MATHEMATICS COACHING FOR A MIDDLE SCHOOL MATHEMATICS TEACHER'S TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE (TPACK) DEVELOPMENT IN GEOMETRY

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ABSTRACT

MATHEMATICS COACHING FOR A MIDDLE SCHOOL MATHEMATICS TEACHER'S TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE (TPACK) DEVELOPMENT IN GEOMETRY

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The aim of this study was to investigate the TPACK (Technological Pedagogical Content Knowledge) development of a middle school mathematics teacher working in a private school in the sixth-grade geometry learning areas of 'Angles and Measurement of Area' through mathematics coaching and the teacher's perception of mathematics coaching regarding TPACK development. The data collection tools for this qualitative research study to monitor changes in the teacher's TPACK across the themes of the TPACK Development Model developed by Niess, Ronau et al. (2009) included 1) TPACK interview before mathematics coaching, 2) observations of teaching practices before mathematics coaching, 3) pre-conferences, 4) observations of teaching practices during mathematics coaching, 5) post-conferences, 6) observations of teaching practices after mathematics coaching, and 7) TPACK interview after mathematics coaching. The teacher's perception of mathematics coaching related to TPACK development was investigated through Mathematics Coaching Interview. The teacher's TPACK development was analyzed using the TPACK Development Model through deductive analysis.

The findings revealed that the teacher progressed to higher levels across the curriculum and assessment theme, the learning theme, the teaching theme, and the access theme and corresponding descriptors of the TPACK Development Model. The findings also indicated that the teacher improved instructional practices by incorporating technology-enriched lessons, with GeoGebra as the main technological tool, through participation in mathematics coaching, and progressed from Pedagogical Content Knowledge (PCK) to TPACK throughout the coaching process. These results emphasize the effectiveness of mathematics coaching in supporting TPACK development, improving teaching practices, and enhancing the integration of technology into classroom instruction.

Keywords: Mathematics Coaching, Professional Development of Teachers, Technological Pedagogical Content Knowledge, GeoGebra, Geometry

ÖZ

MATEMATİK KOÇLUĞU YOLUYLA BİR ORTAOKUL MATEMATİK ÖĞRETMENİNİN GEOMETRİDEKİ TEKNOLOJİK PEDAGOJİK ALAN BİLGİSİNİN (TPAB) GELİŞİMİ

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Bu çalışmanın amacı, özel bir okulda çalışan bir ortaokul matematik öğretmeninin matematik koçluğuna katılımı yoluyla altıncı sınıf 'Açılar ve Alan Ölçme' geometri öğrenme alanlarındaki TPAB (Teknolojik Pedagojik Alan Bilgisi) gelişimini araştırmak ve öğretmenin TPAB gelişimi açısından matematik koçluğuna yönelik algısını incelemektir. Çalışmaya katılan öğretmenin TPAB gelişimini Niess, Ronau vd.'nin (2009) TPAB Gelişim Modelinin temaları boyunca izlemek amacıyla şu veri toplama araçları kullanılmıştır: 1) matematik koçluğu öncesi TPAB görüşmesi, 2) matematik koçluğu öncesi öğretim uygulamalarının gözlemi, 3) ön konferanslar, 4) matematik koçluğu sırasındaki öğretim uygulamalarının gözlemleri, 5) son konferanslar, 6) matematik koçluğu sonrası öğretim uygulamalarının gözlemi ve 7) matematik koçluğu sonrası TPAB görüşmesi. Öğretmenin matematik koçluğu ile ilgili algılarını araştırmak için ise Matematik Koçluğu Görüşmesi yapılmıştır. Öğretmenin TPAB gelişimi, TPAB Gelişim Modeli kullanılarak didaktif analiz yöntemiyle analiz edilmiştir.

Çalışmanın bulguları, öğretmenin müfredat ve değerlendirme teması, öğrenme teması, öğretim teması ve erişim teması boyunca ve bu temaların TPAB Gelişim

Modeli'ne ait tanımlayıcılarda daha ileri seviyelere ilerlediğini ortaya koymuştur. Bulgular ayrıca, öğretmenin matematik koçluğuna katılımı sayesinde, GeoGebra'nın ana teknolojik araç olarak kullanıldığı teknolojiyle zenginleştirilmiş derslerle öğretim uygulamalarını geliştirdiğini ve Pedagojik Alan Bilgisi'nden (PAB) TPAB'ye doğru ilerlediğini göstermiştir. Bu sonuçlar, matematik koçluğunun TPAB gelişimini desteklemede, öğretim uygulamalarını geliştirmede ve teknolojinin sınıf öğretimine entegrasyonunu artırmada etkin olduğunu vurgulamaktadır.

Anahtar Kelimeler: Matematik Koçluğu, Öğretmenlerin Mesleki Gelişimi, Teknolojik Pedagojik Alan Bilgisi, GeoGebra, Geometri

To my father and son

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LIST OF ABBREVIATIONS

MoNE	: Ministry of National Education
METU	: Middle East Technical University
PCK	: Pedagogical Content Knowledge
TPACK	: Technological Pedagogical Content Knowledge

CHAPTER 1

INTRODUCTION

Digital communities worldwide have grown and changed due to the rapid advancement of technology over the years. In parallel, many countries have begun to explore how educational technology can enhance learning (Timotheu et al., 2023). Educational technology has become a powerful tool that can make learning better for today's students and improve how teachers teach (Ertmer & Ottenbreit-Leftwich, 2010; Higgins et al., 2012). Over the past ten years, there have been many new technology projects in education, with governments investing significant money to include ICT (Information and Communication Technologies) in schools (Jewitt et al., 2010; Tamim et al., 2015). Governments aim to make ICT a crucial part of education to improve the quality of education and help teachers and students become more skilled in using new technologies (Fu, 2013).

In this sense, the technological infrastructure of the schools in Türkiye has also been recently enforced. In the national context, the Ministry of National Education (MoNE) of Türkiye declared its vision for ICT in its strategic planning for 2010-2014 as incorporating ICT into the education system, enhancing the system with advancements, continually enhancing the education system through assessments, and facilitating student-centered and project-based learning through the effective use of ICT (MoNE, 2009). In line with these aims, the Ministry of National Education (MoNE) implemented projects to integrate technology into classrooms and enhance student educational opportunities. To exemplify, the FATİH (Movement of Enhancing Opportunities and Improving Technology) was the most crucial project with the largest scale in its MoNe's action plan for 2010–2014. In the scope of the FATİH project, smart boards were set up in classrooms, and fast internet connections were provided. Additionally, tablets were given to students, and hardware and software assistance was provided to schools and teachers (Kurt et al., 2013). MoNE

(2015), within the scope of its 2015-2019 strategic planning, adopted an expansion and development approach, and the distribution of tablets and smart boards was extended to more schools across the country. Moreover, improved digital content and resources were developed and integrated into the curriculum. Furthermore, Professional Development (PD) programs continued to enable teachers to use technology effectively in the classroom. Efforts were intensified to increase the accessibility of educational content through the Education Information Network (EBA) portal. Also, there was also a greater focus on digital literacy and responsible use of technology for students and teachers.

MoNE (2019), within the scope of its 2019-2023 strategic planning, adopted an integration and innovation approach and focused on cloud-based platforms for collaborative learning and resource sharing. Personalized learning tools were introduced to meet individual student needs. Efforts were made to use technology to promote inclusive education for students with special needs. Similarly, MoNE (2019) improved the opportunities of infrastructure to support high-speed internet and more sophisticated technology solutions. Furthermore, the EBA portal was continuously updated with diversified content for various learning needs.

Within the scope of its 2024-2028 strategic planning, MoNE (2024), focused on sustainability and enhanced integration, adopted emerging technologies such as virtual reality (VR), artificial intelligence (AI), and augmented reality (AR) for immersive learning experiences. MoNE (2024) has focused on cyber security to protect educational data and ensure the safe use of technology. The Internet of Things (IoT) has been mainstreamed in learning environments for smart classroom management. The idea of continuous development of digital content and platforms to support lifelong learning was adopted in the 2024-2028 strategic planning of MoNE. Additionally, many other initiatives launched by MoNE, such as EBA TV (Broadcasting channel for tutoring), ÖBA (Network of Teacher-Informatics), MEBBIS (Ministry of National Education Informatics System), MatemaTİK (mathematics-specific interactive website including resources and materials). Considering increasing attention paid and the importance attached to technology took place in educational settings worldwide, the researchers, therefore, concentrated on

the incorporation of technology, pedagogy, and content after realizing the significance of a broader view on teaching via technology (Yenilik ve Eğitim Teknolojileri Genel Müdürlüğü, 2018).

In this century, the requirement for teachers to be more proficient in technology skills and how these skills are seamlessly integrated into instructional practices and enforcing subject matter comprehension has increased (Kaufman, 2014). In response to this obligation, the International Society for Technology in Education (ISTE) developed innovative standards for teachers focused on digital literacy (International Society for Technology in Education, 2017). ISTE (2017) stated that today's teachers cannot just have technical know-how. Still, they must also be leaders who advocate for equal access to educational technology and other web-based resources. They are responsible for ensuring that all learners are equipped with tools and have learning opportunities to fulfill their varied needs (ISTE, 2017). The ISTE's (2017) standards emphasize the importance of being pioneers in the identification, exploration, evaluation, enhancement, and adoption of new digital resources and tools to use even outside the class, among others. ISTE (2017) stated that teachers must construct authentic tasks linked one-on-one with content area expectations if active deep learning occurs. Proficiency in leveraging digital resources and tools, coupled with applying principles regarding instructional design, empowers teachers to forge innovational learning environments in digital settings that affect and accompany students in their educational journey (ISTE, 2017). As stated more clearly in ISTE (2008), teachers are consequently expected to be adept at the following four tasks (ISTE, 2008, p. 1):

1. Design or adjust learning activities using digital tools to boost student learning and creativity.
2. Create technology-rich learning environments that allow all students to pursue their interests, set educational goals, manage their learning, and assess their progress.
3. Use digital resources to customize and personalize learning tasks to fit different learning styles, work methods, and abilities.
4. Provide various assessments that meet both content and technology standards and use the data from these assessments to improve teaching and learning.

