examined preservice teachers' knowledge during practicum was conducted by Singmuang (2002). In his study, four preservice teachers' subject matter knowledge of division with rational numbers was examined, and he found that all the preservice teachers based their lessons on procedures and standard algorithms. In addition to practicum, Arnesen and colleagues (2017) investigated preservice teachers' knowledge of fractions through the KQ Model in lesson plays which was a practice assignment proposed by Zazkis et al. (2009) where preservice teachers planned their teachings by writing scripts in the method course. As a result, Arnesen et al. found that lesson plays helped preservice teachers to improve their mathematical knowledge.

As international studies on teacher knowledge approach this concept from different aspects, research was conducted in Turkey considering this concept. In the following title, national studies on teacher knowledge from different aspects would have covered.

2.1.5.2 National Studies

Like international studies, several studies were conducted in Turkey with in-service and preservice teachers. One of the studies that focused on in-service teachers was conducted by Girit-Yildiz and Akyuz (2019). They aimed to examine in-service teachers' content and pedagogical content knowledge while teaching algebra expressions. In this qualitative study, data was collected from two in-service mathematics teachers through lesson plans, classroom observations, and interviews analyzed by the MKT model. Findings revealed that in-service teachers' SCK about different representations of algebraic expressions was limited. Also, using two concepts (variable and unknown) as if they were the same showed that their CCK about these concepts was insufficient. As a result, the authors concluded that teachers should have a better conceptual understanding of mathematics topics to plan better

lessons and have effective lessons. In order to do so, method courses should emphasize different representations, models, strategies, and effective teaching methods more. Additionally, effectively designing school practice courses could help pre-service teachers improve their SCK.

In another study done with in-service teachers, Tekin-Sitrava (2019) examined in-service mathematics teachers' knowledge of division with fractions and used Shulman's model. In line with this aim, she collected data from 22 in-service teachers through a task with two questions and interviews. She analyzed it considering teachers' solution strategies and the problems they posed. Results suggested that many of the teachers used the same strategy (flipping the second fraction); however, they could not explain why they flipped it because of their rote learning and lack of conceptual understanding of division with fractions. Also, another result indicated that more than 50% of teachers could not write a problem regarding a given operation. As a suggestion, the author pointed out the importance of teachers' adequate knowledge of considering strategies and posing problems since it was closely related to enhancing students' learning.

Additionally, Dogan Coskun et al. (2021) also studied contingent moments in an in-service teacher's lesson on area measurement by analyzing teachers' responses to these unexpected events and the relationship between contingencies and teachers' mathematical knowledge. Data was collected from observation and recordings of five different lessons and interviews following them. Findings showed that the teacher was aware of students' ideas and could use them as an opportunity to integrate them into the lesson. The results of this study could be a guide for future teachers to get familiar with them and benefit from these unexpected events. In addition to this, another study done by Dogan Coskun and Isiksal-Bostan (2018), it was aimed to examine the mathematical knowledge of an in-service teacher regarding area measurement. Even though KQ was developed and generally used to analyze and examine preservice teachers' mathematical knowledge, this study investigated an in-service teacher with a few years of experience. Data was collected through observations, videotaped lessons, and interviews and analyzed with KQ. As

a result, findings showed that it could be said that the teacher was also lacking in handling contingent moments since the teacher preferred to ignore an unexpected moment that could have been used as an opportunity to understand and correct students' thinking. Additionally, considering the use of the textbook, she had resorted to the textbook; however, she could not efficiently use it. She disregarded adapting activities in the book according to the student's needs and levels and preferred to go along with what the book suggested even though that was inappropriate. Also, the teacher lacked examples to challenge students and expose them to formulas. This also showed her insufficient knowledge of the choice of examples.

As well as focusing on in-service teachers, several studies were conducted with preservice teachers. In addition to the study done by Dogan Coskun and Isiksal-Bostan (2022) with in-service teachers regarding measurement, the authors investigated the knowledge of preservice teachers considering the topic of measurement. In this study, they analyzed Turkish and American preservice primary school teachers' knowledge regarding length measurement comparatively. Their study specifically focused on students' errors, misconceptions, and difficulties and how to deal with them. Data were collected from 40 preservice teachers from Turkey and the USA through a test of 4 items in accordance with foundation and transformation dimensions. The results demonstrated that while American preservice teachers were better at pinpointing students' possible misconceptions or errors, Turkish preservice teachers were better at indicating the underlying reason for these errors. Additionally, there was no difference between both countries in dealing with these difficulties because preservice teachers of both countries could not suggest any effective strategy or method to overcome these challenges. However, the authors recommended that teacher educators provide preservice teachers with courses and activities that would enable them to get familiar with possible misconceptions and errors and challenge them to identify causes since this lack of foundational knowledge could cause insufficient transformational knowledge.

Another study that was conducted with preservice teachers was carried out by Isiksal-Bostan and Cakiroglu (2008), and they aimed to investigate pre-service teachers' knowledge of students' misconceptions about division with fractions, the roots of these misconceptions, and strategies to deal with them. In this qualitative study, data were collected through written questions and interviews. As a result, they classified misconceptions into four groups. While these pre-service teachers could identify the roots of misconceptions, they could also develop strategies to handle them. Authors suggested that "content-pedagogy rich courses" for pre-service teachers had a crucial role in developing students' mathematical knowledge of concepts regarding division with fractions (p.181-182).

In short, the literature review considering teacher knowledge showed that it was an essential aspect of teaching mathematics and was studied through different models and frameworks. Researchers analyzed teacher knowledge in terms of addition, subtraction or multiplication (Ball, 1990; Carpenter, 1988; Leavitt, 1998; Rowland, 2007; Turner, 2007; Turner & Rowland, 2008), fraction and its operations (Ball, 1990; Borko et al., 1992; Hutchison, 1997; Isiksal-Bostan & Cakiroglu, 2008; Leavitt, 1998; Tekin-Sitrava, 2019), probability (Maher & Muir, 2014), algebra (Girit-Yildiz & Akyuz, 2019; Li, 2007), measurement (Dogan Coskun & Isiksal-Bostan, 2018; Dogan Coskun et al., 2021; Dogan Coskun & Isiksal-Bostan, 2022; Livy et al., 2012) and problem posing (Tekin-Sitrava, 2019). However, when looking at the studies, it was obvious that studies conducted on teacher knowledge regarding division with whole numbers were limited. Since the concern of the study was to investigate pre-service teachers' knowledge in terms of division with whole numbers, in the following section, division operation and related studies were mentioned in detail.

2.2 Division Operation

NCTM (2000) claims that children in prekindergarten through second grade are expected to comprehend operations, their meanings, and the relationships between

each other. One of these expected operations is division operation. Division operations can be conducted with different number sets, such as whole numbers, integers, fractions, or rational numbers. However, generally, in the early years of formal mathematics education, the focus area is division operations with whole numbers (MoNE, 2018). The current study aims to investigate primary school teachers' mathematical knowledge in teaching regarding the division operation of whole numbers with the Knowledge Quartet Model. Therefore, in the following titles, division operation, its different meanings, and its importance are discussed in detail.

2.2.1 Importance of Division Operation with Whole Numbers

The four fundamental operations (addition, subtraction, multiplication, and division) with whole numbers are one of the most critical objectives in the primary school mathematics curriculum. They are strictly related to each other because of the sequential and cumulative structure of mathematics, and they are crucial for doing higher mathematical tasks (McCallum & Schmitt, 2011). Therefore, teaching and learning the standard algorithms of these operations establish one of the main subjects currently taught in primary schools in different countries. Consequently, a great deal of time is allocated to teaching algorithms.

Considering the Turkish mathematics curriculum of the second grade, where the division operation is first introduced, the sub-learning area 'division operation with whole numbers' has two learning outcomes, and these are:

> M.2.1.5.1. Student will use measurement and partition meanings of the division operation.
>
> a) Include the use of real objects.
>
> b) Make operations with numbers up to 20 with no remainder.

c) Before symbolic representation of division operation, model division operation as repeated subtraction.

M.2.1.5.2. Student will make division operation and use the sign of division operation (÷).

a) Ensure that the student chooses the appropriate operation for the given problem.

b) Emphasize that the concepts of dividend, divisor, quotient and division sign (÷) belong to division operation.

Even though there are only two learning outcomes, they make up 9% of the total allocated time for mathematics in an academic year (MoNE, 2018). They are critical as they stress the conceptual knowledge of the division operation with natural numbers. Also, developing this knowledge without any error or misconception is vital because of the sequential relation between four fundamental operations. They facilitate learning other mathematics subjects in the years to come (Ball, 1990; Yorulmaz, 2018). However, several studies state that children struggle to perform and understand them (Carpenter et al., 1998; NRC, 2001). These difficulties can be caused by a lack of connections between procedures and the relevant conceptual knowledge of the division operation (English & Halford,1995) or students' intuitive thinking of division as equal sharing (Fischbein et al., 1985). However, for students to understand division operation conceptually, they should be aware of the different meanings of division and the relationship between concepts related to division, such as dividend, divisor, and quotient, since the meaning can differ when changing the nature of this relationship (Bryant, 1997).

In short, division operation with whole numbers is an important concept and is affiliated with many other concepts. Additionally, as the studies demonstrated, understanding this concept thoroughly (e.g., what it means and what kind of meanings it has) is crucial. In this sense, as is seen from the objectives above, the curriculum in the second grade in primary school covers this subject and puts

emphasis on its meanings. Thus, in the below section, the definition of division operations and the meanings of division operation are explained in detail.

2.2.2 Division Operation and its Meanings

Learning division operation is required understanding the meanings of division. Division operation is basically "at its foundation, the division has to do with forming groups" (Ball, 1990, p. 452), and it has two different meanings, such as partition or partitive (fair sharing) and measurement quotative (repeated subtraction) (Fischbein et al., 1985; Reys et al., 2014; Van de Walle, 2013). In this study, these terms are used interchangeably. Partition or partitive meaning refers to forming groups that are equal-sized, and their number of elements is known, while the number of these equal-sized groups is unknown. In partition, the number of equal-sized groups gives the quotient, and the size of each equal group is the result. Figure 2.4 exemplifies partition by the operation of $20 \div 4 = 5$. Four represents the number of equal-sized groups, and five represent the size of these groups.

On the contrary, the measurement or quotative refers to forming groups that are equal-sized, and the number of these equal-sized groups is known, while the number of elements in each equal-sized group remains unknown. According to this situation, the size of each equal group is the quotient, while the number of equal-sized groups is the result. Figure 2.5 illustrates the same example with the measurement meaning where four represents the size of the groups, and five represents the number of equal-sized groups.

20 apples in 4 baskets	20 apples, 4 per basket

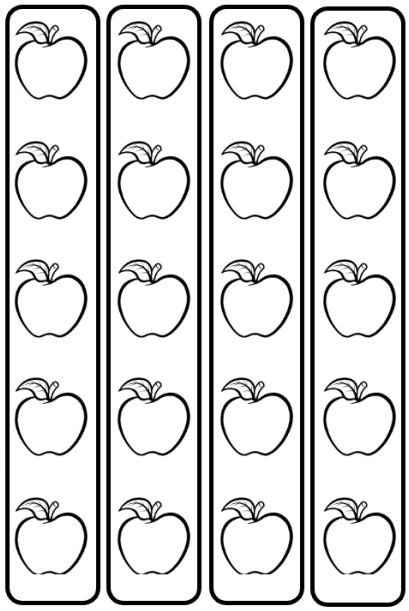
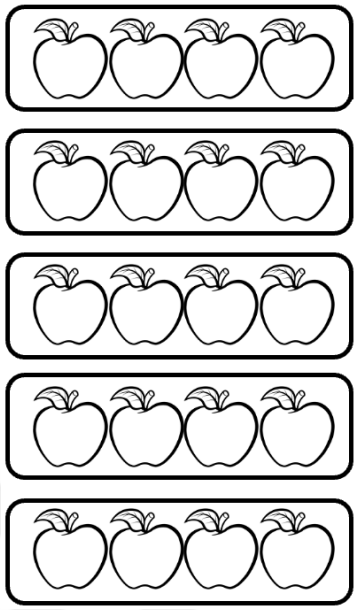

	

Figure 2.4 Partition	Figure 2.5 Measurement

Even though the models represent the same division operation, which is 20 ÷ 4 = 5, their meanings differ. While figure 2.4 illustrates a situation of 20 apples in 4 baskets, where each basket has five apples, figure 2.5 illustrates a situation of 20 apples, where every four apples are in 5 baskets. In the previous section, the studies and the objectives of the second-grade curriculum put emphasis on these different meanings and the importance of conceptualizing these meanings through models. Therefore, the following section holds studies focusing on division operation and how students and teachers conceptualize it.

2.2.3 Studies Related to Division Operation with Whole Numbers

Although studies related to division operations are generally conducted in terms of fractions or rational numbers rather than whole numbers, this section contains the studies done with students, pre-service teachers, and in-service teachers.

A study done with students was conducted by Fischbein et al. (1985) to investigate whether the primitive models of multiplication and division affected students' learning and problem-solving processes negatively. Data was collected through a 42-item test of multiplication and division problems from 628 students who were in different grade levels. The findings showed that the authors were correct in their assumptions. According to the results, children had difficulties doing division operations later in their academic life because of the lack of relationship between the standard algorithm and the primitive models, which they used indeliberately and resistant to change. However, the authors suggested that teachers supporting students' invented and mental strategies was critical since these strategies would enable them to change these primitive models, leading them to confusion while transitioning to the standard algorithm.

As well as studies focusing on students, researchers conducted studies with preservice teachers. For example, a study by Simon (1993) aimed to examine the preservice teachers' knowledge of division operation, its different meanings, and whether they could connect their knowledge with different representations and situations, such as using concrete models and symbolic representations (*connectedness*). Data was collected separately from 33 preservice elementary teachers through open-ended questions and from 8 preservice elementary teachers through interviews. The results showed that preservice teachers tended to think of division first as a partitive situation rather than a quotitive one. When asked to write realistic problems for the operation of $51 \div 4$, where each problem would generate 12, 13, $12\frac{3}{4}$, they could easily come up with problems with partitive context. Still, they had hardship with writing a realistic problem with the answer of 13 since it should have a quotitive context to generate a number more than the exact quotient. Only 17% could write in a quotitive context when required to write a problem. However, the exact opposite situation happened when they had to write a problem for the division of fractions. Preservice teachers could write problems with a quotitive rather than a partitive context. It was also found that even though preservice teachers could divide with a standard algorithm successfully, they could not identify the elements

of the algorithm (dividend, divisor, quotient), distinguish them, and connect the algorithm with one of the meanings of division. As a result, the authors suggested that method courses should be designed more effectively to improve preservice teachers' limited conceptual understanding regarding the division. Also, mathematics courses should emphasize the connections between and within concepts, conceptually understanding the algorithms.

Like Simon's study (1993), Dogan Coskun and Cimen (2019) also found similar results in a study with preservice teachers. The authors examined preservice teachers' problem-solving processes while dealing with division problems. Data were collected from 60 preservice primary school teachers through a test with eight open-ended questions for partition and measurement meanings. Results indicated that many of the preservice teachers were more successful in realistically interpreting the problems with partitive meaning compared to those with quotitive meaning. Also, considering problems with either meaning, they could realistically interpret problems if the remainder was the answer for the problem asked. However, they were less successful if they dealt with problems with *readjusted quotient by partial increment*. In light of these results, the author suggested that asking problems that required preservice teachers to notice the correctness of quantitative answers and whether that answer was appropriate for the context and real life was critical to enhancing preservice teachers' knowledge regarding division operation.

Followingly, another similar study by Tekin-Sitrava et al. (2020) aimed to examine preservice primary school teachers' content knowledge about the meanings of division and their representations of these meanings. In this qualitative study, data were collected from 25 preservice teachers through two open-ended questions and interviews. Findings showed that only eight of twenty-five preservice teachers knew two meanings. Also, many of them could give the correct answer for the problem with a partitive meaning, but only 10 of them could notice that the problem had a quotitive meaning. However, this result was not in line with modeling these problems. While many preservice teachers could model the problem with quotitive meaning, they had difficulties modeling it with partitive meaning.

Apart from the studies on preservice teachers, Roche and Clark (2013) investigated the in-service primary school teachers' Mathematical Knowledge for Teaching (MKT) regarding division operation to examine the professional development program. In this article, they shared the results of one assessment they did for the more extensive study. Regarding this study, 378 in-service primary school teachers were asked a division story which required teachers to indicate two forms of a division operation, draw a picture and write a story problem representing that specific form regarding 12÷3. The findings demonstrated that teachers knew their way around partitive meaning. Still, they had a hard time dealing with quotitive even after participating in the Professional Development program. Also, they had difficulties while writing an appropriate story problem for each meaning. Even though these teachers showed slight improvement after PD, the authors suggested that a deliberate PD program for this subject would effectively enhance teachers' knowledge of division operation and its meanings.

Considering the research on division operation with whole numbers, although they were relatively fewer than other mathematical concepts, most research focused on pre-service teachers (Dogan Coskun & Cimen, 2019; Simon, 1993; Tekin-Sitrava et al., 2020). Also, there were studies on in-service teachers (Roche & Clark, 2013) and students (Fischbein et al., 1985). Regardless of the sample type, almost all the studies demonstrated that teachers' knowledge of division with whole numbers is poor. The number of these related studies is few and limited. Since teacher knowledge is one of the essential aspects of student learning and effective teaching (Ball et al., 2008; Ma, 1999; Monk, 1994; Rowan et al., 1997; Shulman, 1989; Tchoshanov et al., 2008; Wilson et al., 2002), it is significant to examine teacher knowledge regarding division with whole numbers. Thus, this study aimed to investigate the pre-service primary school teachers' knowledge of the division operation.