The theory of instrumentalization helps explain how tools become integral parts of educational practices through their use and adaptation by teachers and students (Rabardel, 1995; Rabardel & Beguin, 2005). It highlights the dynamic and reciprocal relationship between users and tools, emphasizing the importance of customization and reflective practice in maximizing the educational benefits of technological tools (Rabardel, 1995; Rabardel, 2002; Trouche, 2004). Similarly, for teachers' comprehension of how to include technology in the classroom environment, Mishra and Koehler (2006) proposed the TPACK framework, which is shaped by the intricate interactions across three constructs of knowledge: Technology (TK), Content (CK), and Pedagogy (PK). Both theories underscore the dynamic interaction between users and technology, emphasizing the need for integration and adaptation to enhance educational practices. The TPACK framework, which goes beyond the individual knowledge structures it includes, represents an integrated body of knowledge designed for technology integration into instruction. While these knowledge structures—technology, content, and pedagogy—can be analyzed in isolation, they are inextricably linked and inseparable in practical implementation (Angeli & Valanides, 2005, 2009; Mishra & Koehler, 2006; Koehler & Mishra, 2008). This connection mirrors the Theory of Instrumentalization's focus on the interplay between users and tools. Since knowledge of technology cannot be conceived independently from the teaching context, it is crucial to study how technology might be integrated into education in more depth (Koehler & Mishra, 2008). Thus, both frameworks provide valuable insights into the effective incorporation of technology in educational settings, highlighting the necessity for a comprehensive understanding and adaptive approach to technology use.

Since technology integration into education has become fundamental, the research on educational technology has strongly emphasized the teachers' knowledge and skills concerning technology usage (Tondeur et al., 2008). Nevertheless, there is no clear definition of the technological knowledge that teachers should possess or a set procedure for how they should acquire it, even though numerous studies in the field of TPACK have been undertaken (Koehler & Mishra, 2005a). Moreover, several research studies have shown that simply introducing technology into learning and teaching does not lead to a significant change in integrating technology into learning

settings. It is substantial to adopt a comprehensive approach that considers the technology itself, the pedagogical methods, and the teacher's skills in its use (Hughes, 2005; Mishra & Koehler, 2006; Niess, 2005; Schmid et al., 2020). Additionally, it is necessary to evaluate the effectiveness of technology integration regularly and adjust accordingly (Carr et al., 1998). Therefore, it is crucial to emphasize the effective implementation of instructional technologies for teachers rather than solely their learning outcomes. Accordingly, further investigation is required into how technology can be integrated into educational settings, as understanding the teaching context is inseparable from understanding educational technology (Koehler & Mishra, 2005).

After the outbreak of the COVID-19 pandemic, many teachers from various disciplines struggled in terms of technological transformation of their courses while putting emergency remote teaching into practice to maintain students' learning and progress (Bozkurt & Sharma, 2020; Hodges et al., 2020). Most educational professionals were blindsided when this rapid shift to online learning environments from traditional learning settings occurred. Many institutions urgently employed PD programs for effective technology integration into online learning environments to maintain the learning and teaching process (Sato et al., 2023). Even though educational scholars asserted that PD aimed at providing teachers opportunities with an understanding of the dynamic relationship among pedagogy, technology, and content has the potential to enhance online course design, resulting in greater satisfaction among learners and increased confidence in teachers (Ward & Benson, 2010), teachers had no time to gain skills regarding technologies, research and make effective instructional design and practice for emergency remote teaching. All these difficulties experienced by teachers around the world provided evidence that the high proficiency in TPACK might enable teachers to develop new schemas for reconceptualizing content, pedagogy, and technology, thus playing a significant role in designing online learning environments by establishing connections between methods of teaching that are appropriate for a particular subject matter and are enhanced by technology throughout the TPACK development process (Ward & Benson, 2010). Consequently, the COVID-19 pandemic presented a chance and

responsibility to enhance the TPACK of both primary and secondary teachers (Galanti et al., 2020).

Given the insufficiencies and difficulties that teachers had in effective and meaningful technology integration into their lessons during the COVID-19 pandemic, a crucial need for effective and sustainable PD to develop teachers' TPACK arose (Elmaadaway & Abouelenein, 2023). Many research studies indicated that teachers' PD is crucial for the development of their TPACK as it plays a crucial role in the incorporation of technology into technological classroom environments (Harris & Hofer, 2009; Jaipal-Jamani & Figg, 2015; Koehler & Mishra, 2009; Koh et al., 2017; Niess, 2005; Schmid et al., 2020). These initiatives offer in-service teachers opportunities such as making more deliberate, strategic, and diversified choices regarding learning activities and tools, designing student-centered teaching activities, and making thoughtful choices for the responsible use of technology in education (Harris & Hofer, 2009).

Despite numerous PD programs and in-service training to support teachers, many teachers struggle to effectively integrate technology into their teaching practices in a way that enhances student learning (Ertmer & Ottenbreit-Leftwich, 2010; Hew & Brush, 2007; Polly, Mims et al., 2010). This struggle is attributed to existing programs needing to provide the practical and continuous support required for sustainable TPACK development (Desimone, 2009; Garet et al., 2001; Spangenberg & De Freitas, 2019). On the one hand, traditional (regular) PD programs, such as workshops, seminars, lecture series, conferences, and curriculum training sessions, often follow a conventional, structured approach (Darling-Hammond et al., 2017; Lawless & Pellegrino, 2007; Loucks-Horsley, Love et al., 2009). These programs typically focus on delivering content in a formal setting, usually away from the actual teaching environment (Desimone, 2009). Consequently, traditional in-service training often needs to develop teachers' TPACK more effectively since they are short-term and disconnected from daily practice, making it difficult for teachers to apply what they have learned (Niess et al., 2010). Moreover, these traditional approaches often need more personalized, practical, and sustained support for meaningful and lasting improvement in TPACK (Garet et al., 2001; Jaipal-Jamani &

Figg, 2015). On the other hand, non-traditional PD programs, also known as job-embedded PD, such as professional learning communities, coaching and mentoring, lesson study, and action research, are characterized by their flexibility, integration into daily teaching practices, and emphasis on collaboration and reflection (Loucks-Horsley et al., 2009). These methods allow teachers to learn and grow within their teaching environment, making the development process more relevant and immediately applicable (Bybee & Loucks-Horsley, 2000; Loucks-Horsley et al., 1996).

Considering the inadequacies of traditional PD activities for teachers and the effectiveness of innovative PD activities, the literature suggests a need for effective PD programs that focus on the intersection of technology, pedagogy, and content to address these gaps (Darling-Hammond et al., 2017; Garet et al., 2001; Sims & Fletcher-Wood, 2021). Effective PD should include hands-on experiences, collaborative learning opportunities, and personalized and continuous support to help teachers develop and apply TPACK (Harris & Hofer, 2009; Sims & Fletcher-Wood, 2021; Van Veen et al., 2012). Jaipal-Jamani and Figg (2015) suggested that to enhance teachers' TPACK, impactful PD courses should emphasize modeling a technology-enhanced activity as a learning tool. In addition, Sparks and Hirsh (2000) highlighted that efficient PD programs should be long-term, demanding, cumulative, and closely related to what teachers do in the classroom. Teachers must plan, carry out, and evaluate technology-enhanced classes in PD with an eye toward students' comprehension (Niess, Ronau, et al., 2009).

While many studies highlight the benefits of group-based PD, research is scarce exploring the impact of personalized, one-on-one coaching on TPACK development. One-to-one coaching offers tailored support that addresses individual teachers' specific needs and contexts, often missing in broader PD programs (Knight, 2007, 2008; Campbell & Malkus, 2011). Teachers need practice-oriented, face-to-face, and one-to-one training to effectively enhance their TPACK, as these methods provide the practical, hands-on experience and immediate feedback necessary for integrating technology into teaching practices (Darling-Hammond et al., 2017). Considering the characteristics of effective PD, Mathematics coaching stands out in terms of

teachers' PD as having these characteristics (Sims & Fletcher-Wood, 2021). Mathematics coaching has substantial potential to address the deficiencies of traditional and often ineffective in-service training (Bengo, 2013; McGatha et al. (2018). By offering personalized, practice-oriented, face-to-face, and individualized support, mathematics coaching has the potential to significantly enhance teachers' TPACK (Aygün, 2016; Aygün & Işıksal-Bostan, 2019). This approach ensures that technology integration becomes a fundamental aspect of their teaching practice, ultimately leading to improved student learning outcomes (Campbell & Griffin, 2017; Hull et al., 2009).

Moreover, the research studies indicate that coaching has a more significant and lasting impact on teachers' classroom practices than traditional PD (Driscoll, 2008; Gibbons & Cobb, 2017; Knight, 2019). McGatha et al. (2018) stated that mathematics coaching aims to improve teachers' instructional practices as a sustainable and efficient approach to PD. Mathematics coaching, which includes three phases (co-planning of the lesson at a pre-conference, observation of the implementation of the co-planned lesson, and debriefing and discussing the implemented lesson at a post-conference), provides teachers with a unique platform to improve their existing knowledge acquire new skills and knowledge, and gain experiences for classroom practice (Campbell & Malkus, 2011; Desimone & Pak, 2017; Knight, 2007). Teachers who participate in mathematics coaching go through a process to develop their knowledge of how to incorporate technology effectively and adequately into mathematics instruction (Drust, 2013; Hull et al., 2009; Larsen, 2012).

Coaching provides teachers with a one-on-one environment for PD to improve their practice, knowledge, and learning as they strive to integrate new knowledge into their teaching (Joyce & Showers, 1982). Coaching is a form of education that involves close and personalized discussion designed to improve the teacher's knowledge, understanding, and teaching abilities (Knight, 2009). In this personalized context, the coach collaborates and guides, working closely with the teacher to navigate the educational landscape (Showers & Joyce, 1996). The aim is to actively encourage critical thinking, improve teaching methods, and deepen the scope of

educational concepts (Buser, 2018). Coaching becomes a transformative process, where the coach's expertise combines the teacher's objectives and classroom realities in a mutually beneficial relationship. This initiative provides teachers with the required resources, techniques, and understanding to enhance their teaching abilities, thereby enhancing the educational experience for both students and teachers (McGatha et al. (2018).

Fishman and Davis (2006) asserted that developing a strong TPACK is critical for teachers, who should consider it a long-term acquisition. This viewpoint makes it clear that mathematics coaching, as the PD approach of this study, was not a one-day workshop disconnected from classroom practice. Instead, the present study aimed to result in the long-term and permanent development of teachers' TPACK. In this sense, mathematics coaching has the potential to facilitate in-service teachers' acquisition of the skills to incorporate technology into their teaching practice effectively and to assist teachers in integrating theory and practice. In addition, teachers who acquire and implement new pedagogical methods in their classrooms can receive continuous support from the coach (Bengo, 2013). Regarding the importance of the teaching practice for strong TPACK, Jen et al. (2016) stated that teachers' TPACK changes and develops during the actual teaching practice in the classroom. Jen et al. (2016) noted that instead of focusing on what teachers know about successful technology integration, researchers should focus on what happens in teachers' classrooms. Focusing on teachers' concurrent instructional decision-making processes, which are structured around classroom management and assessment, substantially impacts understanding teachers' TPACK (Jen et al., 2016).

Therefore, in this study, focusing on the teacher's TPACK and considering the essentials of effective PD, a mathematics coaching program was implemented to enhance the TPACK of a middle school mathematics teacher in geometry during actual teaching practices in the classroom, which allowed for in-action PD.

1.1. The Purpose and the Research Questions of the Study

This study aimed to investigate the changes in TPACK of an in-service mathematics teacher at the middle school level in the context of geometry through his

participation in the PD program of mathematics coaching. The Mathematics Teacher TPACK Standards and Development Model of Niess, Ronau et al. (2009) was adopted in the current study to determine a teacher's TPACK level before and after his participation in mathematics coaching and to monitor changes in teacher's TPACK during his participation in mathematics coaching. Another purpose of this study was to investigate the teacher's perceptions of mathematics coaching in terms of TPACK development. The following research questions served as a guide for this study:

1. To what extent does attending mathematics coaching influence a middle school mathematics teacher's TPACK in the learning areas of 'Angles and Measurement of Area'?

1. a. How does a middle school mathematics teacher who attends mathematics coaching progress in the Curriculum and Assessment Theme, Learning Theme, Teaching Theme, and Access Theme within the TPACK development model?

2. What are the perspectives of a middle school mathematics teacher regarding the impact of mathematics coaching on the development of TPACK?

1.2. Significance of the Study

With the rising need for technological integration in society and the K-12 education system, it has become imperative for teachers to have strong TPACK (Koehler et al., 2011; Mishra & Koehler, 2006). This specialized knowledge that teachers need to develop is crucial to help them apply technology in the teaching process to create a positive attitude towards learning (ISTE, 2008). Consequently, the significance of this study lies in adding to the knowledge of how mathematics coaching can assist teachers in building their TPACK to improve their teaching practices. Moreover, the theory of instrumentalization, which underlines the importance of teachers' beliefs and perceptions on technology in the process of technology integration into the classroom, can be considered a useful lens to analyze technology integration in education (Rabardel & Beguin, 2005). This theory is in consonance with the TPACK framework, which stresses the need to have a profound understanding of how technology can bring change in teaching and learning processes (Angeli &

Valanides, 2009; Kushner-Benson et al., 2015). Thus, the significance of this study is reinforced by the identification of how mathematics coaching, based on TPACK and the theory of instrumentalization, might help to enhance a teacher's TPACK using GeoGebra as a technological instrument.

Furthermore, related literature shows that research on mathematics teachers' TPACK and its development is still lacking, especially in the national and international contexts (Patahuddin et al., 2016; Zhang and Tang, 2021). Therefore, this study is significant since it aims to fill this gap by examining a middle school mathematics teacher's TPACK development in the context of geometry to provide useful information about the interconnections between TPACK components (Jang & Tsai, 2012; Harris & Hofer, 2009). In addition, the current literature shows that TPACK development differs for pre-service and in-service teachers, with the latter having a higher level of TPACK than the former (Yeh et al., 2014). While most TPACK studies concentrate on the TPACK of pre-service teachers, there are a few studies that have concentrated on in-service teachers' TPACK (Jen et al., 2016; Patahuddin et al., 2016). Therefore, the significance of this study lies in its ability to address this research gap to offer an understanding of TPACK development in an experienced mathematics teacher, which might further help in understanding effective technology use in education.

Also, the majority of TPACK studies have been conducted on digital tools for educational purposes, such as smartboards and graphing calculators, while there is a lack of studies on applications such as GeoGebra (Voogt et al., 2013). Therefore, this study is important as it explores how a middle school teacher implements GeoGebra into mathematics lessons in an effort to understand the change it brings to TPACK in geometry (Hohenwarter & Lavicza, 2007). The results may provide useful insights for further studies on the effectiveness of subject-specific educational technologies in mathematics teaching and learning.

Even though the TPACK framework is one of the most referred models in the field of education to explain teachers' knowledge regarding technology integration, it has been criticized for its lack of subject specificity (Angeli & Valanides, 2009; Graham,

2011). The TPACK Development Model by Niess, Ronau et al. (2009) aimed to resolve this issue since it offers subject-specific descriptors. Thus, in this study, using this model, the TPACK development of a middle school mathematics teacher in geometry is investigated to provide a more detailed picture of TPACK development, particularly in mathematics. The significance of this research lies in its potential contribution to the enhancement of the TPACK framework for mathematics teaching.

Additionally, it can be noted in existing literature that there is limited research that assesses teachers' TPACK in actual classroom teaching, with most studies conducting surveys and interviews (Polly & Brantley-Dias, 2009; Patahuddin et al., 2016). In contrast, this study includes classroom observations in the research design, which provides a richer understanding of how teachers develop and use TPACK in actual teaching practice, thus providing empirical data for the emergent field of mathematics education (Ertmer & Ottenbreit-Leftwich, 2010; Saubern et al., 2020).

Also, the COVID-19 pandemic has thus emphasized the importance of TPACK as teachers have had to teach in both synchronous and asynchronous learning environments (Bozkurt & Sharma, 2020; Trust & Whalen, 2020). Therefore, this study is significant in terms of its potential to shed light on teachers' TPACK and its development to sustain effective teaching in possible future synchronous and asynchronous learning environments (emergency remote teaching or blended teaching situations) and to inform PD programs (Chai et al., 2013).

Furthermore, in-service teachers typically have low levels of TPACK, which points to the importance of focused PD to improve the use of technology in teaching mathematics with suitable pedagogy (Spangenberg & De Freitas, 2019; Niess et al., 2010). Thus, this study is significant since it examines the effects of mathematics coaching as a form of PD, as well as how such interventions might contribute to the improvement of teachers' TPACK (Desimone & Garet, 2015; Polly, 2011).

In addition, coaching as a PD strategy has certain benefits over traditional forms of PD as it includes ongoing and individualized support to teachers (Borko, 2004; McGatha, 2009). Specifically, mathematics coaching sessions that include core

components such as planning lessons together, observing the teaching practices in the classroom, and debriefing sessions that involve reflection help the teachers improve their TPACK (Knight, 2007; McGatha et al., 2018). Therefore, this study is significant since it aims to examine how mathematics coaching may enhance the development of TPACK in the long term, especially in the area of geometry, to provide a model for effective PD in mathematics education (Mudzimiri et al., 2014; Polly & Brantley-Dias, 2009).

In conclusion, this study is significant as it investigates the impact of mathematics coaching on a teacher's TPACK development and perception of mathematics coaching regarding TPACK development while presenting findings that can be used to design effective and sustainable professional development programs concerning technology use in mathematics teaching (Desimone & Garet, 2015; Fishman & Davis, 2006).

1.3. Definitions of Important Terms

Middle School Mathematics Teacher teaches mathematics at the middle school level, comprising grades 5, 6, 7, and 8. The middle school mathematics teacher in this study taught at a private middle school in Türkiye as a part of the primary education system.

Technological Pedagogical Content Knowledge (TPACK) is a framework developed by Mishra and Koehler (2006) that builds upon Shulman's (1986, 1987) concept of Pedagogical Content Knowledge (PCK). TPACK delineates the interplay between teachers' understanding of educational technologies and their Pedagogical Content Knowledge (PCK) to illustrate how teachers can proficiently incorporate technology into their teaching practices (Mishra & Koehler, 2006). In the context of this study, 'TPACK' specifically refers to the appropriate and effective utilization of GeoGebra for teaching geometry topics within the learning areas of 'Angles and Measurement of Area'. TPACK in this study involved integrating technology, pedagogy, and content knowledge to enhance the overall process of teaching and learning mathematics (Mishra & Koehler, 2006).

GeoGebra is a freely available dynamic geometry software that combines Dynamic Geometry System and Computer Algebra System (CAS) into one easy-to-use system. GeoGebra was utilized as the main technological tool of learning and teaching in this study, which the participant teachers integrated into mathematics lessons (Hohenwarter & Lavicza, 2007).

The TPACK Development Model was developed by Niess, Ronau et al. (2009) to determine the TPACK level and monitor changes and development in teachers' TPACK level. This model involves four specific themes: (1) Curriculum and assessment theme, (2) Learning theme, (3) Teaching theme, and (4) Access theme. This study adopted this model to identify the TPACK level and monitor the TPACK development of the participant mathematics teachers in planned and implemented technology-enriched mathematics lessons through GeoGebra as the main technological tool.

Coaching in professional development is used to describe a strategic, supportive process designed to help individuals (the 'coachees') enhance their skills, achieve their professional goals, and improve their performance in a specific area through personalized support, goal setting, skill development, feedback, reflection, and accountability (Hall & Simeral, 2008; Hull et al., 2009; Knight, 2007; West & Staub, 2003).

Mathematics coaching, defined as a professional development approach aimed at improving teaching practices (McGatha et al., 2018), is employed in this study to enhance the TPACK levels of a middle school mathematics teacher, specifically regarding geometry, the learning areas of 'Angles and Measurement of Area' in particular. Mathematics coaching in this study involved engaging the participant teacher in a cyclic process comprising three key stages: Co-planning of lessons during pre-conference meetings, observation of teaching practices, and evaluation of the implemented lessons during post-conference meetings (Bengo, 2013; Liao et al., 2021; McGatha et al., 2018).