2.3 Summary of the Literature Review

The aim of the present study is to examine preservice primary school teachers' knowledge of division operation through Knowledge Quartet Model. As the literature review states that there is a close relationship between their teaching, their students' understanding, and consequently, their students' success (Ball, 1990; Ball et al., 2008; Cohen et al., 1993; Even, 1990; Montes et al., 2018). Since the effect of teacher knowledge is obvious, researchers are encouraged to investigate teacher knowledge and what constitutes it (Ball et al., 2008; Ma, 1999; Monk, 1994; Rowan et al., 1997; Tchoshanov et al.,2008; Wilson et al., 2002). As a result, several teacher knowledge models have been developed (Ball et al., 2008; Carillo-Yanez, 2013; Rowland et al., 2003; Shulman, 1986). Even though all the models examine teacher knowledge in different dimensions or topics, all of them point out the importance of subject matter knowledge and pedagogical content knowledge. However, Rowland's Knowledge Quartet Model indicates that both types of knowledge should be considered as a whole as they affect each other immensely (Rowland et al., 2003). In the section where the Knowledge Quartet Model was explained, it was seen that this model is useful for recognizing the structure of the preservice teachers' knowledge and effective in developing preservice teachers' knowledge.

In parallel to the aim of the current study, when the studies about teacher knowledge have been examined, it was seen that studies investigate teacher knowledge in terms of addition, subtraction or multiplication (Carpenter, 1988; Ball, 1990; Leavitt, 1998; Turner, 2007; Rowland, 2007; Turner & Rowland, 2008), fraction and its operations (Ball, 1990; Borko et al., 1992; Hutchison, 1997; Isiksal-Bostan & Cakiroglu, 2008; Leavitt, 1998; Tekin-Sitrava, 2019), probability (Maher & Muir, 2014), algebra (Girit-Yildiz & Akyuz, 2019; Li, 2007), measurement (Dogan Coskun & Isiksal-Bostan, 2018; Dogan Coskun et al., 2021; Dogan Coskun & Isiksal-Bostan, 2022; Livy et al., 2012) and problem posing (Tekin-Sitrava, 2019).

However, one of the important subjects in primary school mathematics is division with whole numbers. Division operation with whole numbers is an important concept

due to its strict affiliation with many other concepts (McCallum & Schmitt, 2011) and it is the most difficult operation considering the three other operations (Carpenter et al., 1998; NRC 2001). Nonetheless, when the literature has been reviewed, studies on division operation with whole numbers were less examined than other mathematical concepts. Additionally, most research focused on pre-service teachers (Dogan Coskun & Cimen, 2019; Simon, 1993; Tekin-Sitrava et al., 2020). However, there were studies on in-service teachers (Roche & Clark, 2013) and students (Fischbein et al., 1985).

However, looking at international and national studies, it can be seen that studies that examine preservice primary school teachers' knowledge of division operation through the Knowledge Quartet Model are lacking. As the Knowledge Quartet Model suggests, considering subject matter knowledge and pedagogical content knowledge as a whole, the current study aims to investigate preservice primary school teachers' knowledge of division operation in the context of teaching practice course through the Knowledge Quartet Model so that their knowledge would be examined theoretically and practically. This study would contribute to the literature of division operation in regard to teacher knowledge and the Knowledge Quartet. Additionally, this study will be the first study that examines the preservice primary school teachers' knowledge of division operation theoretically and practically in the context of teaching practice courses and would give worthful information to teacher educators and policymakers.

CHAPTER 3

METHODOLOGY

The current study aims to investigate preservice primary school teachers' mathematical knowledge in teaching regarding division operation. Accordingly, this chapter includes the study's overall design, research design, context and participants, data collection procedures, data collection tools, data analysis, the study's trustworthiness and the researcher's role.

3.1 Research Design

The current study aimed to gain an in-depth understanding of preservice primary school teachers' mathematical knowledge in teaching regarding division operation with whole numbers. Considering this aim, the study employed the qualitative research methodology. According to Creswell (2014), qualitative research design should be adopted when the research aims to "obtain a deep understanding" of a multifaceted situation (p.19). Many researchers have identified and described qualitative research and its elements (Bogdan & Biklen, 2007; Creswell, 2015; Denzin & Lincoln, 1994; Fraenkel & Wallen, 2006).

According to Denzin and Lincoln (1994), "Qualitative research is multimethod in focus, involving an interpretive, naturalistic approach to its subject matter. … qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them" (p.2). In qualitative research, the researcher who collects the data from the natural setting is the critical element in the research. In addition, data are based on words instead of numbers and are analyzed inductively (Bogdan & Biklen, 2007; Fraenkel et al., 2012). Given the distinctive features of qualitative research, this research design is well-suited for the current study because its objective is to gain a deep understanding

of preservice primary school teachers' knowledge pertaining to the division operation.

Creswell (2007) lists five different methods for qualitative research design: narrative research, phenomenology, grounded theory, ethnography, and case studies. As a method, the case study was thought to be appropriate for the current study. A case study is an approach to understanding a complicated situation or an issue more profoundly through different methods while considering it in a real-life context (Yin, 2014). Yin (2014) further defines a case study as "an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context" (p. 18) and adds that a case study should be preferred when 'how' and 'why' questions are asked to explain a contemporary phenomenon, and the researcher cannot manipulate or intervene in the phenomenon (p.48-50). The term "case", as previously mentioned, serves as the primary focal point of the research, and it can be a person, a group of people, an institution, an event or a phenomenon (Fraenkel et al., 2012; Green & Thorogood, 2009; Yin, 2014). However, researchers identify case studies as "bounded system" studies (Stake, 2000, p. 436; Creswell, 2002, p.485). For example, Merriam (1998) stresses that "the most single defining characteristic of case study research lies in delimiting the object of study, the case" (p. 27) and in a case study, a case should not only be defined but also be bounded (Yin, 2014). As Creswell elaborates, "Bounded means that the case is separated out for research in terms of time, place, or some physical boundaries" (p.485). Additionally, the scope of a case can be delimited by factors such as time and activity, as noted by Stake (2005), or by definition and context, as outlined by Miles and Huberman (1994). In essence, cases should be delineated within the confines of time, place, definition, or context.

Moreover, a pivotal aspect of the case study method is the identification of the unit of analysis in the research, as emphasized by Yin (2014). Yin (2014) underscores that the unit of analysis is essential in establishing the boundaries of the case, which, as previously mentioned, can encompass a wide range of entities, including individuals, groups, or educational programs. The unit of analysis serves as the

central focal point of the study, representing the subject on which researchers draw their conclusions.

Considering the aim of the current study, which is the investigation of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation, the adoption of the case study approach was deemed suitable.

Furthermore, in this study, the eight steps outlined by Yıldırım and Şimsek (2008) was followed to conduct a case study. The diagram below provides detailed specifications for these eight steps, including case specifications and the unit of analysis for this study.

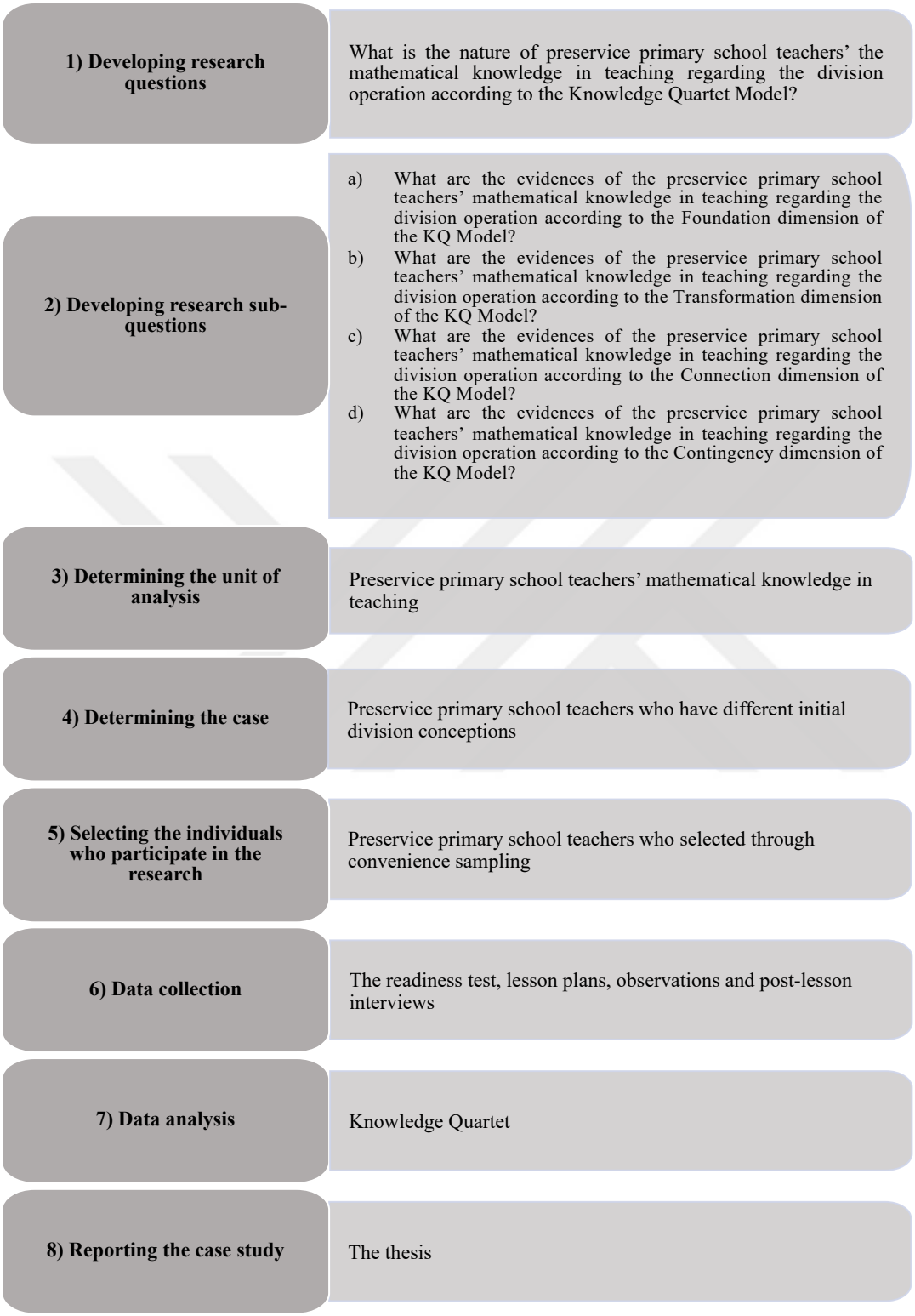


Figure 3.1 Eight Steps to Conduct a Case Study (Yıldırım & Şimsek, 2008)

As depicted in Figure 3.1, the units of analysis for this study were constructed based on preservice primary school teachers' mathematical knowledge in teaching regarding the division operation, utilizing the dimensions of the KQ Model, namely Foundation, Transformation, Connection, and Contingency. Additionally, the cases under investigation in this study were identified as the preservice primary school teachers who have different initial division conceptions. Since it is important to set boundaries for the case (Creswell, 2013; Yin, 2014), in the current study, the cases were bounded by the context of the Teaching Practice Course II at a foundation university in Ankara. It is worth noting that the participants in this study were preservice teachers enrolled in a teacher education program at this institution. A detailed explanation of the context is given in the following section titled Context and Participants.

In addition to these steps, Yin (2014) identifies four different types of design for a case study in terms of the number of cases and the number of units of analysis (holistic or embedded). Figure 3.2 shows the basic types of design for case studies.

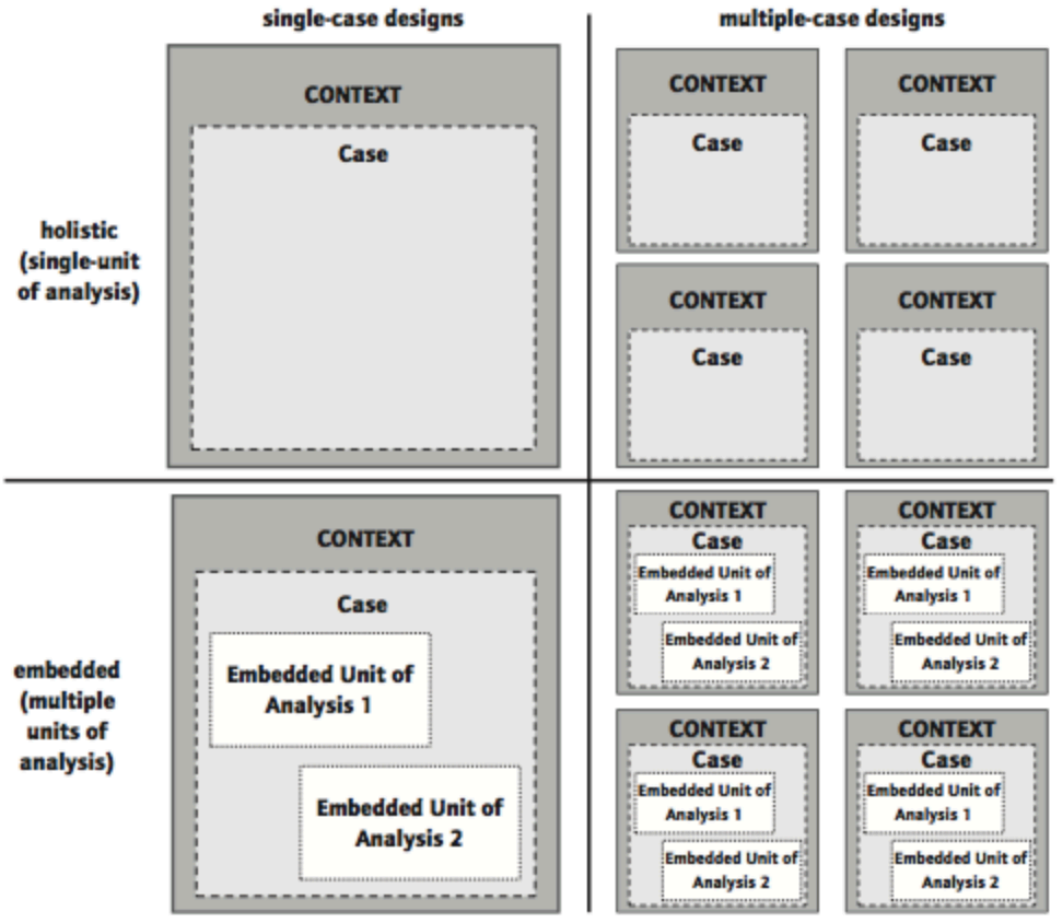


Figure 3.2 Basic Types of Design for a Case Study (Yin, 2014, p.105)

Yin (2014) states that single-case designs could be used if this single case "represents a critical test of existing theory, an extreme or unusual circumstance, a common case, or where the case serves a revelatory or longitudinal purpose" (p. 113). Otherwise, the multiple-case design should be adopted. In the multiple-case design, the cases should present "similar" or "contrasting" results (p.123-124). However, according to Yin (2014), multiple-case design should be chosen if possible since it produces more robust results compared to the single-case design.

In alignment with the types outlined by Yin (2014), this current study sought to achieve an in-depth comprehension of two preservice teachers' mathematical knowledge in teaching two preservice primary school teachers who conducted their instruction in distinct classroom settings. Consequently, the design employed for this

case study falls under the classification of an embedded multiple-case design. Within this design framework, there are four distinct units of analysis, each corresponding to the dimensions of the KQ Model. The customization of the embedded multiple-case design for the current study is given in Figure 3.3.

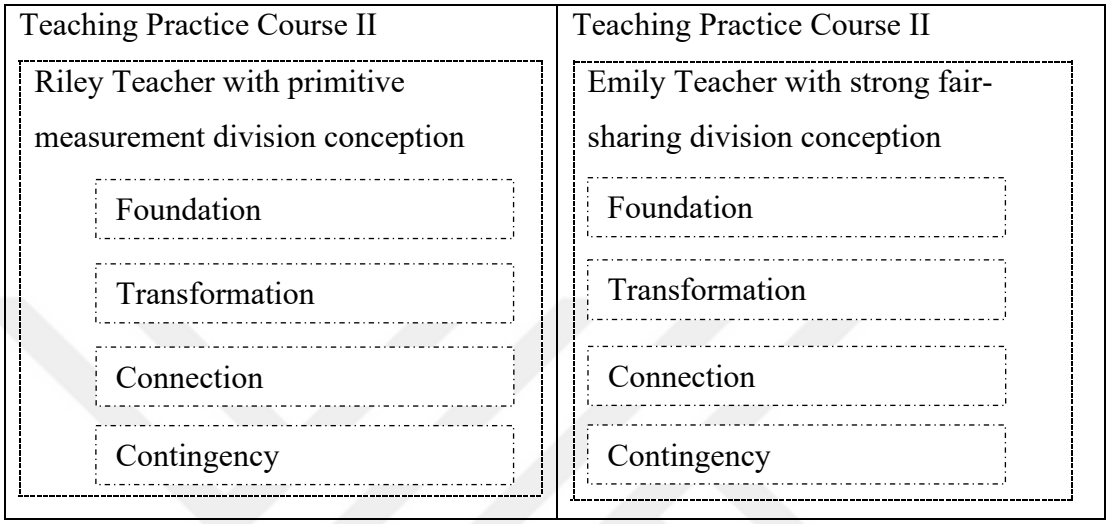


Figure 3.3 Customization of the Embedded Multiple Case Study for the Current Study

In reference to Figure 3.3, for the current study, the cases are the preservice primary school teachers who have different initial division conceptions. The context is the Teaching Practice Course II at a foundation university in Ankara, and the units of the analysis revolve around the examination of preservice primary school teachers' mathematical knowledge in teaching across the dimensions of the KQ Model, including Foundation, Transformation, Connection, and Contingency.

The following section presents the context of the study and the characteristics of the participants.

3.2 Context and Participants

3.2.1 Context

A successful qualitative study relies on the thorough exploration of a specific context, as its primary objective is to attain a deep comprehension of individuals, relationships, or the overall context within a particular setting (Creswell, 2013). Since the current study is a case study that aims to understand preservice teachers' mathematical knowledge in teaching in their natural environment (Yin, 2014), this section gives information about the context of the case.

The current study was conducted with senior students at the Department of Primary Education Program at a foundation university in Ankara within the Spring semester of the academic year of 2021-2022 as these senior students are preservice teachers who would practice their teaching within the scope of the Teaching Practice Course. The 2021-2022 academic year was the first year when lessons were conducted face-to-face again after the COVID-19 pandemic. The study was conducted within the Teaching Practice Course II, with the purpose of scrutinizing the preservice teachers' knowledge through direct observation of their teaching practices in an authentic classroom environment. Before these preservice teachers took this course, they took three other courses related to mathematics education, which are Fundamental Mathematics in Primary School, Mathematics Education I and Mathematics Education II. In these courses, they acquired knowledge in mathematics education and various teaching methodologies applicable to different learning domains. Furthermore, to enroll in the Teaching Practice II course, successful completion of the Teaching Practice I course was a prerequisite. In Teaching Practice I, preservice teachers engaged in classroom observations led by in-service teachers in a range of school settings, including both public and private schools.

The present study aimed to investigate the preservice primary school teachers' mathematical knowledge in teaching regarding division operation. In the following

section, each preservice teacher who volunteered for the study was introduced, and their individual characteristics were provided.

3.2.2 Participants

This study was conducted to investigate preservice primary school teachers' mathematical knowledge in teaching regarding division operation with two different preservice primary school teachers who were studying at a foundation university. According to Stake (1995), in a case study, the goal is not sampling and making generalizations but understanding a case in detail. Consequently, in this study, the selection of participants was guided by a convenience sampling method, aligning with the research design and methodology. Given that the current research seeks to investigate the preservice primary school teachers' mathematical knowledge in teaching in relation to the division operation while also observing their teaching practices, certain criteria were considered crucial for participant selection. Therefore, it was imperative that the participants be senior students, as their preservice teaching experiences typically occur during this period. Additionally, their completion of mathematics method courses was a critical requirement. Given that the participants had successfully fulfilled these prerequisites, their inclusion in the study was deemed appropriate. In addition to that, after the readiness test was applied, it was observed that each preservice primary school teacher had different initial division conceptions. These differences will be explained in the following title.

Furthermore, the case study approach necessitates an in-depth exploration of real-life contexts. In this regard, the preservice teachers conducted their teaching practices in their designated schools as part of their coursework. The distinctive traits of these preservice teachers have been detailed in the upcoming section. To uphold ethical standards and protect privacy, pseudonyms have been employed for the preservice teachers' names.

3.2.2.1 Riley Teacher

Riley Teacher is a preservice primary school teacher enrolled in the primary education department at a foundation university in Ankara. Prior to her teaching implementation, she had successfully completed all requisite mathematics method courses, earning an outstanding grade of AA. Her academic performance reflected a commendable CGPA of 3.74. Notably, she pursued a double major in the Department of Psychological Counseling and Guidance, demonstrating her dedication to a comprehensive education. As a part of her double major program, she had to complete some courses such as Field Practice in Individual Counseling, Developmental Psychology, Psychology of Learning and so on.

Furthermore, within the scope of her teacher education, Riley Teacher took various elective courses offered by the Faculty of Education. These courses encompassed Microteaching, Textbook Analysis, and Disaster and Disaster Management, showcasing her commitment to a well-rounded pedagogical foundation.

Additionally, Riley Teacher stated that she liked mathematics and that she was successful at mathematics. Also, she had a positive attitude toward teaching mathematics and felt relaxed and at ease before the teaching practice. She also mentioned that as a preservice primary school teacher, she preferred to teach mathematics instead of teaching other subjects such as science and Turkish. She believed using material, storytelling and games is important for teaching mathematics, especially in the first two grades.

On top of that, considering the teaching practice courses in the teacher education programs, Riley Teacher gained valuable experience as a preservice teacher for one semester. The school where she implemented her teaching was a public school, and the class she taught had 24 students. The specific mathematical topic under investigation in the current study, the division operation, is typically introduced to second-grade students in primary school, and Riley Teacher conducted her teaching in a second-grade classroom as well. Riley Teacher stated that she taught division

operation with remainder to the third graders in a course within the teaching practice for a 40-minute lesson.

Lastly, considering the readiness test, it was observed that even though Riley Teacher focused on fair-sharing while explaining division operation, in some cases, she considered measurement meaning. For example, when she was asked to write and solve a question with a different meaning, she wrote a question with a measurement context, and she labeled that meaning as "subtraction". However, her definition of the division operation did not include this meaning; therefore, Riley Teacher could be considered a preservice teacher with a primitive measurement division conception.

3.2.2.2 Emily Teacher

Emily Teacher is a preservice primary school teacher in the primary education department at a foundation university in Ankara. Before she implemented teaching, she had completed all the required mathematics method courses with a grade of AA, and her CGPA was 3.76. She also pursued a double major in the Department of Early Childhood Education. In addition, within the scope of teacher education, she took several elective courses from the Faculty of Education, including Project Design in Education and Textbook Analysis.

Emily Teacher stated that she loved mathematics. As a preservice teacher, she had no negative attitudes toward teaching mathematics, and courses about teaching mathematics were her favorite courses in her teacher education. She believed that mathematics is the basis of every other subject such as science and Turkish, and according to her, it is strictly related to daily life.

Similar to Riley Teacher, Emily Teacher also had experience as a preservice teacher for one semester. The school where she implemented her teaching was a public school, and the class she taught had 26 students. She stated that she had never taught division operation in a course within the teaching practice or in an educational

institution before. Unlike Riley Teacher, she did not have any worries about the topic and the class she would teach.

Lastly, in the readiness test, unlike Riley Teacher, it was evident that Emily Teacher had a strong focus on fair-sharing meaning, and in any part of the readiness test. She did not mention measurement meaning or a construct that could be related to measurement. Hence, Emily Teacher was a preservice teacher with a strong fair-sharing division conception. The following section presents how the data were collected from the participants.

3.3 Data Collection Procedure

3.3.1 Data Collection Tools

A case study is designed to achieve a deeper understanding and thorough explanation of a specific situation. Consequently, it encourages the utilization of multiple data collection tools, enabling researchers to gather a comprehensive wealth of information (Merriam, 1998; Yin, 2014; Creswell, 2007). These data collection tools include interviews, observations, documents, etc. (Merriam, 1998). In the current study, the purpose was to investigate preservice primary school teachers' mathematical knowledge in teaching about division operation. Thus, multiple data collection tools involving readiness tests, lesson plans, lesson observations and post-lesson interviews were utilized. These tools were aligned with the KQ Model and its dimensions, in accordance with the research questions and sub-questions. The following table presents an overview of the data collection tools in relation to the corresponding research questions, sub-questions, and dimensions.

Table 3.1 Research Question, Sub-Research Questions, Related Dimension and Related Data Collection Tool

Research Question	**Research Sub-questions**	**Related Dimension of KQ Model**	**Data Collection Tool**
What is the nature of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Knowledge Quartet Model?	What are the evidences of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Foundation dimension of the KQ Model?	Foundation	Readiness Test
	What are the evidences of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Transformation dimension of the KQ Model?	Transformation	Lesson Plans, Observation and Post-lesson Interview
	What are the evidences of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Connection dimension of the KQ Model?	Connection	Lesson Plans, Observation and Post-lesson Interview
	What are the evidences of the preservice primary school teachers' mathematical knowledge in teaching regarding the division operation according to the Contingency dimension of the KQ Model?	Contingency	Observations and Post-Lesson Interview

All the data collection tools were explained in detail below.

3.3.1.1 Readiness Test

Readiness means “the state of being ready or prepared” (Cambridge Dictionary, n.d.). As Rowland states, the Foundation Dimension pertains to the theoretical background and knowledge acquired by preservice teachers during their teacher education (2005). Since one of the aims of this study is to investigate the nature of preservice primary school teachers’ mathematical knowledge in teaching concerning division operation, a readiness test was administered to assess preservice teachers' readiness and gauge their theoretical background and knowledge regarding the division operation. This test was specifically designed to align with the Foundation dimension.

The readiness test utilized in the current study comprises two sections, containing total nine questions. The initial section encompasses five questions that pertain to the division operation and its underlying concepts. The table below illustrates these five questions from Section 1 of the readiness test.

Table 3.2 Questions in Section 1

Questions
1) What is division operation? How can you explain it?
2) What is the meaning(s) of the division operation? How can you explain it?
3) Write a problem that requires division operation and solve it.
4) Which meaning did you use in the problem you have written?
5) Write a problem that requires division operation with a meaning that you did not use in the previous problem and solve it. (Please state which meaning you used.)

The first question aimed to elicit how preservice teachers conceptualized the division operation, while the second question assessed their awareness of various interpretations of division operation and their ability to articulate them. In the third question, they were asked to write a division operation problem, and subsequently,

in the following question, they were asked to identify the interpretation they had applied. Finally, the fifth question required preservice teachers to create another division operation problem utilizing a different interpretation and specify the interpretation they used.

Following the administration of Section 1 of the readiness test, Section 2, comprising four questions, was introduced. Section 2 centered around students' solutions and delved into the understanding of the division operation through three sub-questions. Table 3.3 provides an overview of the four questions in Section 2.

Table 3.3 Questions in Section 2

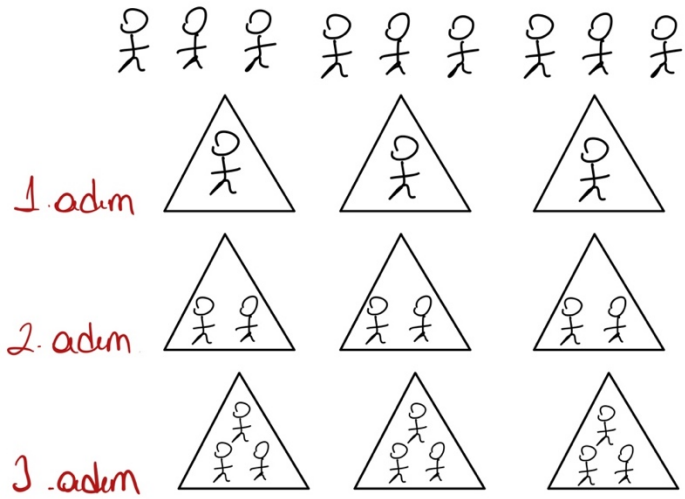
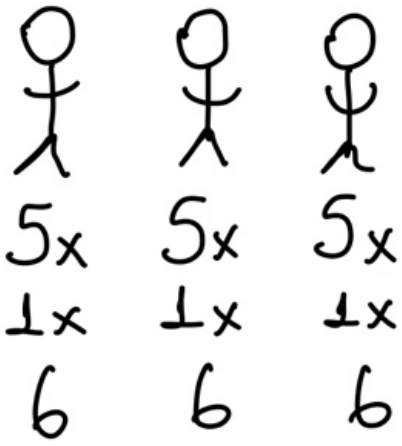
Questions
1) *Nine friends, who go camping, will spend the night in three tents and there will be an equal number of people in each tent. How many are there in one tent?* Gökçen solved the problem as shown below, and the result she found was 3.  • Do you think Gökçen's solution is correct? Why? • Explain how Gökçen solved the question. • Does this solution match with the meanings you previously listed. Explain it.

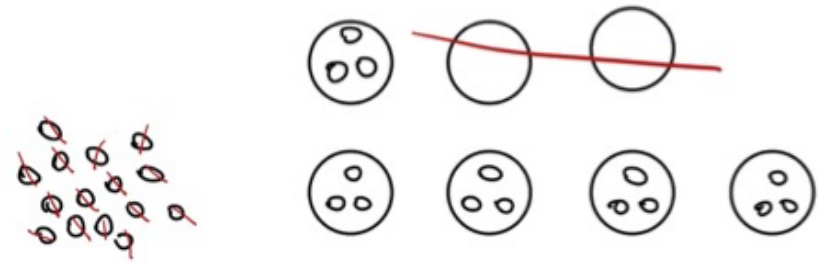
Table 3.3 (Continued)

2) *Three people will share 18 balloons and each person will get an equal number of balloons. How many balloons will each person get*?
Ada solved the problem as shown below, and the result she found was 6.



- Do you think Ada's solution is correct? Why?
- Explain how Ada solved the question.
- Does this solution match with the meanings you previously listed. Explain it.

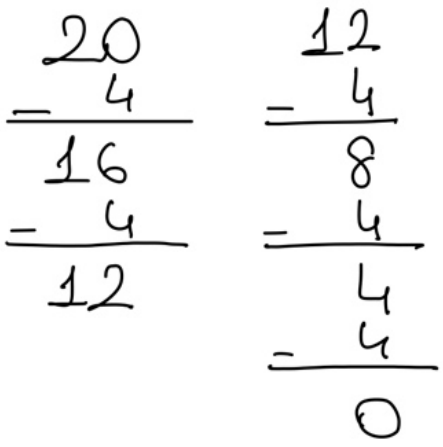
3) *Sevim will place 15 cookies on plates so that each plate contains 3 cookies. How many plates does Sevim need?*
Elif solved the question as shown below, and the result she found was 5.



- Do you think Elif's solution is correct? Why?
- Explain how Elif solved the question.
- Does this solution match with the meanings you previously listed. Explain it.

Table 3.3 (Continued)

4) *A group of friends will share 20 balls and each friend will get four balls. How many friends are there in this group?*
Defne solved the question as follows, and the result she found was 5.



- Do you think Defne's solution is correct? Why?
- Explain how Defne solved the question.
- Does this solution match with the meanings you previously listed. Explain it.

Every question in Section 2 involves students' solutions pertaining to various interpretations of the division operation. Preservice teachers were asked to determine the correctness of these solutions, elucidating the students' thought processes and identifying the meaning of the division operation problem presented. Notably, the first two questions centered around the concept of fair-sharing, while the last two questions revolved around measurement interpretation.

After the administration of the readiness test, the preservice teachers were asked to prepare lesson plans, and their implementations of the lesson plans were observed. These lesson plans and the corresponding observations constitute two essential data collection tools, and their detailed descriptions are provided in the subsequent section.

3.3.1.2 Lesson Plans and Observations

According to Wong and Wong (2009), "Effective teachers have lesson plans and procedures that produce student learning" (p. 18). A lesson plan is a comprehensive instructional guide created by teachers for themselves, outlining the learning objectives for students and detailing how the teaching process will unfold during the lesson (Milkova, 2012).

In the current study, each preservice teacher was asked to plan two lessons for their teaching implementations. Each lesson plan focused on one meaning of the division operation, such as fair-sharing and measurement. The lesson plans of preservice teachers were essential to understand their subject matter knowledge and pedagogical content knowledge regarding division operation with reference to the Transformation and Connection dimensions of the KQ Model, where the Transformation dimension addresses how preservice teachers transform their theoretical background and knowledge into lesson, while the Connection dimension delves into how this transformation shapes a cohesive and meaningful instructional experience (Rowland, 2003).

In the current study, the preservice teachers were provided with a template for lesson planning (see Appendix A). However, they were free to make any changes to it. The template is a frequently used lesson plan template in Teaching Practice courses and was adapted by the researcher for the preservice teachers in the study.

Another data collection tool was observation, and it is one of the critical data collection tools in a qualitative study (Murphy & Dingwall, 2007). Observing individuals in their natural behaviors helps mitigate potential issues encountered with self-reported tools. Moreover, it offers an opportunity to examine the innate behaviors of individuals, including actions they may not consciously recognize themselves (Mays & Pope, 1995; Furlong, 2010). Observations, a common data collection tool in case studies as noted by Merriam (1998), offer researchers the opportunity to study individuals and their behaviors within their genuine

surroundings. In the context of the present study, which seeks to assess preservice teachers' comprehension of the division operation, these teachers executed their lesson plans in actual classroom settings, facilitating observations during their teaching processes within the authentic classroom environment.

The significance of observations in this study is two-fold. Firstly, it allows for an evaluation of the preservice teachers' mathematical knowledge in teaching with regard to the Transformation and Connection dimensions, shedding light on how they translate their theoretical understanding into practical lessons. Secondly, it enables the identification of contingent moments and deviations from the planned agenda, thus facilitating the examination of their knowledge within the framework of the Contingency dimension.

Furthermore, in qualitative studies, there are different degrees of observer participation, such as participant observation and non-participant observation (Fraenkel et al., 2012). In participant observation, the observer is active during the observation, whereas in non-participant observation, the observer is passive and only watches the events happening during the observation process.