Mathematics coach is a knowledgeable professional proficient in both mathematics content and pedagogy (Campbell & Malkus, 2011). A mathematics coach

collaborates directly with classroom teachers to improve their teaching practices and enhance student learning in mathematics (McGatha et al., 2018). In this study, the researcher acted as the mathematics coach who conducted mathematics coaching cycles with the participant teacher.

CHAPTER 2

LITERATURE REVIEW

The primary purpose of this study was to investigate the changes in TPACK of a middle school mathematics teacher in the 'Angles and Measurement of Area' learning domains of geometry. The study examined how a teacher's TPACK is impacted by his involvement in mathematics coaching. This study also examined the teacher's perceptions of mathematics coaching related to TPACK development. Accordingly, this chapter aims to provide an understanding of the Theoretical Framework of TPACK, TPACK Development, and Mathematics Coaching. Therefore, this chapter is organized under four sections: 1) The Theoretical Framework of TPACK, 2) The TPACK Development Model of Niess, Ronau, et al. (2009), and 3) Professional Development Approaches to Mathematics Teachers. A summary of the literature review appears at the end of the chapter as the fourth section.

2.1. Theoretical Framework of TPACK

This section presents information about the TPACK framework (Mishra & Koehler, 2006), used in this study to evaluate the participant teacher's knowledge regarding integrating technology into teaching practice with proper pedagogy.

In 1986, Lee Shulman developed a groundbreaking concept to comprehend the knowledge teachers need to instruct using a construct. He termed this knowledge 'Pedagogical Content Knowledge (PCK).' This new approach mandated that teachers integrate their primary content and pedagogical knowledge to teach effectively. Knowledge about teaching and learning is central to PCK. PCK, situated at the intersection of PK and CK, is described as the ability to formulate and communicate the knowledge of the subject matter comprehensibly for the students (Shulman, 1986,

1987; Wilson et al., 1987). Consequently, modern digital technologies are now recognized as valuable tools for learning and teaching as this understanding of PCK has developed. Shulman's idea was expanded by Mishra and Koehler (2006) by adding technology and a 'T' to the framework of PCK. Mishra and Koehler (2006) incorporated the element of 'T' into Shulman's Pedagogical Content Knowledge (PCK) framework to encompass essential qualities of knowledge necessary for teachers to effectively integrate technology into their teaching, recognizing the intricate, multifaceted, and context-dependent nature of teacher knowledge. Consequently, expanding the Pedagogical Content Knowledge (PCK) model was deemed appropriate to comprise technology as a core component (Mishra & Koehler, 2006).

TPACK framework concentrates on teachers' content, technology, and pedagogy knowledge. A teacher's comprehension of how to instruct a particular subject in the ways of presenting and articulating a subject in a manner that makes it understandable to others is known as PCK. Put differently, PCK is how to modify certain content so that it can be taught (Eichelberger & Leong, 2019).

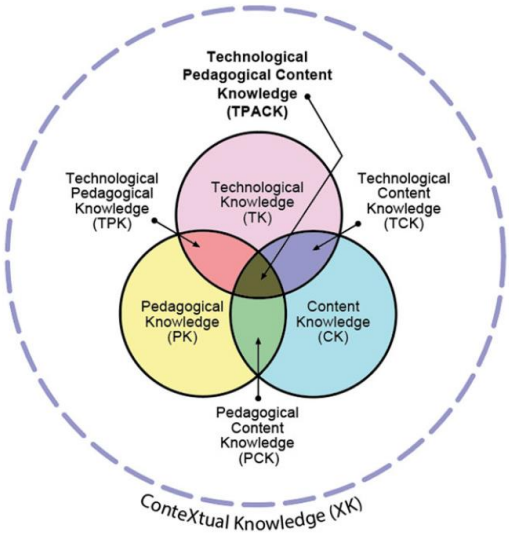


Figure 1. Upgraded diagram of TPACK Framework with the addition of XK (Mishra, 2019, p. 77)

How pedagogy, content, and technology interact with one another produces seven discrete subsets, which are illustrated in Figure 1: Technological Knowledge (TK),

Pedagogical Knowledge (PK), Technological Content Knowledge (TCK), Content Knowledge (CK), Technological Pedagogical Knowledge (TPK), and Technological Pedagogical Content Knowledge (TPACK). Based on a study by Mishra and Koehler (2006), the connections or interactions among the various subsets carry equal significance in the TPACK framework. The TPACK diagram has undergone a revision in 2019. Previously named "Contexts," Mishra (2019) has changed the name of the outermost circle to "Contextual Knowledge (XK)" (i.e., the comprehension of the teacher regarding the context).

As one of the three primary domains, *Content Knowledge (CK)* pertains to the understanding of the specific subject matter being taught or learned. This domain focuses on discipline-specific thinking patterns that are characteristic of each field. CK comprises ideas, theories, concepts, organizational frameworks, proofs, evidence, and established approaches and practices for cultivating such knowledge (Mishra & Koehler, 2006). The other primary component is *Pedagogical Knowledge (PK)*, which refers to teachers' understanding of instruction methods, strategies, and approaches. This knowledge encompasses a variety of topics, including broad educational objectives, values, and purposes. PK includes techniques for evaluating students' comprehension as well as information on the characteristics of the intended audience (Mishra & Koehler, 2006). *Technological knowledge (TK)* is described as the knowledge of both more basic technologies (e.g., blackboard with chalk, overhead projector, and textbooks) and more sophisticated technologies (e.g., a projection device, a device with an internet connection, the digital video), as well as the skills required to use specific technologies. Moreover, the ability to pick up new skills and adjust is especially emphasized because of how quickly technology changes (Mishra et al., 2009). *Pedagogical Content Knowledge (PCK)*, as proposed by Shulman in his PCK framework, recognizes that teaching effectively goes beyond mere comprehension of content and pedagogy. Instead, it is knowledge of pedagogy that makes content-area education remarkably easier.

The core components of PCK are learning, teaching, assessment, course content, and reporting; they include learning-enhancing circumstances and the connections between curriculum, pedagogy, and assessment (Mishra & Koehler, 2006).

Technological Pedagogical Knowledge (TPK) is described as the comprehension of how learning and teaching might be influenced by technology when used in specific ways. TPK requires an awareness of the pedagogical restrictions and advantages offered by numerous technological tools concerning developmentally and disciplinarily suitable pedagogical methods and designs, as stated by Mishra and Koehler (2006). *Technological Content Knowledge (TCK)* refers to understanding how a teacher comprehends the content-technology relationship and its limitations. Since TCK acknowledges technology's impact on knowledge and practices within a specific discipline, it is crucial to create suitable technological tools for instructional purposes (Mishra & Koehler, 2006). *Technological Pedagogical Content Knowledge (TPACK)* is an emerging type of knowledge that encompasses the intricate connections among the three fundamental components: Technology, content, and pedagogy. Mishra and Koehler (2006) asserted that the TPACK framework suggests that successful teaching with technology requires a grasp of how concepts are conveyed through technologies, the utilization of pedagogical strategies that employ technologies for content instruction, an awareness of the obstacles and supports for learning particular concepts, and the capability of technology to address students' difficulties in learning. According to Mishra and Koehler (2006), TPACK also includes an awareness of students' prior understandings and epistemological conceptions and how technologies can enhance or construct new epistemologies.

This study employed the TPACK framework Mishra and Koehler (2006) introduced as a theoretical perspective to examine how a middle school mathematics teacher integrated technology into mathematics classrooms.

2.1.1. Integrative vs Transformative Perspectives of TPACK

In the evolving landscape of educational practices, the TPACK framework has been instrumental in guiding the integration of technology into teaching (Mishra & Koehler, 2006). However, varying interpretations of this framework have emerged, leading to the development of two primary perspectives: integrative and transformative.

In examining the literature, two distinct perspectives define the TPACK framework - the *integrative* and *transformative* perspectives. Angeli and Valanides (2009) stated that *the integrative perspective* of TPACK is rooted in Shulman's (1986) PCK model, amalgamating knowledge of content, pedagogy, and technology that teachers utilize in the instructional process. However, Angeli and Valanides (2009) asserted that the integrative model is uncertain due to incorporating diverse definitions for TPACK components. They stress the need to construct a new theory that is better defined and less open to interpretation. Due to the constant evolution of information in this digital era, teaching practices must be updated by integrating various technological tools while considering teachers' contextual learning environment and the TPACK framework rather than solely adopting segmented component definitions (Angeli & Valanides, 2009).

The transformative perspective relies on the interaction and transformation of the knowledge related to content, technology, and pedagogy (Canbazoğlu-Bilici et al., 2016). This view of TPACK helps to extend the idea of technology integration in the teaching and learning process. Instead, it encourages them to rethink their teaching and course design in a manner revolutionized by technology, improving traditional teaching methods. This approach does not perceive technology as another tool that can be added to the teacher’s toolbox but as a tool that revolutionizes teaching and learning, forcing teachers to transform and recreate their instructional practices to make the best use of the available technological tools (Canbazoğlu-Bilici et al., 2016). A review was conducted by Koehler and Mishra (2008), acknowledging that the TPACK components should not be evaluated in isolation. When integrating technological tools with all other constituent and related components, instructional processes must occur. The components of TPACK, namely content knowledge, pedagogical knowledge, and technological knowledge, are not considered distinct and unrelated. Instead, Koehler and Mishra (2009) depicted TPACK as situated within the context of the learning environment in the classroom, characterized by active, multifaceted, and changeable elements linked to teaching with technology. Thus, technology-enhanced teaching practices go beyond the components of TPACK as a separate set of skills and teaching methods (Kurt et al., 2013). Essentially, this framework undergoes a transformational process to articulate its structure and

connections clearly and concisely. As Angeli and Valanides (2009) confirmed, the additional elements complementing TPACK in terms of context make it a distinct form of knowledge, diverging from the mere combination of its knowledge domains. Moreover, Rosenberg and Koehler (2015) stated that TPACK transcends the mere amalgamation of its knowledge domains; it embodies a transformative body of knowledge that appears when teachers integrate technology, pedagogy, and content into teaching. Thus, the measurement of TPACK has become an increasingly crucial issue due to its situated and sophisticated nature (Jen et al., 2016).