In the context of the current study's case study design, it was imperative to observe preservice primary school teachers within their real classroom environments; therefore, the researcher of the current study remained a non-participant observer in the teaching process, sat in the back of the classroom, took notes, and compared the executed lesson plans with the unfolding events in the classroom, paying particular attention to contingent moments.

3.3.1.3 Interviews

One of the critical data collection tools in qualitative studies, especially in case studies, is interviews (Merriam, 1998; Yin, 2014). There are three types of interviews as structured, semi-structured, and unstructured (Merriam, 1998). In structured interviews, the questions, their order and wording are pre-determined, whereas semi-

structured interviews are more flexible in terms of questions, their order and wording (Fraenkel et al., 2012). This study used a semi-structured interview to investigate preservice primary school teachers' mathematical knowledge in teaching.

Following the completion of their teaching sessions, the preservice primary school teachers participated in semi-structured interviews. These interviews were thoughtfully designed to align with the Contingency Dimension of the KQ Model, aiming to probe into the preservice primary school teachers' capacity to navigate decisions and respond to unexpected scenarios and ideas. While certain core questions were pre-established, the semi-structured nature of the interviews allowed the researcher to adapt and introduce spontaneous questions as warranted by the specific circumstances. It is worth noting that the questions were posed separately for each day the preservice teachers conducted their teaching. Here are some illustrative questions from the post-lesson interviews in Table 3.4.

Table 3.4 Example Interview Questions from the Post-Lesson Interview

Example Questions
How was your teaching implementation? How can you evaluate it?
Do you think you reached your goals and objectives?
Why did you prefer to begin your lesson with this example?
What criteria guided your selection of the numbers used in your examples?
Why did you prefer to use manipulatives?
Why did you prefer to teach the fair-sharing meaning first?
Were there any contingent moments such as deviation from your lesson plan?
What was your reaction to these contingent moments?
If you had the opportunity to reteach this lesson, would you change anything?

3.3.2 Data Collection Procedure

A case study requires collecting the data in the natural settings of the cases (Merriam, 1998; Yin, 2014; Creswell, 2007). Hence, the data were collected in the school where the study participants, the preservice primary school teachers, carried out their teaching practice. The data of the study were collected in May of the 2021-2022 academic year for three weeks. The process of how the data were collected was demonstrated in the below figure.

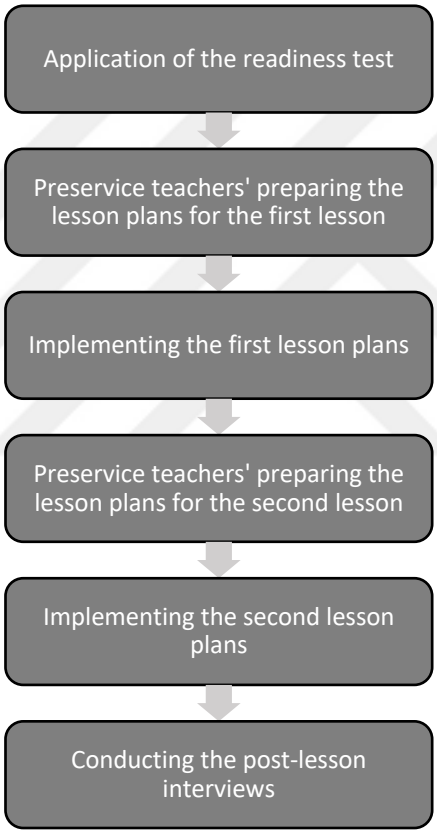


Figure 3.4 Data Collection Procedure

In order to investigate the preservice primary school teachers' mathematical knowledge in teaching about division operation, a readiness test was prepared to gauge their understanding of the topic. After the readiness test, the preservice teachers were given one week to plan their lessons. The following week, the preservice primary school teachers implemented their teaching about division

operations according to their lesson plans. After their teaching implementation, post-lesson interviews were conducted, and the data collection procedure was finalized. The following section gives details about how the collected data were analyzed.

3.4 Data Analysis

According to Merriam (1998), data analysis is "the process of making sense out of the data and making sense out of data involves consolidating, reducing, and interpreting what people have said and what the researcher has seen and read – it is the process of making meaning" (p. 178). In a qualitative study, Straus and Corbin (1990) identify two analysis types: descriptive analysis and content analysis. In the current study, which aims to investigate the knowledge of preservice primary school teachers about division operation, descriptive analysis was used. In the descriptive analysis, data are classified according to predetermined themes, and the findings related to data are summarized and interpreted by the researcher. In the current study, the KQ Model was adopted as the theoretical and analytical framework, and its dimension and codes were used to analyze the data. Before the analysis, both interviews conducted with the preservice primary school teachers were transcribed by the researcher. Then, the data were analyzed according to the KQ Model. Below, the dimensions of the KQ Model and their related codes are given.

Table 3.5 The Knowledge Quartet Model's Dimensions and Codes

Dimension	Codes
Foundation	Awareness of Purpose
	Identifying Pupil Errors
	Overt Display of Subject Knowledge
	Theoretical Underpinning of Pedagogy
	Use of Mathematical Terminology
	Adherence to Textbook
	Concentration on Procedures

Table 3.5 (Continued)

Transformation	Choice of Examples
	Teacher Demonstration
	Choice of Representation
	Use of Instructional Materials
Connection	Making Connections Between Procedures
	Making Connections Between Concepts
	Anticipation of Complexity
	Decisions About Sequencing
	Recognition of Conceptual Appropriateness
Contingency	Responding to Students' Ideas
	Deviation from Agenda
	Teacher Insight
	Responding to (Un)Availability of Tools and Resources

3.5 Trustworthiness

In research, validity signifies that the inferences drawn by the researcher are meaningful and pertinent, whereas reliability underscores that these inferences remain consistent regardless of when and where they are applied (Fraenkel et al., 2012). However, in contrast to quantitative research, the concepts of validity and reliability exhibit variations in a qualitative research study. According to Seale (1999), "trustworthiness of a research report lies at the heart of issues conventionally discussed as validity and reliability" (p. 266). Specifically, trustworthiness ensures that the deduction made by the researcher has credibility. According to Lincoln and Guba (1985), trustworthiness in a qualitative study includes credibility, transferability, dependability, and confirmability. Shortly, credibility refers to the truthfulness of the findings. In other words, it is the "quality of data and the soundness of the reasoning that has led to the conclusion" (Bakker & Van Eerde,

2015, p. 444). One of the ways to ensure credibility is to have triangulation (Stahl & King, 2020). Triangulation is the usage of multiple datasets, data sources, investigators, and theories to understand the phenomena being researched (Patton, 1999). Patton (1999) defines four types of triangulation: method triangulation, investigator triangulation, theory triangulation, and data source triangulation.

Method triangulation refers to using multiple methods to collect the data, such as interviews or observations. Theory triangulation indicates using multiple theories for the data analysis. Data source triangulation, on the other hand, refers to collecting data from different individuals, locations or times (Denzin, 1978). In the current study, Patton's recommended approach of triangulation was employed to enhance credibility. Methodologically, triangulation was achieved through diverse data collection tools, including readiness tests, lesson plans created by preservice teachers, and observations and interviews conducted with each preservice teacher. This multifaceted approach contributed to the overall trustworthiness and credibility of the study.

The second construct that provides trustworthiness is transferability, which refers to the extent to which the study's findings can be applied to other contexts. Essentially, it parallels the concept of external validity in quantitative research (Lincoln & Guba, 1985), although it is worth noting that making broad generalizations with a small sample size, as observed by Merriam (1998), can be challenging. To ensure the transferability of findings to different contexts, explaining the research process from beginning to end, including details about participants, data collection tools, data collection sources and data analysis, is often sufficient (Merriam, 1998; Mishra, 2021). In the current study, the thorough explanation of the entire data collection process in the title of the data collection procedure, along with a clear description of the data collection tools used, serves to establish the transferability of the study.

Thirdly, dependability indicates consistency within the findings and in a quantitative study, it corresponds to reliability (Merriam, 1998; Shenton, 2004). In order to ensure dependability, as mentioned before, the data were coded by two people, including

the researcher until they reached a consensus. Moreover, as noted by Lincoln and Guba (1985), dependability and credibility are closely interconnected, with credibility ensuring dependability. Consequently, it can be inferred that the current study, by establishing credibility, inherently contributes to the dependability required for trustworthiness.

The final concept of trustworthiness is confirmability, which refers to objectivity in the study, specifically in the data collection procedure. A researcher should be neutral and not be affected by biases (Shenton, 2004). To ensure confirmability, the strategy of data triangulation can be effectively employed (Lincoln & Guba, 1985). As previously highlighted, the study has implemented data triangulation, thus bolstering the confirmability of the research. Furthermore, to uphold objectivity, the role of the researchers is meticulously elucidated in the subsequent section.

In summary, it can be affirmed that in the context of the current study, all essential components — credibility, transferability, dependability, and confirmability — necessary for establishing trustworthiness have been satisfactorily addressed.

3.6 Researcher's Role

In a qualitative study, the researcher is one of the important elements of the data collection procedure and is also critical in ensuring the credibility of the study (Creswell, 2012). In the current study, the researcher took an active role in designing the study, preparing data collection tools, collecting the data and analyzing the collected data under the supervisor's guidance. While designing the study, the researcher worked closely with her supervisor and identified the gaps and deficiencies in the literature regarding division operation. Then, the problem and research questions were generated, and the design of the study was constructed. Thereafter, the researcher prepared the two data collection tools, which are the readiness test and the post-lesson interview, and she asked two experts in mathematics education to evaluate the readiness test and interview questions and

whether these tools are related to KQ Model precisely. After preparing these data collection tools, the researcher submitted her plan to conduct this study to the Human Subject Ethics Committee of Middle East Technical University and obtained the necessary permissions to conduct the current study (see Appendix B). After obtaining permission, the researcher met the assistant professor, who was the teacher of a teaching practice course, informed her about the study and asked permission to explain and invite her seven senior students who were preservice primary school teachers to participate in the current study. Later, the researcher explained the current study, its aims and the process of the study to seven preservice teachers. Among these teachers, two were willing to participate in the current study.

The researcher conducted a private meeting with the volunteer preservice teachers and ensured the preservice teachers that their participation and events that occurred in the process of the study would not have any effect on their lesson grades. Following this meeting, the preservice teachers took the readiness test, administered by the researcher in two consecutive sections. Then, the researcher gave preservice teachers one week to plan a lesson, and she provided the objectives the preservice teachers should use in their lesson plans. Apart from providing objectives, the researcher did not intervene in the lesson plans the preservice teachers planned. She did not intervene in the preservice teachers' choice of the sequence in which they would teach the different meanings of division. While the preservice teachers were allocated one week for their lesson planning, the researcher informed both the school administration and the teachers at the public school where the preservice teachers would conduct their teaching. It's important to note that the public school had already been assigned to the preservice teachers by the university administration as part of their teaching practice course. Consequently, the necessary permissions for the researcher to observe the teaching implementations had already been secured from the Ministry of National Education (MoNE) by the university administration. Subsequently, the preservice teachers executed their lesson plans over the course of four teaching hours, and the researcher observed these teaching implementations as a non-participant observer, took notes regarding the events and contingent moments

that occurred in the lesson, and did not intervene before, during, after the lesson or in the breaks between two lessons.

Immediately after the teaching implementations, the researcher conducted the post-lesson interviews with the preservice teachers individually in private meetings and audio-recorded the sessions by obtaining permission from the preservice teachers to ensure the objectivity of the data. During the post-lesson interview, which followed a semi-structured format, the researcher exercised caution when introducing new questions, ensuring that they did not steer or influence the preservice teachers' responses. Furthermore, the researcher explicitly conveyed to the preservice teachers that there were no right or wrong answers, fostering an atmosphere of ease and open communication during the interview process.

Following the post-lesson interviews, which marked the culmination of the phase where participants played an active role, the researcher embarked on the transcription of the audio recordings from these interviews. Subsequently, the researcher diligently organized the collected data derived from various sources, including the readiness tests, lesson plans, observations, and post-lesson interviews. The researcher began to code the data by using the Knowledge Quartet Model by Rowland, its dimensions and their related codes. To ensure the credibility of the study, an independent colleague of the researcher, who also worked in the field of mathematics education, coded the data collected by different collection tools. This approach aligns with investigator triangulation, further enhancing the rigor of the research. After completing the coding of the data, the researcher analyzed the data and reported the results. The fact that the researcher graduated from the undergraduate program of elementary mathematics education, took part in teaching in a classroom setting as a part of the undergraduate study, was continuing with her graduate study in the same field and working as a research assistant at a university in Ankara provided convenience for the researcher to analyze the data and make sense of the results of the study.

In the next chapter, the findings of the study are presented.

CHAPTER 4

RESULTS

The study aims to examine the pre-service primary school teachers' knowledge of the division operation through Knowledge Quartet Model. Findings are presented with separate headings for each preservice teacher. Each title consists of the examination of their knowledge in accordance with Knowledge Quartet and its four dimensions.

4.1 Riley Teacher

The number of observations in the codes under the Knowledge Quartet Model dimension in Riley Teacher's lessons is given in the table below.

Table 4.1 Emerging Codes of Knowledge Quartet Model in Riley Teacher's Lesson

Dimension	Codes	Number of Observation
Foundation	Awareness of Purpose	11
	Identifying Pupil Errors	0
	Overt Display of Subject Knowledge	7
	Theoretical Underpinning of Pedagogy	11
	Use of Mathematical Terminology	11
	Adherence to Textbook	0
	Concentration on Procedures	4
Transformation	Choice of Examples	9
	Teacher Demonstration	9
	Choice of Representation	2
	Use of Instructional Materials	1

Table 4.1 (Continued)

Connection	Making Connections Between Procedures	3
	Making Connections Between Concepts	3
	Anticipation of Complexity	3
	Decisions About Sequencing	6
	Recognition of Conceptual Appropriateness	11
Contingency	Responding to Students' Ideas	3
	Deviation from Agenda	12
	Teacher Insight	1
	Responding to (Un)Availability of Tools and Resources	1

The above table indicates that in the whole process (before, during and after teaching), there is evidence of all the codes except the code of *adherence to textbook* and *identifying pupil errors*. Throughout the process, Riley Teacher never looked to or made reference to the textbook or recognize the errors or mistakes that students could make. In the whole process, the most observed code was the *deviation from agenda,* which was observed 12 times, followed by the codes of *awareness of purpose, theoretical underpinning of pedagogy, use of mathematical terminology* and *recognition of conceptual appropriateness,* which were observed 11 times. The least observed codes *use of instructional materials, teacher insight* and *responding to (un)availability of tools and resources*, which were observed once.

In the following section, Riley Teacher's knowledge was investigated through the codes on the basis of four dimensions: Foundation, Transformation, Connection and Contingency.

4.1.1 Foundation Dimension

As mentioned, Foundation Dimension consists of pre-service teachers' whole knowledge, beliefs, and understanding they obtain in their undergraduate study (Rowland, 2007). It covers the codes of *awareness of purpose, identifying pupil errors, overt display of subject knowledge, theoretical underpinning of pedagogy, use of mathematical terminology, adherence to textbook* and *concentration on procedures.*

In terms of *overt display of subject knowledge,* it was seen that, before conducting the lesson, Riley Teacher's definition of division operation was ill-explained while defining division operation. In her definitions, she mostly focused on partitive meaning rather than quotitive meaning. In the following examples, these definitions for division operation can be seen.

> *"To break an amount, we define as whole into pieces. At the primary level, it separates an amount into smaller and equal pieces."* (Readiness test).

Also, she added

> *"When I think of the division operation, the first thing that comes to my mind is separating into equal pieces. A whole is given, and I divide it into equal parts by grouping."* (Readiness test)

Additionally, it was observed that although Riley Teacher did not include measurement meaning in her definitions when she was asked to write and solve a question in the readiness test, she was able to write and solve questions with measurement context. Yet, she did not integrate this meaning in her definitions of division operations as another meaning of the division operation.

However, considering her *overt display of subject knowledge* after conducting the lesson, there was a difference regarding division operation. Riley Teacher approached the subject more critically and explained division operation in a more conceptual way rather than the superficial explanation she did before. Before, she

overlooked the quotitive meaning; now, when discussing division, she emphasizes this meaning. Additionally, she explained some terms and why they are essential for division operation. In the following example, an excerpt of her explanation of division operation can be seen.

> *Interviewer: Why is the word 'group' important for division operation?*
>
> *Riley Teacher: Identifying that they are in different places is important. Completing a group means completing an object that will be subtracted from the division operation. In fact, they found the repeated subtraction by themselves. If there is a group of 5, they subtracted five and tried to get something. They tried to get something, pack it in, and remove it. The word 'group' is important to indicate the operation's completion.* (Post-lesson interview)

In terms of *awareness of purpose,* as a pre-service primary school teacher, it could be said that Riley Teacher always recognized objectives, goals, and purposes. While describing division, she emphasized the teaching level she would teach and was aware of what primary school entailed regarding division operation, although it was lacking and ill-defined. The following quotation could exemplify this situation:

> *"...In the primary level, it (division operation) is to separate an amount into smaller and equal pieces"*. (Readiness test)

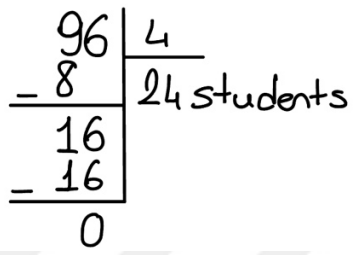
Likewise, it is clear that Riley Teacher planned her lesson while being aware of the objectives and aims of the lesson. She planned examples to give in case of no answer to a question, or she asked questions in accordance with her plans and aims. For example, she deliberately asked questions as a transition to the lesson's objective or as a reminder and connected the previous lesson with the ongoing one. Followingly, there are two examples to illustrate this situation.

> *"What if we knew the amount of money Ahmet paid for the books and the cost of one book? Could we find the number of the book he bought?"* (Lesson plan)

"Why do we make this grouping? Where do we use it? Can you use daily life examples for it?" (Lesson plan)

Considering *concentration on procedure,* it could be observed that Riley Teacher depended on procedures and algorithms before or during the lesson implementation. For example, in the readiness test, when she was asked a question, she only used and focused on the standard algorithm rather than explaining it or using other methods such as drawing, sharing equally or any other representations. To illustrate the situation, the following was Riley Teacher's answer to a question she wrote.

"In a school, there are ninety-six 3rd-grade students. If there are four sections of 3rd grade, how many students will be in each section equally?"



(Readiness Test)

Regarding *use of mathematical terminology,* Riley Teacher' usage was limited to only a few specific terms. Riley Teacher put greater emphasis on these mathematical terms and believed this emphasis was important to teach division operation; however, her usage of mathematical terms was few in number. For example, Riley Teacher constantly repeated the word 'group,' and she believed that it was important to describe how the operation worked. Her explanation of this insistence can be seen below.

Identifying that they are in different places is important. Completing a group means completing an object that will be subtracted from the division operation. ... The word 'group' is important to indicate the operation's completion. (Post-lesson interview)

Apart from the word group, she repeatedly used terms such as 'sharing' and 'division.' Nevertheless, as stated previously, her use of these terms included these

particular terms and did not use alternative words to describe the operation and procedures.

Lastly, in terms of *theoretical underpinning of pedagogy,* Riley Teacher had opinions of which circumstances would be better for students to learn. She tried to plan the lessons from her own perspective about better circumstances for students. The below example could be evidence of her knowledge. In her lesson plan, she planned to repeat a particular question for her students to better understand the concept of division according to her students' levels.

> *"What if we knew the amount of money Ahmet paid for the books and the cost of one book? Could we find the number of the book he bought?"* (Lesson plan)

Another example that showed her knowledge was about worksheets. Riley Teacher planned to distribute worksheets after reminding them of the first lesson. She explained her reasoning below, and it demonstrated Riley Teacher's view of how to teach mathematics best.

> *"Teacher gives time to students to complete the worksheet. If students do it slower, the teacher can help by drawing on the board".* (Lesson plan)

Additionally, during the lesson implementation, Riley Teacher preferred using manipulatives, and she said that she believed learning by touch and by sense was much more beneficial. As a result, it can be said that Riley Teacher had some perspective on how students learned, and she acted accordingly.

Consequently, regarding Foundation Dimension, Riley Teacher's knowledge before and after lesson implementation provides evidence for the following codes: *awareness of purpose, overt display of subject knowledge, theoretical underpinning of pedagogy, use of mathematical terminology,* and *concentration on procedures.* Considering the *awareness of purpose,* it was seen that Riley Teacher was aware of the purpose and objectives. Additionally, about *theoretical underpinning of pedagogy,* her teaching and her lesson plans provided evidence of strong examples

due to her attending to planning and delivery of the lesson. Regarding *overt display of subject knowledge,* before lesson implementation, she could not fully describe what division operation meant and which meanings it had; however, after lesson implementation, she was able to explain the division operation and its meanings well. On the other hand, in regard to *concentration on procedure,* it was observed that Riley Teacher rested on standard algorithms and procedures without considering alternative approaches. Lastly, considering *use of mathematical terminology*, her usage of terms could be evidence for weak examples since it lacked diversity.

After analyzing Riley Teacher's Foundation Knowledge, which dealt with what the pre-service teacher brought from her undergraduate study or experiences, the following section was about how Riley Teacher transferred her knowledge in her teaching implementation.

4.1.2 Transformation Dimension

In the KQ Model, the Transformation Dimension refers to pedagogical ways of teachers transforming their knowledge for their students. This dimension contains four codes: choice *of examples, teacher demonstration*, *choice of representation* and *use of instructional materials* (Rowland, 2007; Rowland, 2013).

In terms of *choice of examples*, although Riley Teacher selected her examples and questions deliberately per her aims and there was no strong justification for why these examples and questions were appropriate. The following two examples demonstrated two different situations. In the first example, she explained why she began her second lesson implementation with a multiplication question. It was clear that she chose her examples deliberately to connect and introduce division according to her instructional purposes. However, in the second example, which was an excerpt from the interview after the lesson, Riley Teacher could not explain why she chose some specific examples, which was why she preferred to remind the measurement meaning.

"They completed multiplication in the previous week. I just wanted to refer a bit to multiplication. Even though they did not know that we find factors with the help of division operation, I wanted them to make sense of factors resulting from the division operation. That is why multiplication is the inverse of division and vice versa. I started the lesson with an example to get the students to make sense of this." (Post-lesson interview)

Interviewer: Why did you prefer to remind the grouping (measurement) for the second day? Why did you do that? Why did you plan in this way?

Riley Teacher: Because I did not want to say, "Let us divide these numbers." Division and grouping have the same meaning, as I explained in the previous objective. (A long silence). I do not know; it had to be so.

Interviewer: So, was it necessary to explain fair sharing (partition) meaning?

Riley Teacher: Yes, but I do not know how to explain that. (Post-lesson interview)

Additionally, regarding this code, in some instances, Riley generated her examples randomly according to students' ideas, and in one case, getting ideas from students to generate examples did not serve the purpose of the lesson and had the possibility to encourage misconceptions. The example for this instance will be mentioned in the Contingency dimension.

Considering *teacher demonstration,* Riley Teacher mainly focused on only two demonstrations and did not use any other way to introduce and teach division. For example, while introducing division, she preferred to use cubes and illustrations to introduce and teach division; she depended on only these two demonstrations and did not use any other way to teach. Alongside cubes, in the second lesson plan, Riley Teacher sharing carrots with rabbits one by one also illustrated her preference to explain the meaning of equal sharing meaning. However, besides cubes and illustrations, Riley Teacher did not prefer any other demonstration to teach division.

Also, followingly, regarding *choice of representation,* Riley Teacher's options for different representations were limited. In her first lesson plan, there was a part where Riley Teacher would group students in pairs. However, instead of repeating two in her plan, she preferred to use rhythmic counting, such as *"Now, I separate you into groups. Listen to me carefully and count with me. 2,4,6…"* Her reasoning for this choice was, *"In this way, they both make rhythmic counting by two, and the teacher will have separated the groups."* Also, in her second lesson plan, Riley Teacher used another problem with sharing six loaves of bread with three friends. Nevertheless, in this question, she chose to use repeated subtraction by three instead of sharing one by one. Apart from these, she did not use different representations.

Regarding the last code, *use of instructional materials,* it can be seen that she preferred to use some materials, such as unit cubes and activity sheets, in her lesson plans and lesson implementations. However, considering the subject, her choice of materials was relatively conventional and few.

Consequently, considering Transformation Dimension, Riley Teacher's knowledge before and after lesson implementation provides evidence for the following codes: *choice of examples, teacher demonstration, choice of representation and use of instructional materials.* As it is seen, this dimension is mostly about how teachers transfer their knowledge to their students through different ways, such as examples, demonstrations, materials and representations. Although Riley Teacher used different methods, her methods were limited and few in number; Her invariant and few methods, materials and representations, and random selection of examples could provide evidence of weak examples considering all the codes of Transformation dimension, such as *choice of examples, teacher demonstration, choice of representation and use of instructional materials.*

After the Transformation dimension of her knowledge, the following section was about Riley Teacher's decisions and choices to make her lessons understandable.

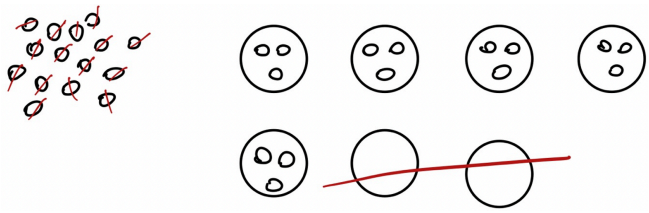
4.1.3 Connection Dimension

The third dimension of the KQ Model is the Connection Dimension, which is the knowledge of making teaching more coherent through specific choices and decisions (Rowland, 2013). The contributory codes for this dimension are *making connections between procedures, making connections between concepts, anticipation of complexity, decisions about sequencing,* and *recognition of conceptual appropriateness.*

Considering *anticipation of complexity,* in the whole process, it was observed that Riley Teacher considered possible limitations and complications for students to learn division. Below there was an example of how Riley Teacher could recognize possible mistakes through a student solution; even if Riley Teacher was not asked to determine a possible mistake, she kept it in her mind.

Sevim will place 15 cookies on plates, where each plate will have 3 cookies. How many plates does Sevim need?

Elif solved this problem and found 5 as a result.



a) Do you think Elif's solution is correct?

(Readiness Test)

Riley Teacher answered this question as, *"Yes, Elif distributed the cookies one by one. However, I would like to know why she deleted the other plates. Drawing seven plates would lead her to mistake."*

This answer is the evidence for the code of *anticipation of complexity* since she was aware of possible difficulties for students to understand. Although Riley Teacher was

successful at recognizing possible complexities, sometimes, when these complexities arose, she could not manage these kinds of instances effectively. The below example showed Riley Teacher explanation of an instance where she wanted to change an indivisible number that a student chose for a division problem with another divisible number.

> *"They did not know division with remainders. Actually, they did not know the division at all; they just knew grouping."* (Post-lesson interview)

Though, during the teaching implementation, Riley Teacher could not handle the situation effectively, this case demonstrated that she was aware that using a prime number would lead students to confusion. In brief, as a pre-service teacher, Riley Teacher could recognize and identify complexities for students, even though sometimes she had the hardship of dealing with them.

Regarding the code of *decisions about sequencing,* it can be said that Riley Teacher's decisions did not give the desired outcome. For instance, Riley Teacher had two main concepts to teach, such as fair sharing and measurement meanings of division. She preferred to teach measurement meaning first. Nevertheless, in her post-lesson interviews, she indicated that she would do the opposite and teach fair sharing first. The below excerpt from the post-lesson interview exemplified her explanation of this choice.

> *Interviewer: .... So, do you think you could have taught the meaning of partition (fair sharing) first?*
>
> *Riley Teacher: Yes, I could have taught partition meaning first. Because a student could have thought, 'According to what will we group the objects?' and 'All in all, groups do not have to be equally sized.' That is why I plan to switch the order of these two meanings in the following lessons.*
>
> *Interviewer: So, you say first partition meaning and then measurement?*
>
> *Riley Teacher: Yes, first partition. Since these students knew it, we had no difficulty. However, if they would not know, I had to allocate 30 more minutes*

to this concept and the reasons, even if there were no misconceptions. (Post-lesson interview)

Consequently, it can be stated that Riley Teacher could plan sequencing based on conscious decisions, even though she could not show this in her current lesson plans and implementations. However, it was vital that she could plan her following lessons according to these conscious decisions. Therefore, this instance was evidence for *decisions about sequencing.* Additionally, although Riley Teacher did not specify any reason to justify this, she also planned her activities, tasks and questions in a determined order. For example, a section from her lesson plans was as below.

> *"Teacher says to students, 'Let us group the objects we have, now.' The teacher asks students to give object names to the manipulatives (colored cubes), such as trees, pencils, etc. The teacher says, 'Let us assume we have 15 pencils. Can you separate these pencils into groups of 3.'. Then, the teacher calls for a volunteer to group the objects with manipulatives. Afterward, the teacher groups the students in pairs and says, 'Now, I separate you into groups. Listen to me carefully and count with me. 2,4,6…' Thus, they will have both counted rhythmic counting by two, and the teacher will have separated the class into groups. Then, she distributes the different materials."* (Lesson plan)

Although the reason behind such a sequencing was not uttered, it demonstrated Riley Teacher's choice of how to order tasks or questions within one class hour.

Considering *making connections between concepts* and *making connections between procedures*, it was observed that Riley Teacher could make relations between concepts regarding division operation. In her lesson plan, Riley Teacher determined the prerequisite knowledge and skills, such as counting objects, rhythmic counting, and multiplication operation. Identifying these concepts as necessary knowledge showed her making a connection between them. Also, regarding the multiplication operation, she chose to start her first lesson with a multiplication operation since she knew that the students dealt with this subject in their previous lesson. Being able to

connect lessons was also another indicator that she could make connections. Additionally, in her lesson plan, Riley Teacher used equal sharing one by one. This action was proof of her establishing a relationship between the procedure. Besides equally sharing one by one, Riley Teacher used the repeated subtraction procedure even though she did not plan to use it in the first place. She said that she made this decision according to students' strategies. When she noticed that students tended to do subtraction, she utilized this opportunity to connect the two procedures. Looking at these instances, Riley Teacher could make these connections between procedures and concepts.

Lastly, regarding *recognition of conceptual appropriateness,* Riley Teacher planned her lesson plan by considering students' developmental levels and prerequisite knowledge. In her teaching, it was observed that Riley Teacher chose to share objects one by one, and she justified this decision as below.

> *"Since they were second graders when they learned multiples better, they can share the objects two by two."* (Post-lesson interview)

This answer was evidence for the code of *recognition of conceptual appropriateness* could be related to the instance.

As a result, considering the Connection Dimension, Riley Teacher's knowledge before and after lesson implementation provides evidence for the following codes: *making connections between procedures, making connections between concepts, anticipation of complexity, decisions about sequencing,* and *recognition of conceptual appropriateness*. Considering *making connections between procedures* and *making connections between concepts,* she was able to make connections between concepts and between procedures. Also, it was evident that Riley Teacher organized the flow of the lesson consciously and based on a reason in regard to *decisions about sequencing.* Regarding *anticipation of complexity,* she was aware of possible confusion coming from students. Lastly, Riley Teacher paid regard to the development levels and needs of students considering the code of *recognition of conceptual appropriateness.*

After the Connection dimension of her knowledge, the following section was about Riley Teacher's reactions to unexpected moments in her teaching.

4.1.4 Contingency Dimension

Contingency Knowledge refers to teachers' responses and actions to unplanned or unanticipated situations (Rowland, 2013), and the preservice teachers' nature of knowledge in the contingent moments could be best seen in the execution of their lesson plans and interviews. Considering *responding to students' ideas*, Riley Teacher's responses were affected by her inexperience. An excerpt from the post-lesson interview can be found below.

The scene was that one of the students proposed 19 as a dividend, and Riley Teacher wanted the student to change her/his number and the interviewer asked the following questions regarding this issue.

> *Interviewer: There was an instance where you did not accept 19 as a dividend. However, then, you decided to go back to handling 19. Why did that happen?*
>
> *Riley Teacher: It was because I thought this instance would gnaw away at their minds. First, I wanted to complete the previous problem and then deal with why 19 could not be used. I changed the number to 18 so that they would see that this number worked. Then we could prove the other one. I thought it would be less complicated if we used the deduction method. Also, dealing with 19 later did not affect other students except the one who asked why it could not be used.*
>
> *Interviewer: So, why did you not want to accept at first? What you said is about why you went back. However, why did you refuse at first?*
>
> *Riley Teacher: Because they did not know division with remainders. Actually, they did not know the division at all; they just knew grouping. However, they*

> *could say we can divide an object into two this time. If this object were bread, bread could be divided into two. Then, they would ask again. First, I explained why we had to divide without the remainder; that is why I refused it. Later… (silence). Actually, the reason why I refused first was not that I did not want to explain. It was to go back to it as being clearer after completing the previous one and also gain time for me to think.* (Post-lesson interview)

This scene showed how Riley Teacher attended to an unexpected student idea. As an inexperienced teacher, she preferred to pass it since she did not have a ready answer, and consequently, she gained time to think about it. However, she also saw the possible confusion and returned to the question. Also, this unexpected student idea created an opportunity for Riley Teacher to explain a prime number as a dividend. This scene also showed her ability to use opportunities even though she dealt with them late due to her lack of a ready answer.

Regarding *deviation from the agenda* and *responding to (un)availability of tools and resources,* Riley Teacher was able to change the lesson because of an unanticipated event. For example, while Riley Teacher planned her first lesson, she decided to use a worksheet but could not execute this plan. She justified this as below.

> *"Since I extended lesson time while posing questions for students to understand better, I could not distribute worksheets. Therefore, I changed this worksheet as a reminder for the next lesson."* (Post-lesson interview)

While this scene was evidence of the code of *deviation from the agenda*, it was also evidence of the code of *responding to (un)availability of tools and resources* since Riley Teacher effectively changed the lesson's flow and the activity's function rather than removing this activity. How Riley Teacher deviated from her lesson plan and how she integrated a nonfunctional activity could demonstrate evidence for these codes.

Lastly, considering *teacher insight,* when Riley Teacher saw a deficiency, she tried to rectify this deficiency throughout the lesson. For example, during the instruction,

although Riley Teacher did not plan to teach repeated subtraction in the first lesson plan, she explained it in the lesson implementation. She explained herself as below:

> *"The reason why I taught repeated subtraction was the feedback from the students. They learned grouping. When I asked them to group 16 into 4, they removed four and counted the remaining objects. When I got this message, I noticed they directed to subtraction. Also, when students subtracted again, they discovered it on their own. I mentioned it in every example when I realized they would understand it."* (Post-lesson interview)

Looking at this scene, it is clear that she could identify a deficiency and overcome this problem by integrating the absent process instead of neglecting repeated subtraction in her lesson plan, and this showed her *teacher insight*.