In the current research, the transformative view of the TPACK model, as described by Canbazoğlu-Bilici et al. (2016), was used because it is suitable for changing teachers' content, pedagogical, and technological knowledge through mathematics coaching. This perspective considers technology as another tool and a mechanism to transform current teaching practices to make lessons more interesting, student-oriented, and characteristic of good mathematics teaching (Canbazoğlu-Bilici et al., 2016). With this view in mind, this study intends to explore how the involvement in mathematics coaching affects the TPACK of a middle school mathematics teacher in the context of integrating critical knowledge TPACK domains into teaching and learning.

2.1.2. Research Studies on TPACK

The TPACK framework is essential to explain how technology can be appropriately incorporated into the teaching practice (Mishra & Koehler, 2006). This section presents the literature that focuses on the effect of TPACK on teaching practices, emphasizing mathematics education and the challenges and solutions associated with the growth of TPACK in teachers. These are important to guide the professional development that seeks to improve on the TPACK of the teachers.

Even though technology has been incorporated into the teaching-learning process across the globe, research has indicated that teachers incorporate it to a limited extent or for simple tasks (Ertmer & Ottenbreit-Leftwich, 2010; Howard et al., 2015; Inan & Lowther, 2010). Accordingly, technology is gradually being integrated into the

teaching and learning process; for example, teachers use it to set tasks and ask students to work on them on computers (Scherer et al., 2015; Olofson et al., 2016). However, several research studies have revealed challenges hindering teachers from effectively integrating technology into teaching. These include inadequate teaching experience of the teacher, misconceptions about technology, limited time and resources, and high expectations from students regarding the use of technology (Ismail et al., 2020). Other barriers include the differences in teaching approaches argued by external sources (Ertmer et al., 2012) and the poor assessment to determine students' technological learning achievements (Tondeur et al., 2017). Most of these problems can be solved if teachers are more eager to use technology in their teaching rather than innovatively (Ottenbreit-Leftwich et al., 2010; Kim et al., 2013).

As a result of these challenges, the theoretical model known as TPACK has become the most current and practical approach for implementing technology into the teaching process. TPACK is the set of knowledge that teachers accumulate by using technology in the teaching and learning process (Koehler & Mishra, 2005; Mishra & Koehler, 2006). It is a comprehensive repository of information that teachers can use to combine the different facets of teaching (e.g., material, learners, and provisions) in a sequence of knowledge transformations (Yeh et al., 2014).

Mishra and Koehler (2006) explained the perception that teachers and graduate students had towards TPACK. The authors documented the shifts in TPACK levels of the participants in their 'Learning Technology by Design' course and noted that TPACK and technology knowledge enhanced progressively throughout the semester. The given research results revealed that TPACK is an independent factor that depends on technology, pedagogy, and content, as proposed by Koehler and Mishra (2005). Based on this, the role of TPACK in improving teaching practices is also underlined in the work of Jang and Tsai (2012), who studied TPACK among Taiwanese mathematics and science teachers. Their work highlighted why TPACK is helpful for the enhancement of teaching and learning with technology as a tool to enhance the quality of teaching and learning. Mishra, Koehler, and Cain (2013) also underlined that pre-service teachers should have a well-developed TPACK because understanding the relationships between pedagogy, content, and technology is crucial

for implementing technology into educational practices. Guerrero (2010) also supported the importance of TPACK in mathematics education in integrating technology. In this regard, Guerrero (2010) claimed that having a well-defined TPACK framework is crucial to enable technology to improve students' engagement, achievement, and confidence in mathematics.

With the growing realization of the significance of TPACK, the need to properly assess it to evaluate teachers' readiness and potential gaps became inevitable. This need was underlined by Archambault and Barnett (2010), who assessed the factors affecting the validity of instruments used to assess TPACK. The authors' work revealed the difficulties of measuring TPACK since the model elements are interconnected. Similarly, Agyei and Voogt (2012) also discussed ways of assessing TPACK in pre-service teachers, including surveys, interviews, and observations. They observed that surveys are essential, but they may not accurately measure how TPACK is applied in real classrooms; thus, the use of multiple instruments was recommended. The attention given to the enhancement of teachers' TPACK is also evident in the literature. Niess (2011) developed a professional development project for in-service mathematics teachers to enhance the TPACK of those teachers. Harris and Hofer (2011) elaborated on how TPACK could support the planning of instructions and improve the teachers' TPACK in action during teaching procedures. They stated that professional development anchored on the TPACK model enhances teachers' planning regarding course delivery, enhancing the quality of teaching.

Based on the research studies in the related literature, the present study is significant given its attempt to explore the changes in TPACK of a middle school mathematics teacher within the 'Angles and Measurement of Area' learning domains of geometry. The present study focused on the change in the teacher's TPACK due to mathematics coaching, a specific professional development measure. The focus on the PD and the role of TPACK, which was discussed in the literature, corresponds well with the goals of the current research. Thus, this study aimed to extend the understanding of how mathematics coaching influenced TPACK development from the teacher's perspective and how it specifically affected the teaching of geometry using technology.

2.1.3. Research Studies on TPACK Focusing on Geometry

This section illustrates the importance of TPACK in geometry teaching by reviewing related studies. Technology integration in mathematics teaching, especially teaching geometry, has become one of the significant areas of concern in research for the last two decades. The TPACK framework has provided a powerful instrument that illustrates how teachers can enhance the use of technologies to provide effective learning environments in teaching practices. In particular to geometry teaching, the TPACK framework assists the teacher in relating the content, pedagogy, and technology knowledge to enhance the student's learning process (Archambault & Barnett, 2010). These intersections of knowledge domains are crucial because they enable teachers to do more than lecture geometrical concepts; they also improve students' conceptual understanding in technology-enriched learning environments.

When using technology to teach geometry, students can experiment with ideas and hypotheses and learn about geometrical shapes' properties, which is one of the most important benefits. According to Jones (2002), students must be allowed to use technology, especially in a dynamic geometry environment, to recognize, estimate, invent, and explain geometric concepts. According to the TPACK framework, the above-identified technological advantages reveal the significance of the teacher's knowledge of the most suitable ways of content and pedagogical knowledge in combination with technological knowledge. In this shifting learning environment, students can create geometric figures that a mouse may move and drag around the screen. Also, all the geometric relations incorporated during the construction of the geometric object are maintained when the students manipulate and transform the object in size. This is because the changes made to the shape are systematic in a way that each shape transformation adheres to specific geometrical rules of the initial shape (Laborde et al., 2006). This kind of environment enables the creation of geometric objects and the comprehension of the relationships between these objects.

From the analysis of the above studies, it can be concluded that TPACK is vital in improving the teaching of geometry lessons provided through technological tools. For instance, Saralar et al. (2018) carried out a multiple case study to explore the

TPACK of a preservice mathematics teacher when teaching three-dimensional shapes using GeoGebra, a dynamic geometry software. This research attempted to ascertain the impact of integrating technology, pedagogy, and content knowledge on teachers' performance in teaching complex geometry concepts. In addition to enriching the teacher's content knowledge of 3D shapes, the researchers established that GeoGebra offered creative ways of presenting and transforming these shapes, thereby helping the students to develop a deep understanding of the concept. The study insisted that pre-service teachers need TPACK to teach geometry because visual and interactive aids effectively enhance learners' understanding. On a similar note, Guerrero (2010) examined the use of TPACK in the mathematics class, especially in geometry. The study outlined four central components of TPACK necessary for effective technology integration in mathematics instruction: The subcategories included teacher's understanding of the conception and application of technology, technology-integrated mathematics teaching, classroom management, and depth and width of mathematics content knowledge. The study stressed the significance of teachers deciding when and how technology can help teach geometrical concepts to the students.

In summary, the related literature on TPACK and geometry teaching and learning indicates that technology is essential to contemporary mathematics teaching. Therefore, teachers must build strong TPACK knowledge to tap into technology fully in teaching geometry. This entails not only the knowledge of the subject and how geometry content should be taught but also how to effectively and efficiently incorporate technological tools into the teaching and learning process to improve the student's understanding and interest in the subject. Thus, the analysis of TPACK's potential for geometry education is essential in the process of preparing teachers for the challenges of 21st-century classrooms.

Based on the role of TPACK in geometry teaching, this study aimed to examine the changes in a middle school mathematics teacher's TPACK during a mathematics coaching professional development program. In this way, the study intended to shed light on the viability of such programs in promoting technology usage in geometry classes.

2.1.4. Dynamic Geometry Software

Dynamic geometry software (DGS) has introduced environments enhanced with dynamic features for learning geometric concepts in the teaching and learning of mathematics. Several DGS tools are widely used today, including GeoGebra, Geometer's Sketchpad, Cabri, Geometry Expressions, and Cinderella. Each of these tools is described by some features and goals that are focused on supporting the effective teaching and learning of mathematics (Hohenwarter & Lavicza, 2007; Jackiw, 1991; Laborde, 2000; Richter-Gebert & Kortenkamp, 1999; Todd, 2006). Geometer's Sketchpad is a software product created by Jackiw in 1991. It lets the user draw and transform geometric figures on the computer screen and study their properties. Likewise, as documented by Laborde (2000), Cabri presents interactive geometry tools that assist students in enhancing their understanding of geometry and its features. Geometry Expressions, as pointed out by Todd (2006), is an application that incorporates both symbolic algebra and dynamic geometry where users can construct geometric objects and get their equivalent algebraic representations, hence closing the geometry algebra gap. Cinderella, introduced by Richter-Gebert and Kortenkamp (1999) can support both Euclidean and non-Euclidean geometry and add physics, making it a handy tool for teaching mathematics and science. All these tools provide specific features that enhance mathematics learning in a more interactive and fun way.

The subsequent sections provide the rationale for adopting GeoGebra in this research, information on GeoGebra, and various research studies investigating the effects of professional development interventions with GeoGebra.