As a result, regarding the Contingency Dimension, Riley Teacher's knowledge before and after lesson implementation provides evidence for the following codes: *responding to students' ideas, deviation from agenda, teacher insight, and responding to (un)availability of tools and resources.*

Considering *responding to students' ideas and deviation from agenda,* Riley Teacher's behaviors provided strong examples for these codes since she acknowledged the idea and successfully deviated from the lesson plan. Accordingly, regarding *teacher insight,* the fact that she identified a deficiency and took action was a strong example for this code. Lastly, her being able to re-integrate a task was a strong example for the code of *responding to (un)availability of tools and resources.*

In the following title, the knowledge of Emily Teacher, the other pre-service teacher, was investigated through all four dimensions.

4.2 Emily Teacher

The number of observations in the codes under the Knowledge Quartet Model dimension in Emily Teacher's lessons is given in the table below.

Table 4.2 Emerging Codes of Knowledge Quartet Model in Emily Teacher's Lesson

Dimension	Codes	Number of Observation
Foundation	Awareness of Purpose	8
	Identifying Pupil Errors	0
	Overt Display of Subject Knowledge	6
	Theoretical Underpinning of Pedagogy	4
	Use of Mathematical Terminology	3
	Adherence to Textbook	5
	Concentration on Procedures	2
Transformation	Choice of Examples	5
	Teacher Demonstration	3
	Choice of Representation	3
	Use of Instructional Materials	3
Connection	Making Connections Between Procedures	1
	Making Connections Between Concepts	5
	Anticipation of Complexity	1
	Decisions About Sequencing	2
	Recognition of Conceptual Appropriateness	3
Contingency	Responding to Students' Ideas	1
	Deviation from Agenda	2
	Teacher Insight	1
	Responding to (Un)Availability of Tools and Resources	1

The above table indicates that in the whole process (before, during and after teaching), there is evidence of all the codes except the code for *identifying pupil errors*. Throughout the process, Emily Teacher did not recognize the errors or mistakes that students could make. In the whole process, the least observed codes were *making connections between procedures, anticipation of complexity, responding to student's ideas, teacher insight* and *responding to (un)availability of tools and resources,* which was observed once. The most observed code was *awareness of purposes*, which was observed 8 times, followed by the code of *overt display of subject knowledge,* which was observed 6 times.

In the following section, Emily Teacher's knowledge was investigated through the codes on the basis of four dimensions such as Foundation, Transformation, Connection and Contingency.

4.2.1 Foundation Dimension

Considering *overt display of subject knowledge,* Emily Teacher's definition of division operation was ill-explained, and she could not properly list the meanings of the division operation. She considered division mainly fair sharing (partition) and neglected the measurement (quotitive) meaning. For example, below her definition of the division operation can be seen.

> *"Division is sharing a certain number of objects with a certain number of people. There is a need to limit this to objects or people. What is important is distributing or sharing equally."* (Readiness test)

As mentioned before, she could not utter the correct meanings of the division operation, and she mostly repeated the fair-sharing meaning. To illustrate, her list of meaning was as follow.

> *"Sharing, sharing out, distributing."* (Readiness test)

All these answers showed that Emily Teacher's definition of division was restricted to only fair-sharing. However, after lesson implementation, there was a change considering her *overt display of subject knowledge*. Before, she ignored repeated subtraction and ruled out this method for division. Nonetheless, after the lesson implementation, when Emily Teacher talked about what she paid attention to while describing division operation, it was apparent that she included repeated subtraction, as shown below.

> *"While sharing, I made a point of sharing and distributing equally. While grouping, if I made a group of 3, I continued to do that until I ran out of objects. ... For instance, if I did repeated subtraction, how many times I did this operation gave the result of the division. In fact, all of this is related to division."* (Post-lesson interview)

Here, it was clear that Emily Teacher could give a more accurate description of division and was also aware of different meanings of the division.

Regarding the code of *awareness of purpose*, although there was no evidence in the readiness test, both in her lesson plans and lesson implementation, Emily Teacher was aware of the objectives and the purposes. Emily Teacher prepared her lesson plans, and she chose these examples while recognizing the purpose. For instance, she explained why she included interrogative questions in her lesson plans as follows.

> *"Because if I told them, 'Division is this, it has two meanings: fair sharing and measurement.' They would not learn it. Actually, they would have memorized it. That is why I tried to explain the sense of fair sharing. In fact, I allowed them to interrogate this. This is the reason for these interrogative questions"* (Post-lesson interview)

Additionally, it was clear that Emily Teacher was aware of the objectives. An excerpt from this instance can be found below.

> *Interviewer: What did you consider when you selected the examples' numbers?*

> <u>Emily Teacher:</u> *... Objectives included the numbers until 20. That is why I preferred to use numbers from 10 to 20.* (Post-lesson interview)

Like Riley Teacher, Emily Teacher had an awareness of objectives and goals, which led to the emergence of the code of *awareness of purpose*.

Regarding *concentration on procedure,* it was clear that Emily tended to use standard and traditional algorithms and methods. For example, when Emily Teacher was required to write and solve a division problem, she immediately solved her question by the standard algorithm. Below, her problem and way of solution can be seen.

> *Emily Teacher's problem: "There are 45 people who will participate in a school trip. The school will rent buses for the trip. If each bus is for 15 people, how many buses should be rented for 45 people?"*

Emily Teacher's solution:



(Readiness test)

That Emily Teacher solved this problem using a standard algorithm rather than drawing or another representation showed her reliance on conventional procedures.

Although Emily Teacher did not plan it, she also talked about division operation and its elements in her lesson implementation. The interviewer's interrogation of this instance was as follows.

> <u>Interviewer:</u> *There was an instance between two questions when you had no such thing in your plan; you talked about the division and its elements. For*

example, you asked, "How many 7s are there in 7?". You tried to do some things there, but then you gave up. What happened there?"

Emily Teacher: That instance was due to inexperience, I believe. Suddenly, I went there, and I was surprised as well. I needed an operation, some numbers. For some reason, I needed that. I thought they would say, "We do that, but what is happening? (Post-lesson interview)

Although Emily Teacher related this situation to her inexperience, this excerpt showed that considering the code of *concentration on procedure,* she had a tendency to base her lessons on procedures and algorithms; however, she could not successfully integrate the standard algorithm into her teaching.

Regarding *use of mathematical terminology*, Emily Teacher's usage of mathematical terms was restricted to only a few terms. For example, in the readiness test, Emily Teacher answered almost all questions by emphasizing terms such as equal, equal sharing, and equal distributing. Accordingly, in her lesson plans, the questions she would ask students included related terms such as fair sharing and groups. For example, one of the questions was, "While we do job-share, how do we determine the number of groups?" or "What does dividing into groups give us?". Furthermore, in the post-lesson interview, as mentioned previously, she especially emphasized equal sharing or grouping while she described the division operation. Nevertheless, considering her use of terms, it could be observed that her use of terms was limited to only equal sharing and groups.

Considering the code of *adherence to textbook,* while Emily Teacher did not use the textbook in her first lesson plan, she based her second lesson mainly on the textbook due to textbooks' availability. It can be said that when she ran into trouble, such as a lack of time for preparing a lesson, she knew that she had material she could use. An excerpt of Emily Teacher's reasoning for this instance was as below.

Interviewer: While you did many activities in the first lesson, you relied on the textbook in the second. Why did it happen?

Emily Teacher: Because I did not see satisfactory examples for the first lesson. However, for the second lesson, there were some good examples. Another reason is that I had enough time to prepare for the first lesson. However, for the second lesson, I had two days. Since I liked the examples, I decided to use the textbook for the second lesson.

Interviewer: If you compare two lessons, which was better? Activity-based or textbook-based?

Emily Teacher: I think activities with concrete materials were more permanent, but the textbook also supported me. The textbook was there and ready. I could use it however I want. Since I needed to solve more questions, it met my needs. (Post-lesson interview)

Here, it was clear that although her first choice was not using textbooks, its benefits were obvious for Emily Teacher. Since it was pre-prepared and time-saving, Emily Teacher preferred to use the textbook. However, while Emily Teacher used the textbook, she did not analyze the problems and modify them according to her lesson, and she directly integrated them into her lesson.

Lastly, regarding the code of *theoretical underpinning of pedagogy*, it can be said that Emily Teacher's view of which circumstances were better for learning shaped her lesson planning. Her answers from the post-lesson interview supported her view on how to teach mathematics. For instance, her reasoning of why she used "one for you, one for me" while dividing was *"Because it was from daily life. Children… At least, that was the case for me. If I would share something with my sibling, I could not mentally say, 'There are ten objects; 3 of them would be mine, and 3 of them would be yours.' This was a case of going off half-cocked."* (Post-lesson interview)

Additionally, comparing the activity-based or textbook-based lesson, she expressed that learning with concrete materials was more permanent. Accordingly, she planned her lessons and executed them with respect to her view of the best circumstances.

Consequently, regarding Foundation Dimension, Emily Teacher's knowledge before and after lesson implementation provides evidence for the following codes: *awareness of purpose, overt display of subject knowledge, theoretical underpinning of pedagogy, use of mathematical terminology, adherence to textbook* and *concentration on procedures.*

Considering the *awareness of purpose,* it was seen that Emily Teacher was aware of and recognized the goals and objectives as well. Additionally, about *theoretical underpinning of pedagogy,* the fact that she cared about the planning and delivery of the lesson provided evidence of strong example. Regarding *overt display of subject knowledge,* before lesson implementation, her definition of division operation was ill-explained; however, after lesson implementation, she could define the division operation and its meanings well. In contrast, in regard to *concentration on procedure,* Emily Teacher clung to standard algorithms and procedures and also could not integrate algorithms into her lessons. In addition, considering *use of mathematical terminology*, her usage of terms could be evidence for weak examples since it lacked diversity. Lastly, the fact that Emily Teacher used the textbook without adapting and integrating it directly showed evidence of a weak example of the code of *adherence to textbook.*

After analyzing Emily Teacher's knowledge in regard to Foundation dimension, which was about her undergraduate study or experiences, the following section was about how Emily Teacher used her knowledge in her teaching.

4.2.2 Transformation Dimension

Regarding the code of *choice of example*, Emily Teacher selected the examples she used in her teaching with a goal in mind; however, her examples did not serve to develop a conceptual understanding of division. In none of the instances, Emily Teacher's selection of examples could be related to developing meanings of division. Specifically, her examples were mostly related to her teaching and the

circumstances. Additionally, she thought that in order for students to internalize division operation, she should use bigger numbers. For instance, the excerpt below showed the interrogation of Emily Teacher's selecting specific numbers for her examples. She explained that she chose numbers so that students would engage with the example more. However, she did not think that using smaller numbers was also appropriate for teaching division operation.

> *Interviewer: What did you consider when you selected the examples' numbers?*
>
> *Emily Teacher: .... Objectives included the numbers until 20. That is why I preferred to use numbers from 10 to 20. I preferred to use numbers from 10 to 20. Because, for example, I wanted the students to get smaller numbers while they shared 18 bones to 6 dogs.*
>
> *Interviewer: Why did you want to get smaller numbers?*
>
> *Emily Teacher: To challenge them more. How can I say? For instance, if there were 4, and they shared them with two dogs, it would take less time. I wanted them to engage with it more, and I wanted them to internalize it more.* (Post-lesson interview)

In addition, she integrated an activity whose examples helped her get to know the class and her selection of these examples was not related to the subject at hand. The below excerpt demonstrated her way of thinking.

> *Interviewer: If you consider your activity with seven questions, did it go to plan?*
>
> *Emily Teacher: It absolutely did. Since I did not know the students and their names, I needed to plan a lesson where they would be more active. I used the terms for this, but I forgot them now.*
>
> *Interviewer: For example, you said, "People who are responsible for the orange dogs in parks and streets, let us share 16 bones to 4 dogs.*

> *Emily Teacher:* *I aimed to see all students active in the activity. Since I did not know their names, I needed to use a different method. When I distributed the terms in that sentence to students, these four terms [orange, dog, street and park] would form a group of 4. They would solve the question together. In this way, I could see them active in the activity, and not knowing their names was not an issue for me. Ultimately, I reached my goal, and they [students] studied together.* (Post-lesson interview)

This instance showed that Emily Teacher reasoned her way of choosing examples according to her needs for teaching. However, she did not emphasize division operation while choosing the examples.

Considering *teacher demonstration,* Emily Teacher's demonstration was limited, and it relied on mostly traditional demonstrations such as using a blackboard to illustrate a sharing or grouping. On only one occasion, Emily Teacher used a different demonstration to introduce division operation in her first lesson after the interrogative question in the introduction part of the lesson and used this demonstration to move into division. Here, she had nine pizza slices and stuck this material to the blackboard. After that, she invited students to share these slices with three people by inviting students to the blackboard. Below, the demonstration she used could be found.



(Lesson Plan)

Besides this demonstration, she used illustrations to share objects on the blackboard, but her demonstrations were few and limited except for these two demonstrations.

Regarding *choice of representation,* Emily Teacher used few conventional representations. She did not use any other representation besides sharing one-by-one and repeated subtraction. Once she attempted to represent division in standard form, she abandoned this representation, as was mentioned in the previous section.

Lastly, considering the code of *use of instructional materials*, Emily Teacher's usage of different instructional materials was restricted to only the pizza model, activity sheet, and textbook.

Consequently, considering Transformation Knowledge, Emily Teacher's knowledge before and after lesson implementation provides evidence for the following codes: *choice of examples, teacher demonstration, choice of representation and use of instructional materials.* Even though Emily Teacher tried to use different methods, materials and representations, they were limited, few in number and conventional, and they did not serve the purpose of the lesson. These characteristics could provide

evidence of weak examples considering all the codes of the Transformation dimension, such as *choice of examples, teacher demonstration, choice of representation and use of instructional materials.*

After this dimension, the following section was about Emily Teacher's decisions and choices to make her lessons more coherent.

4.2.3 Connection Dimension

Considering the code of *making connections between concepts* and *making connections between procedures,* even though Emily Teacher could connect some concepts and procedures, these connections were few and unvaried. For example, she mentioned 'subtraction with the whole number' in the first lesson plan as a prerequisite knowledge. However, except for the subtraction operation, there was not any other connection between other concepts, such as multiplication, rhythmic counting, or previous lessons. However, after teaching implementation, she mentioned fractions when she explained the reasoning behind determining numbers. Her way of thinking could be found below.

> *"I did not apply a strategy when selecting them, but I should have. Because you know, in fractions, you divide a whole into halves, then continued to divide by 2…*" (Post-lesson interview)

Although this instance was not well-explained, it demonstrated some sense of her connecting division and fractions. Also, even though there was no evidence in her lesson plan regarding this issue, while she explained why she preferred to teach fair sharing before measurement, she said, *"I thought that if I began with sharing, I would move onto grouping easily. Because if one shared, it was equal sharing. Let us assume there are three people, each with three slices of pizza. Here, these three pizza slices are just like a group. Three groups of three. First, let me share; then they would understand that these are groups."* Here, again, it could be said that Emily Teacher was able to connect these two meanings of division.

Additionally, her connections were restricted to only sharing one-by-one and repeated subtraction. However, although Emily Teacher taught repeated subtraction, her lesson plan did not include this procedure. She explained this as

> *"Repeated subtraction was already one of the objectives. I particularly did not include it in my lesson plans, but it is already an example of the division operation. While I prepared my lesson plan, I specifically wrote the page numbers where the repeated subtraction was conducted."* Here, it was apparent that she perceived repeated subtraction as a procedure for division even though she did not express it clearly in the lesson plan. Another procedure she connected to the division was sharing one by one. She could connect these procedures. Nonetheless, they were limited and not varied considering the subject at hand.

Regarding the code of *anticipation of complexity*, Emily Teacher paid regard to possible confusion or complications of students when she prepared her lessons, although her explanation was poor regarding the conception of division operation and developing conceptual understanding. For example, the excerpt below showed that Emily Teacher's reasoning for a saying she used in the lesson revealed that it was affiliated with an expected complication.

> *Interviewer: You especially emphasized "one for you, one for me" in the lesson. Why did you emphasize it?*
>
> *Emily Teacher: Because it was from daily life. Children… At least that was the case for me; if I would share something with my sibling, I could not do mentally that "There are ten objects, 3 of them would be mine, 3 of them would be yours. It skinned the bear at once. [It happened] in order to take it from the top. Let me give them one by one until I run out of them so that they would be equal. For example, I shared two by two what would happen if I had one or three objects when I ran out of [objects]. Students will ask, "Why did we share two by two?" That is why, one by one. To teach easily.* (Post-lesson interview)

In this case, it was evident that Emily Teacher recognized a possible complexity and planned her lesson accordingly. In addition to that, her choice to start with the equal sharing meaning to teach the division was another evidence that she could recognize the complexities. This instance will be mentioned in detail in the following code.

Additionally, regarding *decisions about sequencing*, it was evident that she based the lesson flow on a foundation. To illustrate, when she reasoned about why she initiated her lesson with fair sharing meaning, she said,

> *"I thought that if I began with sharing, I would move onto grouping easily. Because if one shared, it was equal sharing. Let us assume there are three people, each with three slices of pizza. Here, these three pizza slices are just like a group—three groups of three. First, let me share, then they would understand that these are groups"* (Post-lesson interview).

Besides the meanings, she also ordered the activities based on a reason, and the below excerpt indicated this case.

> *Interviewer: You conducted an activity with counters. Why did you prefer to do so?*
>
> *Emily Teacher: … When I looked at the textbook, I saw that activity. I said, "Oh, I have counters. It will be good if I used them [counters]". I would have livened up [the lesson], what is more, they would have worked together…* (Post-lesson interview)

When looking at decisions about sequencing, it was clear that she deliberately ordered the sequencing.

Lastly, considering *recognition of conceptual appropriateness*, while Emily Teacher planned and implemented her lessons, she would be able to identify conceptual appropriateness for the students concerning her view. For example, according to her, she needed the interrogative questions at the beginning because she believed that if she said to students, *"Division is this; it has two meanings such as fair sharing and measurement.", they would have tried to memorize it. That is why she preferred to*

begin with questions. In addition, she explained why she began with the pizza model such as "It was parallel to reason behind me starting with interrogative questions. [I begin with that] to carve division in stone. They are 2-graders who understand [concepts] concretely and could not perceive abstractly." Thereafter, while Emily Teacher planned her lesson, she noticed students' developmental levels and needs.

As a result, considering the Connection Dimension, Emily Teacher's knowledge before and after lesson implementation provides evidence for the following codes: *making connections between procedures, making connections between concepts, anticipation of complexity, decisions about sequencing,* and *recognition of conceptual appropriateness.*

Considering *making connections between procedures* and *making connections between concepts,* her connections between concepts and between procedures were not varied, and they were few in number. Also, it was evident that Emily Teacher made conscious decisions while organizing the flow of the lesson regarding *decisions about sequencing.* About *anticipation of complexity,* she was aware of possible confusion coming from students and planned her lessons accordingly. Lastly, Emily Teacher recognized the development levels and needs of students while planning the lesson and implementing it.

After the Connection dimension of her knowledge, the following section was about Emily Teacher's responses to unexpected situations during the lesson.

4.2.4 Contingency Dimension

Regarding the code of *responding to students' ideas,* even though this code refers to students' unexpected ideas, there was an instance in the lesson where Emily Teachers had to respond not to students' ideas but to the students' expressions. It was enough to say that Emily Teacher could not attend to students' ideas or challenges, and her ability was lacking. An excerpt that interrogated this instance was given below.

Interviewer: There was an instance such that between two questions when you had no such thing in your plan, you talked about the division and its elements. For example, you asked, "How many 7s are there in 7?". You tried to do some things there, but then you gave up. What happened there?"

Emily Teacher: That instance was due to inexperience, I believe. Suddenly, I went there, and I was surprised as well. I needed an operation, some numbers. For some reason, I needed that. I thought they would say, "We do that, but what is happening?" ... Then this question came: How many 7s are there in 7? But the students had blank expressions. I kept explaining, but it led nowhere. Then I noticed this did not exist [in the plan and objectives] ... (Post-lesson interview)

After this case, she gave up teaching this and moved on to another question. Even though this case related to the code of *responding to students' ideas*, considering this code, it was enough to say Emily Teacher was not able to handle students' ideas or their challenges.

Additionally, regarding the code of *deviation from agenda*, while Emily Teacher could successfully deviate from the lesson on some occasions, in other cases, she could not. The previously mentioned case could be considered evidence since this deviation came to nothing and it represented a weak example for this code because it displayed a mathematical uncertainty. One of the other cases where she deviated from the lesson plan was using the textbook even though it was not a part of the first lesson plan. In this case, when Emily Teacher completed the lesson plan, she used the textbook. She explained this instance as follows.

"Everything that I should complete was done at that point. When I checked, there was plenty of time. So, I thought we should go back to the textbook. Here, our teachers should be more practical. My plan is finished, and I am not the one who will set students loose. I thought there was a ready book; let me look at it. I solved one question from there…" (Post-lesson interview)

Here, it was clear that Emily Teacher's deviation was not always fruitful, but in some cases, she thought it was practical so that she could change the flow of the lesson.

Followingly, considering the code of *responding to (un)availability of the resources*, Emily Teacher's being able to integrate the textbook even though it was not a part of the lesson plan showed that Emily Teacher was able to respond to available resources.

Lastly, regarding the code of *teacher insight*, although Emily Teacher recognized a deficiency in her plan and deviated from her agenda accordingly, her reason for doing so was uncontemplated and spur of the moment. The following excerpt could be evidence of her knowledge even though it happened due to external resources.

> *Interviewer: You asked students, "If I group 15 by 5, there will be three groups. Then, how many 3s are there in 15?" However, this did not exist in your lesson plan. Did you really need to say that? Why did you do it?*
>
> *Emily Teacher: Actually, I was not sure if I should say that or not. However, when I looked at the textbook, I saw they wrote at the end of the pages. Then, I thought that I should say it. Because it [textbook] belongs to MoNE, that is why I thought I had to say it.* (Post-lesson interview)

This case was an example that Emily Teacher could identify a deficiency in her plan and she deviated from her agenda accordingly. However, apparently, her doing so was not well-founded.

In brief, considering the Contingency Dimension, Emily Teacher's knowledge before and after lesson implementation provides evidence for the following codes: *responding to students' ideas, deviation from agenda, teacher insight*, and *responding to (un)availability of tools and resources*.

Considering the code of *responding to (un)availability of tools and resources,* the fact that Emily Teacher could integrate the textbook into her lesson was a strong example of this code. However, in terms of the codes *responding to students' ideas and deviation from agenda,* Emily Teacher's behaviors provided weak examples for

these codes since she ignored the students' confusion, which was a result of her unsuccessful deviation from the lesson plan. Lastly, regarding *teacher insight,* Emily identified a deficiency and took action; however, the fact that it was not planned, and it did not serve the lesson presented a weak example for this code.

4.3 Summary of the Results

The aim of the study is to investigate the nature of preservice primary school teachers' mathematical knowledge in teaching regarding division operation and what kind of evidence there is in relation to their knowledge regarding the dimensions of the Knowledge Quartet.

Regarding the Foundation Dimension, the code of *identifying pupil errors* was not observed in both preservice teachers before and after teaching implementation. Additionally, there was a positive change in both preservice teachers' definition of a division operation, which was evidence for the code of *overt display of subject knowledge,* before and after they prepared a lesson plan and implemented their lessons. Regarding the code of *awareness of purpose,* there was evidence for that both preservice teachers recognized the objectives and prepared their lessons accordingly. However, it was evident that they were highly dependent on procedures and algorithms regarding the code of *concentration on procedures.* Also, their *use of mathematical terminology* was restricted to a few words such as grouping, equal sharing, sharing etc. Followingly, both preservice teachers planned their lessons and implemented the teaching by considering the conditions in which the students learn best considering the code of *theoretical underpinning of pedagogy*. Lastly, only one preservice teacher (Emily Teacher) used the textbook during her planning and implementation; however, she directly used it without making any adaptations regarding the code of *adherence to textbook*.

Considering the Transformation Dimension, preservice teachers' representation, examples, demonstrations and usage of instructional materials were unvaried, few

and conventional, and they were evidence for their knowledge according to the Transformation dimension in regard to codes of *choice of examples, teacher demonstration, choice of representation* and *use of instructional materials.* Specifically, for the code of *choice of examples,* even though both preservice teachers mostly chose their examples consciously, one preservice teacher could not justify her choices in some cases and also generated instantaneous examples according to students' ideas, and some of the examples had the possibility to cause misconceptions. Also, one preservice teacher's (Emily Teacher) examples were chosen deliberately, but most of them were not related to division operation.

Followingly, considering the Connection Dimension, it was evident that both preservice teachers could recognize possible confusion that could occur in regard to the code of *anticipation of complexity;* however, while one preservice teacher could not handle this confusion, the other preservice teacher's explanation of how to handle possible confusion was poor regarding conceptual understanding. Followingly, both preservice teachers could recognize the levels of the students, and they planned their lessons accordingly. Considering *decisions about sequencing,* it was evident that both preservice teachers made conscious decisions while they ordered the task, activities or explanation, even though, in one case, one preservice teacher (Riley Teacher) said that she would change her current order. Lastly, in terms of *making connections between procedures* and *making connections between concepts,* while one preservice teacher could easily make connections between procedures and concepts, the other one's connection was limited to basic and few procedures and concepts such as equal sharing, multiplication etc.

Lastly, regarding the Contingency Dimension, one preservice teacher (Riley Teacher) could deal with contingent moments in regard to all four codes of *responding to students' ideas, deviation from agenda, teacher insight and responding to (un)availability of tools and resources;* her responses and actions were affected by her inexperience. At the same time, the other preservice teacher's (Emily Teacher) actions and responses were spur of the moment, uncontemplated and

affected by her inexperience in terms of the codes of *responding to students' ideas, teacher insight* and *deviation from agenda.*

In the following table, it can be seen that the similarities and differences between the two preservice teachers' actions in regard to all codes of the four dimensions of KQ.

Table 4.3 Similarities and Differences Between Preservice Teachers Regarding All Codes

Preservice Teachers / **Codes**	**Riley Teacher (with primitive measurement division conception)**	**Emily Teacher (with strong fair-sharing division conception)**
Overt display of subject knowledge	Positive changes in definitions	Positive changes in definitions
Awareness of purpose	Aware of the objectives	Aware of the objectives
Use of mathematical terminology	Limited use of mathematical terminology	Limited use of mathematical terminology
Concentration on procedures	Focused on standard algorithms	Focused on standard algorithms
Theoretical Underpinning of pedagogy	Taking into consideration of the conditions which the students learn best	Taking into consideration of the conditions which the students learn best
Choice of examples	Traditional and limited variety of examples	Traditional and limited variety of examples
Teacher demonstration	Traditional and limited variety of demonstrations	Traditional and limited variety of demonstrations
Choice of representations	Traditional and limited variety of representations	Traditional and limited variety of representations
Use of instructional materials	Traditional and limited variety of instructional materials	Traditional and limited variety of instructional materials

Table 4.3 (Continued)

Anticipation of complexity	Being able to anticipate complexities	Being able to anticipate complexities
Decisions about sequencing	Designed the flow based on conscious decisions	Designed the flow based on conscious decisions
Responding to (Un)Availability of Tools and Resources	Being able to reintegrate resources into lessons	Being able to reintegrate resources into lessons
Adherence to textbook	Not using textbook	Used textbook for a whole lesson
Making connection between concepts	Being able to make connections	Not being able to make connections
Making connection between procedures	Being able to make connections	Not being able to make connections
Responding to Students' Ideas	Acknowledging but sidelining	Ignoring
Deviation from Agenda	Being able to deviate from the lesson plan	Not being able to deviate from the lesson plan
Teacher Insight	Successfully managing deficiencies	Unproductive managing deficiencies

In brief, both preservice teachers had similarities in several codes, such as *overt display of subject knowledge, awareness of purpose, use of mathematical terminology, concentration on procedures, theoretical underpinning of pedagogy, choice of examples, teacher demonstration, choice of representations, use of instructional materials, anticipation of complexity, decisions about sequencing,* and *responding to (un)availability of tools and resources.* However, there were differences in some codes, such as *adherence to textbook, making connections*

between concepts, making connections between procedures, responding to students' ideas, deviation from agenda and *teacher insight*.

In the following section, the findings at hand were explored in terms of what they indicate and mean by referring to relevant literature.

CHAPTER 5

DISCUSSION

The current study aimed to examine the preservice primary school teachers' mathematical knowledge in teaching division operation through the Knowledge Quartet model. This chapter includes a discussion of the findings of the current study based on the dimensions of the KQ Model. Furthermore, implications for practice and recommendations for further studies are presented in the chapter.

5.1 Discussion of the Results

Consistent with the study's objectives, this section delves into the study's findings, organized according to the dimensions of the KQ Model: Foundation, Transformation, Connection, and Contingency, in correlation with the research sub-questions.

5.1.1 Foundation Dimension

In line with the first research sub-question, which inquired about the evidence regarding preservice primary school teachers' mathematical knowledge in teaching within the Foundation dimension of the KQ Model, this section discusses the study's findings through the lens of the observed codes in the teaching practices of the preservice primary school teachers. Rowland (2003; 2013) presents seven codes related to the Foundation dimension of the KQ Model: *awareness of purpose, overt display of subject knowledge, identifying pupil errors, theoretical underpinning of pedagogy, adherence to textbook, concentration on procedure,* and *use of mathematical terminology*. In the context of the current study, a comprehensive

examination of two preservice primary school teachers' mathematical knowledge in teaching unveiled all codes except for *identifying pupil errors*.

Firstly, concerning the code of *awareness of purpose*, both preservice teachers meticulously structured their lessons by identifying clear objectives aligned with the second-grade curriculum. They posed pertinent questions and thoughtfully designed activities that harmonized with these objectives. Moreover, one preservice primary school teacher (Riley Teacher) answered some pre-lesson questions by considering the objectives of different grade levels. Objectives and goals are important for the process of teaching and successful assessment (Bümen, 2006). According to Bloom's Taxonomy, the two objectives in the current study belonged to the level of application. The fact that both preservice teachers were aware of the objectives and planned their lessons accordingly aligns with the literature (Altayli Ozgul & Polat, 2021). It concurs with the notion that preservice teachers typically excel in recognizing apply-level objectives and can readily devise pertinent examples and questions at that cognitive level. The reason why the preservice teachers can recognize objectives and be aware of them might be that the level of these objectives does not require a higher level of mathematical thinking. Additionally, it's possible that they encountered apply-level objectives more frequently throughout their prior academic experiences before university, compared to objectives at higher cognitive levels such as evaluate or create. Furthermore, the fact that they took classes such as Instructional Principles or Mathematics Method courses during their teacher education might have helped them recognize the objectives and how to use these objectives in their lesson plans.

Regarding the code of *overt display of subject knowledge,* both preservice teachers' definitions of division operation were ill-explained before the lesson implementation. Their definitions of division operation were limited to only fair sharing meaning. Also, when they were trying to write a problem with a division context, they used fair sharing meaning; they did not think of measurement meaning at all. However, after implementing their lesson plans, their definitions significantly changed. Rather than focusing only on fair sharing meaning, they started to include

measurement meaning in their explanation of division operation, and they were able to prepare activities and problems that involved measurement meaning.

Considering their definition before implementation, this result is in line with the literature since fair-sharing is the primitive meaning of division operation, and it is the first concept that comes to mind when thinking of division operation (Fischbein et al., 1985; Simon, 1993; Sitrava et al., 2020). Firstly, students have a tendency to associate an operation with the model they were initially introduced to. Secondly, primitive models, such as fair sharing in the case of a division operation, are resistant to change and exert a direct influence on students' behaviors. In the context of the current study, both preservice primary school teachers were once students themselves, and it's plausible that they continue to perceive division operation primarily as fair sharing, considering it was the initial and fundamental model they encountered during their early years of schooling. This perception may persist despite their exposure to various mathematics method courses that addressed division operation in greater depth.

However, after implementing their lesson plans, significant changes were observed in their definitions. Rather than focusing only on fair sharing meaning, they started to include measurement meaning in their explanation of division operation, and they were able to prepare activities and problems that involved measurement meaning. This result is also expected and consistent with the literature since “practical knowledge,” i.e., the teachers’ activities and actions in the class, is helpful in modifying real-life situations (Chapman, 2004, p.192). Considering the current study, this real-life situation can refer to preservice teachers’ planning a lesson on division operation and implementing this lesson plan in their practicum, consequently, the alteration in their definitions. In other words, the reason why participant preservice teachers’ definitions changed might be the fact that they modified their definitions after their activities and plans during the lesson implementations. Moreover, the challenges they face in imparting their subject in a manner that resonates with their students may serve as a catalyst for the preservice

teachers to enhance their understanding of students, the learning context, the curriculum, and pedagogical approaches, as noted by Ball and McDiarmid (1990).

Considering the *concentration on procedure,* both preservice teachers in the present study heavily relied on standard algorithms or conventional ways. This result aligns with the literature as Singmuang (2002) states that following specific steps or using formulas is easier; therefore, teachers do not try to find different representations for a particular problem. Since the standard algorithm for division operation functions no matter what the situation is and as the literature suggests standard algorithms are easier to follow, the pre-service teachers of the present study might have tended to use the standard algorithm before and during teaching. In addition, Simon (1993) suggests that preservice teachers' knowledge of division operation is already based on procedures and argues that it stems from their beliefs "m What Simon proposes regarding how preservice teachers perceive mathematics and its functions might offer another explanation for why the preservice teachers in the current study exhibited a strong emphasis on procedures.

Considering the code of *use of mathematical terminology,* in the current study, preservice teachers' vocabulary was restricted only to words of 'division,' 'group,' and 'equal sharing.' This could be caused by the fact that in the present study, preservice teachers' primary focus may not have been on employing precise vocabulary and terminology correctly. Instead, they may have primarily strived to elucidate the meanings of the division operation. This contrasts with the idea put forth by Ulusoy (2021), which suggests that consciously employing vocabulary and terminology directly contributes to the accurate definition of concepts.