2.1.4.1. Dynamic Geometry Software of GeoGebra

GeoGebra is a versatile and powerful software that contains geometry, algebra, statistics, and calculus among its features, and that is why it can be used at all levels of education since it is easy to use and has a large community of users (Hohenwarter & Lavicza, 2007). There are other DGSs like Geometer's Sketchpad, Cabri-Geometre, Geometry Expressions, and Cinderella, which also have unique features

but have their weaknesses compared to GeoGebra. For example, Geometer's Sketchpad and Cabri-Geometre are not freeware, and Geometry Expressions is mainly based on the algebraic representation of geometric objects, which is not always easy to understand (Hölzl, 2001; Jones, 2012; Richter-Gebert & Kortenkamp, 2012).

GeoGebra was chosen as the primary technological tool in this study for several reasons: First, it is free of charge and open source, making it accessible to schools with low or no funding (Hohenwarter et al., 2008). Second, it encompasses all the branches of mathematics, thus making it suitable for use at different levels of education (Hohenwarter et al., 2009). Third, it has a user-friendly interface that would reduce the time teachers and students would take to learn how to use it (Zulnaidi & Zakaria, 2012). Also, GeoGebra is backed by many users and developers who provide materials such as lesson plans and tutorials, and the software is compatible with many operating systems to enhance its usability in different classrooms (Hohenwarter & Lavicza, 2007). GeoGebra has been continually updated, and new features have been added to it from time to time depending on the feedback received from the users, which makes it very useful in educational contexts (Fahlberg-Stojanovska & Stojanovski, 2009). Studies have also indicated that GeoGebra enhances mathematics learning among students and their interest in the subject because of flexibility, ease of use, and extensive community support (Hohenwarter et al., 2009). Finally, it can be stated that GeoGebra will be the most appropriate dynamic geometry software for this research due to its easy accessibility, flexibility, understandability, and strong support from the community, which makes it an ideal tool for improving mathematics education at different levels.

In a nutshell, all the above reasons make GeoGebra the most appropriate tool for dynamic geometry software in the study due to its effective and enhanced tools, ease of use, and frequent updates (Hohenwarter & Preiner, 2007).

2.1.4.2. GeoGebra in TPACK Studies

This section presents various research studies that examined the effects of professional development activities employing GeoGebra, the main technological

tool in this research, on the development of teachers' TPACK in geometry teaching and learning.

Relating to the focus and objective of the present study, Kartal and Çınar (2022) conducted a three-semester longitudinal, two-phase, multiple case study to examine the development of pre-service elementary mathematics teachers' (PSTs) TPACK while teaching polygons using GeoGebra through Microteaching Lesson Study (MLS). The study aimed to illustrate and explain how polygons can be represented and taught with GeoGebra using the transformative approach of the TPACK framework. Participants of MLS were to plan, implement, and assess the technology-integrated pedagogy. The research data was gathered from lesson plans, observation of teaching practices, and interviews. First, 33 PSTs were pre-tested via the TPACK-Self Assessment Scale, TPACK survey, and a polygon content knowledge questionnaire. Six PSTs were chosen for the study, and the four cases were taught using technology-based interventions. They were exposed to 15 hours of TPACK workshops that showed the integration of technologies in teaching learning topics. The TPACK model by Niess, Sadri, and Lee (2007) was employed to analyze participants' TPACK levels. PSTs had challenges with the use of technology, but they were able to develop their TPACK after reflecting on their teaching practices. The study findings showed that technology implementation was enhanced as the teachers' experience and confidence increased in using GeoGebra. One of the participants did not show improvement, which was linked to attitudes towards technology (Kartal & Çınar, 2022).

Similarly, in 2021, a study led by Benning focused on uncovering the essential strategies for effectively teaching mathematics with GeoGebra. In a 12-month professional development program, eleven in-service mathematics teachers developed lessons based on GeoGebra, which they eventually shared with their peers and students. Teachers' behaviors and perceptions of using GeoGebra to implement mathematics classes were examined to identify the essential elements of successful mathematics pedagogy. Benning's (2021) study employed focus groups, interviews, lesson plans, and observations to gather data. The results demonstrated that teachers could, to varied degrees, use fundamental methods in five crucial areas of successful

mathematics instruction via GeoGebra: (1) Establishing a mathematical environment, (2) Providing students with challenging tasks, (3) Facilitating mathematical discussions, (4) Establishing connections in mathematics, and (5) Assessing the learning of students. The findings showed that professional development programs are essential to developing TPACK among teachers. Such programs are beneficial in helping the teachers enhance their understanding of the technology that can be used in the classroom, such as GeoGebra, to foster better teaching and learning of mathematics. The study showed that GeoGebra can enhance effective teaching and learning of mathematics. However, the findings point towards the need for more systematic and comprehensive support for GeoGebra to engage students effectively in the learning process.

Continuing the review on the professional development with GeoGebra, Bueno et al. (2021) carried out a qualitative study to explore the development of mathematics teachers' TPACK (both pre-service and in-service teachers) via an online course that was aimed at designing educational applications and puzzles with the help of GeoGebra. This course was designed and implemented within six weeks. It was divided into four stages of the TPACK development model proposed by Niess, Ronau et al. (2009) and named 'Development of Digital Logic Games and Apps through GeoGebra.' A total of 30 participants were recruited, and 19 of them finished the course and filled out the survey. The course was designed to improve the teachers' technological pedagogical content knowledge in GeoGebra and to motivate the teachers to design engaging and interactive GeoGebra resources to enhance students' engagement and self-regulation. As the course developed, the participants raised issues of providing multiple ways of learning through ICT and recognized the importance of maintaining continuous professional development. The study revealed that using GeoGebra boosted the participants' confidence and comer. Thus, they made a significant improvement in their TPACK. This progress was not limited to using GeoGebra only; it also included the creation of ICT lessons and activities and shifting the focus towards students' learning. In addition, it was observed that participants endeavored to integrate technology, pedagogy, and content in their teaching practices.

Açıkgül and Aslaner (2020) performed a study to investigate the effects of the Game-based TPACK Practices (GBTP) and GeoGebra-supported Micro Teaching Applications (GSMTA) on TPACK efficacy and self-confidence perceptions of the preservice teachers in polygons. In the study, eighty-eight prospective mathematics teachers were equally selected as participants and assigned to four treatment groups. The first set of teachers in the study received training in Micro Teaching Applications and GBTP, while the second set received training in GSMTA, the third set received training in Micro Teaching Applications only, and the fourth set of teachers received training in both GSMTA and GBTP. The TPACK Scale for Geometry was administered to assess pre-service teachers' self-efficacy on the TPACK. In addition, teachers' efficacy was assessed based on the lesson plans and participant forms. The research established that GeoGebra was central and versatile in the growth of the TPACK of prospective mathematics teachers. It served as an instrument and a resource material that enhanced the TPACK to a great extent, especially in terms of technological content knowledge (TCK). The use of GeoGebra-supported micro-teaching applications in this study resulted in a higher post-test efficacy score, thus supporting the use of technology in teaching and learning. Implementing GeoGebra helped teachers better accomplish learning objectives, especially in teaching polygons, because GeoGebra offered good tools for lesson planning and instructional design. Still, GeoGebra applied with TPACK game practices showed the most significant improvements in the TCK and TPACK, yet with a relatively minor effect on the CK and PCK. This can be seen as an essential aspect of GeoGebra in enhancing the use of technology in teaching and learning practices.

The findings of the studies described in this section suggest the importance of professional development and particular approaches such as Microteaching Lesson Study, Game-based TPACK Practices, and GeoGebra-supported workshops for enhancing the teachers' TPACK. These findings align with the objectives of the current study, which was to assess the changes in TPACK of a middle school mathematics teacher in geometry, particularly the 'Angles and Measurement of Area.' Therefore, in this study, the research question was to assess the changes in a teacher's TPACK in geometry through collecting comprehensive data during

mathematics coaching sessions where the participant teacher was involved in designing and delivering lessons mainly using GeoGebra as the primary technology tool.

2.2. TPACK Development Model and Standards for Mathematics Teachers

The following methods and models for TPACK development are discussed in this section. The following sections describe the rationale and reasons for using the TPACK development model of Niess, Ronau et al. (2009) to track the changes in TPACK of the middle school mathematics teacher in this study. Subsequently, the Mathematics Teacher TPACK Standards of Niess, Ronau et al. (2009) are described in detail. The following are the TPACK development approaches and models by various authors in the existing literature, as discussed in Zhang & Tang (2021).

Various studies showed that the teachers who have yet to participate in the specific professional development programs related to TPACK have lower levels of TPACK knowledge (Koh et al., 2017; Schmid et al., 2021). In response, educational researchers have developed different models and methods to support the development of TPACK to enhance teachers' use of technology in their teaching (Tondeur et al., 2017). Various models and methods have been suggested for unpacking teachers' TPACK growth. The most popular methods of TPACK development are the Learning by Design Approach, Lesson Study, Microteaching, Microteaching Lesson Study (MLS), and TPACK development models that include the TPACK-Practical, TPACK-COPR, SQD Model, TPACK-COIR, TPACK-IDDIRR, and Mathematics Teacher TPACK Standards and Development Model (Niess, Ronau et al., 2009).

The TPACK development methods include the Learning Design Approach (LDA), a collaborative method of developing TPACK in light of actual teaching and learning challenges. LDA offers teachers a model that fosters them in developing solutions to professional learning and teaching issues in teams and, therefore, fosters the integration of content, pedagogy, and technology (Koehler & Mishra, 2005a, 2005b). Koehler and Mishra (2005a, 2005b) showed how LDA can assist teachers in learning

about educational technology and building their TPACK while engaging in collaborative design processes, such as designing an online course (Koehler et al., 2014). Lesson Study and Microteaching Lesson Study (MLS) are other forms of practice-based professional development that are more popular in the Japanese context (Fernández, 2010). Lesson Study is a form of participatory teaching practice that entails planning, observing, and reflecting on a 'Research lesson,' it has been employed in many TPACK types of research (Cavin, 2007; Kurt, 2016). MLS integrates micro-teaching with lesson study, which provides a well-organized setting for preservice teachers to refine their teaching and content knowledge (Fernández, 2005; Murtafiah & Lukitasari, 2019).