Regarding *adherence to textbook,* one of the preservice teachers (Riley Teacher) did not use the textbook or similar learning materials, while the other (Emily Teacher) used the textbook in her second lesson mainly because of the textbook's availability. When she did not have enough time to prepare a lesson, she chose to use the textbook. However, she adhered to the textbook and its problems without engaging in critical analysis or adapting these problems to better suit the needs of her students

(Ball et al., 2008; Rowland et al., 2009; Stein et al., 2007). This finding aligns with existing literature, as it is common for many preservice teachers to resort to textbooks or written materials for similar reasons, including constraints related to availability, limited time, or lack of guidance (Nicol & Crespo, 2006). Additionally, preservice teachers often encounter challenges when it comes to selecting or adapting problems and activities, particularly in terms of effectively utilizing the textbook as a teaching resource (Ball et al., 2008; Evertson & Smithey, 2000; Stein et al., 2007). The reasons why the preservice teacher (Emily Teacher) in the current study adhered to the textbook without adapting it could be her inexperience in lesson planning and teaching and her limited knowledge of how to select and adapt tasks and problems.

5.1.2 Transformation Dimension

Consistent with the second sub-question, which inquired about the evidence pertaining to preservice primary school teachers' mathematical knowledge in teaching within the Transformation dimension of the KQ Model, this section explores the study's findings through the lens of the observed codes within the teaching practices of the preservice primary school teachers. Rowland (2003; 2013) presented four codes related to the Transformation dimension of the KQ Model: *choice of examples, teacher demonstration, choice of representation* and *use of instructional materials*. In the current study, all the codes emerged while examining two preservice primary school teachers' knowledge.

In terms of *choice of examples*, both preservice teachers predominantly opted for conventional and limited variations. Additionally, one preservice teacher generated spur-of-the-moment examples during the teaching process, but in one instance, this impromptu approach didn't align with the intended lesson objectives. This is in line with the literature as Rowland and his colleagues (2009) suggest that while teachers generate examples, they can use pointless numbers or create spontaneous examples, and consequently, they generate examples that do not fit the purpose of the lesson. The reason why this instance occurred in the current study might be related to

preservice teachers' inexperience and to the lack of activities and tasks related to division operation in mathematics method courses.

Regarding *teacher demonstration*, both preservice teachers used specific demonstrations, such as pizza model or cubes; however, both demonstrations in the current study were used only to elucidate procedural aspects such as fair sharing or grouping. Charalambous et al. (2011) state that demonstrations should focus on a specific concept and rest on students' prior knowledge, and they should lead students from their prior knowledge to the subject at hand. Furthermore, it is crucial to recognize that demonstrations extend beyond mere instructional aids; they should also underscore the underlying meaning of the concepts being taught. Both preservice teachers employed demonstrations primarily to illustrate the aforementioned procedures. In light of this observation, it aligns with patterns identified in the literature, which argues that preservice teachers use demonstrations to mainly explain the procedure (Borko et al., 1992; Kinach, 2002). Kinach explains why preservice teachers tend to use demonstration to simply explain a procedure. It is suggested that these tendencies stem from preservice teachers' beliefs about teaching mathematics, wherein there may be a prevailing emphasis on "telling-math" explanations, which involve merely describing procedures rather than prioritizing "teaching-for-understanding" approaches (Charalambous et al., 2011, p.444). This rationale could indeed hold true for the present study, shedding light on why the preservice teachers predominantly employed demonstrations to illustrate procedures rather than placing a stronger emphasis on teaching for understanding.

Considering the code of *choice of representation*, both preservice teachers used a limited number of representations, which are repeated subtraction or sharing one-by-one. However, as many researchers suggest, teachers should use various representations to teach a subject so that students can establish relationships between representations and learn the subject in a more conceptual manner (Elia et al., 2009; Kaput, 1992; Moru, 2006). The reason why the preservice teachers have a limited choice of representations may be caused by their lack of subject matter knowledge regarding division operation or lack of practice in terms of teaching. Moreover,

according to Sitrava et al. (2020), lack of encounter with division operation in different contexts may cause preservice teachers not to use different representations. This insight could also be applicable to the current study, helping to explain why the preservice teachers did not utilize a broader range of representations.

Lastly, when examining the code of *use of instructional material*, it becomes evident that the preservice teachers made use of a limited range of materials, including cubes, activity sheets, and pizza models. This practice of preservice teachers relying on a restricted array of instructional materials is consistent with the findings from existing literature (Castro, 2006). As stated in the literature, one of the reasons for this may be their inexperience or their own experience when they learned this subject (Castro, 2006; Lampert & Ball,1998). Given that the preservice teachers in the current study were once students, they may have some static beliefs about how this subject can be taught even though they completed the mathematics method courses.

5.1.3 Connection Dimension

Consistent with the third research sub-question, which inquired about the evidence regarding preservice primary school teachers' mathematical knowledge in teaching within the Connection dimension of the KQ Model, this section delves into the study's findings through the lens of the observed codes within the teaching practices of the preservice primary school teachers. Rowland (2005; 2013) presented five codes related to the Connection dimension of the KQ Model: *making connections between procedures*, *making connections between concepts, anticipation of complexity, decisions about sequencing* and *recognition of conceptual understanding*. In the context of the current study, a comprehensive examination of two preservice primary school teachers' knowledge uncovered all relevant codes.

Considering the code of *anticipation of complexity,* both preservice teachers could recognize some complexities that could present difficulties for students, and they planned their lessons accordingly. Many researchers suggest that it is crucial for

teachers to be aware of students' thinking and proceed in accordance with students' thinking (Arnesen et al., 2017; Lewis, 2000; Yoshida, 1999). In this context, students' thinking may refer to complexities, and preservice teachers' awareness of complexities can direct them when they plan their lessons. The reason why preservice teachers considered possible complexities may be the lesson plan template provided by the researcher. In the lesson plan template, there were some prompts that directed the preservice teachers to think about possible confusion.

In terms of *decisions about sequencing*, both preservice teachers approached the planning of their lessons with careful consideration. While one of the preservice teachers (Riley Teacher) mentioned a willingness to alter the sequencing in her subsequent lessons, the fact that they were attentive to the situations and made such conscious decisions highlighted their thoughtful approach throughout the lessons. Sequencing decisions made by teachers, including the order in which concepts are introduced, as well as the selection of tasks and activities, "requires coordination between the mathematics at stake and the instructional options and purposes at play" (Ball et al., 2008, p.401). The reason why the preservice teacher thought that she could or would change the order may be her opportunity to make a reflection on her teaching, and it may have directed her to think about and alter the order of the lessons and tasks.

Lastly, considering *recognition of conceptual appropriateness*, both preservice teachers paid attention to students' developmental levels and their possible needs and planned their lessons accordingly in terms of choice of examples and tasks. Parallel to what many researchers suggest, it is important to know the students and their characteristics, and teachers should modify their lessons according to their needs and characteristics (Ball et al., 2005; Rowland et al., 2003). In the study, one of the preservice teachers was studying in the double major program of psychological guidance and counseling, which might have a positive effect on her ability to recognize students' levels and needs.

Regarding the code of *making connections between procedures* and *making connections between concepts*, while one preservice teacher (Riley Teacher) could make better connections between procedures and between concepts, the other could not. For example, she did not try to connect her current lessons with the previous ones. However, many researchers state that making a connection with prior knowledge is vital for learning (Ausubel, 1968; Askew et al., 1997; Ball et al., 2008; Hiebert & Carpenter, 1992; Sleep & Ball, 2009). In addition, being able to make connections requires strong subject matter knowledge in terms of procedures and concepts (Rowland et al., 2009). In terms of the preservice teachers' proficiency in making connections, it is plausible that their limited exposure to tasks and activities during their method courses, which are designed to foster the ability to establish links between procedures and concepts, combined with their relative lack of teaching experience, may have contributed to their lower proficiency in making these connections, as observed in the current study. Also, the fact that Riley Teacher was better at making these connections may have been caused by the different elective courses they took during their teacher education program. For example, unlike Emily Teacher, Riley Teacher took a Microteaching course, which only focused on preparing lesson plans and activities, evaluating these lesson plans and implementing teaching practices. This course might have helped Riley Teacher to make these connections.

5.1.4 Contingency Dimension

Consistent with the fourth research sub-question, which sought evidence pertaining to the preservice primary school teachers' mathematical knowledge in teaching within the Contingency dimension of the KQ Model, this section delves into the study's findings through an exploration of the observed codes within the teaching practices of the preservice primary school teachers. Rowland (2005; 2013) presented four codes related to the Contingency dimension of the KQ Model: *responding to students' ideas, use of opportunities, deviation from agenda, teacher insight* and

responding to (un)availability of tools and resources. In the context of the current study, a comprehensive examination of the knowledge of two preservice primary school teachers uncovered all relevant codes.

Concerning the code related to *responding to students' ideas*, it was observed that both preservice teachers initially tended to avoid addressing situations that they found challenging to respond to. While one preservice teacher directly moved on to another question (Emily Teacher), the other preservice teacher (Riley Teacher) initially altered the direction of the discussion, although she eventually circled back to address the question. This is in parallel with the literature, which states that there are three ways to respond when unexpected moments are encountered: "ignore," "acknowledge but sideline," and "respond and incorporate" (Rowland et al., 2009, p.137). One preservice teacher (Emily Teacher) decided to ignore the challenge, while the other one (Riley Teacher) acknowledged but sidelined it. This situation can be caused by their limited knowledge of the subject at hand, by focusing on the correct answers (Moyer & Milewicz, 2002) or by preservice teachers' inexperience in managing this kind of contingent moment.

Considering the code of *deviation from agenda*, in accordance with the previous code, one preservice teacher (Riley Teacher) ignored the students, moved on, and displayed a mathematical uncertainty (Weston et al., 2013). Her unsuccessful deviation may have been caused by her inexperience in teaching. Nevertheless, it is worth noting that the other preservice teacher, who initially chose to acknowledge but sideline the issue, ultimately set a strong example for this code. Despite her initial preference for sidelining the problem, she demonstrated flexibility by deviating from her original agenda and returning to the issue at hand, successfully addressing the students' confusion. However, her initial reaction of sidelining the problem could also be attributed to her inexperience in managing such situations and her reluctance to deviate from the planned lesson, as highlighted in the literature by Rowland and Zazkis (2013).

Regarding the code of *teacher insight*, both preservice teachers could see some deficiencies in their teaching and lesson plans, and they took action accordingly. However, while one preservice teacher (Riley Teacher) easily and successfully managed these deficiencies, the other preservice teacher's (Emily Teacher) handling of these deficiencies seemed impromptu and unproductive. Considering teacher insight, Kula (2014) listed six different reactions of teachers: "explaining," "reminding," "repeating the solution," "simplifying the process," "changing the content," and "submitting it to approval or voting" (as cited in Kula, 2018, p. 691). While the first preservice teacher decided to change the content, the other one decided to explain. However, this can also be caused by the differences between expert and novice teachers since expert teachers can easily detect and overcome problems, while novice and preservice teachers cannot detect and manage these problems in lessons, as Rowland and his colleagues also stated (2011).

Additionally, compared to Emily Teacher, Riley Teacher showed strong examples for some codes regarding the Contingency dimension, such as deviation from agenda and teacher insight. This divergence in their abilities could potentially be attributed to the different elective courses they took, including Microteaching, with Riley Teacher having some teaching practice experience in this course. Another contributing factor may be Riley Teacher's status as a double major student in the Guidance and Psychological Counseling program. During her education in this program, she had to do some sessions with individuals in the scope of Field Practice in Individual Counseling, where she had to be attentive and flexible during some contingent and unplanned moments in the sessions. Having this kind of experience might have helped Riley Teacher successfully deviate from her agenda and also make instant decisions to change the plan throughout the lesson.

5.2 Implication of the Study

In the current study, the preservice primary school teachers' mathematical knowledge in teaching the division operation has been investigated using the

Knowledge Quartet Model. The findings of the study showed that preservice primary school teachers' definitions of division operation have changed positively after planning a lesson and implementing it. In this respect, providing teaching settings, especially real-classroom settings, and encouraging preservice teachers to prepare lessons would be valuable. This study gives worthy feedback to policymakers who prepare curricula for Faculties of Education to include more practicum courses in the curriculum. In addition to that, considering the effect of the Teaching Practice course on the current study's participants regarding their knowledge, enabling preservice teachers to take this course in the early years of their teacher education would be valuable to see the positive change in their knowledge without further delay. Furthermore, teacher educators can benefit from this study's findings to create settings for teaching in their own classes that support preservice primary school teachers' experience in teaching.

On another note, the model approaches teacher knowledge beginning from theoretical knowledge to methods and techniques to practices of the teachers while considering the pedagogy of learning and teaching. Therefore, considering the holistic approach of the framework of the current study, which is the KQ Model, teacher educators can use and base upon this model while structuring the courses in the teacher education program.

On top of that, looking at the curriculum of the primary education program, due to the limited number of teaching practice courses, preservice teachers may not have enough time to practice mathematics teaching since they have to teach other subjects such as science, language etc. Therefore, encouraging and enabling preservice primary school teachers to practice more mathematics teaching in the teaching practice courses would be valuable, considering the current study's findings.

Additionally, considering the Transformation Dimension and Connection Dimension, which deal with how teachers transform their knowledge to teach their students and make these lessons coherent, it was observed that the preservice primary school teachers used unvaried and limited examples, demonstrations and

representations. This study gives valuable feedback to teacher educators to add different tasks and activities in the method courses that support preservice primary school teachers' use of varied examples, representations and demonstrations that support conceptual understanding.

In addition to that, there are possible effects of participants' double majoring in different fields on their knowledge, behaviors and reactions. Consideringly, this study's findings would give valuable feedback to policymakers and curriculum developers to enable preservice teachers to encounter different fields and benefit from different viewpoints during their teacher education by providing double major opportunities or adding several elective courses from different fields.

It was further observed that preservice primary school teachers had difficulties when they encountered contingent moments. In this respect, teacher educators can plan their lessons in a way that provides opportunities for preservice teachers to encounter contingent moments and to deal with these moments. Furthermore, during the class sessions of the Teaching Practice courses and method courses, teacher educators could incorporate real classroom videos or present teaching cases to the preservice teachers. This approach would encourage the preservice teachers to engage in discussions about these videos and cases, prompting them to reflect on how they would handle similar situations and moments if they were the teacher featured in the video or case.

5.3 Limitations and Recommendations

In this study, the preservice primary school teachers' mathematical knowledge in teaching the division operation has been investigated using the Knowledge Quartet Model. The research design adopted for this study was a case study focusing on two preservice primary school teachers. The findings have illuminated the significant influence of preservice primary school teachers' mathematical knowledge on their lesson plans and teaching implementations. To enhance the generalizability of these

findings, future research endeavors could consider expanding the sample size to encompass a larger cohort of preservice primary school teachers. Additionally, the current study was conducted with preservice teachers who have different conceptualizations and different backgrounds, such as double majoring or study at a foundation university. For further research, the current study could be replicated with preservice teachers with the same conceptualizations of division operation or those who study at a state university.

The current study deals with the preservice primary school teachers' mathematical knowledge in teaching the division operation within the context of teaching practice. To further enrich our understanding, future research endeavors could consider conducting longitudinal studies that track the development of preservice teachers throughout their entire teacher education journey, considering different mathematics topics and grade levels.

Also, in the current study, considering the KQ Model, some codes were not observed in the process. Investigating the knowledge of preservice teachers regarding not observed codes could be considered for further research for a complete perspective of preservice teachers' mathematical knowledge in teaching.

Lastly, since division operation is important and has a strict relationship with other operations and other mathematical concepts due to the cumulative structure of mathematics, the present study could be replicated by taking into consideration other operations and other mathematical concepts, which could provide a more comprehensive perspective. This expanded scope could offer valuable insights into how preservice teachers navigate the complexities of teaching various mathematical concepts.

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APPENDICES

A. Lesson Plan Template

Your name: Date:

Topic/Grade Level:

Big mathematical ideas/learning goals (What do you want your students to learn):

Instructional Objectives:

Duration:

Prerequisite knowledge and/or skills (What prior knowledge will your students need to successfully complete the task(s)?):

Key terms or Vocabulary:

Materials (including manipulatives and technology):

References:

Short description of the activity:

Time/Task	Instructional Moves
Opening—setting expectations and giving access • What will you say to the students prior to starting your lesson? • How will you introduce students to the task(s) or activity so as to provide access to all students while maintaining the cognitive demands of the lesson? • How are you planning to motivate/engage students? • How are you planning to activate/connect students' previous knowledge?	

Middle—addressing goals, anticipating, eliciting, and representing student thinking • What questions do you plan to ask to address your mathematical goals? • What do you anticipate the students would do (the strategies they would use)? • What are your expectations for students as they work on and complete this task? (How will the students work—independently or in groups? How will students record and report their work?) • What questions do you plan to ask to elicit students' thinking and reasoning? • What are the challenges do you anticipate with this activity? • How are you addressing equity in the lesson (language learners, student agency/participation, different abilities)? • What will you write on the board or use as students share their strategies/thinking?	
Closing—assessing students' thinking, summarizing • What do you expect to see or hear that lets you know about students' understandings of mathematical ideas that you intended for them to learn? • What will you say or do at the end in order to assess students' learning and close the activity?	

B. Approval of the Human Subject Ethics Committee of METU

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20 MAYIS 2022

Konu : Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi : İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof. Dr. Mine Işıksal BOSTAN

Danışmanlığını yürüttüğünüz Ekin BALCI'nın "Sınıf Öğretmeni Adaylarının Bölme İşleminin Anlamları Konusundaki Alan ve Alan Öğretimi Bilgilerinin İncelenmesi" başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **0325-ODTUİAEK-2022** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız.

[redacted]
Prof.Dr. Mine MISIRLISOY
İAEK Başkan