In terms of the TPACK development models, several models have been suggested to assist teachers in incorporating technology into their teaching and tracking the TPACK's progression. For instance, the TPACK-Practical (TPACK-P) model proposed by Yeh et al. (2014) focuses on enhancing TPACK through practical teaching experiences. As another model, Jang and Chen (2010) developed the TPACK-COPR model, which incorporates TPACK understanding, observation, teaching practice, and reflection to improve the TPACK of preservice science teachers. Moreover, the SQD Model, identified by Tondeur et al. (2012), focuses on six key themes: learning technology through design, scaffolding authentic experiences, role models, peer collaboration, reflecting on technology's role, and ongoing feedback. The TPACK-COIR model, also by Jang and Chen (2010), focuses on integrating Interactive Whiteboards (IWB) into teaching to build TPACK through comprehension, observation, instruction, and reflection. Lastly, the TPACK-IDDIRR model by Lee and Kim (2014) is a five-stage process tested in a technology integration course, revealing teachers' challenges in developing pedagogical knowledge and applying TPACK effectively. These methods and models underscore the importance of structured, reflective, and collaborative approaches in developing teachers' TPACK, highlighting the need for continuous professional development and integrating technology into teaching practices.

Even though there are several methods and models for TPACK development, as mentioned above, they are not specific to a particular subject matter except the

TPACK development model of Niess, Ronau et al. (2009). Consistently, Grandgenett (2014) discussed TPACK when applied to mathematics, noting that Mishra and Koehler's (2006) framework lacks precise definitions for the components of TPACK and offers a more generic interpretation. Niess (2005) suggested a pedagogical model specific to the subject matter to capture the outcomes of TPACK growth within a teacher education program, enriching the four-component model of Pedagogical Content Knowledge (PCK) expanded by Grossman in 1990 upon Shulman's (1986) concept of PCK. Niess (2005) added a component of the teacher's understanding of incorporating technology into mathematics teaching, represented by 'T,' and proposed the TPCK framework. The standards' themes are based on TPCK concepts Niess (2005) borrowed from Grossman's four PCK aspects. These themes consider teachers' comprehension of the use of technology to teach mathematics and their corresponding knowledge and beliefs that align with the following objectives: (a) to have an overall view of the rationale for integrating technology in the teaching of mathematics; (b) to understand how students perceive, think, and learn mathematics through the use of technology; (c) content knowledge and pedagogical knowledge concerning the subject of mathematics, including technology; and (d) knowledge of the use of instructional strategies and visualization for mathematics teaching with technology (Niess, Ronau, et al., 2009).

The TPACK standards developed by Niess, Ronau, et al. (2009) concerning the teaching and learning of mathematics share a close resemblance with Mishra and Koehler's (2006) TPACK model. It is suggested that mathematics teacher training should focus on improving teachers' knowledge and thinking patterns, involving nurturing a comprehensive understanding of teaching with technology, pedagogical approaches, and content knowledge within the TPACK conceptual model's four components, according to Niess (2005). This framework can be used as a reference for practices that align with improving the teaching and learning of mathematics. Based on the TPACK standards for mathematics teachers, Niess, Ronau, et al. (2009) defined the themes, levels, and descriptors of the levels within the Mathematics Teacher TPACK Standards and Development Model. The study of Niess, Ronau et al. (2009) aimed to outline teachers' progression in cultivating this specialized knowledge.

Given the comprehensive delineation of teachers' TPACK development in mathematics, among the various models mentioned above, the TPACK development model of Niess, Ronau et al. (2009) was chosen to monitor the changes in the TPACK of a middle school mathematics teacher in this study as this model was deemed appropriate for mathematics coaching due to its alignment with the nature, scope, context, and data collection procedures of this study.

Although the TPACK framework introduced an innovative perspective to the idea of teacher knowledge, it was intended for diverse disciplines within the educational field. Therefore, more was needed to track the professional growth of teachers in TPACK. In other words, the depiction of TPACK is not specific to the subject but rather general. Niess et al. (2007) introduced the TPACK development model regarding mathematics teaching to address the gap in mathematics education. The Mathematics Teacher TPACK Standards provide a framework for understanding and applying TPACK (Niess et al., 2007). The Mathematics Teacher TPACK Standards outlined by Niess, Ronau, et al. (2009, pp. 18-19) include:

1. **Creating and Developing Digital Learning Environments:** Teachers design and create authentic learning settings and experiences using suitable digital tools and resources to enhance mathematical learning within context (Niess, Ronau, et al., 2009, p. 18).
2. **Instruction, Learning, and Mathematics Curriculum:** Teachers execute curriculum plans incorporating methods and strategies for utilizing appropriate technologies to optimize student learning and foster creativity in mathematics (Niess, Ronau, et al., 2009, p. 18).
3. **Evaluation and Assessment:** Teachers use technology to support a range of practical assessment and evaluation methods (Niess, Ronau, et al., 2009, p. 18).
4. **Efficiency and Professional Practice:** Teachers leverage technology to improve their productivity and professional practices (Niess, Ronau, et al., 2009, p. 19).

Niess et al. (2007) proposed the TPACK model based on Everett Rogers's (1995) innovation-decision process model, which was initially developed in 1962 to

describe the process of the diffusion of innovations in society. Rogers (1995) described this process in five sequential stages, which define how an individual accepts or rejects an innovation. Niess et al. (2007) defined the process as mathematics teachers learning to implement a new technological tool they had not used to teach and learn mathematics before. The Mathematics Teacher Development Model presents the process of developing TPACK to meet the Mathematics Teacher TPACK standards. The TPACK Development Model developed by Niess et al. (2007) comprises four key themes: (1) Curriculum and Assessment, (2) Learning, (3) Teaching, and (4) Access. *The Curriculum and Assessment Theme* pertains to the pedagogy of a particular subject and the evaluation of students' comprehension concerning that subject matter. Curriculum and Assessment were grouped in a theme to emphasize the link between decision-making processes in curriculum and assessment. The emphasis of the *Learning Theme* is on conceptualizations of how students learn and how their thinking skills improve. The *Teaching Theme* is pertinent to strategies for instruction, learning setting, and professional development. *The Access Theme* centers around issues such as the availability of technology to the students in the classroom and the ways that teachers overcome obstacles to integrating technology. Also, this theme focuses on how technology increases access to mathematics for discovering mathematical ideas and concepts (Niess, Ronau et al., 2009).

The TPACK Development model, in addition to its themes, contains a sequential process in which teachers make progress through five levels: (a) *Recognizing*, (b) *Accepting*, (c) *Adapting*, (d) *Exploring*, and (e) *Advancing*. Teachers advance through the TPACK Development Model as their TPACK level increases. As teachers with strong Pedagogical Content Knowledge (PCK) advance along the sequential levels, their level of TPACK also develops. There is a transition from 'recognizing' to 'advancing' for each of the four main themes. As teachers advance through the TPACK Development model, there is a convergence and expansion of the conceptions of CK, TK, and PK, as outlined by Niess, Ronau et al. (2009).

As shown in Figure 2, The content, technology, and pedagogy knowledge domains overlap as the teacher progresses through the levels (from recognizing to advancing)

of Niess, Ronau et al.'s (2009) TPACK development model. Teachers at the recognizing (knowledge) level can use technology and be aware of its applications in mathematics. However, they do not incorporate it into their teaching practice. Teachers at the accepting (persuasion) level develop a positive or negative attitude towards mathematics teaching using appropriate technology (Niess, Ronau, et al., 2009). Teachers at this level will likely have difficulty properly incorporating technology into their mathematics teaching practices. In the adapting (decision) level, teachers not only start realizing the advantages of utilizing suitable technology as a tool for teaching and learning but also become involved in actions that lead them to decide to utilize or reject an appropriate technology for specific content with pedagogy (Niess, Ronau, et al., 2009). Teachers at the exploring (implementation) level actively, effectively, and sufficiently incorporate appropriate technology into mathematics teaching and learning. Teachers at the exploring level create and revise activities and learning materials in compliance with their curriculum. As their TPACK develops, teachers engage their students in more intricate activities, such as mathematical investigations within the classroom or problem-solving, using technology as an essential tool for learning and teaching (Niess, Ronau, et al., 2009). Teachers at the advancing (confirmation) level change the curriculum as necessary to use technology's convenience to evaluate the consequences and outcomes of the decision to incorporate suitable technology into mathematics teaching and learning (Niess, Ronau, et al., 2009).

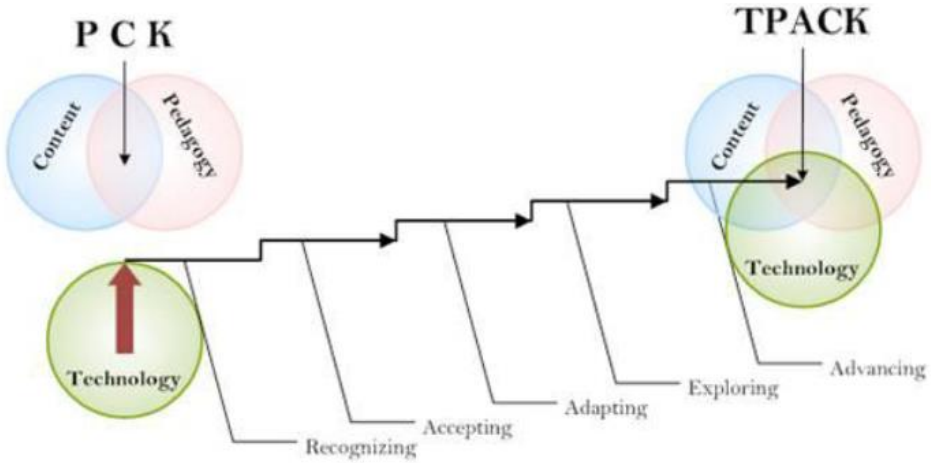


Figure 2. TPACK Development Model for Mathematics Teachers
(Niess, Ronau et al., 2009, p. 10)

In this study, to provide a detailed description of a middle school mathematics teacher's progress in developing subject-specific knowledge, the TPACK development model of Niess, Ronau et al. (2009) as a specific model for mathematics teachers was adopted for monitoring the changes in the teacher's TPACK levels in themes and related descriptors and assessing the teacher's progress in the development of TPACK while designing and implementing technology-enriched lessons through GeoGebra.

2.2.1. Research Studies on TPACK Development of Mathematics Teachers

Research studies have consistently demonstrated the importance of enhancing TPACK as an integral aspect of teacher development. This applies to in-service and prospective teachers who should be provided opportunities to enrich their knowledge and practical experience in technology integration. For instance, Niess (2005) underscored the importance of TPACK development, emphasizing it as a crucial body of knowledge for effective mathematics teaching. This highlights the need to integrate TPACK development into teaching and learning coursework and courses to improve mathematical knowledge (Niess, 2005). Consequently, Niess (2005) recommended that both in-service and prospective teachers should be proficient in integrating technology into mathematics learning and teaching.

Building on the importance of practical experience in technology integration, Mutlu et al. (2019) investigated the impact of micro-teaching lesson study (MLS) experiences utilizing the VuStat software on the TPACK of ten preservice mathematics teachers. Over two weeks, these teacher candidates participated in MLS sessions that demonstrated a positive increase in their ability to integrate technology into teaching. The findings showed that MLS contributed significantly to developing TPACK, particularly in using smartboards, which improved TPACK self-esteem among the participants (Mutlu et al., 2019).

Similarly, Murtafiah and Lukitasari (2019) conducted a study to assess the PCK of prospective mathematics teachers via the implementation of lesson study within microteaching. Their analysis revealed notable enhancements in mathematics

teaching, including lesson planning, learning activities, and content knowledge. The progression from average to good levels across these areas highlighted the effectiveness of MLS in developing prospective teachers' TPACK, particularly in terms of content mastery and pedagogical knowledge (Murtafiah & Lukitasari, 2019). Expanding on these findings, Koştur (2018) conducted a single case study aimed at enhancing and exploring the development of prospective middle school mathematics teachers (PMTs) in TPACK-Practical through an undergraduate course named 'Technology Use in Mathematics Education (TUME).' The TUME course, designed to foster TPACK-Practical in three critical areas of pedagogy—planning and design, implementation, and assessment—proved effective in advancing PMTs' development in TPACK-P. The study outcomes affirmed the success of practice-based learning in improving PMTs' abilities to integrate technology into mathematics education (Koştur, 2018).

Contributing to the literature on lesson study and microteaching, Meng et al. (2013) examined how implementing Geometer's Sketchpad (GSP) in mathematics teaching through lesson study could improve the TPACK of secondary teacher candidates. The research established enhanced teachers' TPACK in all the subscales, indicating that incorporating TPACK into lesson studies in teacher preparation programs enhances the prospective teachers' technological skills in teaching (Meng et al., 2013). Cavin (2008) also investigated preservice teachers' TPACK growth through a Microteaching Lesson Study (MLS). The research centered on the choice-making activities incorporating technological aid into content lessons. The results indicated that MLS helped preservice teachers to shift from the procedural to the conceptual level of thinking concerning technology integration in mathematics teaching, which is consistent with the utility of MLS in TPACK improvement (Cavin, 2008).

In addition, Birel and Çakıroğlu (2018) investigated the development of preservice mathematics teachers' TPACK in teaching statistics using virtual manipulatives in a microteaching lesson study (MLS). The conceptual foundation of the study relied on the TPACK model developed by Koehler and Mishra in 2009. The MLS group of 5 learners observed and taught a research lesson. Birel and Çakıroğlu (2018) concluded that the TPACK of the pre-service mathematics teachers transformed and evolved

with MLS. Based on the analysis, they showed some developments in some aspects of the TPACK knowledge domain. In another study on TPACK development, Uygun (2013) explored TPACK development through Learning by Design (LBD) tasks in an undergraduate course. The study indicated that pre-service teachers from different disciplines used various methods to develop TPACK, and LBD modules used in the study were found to be efficient tools for developing TPACK by providing students with real-life, practical, design-based experiences that are relevant to teaching (Uygun, 2013). Mudzimiri (2012) also added to this line of research by investigating the growth of preservice teachers' TPACK for 15 weeks using collaborative courses. Her study showed that TPACK development is complex and depends on teachers' experience and perception of technology use in math teaching. Thus, the TPACK formation process proves that such model development requires more than a single course or several courses during teacher education (Mudzimiri, 2012).

As a similar study to the present study in terms of monitoring teacher's TPACK development using the model of Niess, Ronau et al. (2009), Çetin's (2017) research was a mixed-methods study that explored the changes in TPACK of secondary school mathematics teacher candidates concerning the 10th-grade mathematics curriculum. The population of the study consisted of 33 teacher candidates attending a state University who were enrolled in a pedagogical formation program and who had been trained in lesson planning based on TPACK, use of educational software (GeoGebra, Cabri 3D), interactive sites (Desmos), and manipulatives (National Library of Virtual Manipulatives) for seventeen weeks. To gauge the TPACK's progress, Çetin (2017) drew on several tools, including a TPACK Competency Scale designed for mathematics teachers. This scale was aligned with the TPACK development model proposed by Niess, Ronau et al. (2009), which outlines a five-level process for TPACK growth: These include recognizing, accepting, adapting, exploring, and advancing. Thus, Çetin made sure that the scale related to these levels has an instrument that would allow mapping the stages of TPACK development, which is reflected in the improvement of teacher candidates' ability to integrate technology into teaching. Also, the qualitative data were gathered using interviews, focus groups, TPACK rubrics, videos, lesson plans, and digital resources, which helped to obtain a broad picture of TPACK development. Statistical analysis showed

that teacher candidates' performance had a statistically significant increase in all the subdomains of TPACK; the most remarkable progress was noticed in the PCK sub-component than other TPACK sub-components, including TPK, TCK, PK, and TK. The qualitative data provided evidence for these findings in that the teacher candidates moved from pre-TPACK adaptation to a level of adaptation in technology integration skills in mathematics teaching. The researcher established that, depending on the level, several teacher candidates showed progress in understanding how to use technology for teaching and learning purposes, thus attaining the exploring and adapting levels.

Further studies have also been conducted to determine the difficulties and attitudes of in-service mathematics teachers towards integrating instructional technologies, thus underlining the issue of building TPACK in this subject area. To provide an essential insight into the present study, Taylor (2011) conducted qualitative research on the perceptions and enactment of secondary mathematics teachers regarding integrating instructional technologies, with TPACK as the theoretical foundation. The current research established a stark contrast between the teachers' levels of TPACK, as reported by the teachers themselves, and their demonstrated practices in the classroom. Several participants demonstrated a somewhat naive knowledge of the instructional technologies, defining the former as the latter, more complex, and integrated technological resources. The results of this study corresponded to the TPACK Development Model described by Niess, Ronau, et al. (2009), which depicts the process of developing technology-enhanced teaching from the perception of technology as an object to the perception of technology as an integral part of the teaching process. Taylor's (2011) study pointed to the process of this development, showing that most teachers are still in the first stage of the model, where technology is considered an integrative tool. This study also underlined the need to focus on targeted professional development to address those gaps, especially as it helps to move teachers from a beginner's level of TPACK knowledge, as described by Niess, Ronau, et al. (2009). Although teachers recognize that technology is a crucial aspect of mathematics teaching and learning, a considerable gap must be addressed to enhance teachers' TPACK and improve technology integration in teaching and learning.

In conclusion, the research studies reviewed in this section underscore the critical role of targeted professional development programs in enhancing the TPACK of mathematics teachers. Approaches such as Mathematics Coaching, Microteaching Lesson Study, Lesson Study, and Learning by Design have been shown to significantly impact teachers' ability to integrate technology effectively into their teaching practices. These findings align with the objectives of the present study, which examines changes in a middle school mathematics teacher's TPACK within the context of geometry, specifically in the 'Angles and Measurement of Area' learning domains. The evidence from prior research highlights the necessity of ongoing, context-sensitive professional learning opportunities, such as mathematics coaching, to support teachers' advancement through the TPACK levels. Given the low TPACK levels identified in previous studies, it is evident that teachers require specific professional development to strengthen their ability to use technology effectively in teaching. The following paragraphs describe activities that can support teachers in this professional growth.

2.3. Professional Development Approaches to Mathematics Teachers

Teacher professional development (PD) is one of the most critical factors that can contribute to enhancing teachers' performance and, in turn, the achievements of the students. According to Desimone (2009), the general goal of PD is to assist teachers in enhancing their professional growth, increasing content knowledge, improving teaching methods, and fostering reflective practice in teaching. In mathematics education, teachers require effective PD because of the dynamic nature of mathematics education and the new techniques needed to teach students. Effective PD prepares teachers for new challenges and ensures they adopt the best practices in teaching mathematics (Darling-Hammond et al., 2017).

The methods and techniques commonly employed in imparting PD to mathematics teachers include traditional approaches, as well as on-the-job training or what is commonly referred to as job-embedded professional development. Traditionally, workshops, seminars, conferences, and lectures have introduced new concepts and strategies. For example, workshops engage participants in activities structured within

a specific time frame and targeted at certain aspects that require immediate solutions and techniques that can be used in the classroom (Guskey, 2000). In the same way, seminars and lectures provide huge and encompassing information on the concepts and practices of education to equip teachers with new knowledge and trends (Kennedy, 2016). Conferences allow the participants to get to know other people and find out what works and what does not (Desimone & Garet, 2015).

Unlike traditional approaches, modern and more interactive approaches such as coaching, lesson study, action research, and Professional Learning Communities (PLCs) are also used. These methods stress ongoing, collaborative work squarely wedded to the classroom context. For instance, coaching involves receiving guidance from qualified tutors who give guidance immediately and give feedback (Bonghanoy et al., 2019). Lesson study is a process of group lesson planning, teaching, and post-teaching discussion and reflection to enhance decision-making in the lesson (Lewis, 2002). Action research is a form where the teacher looks at a particular practice, makes changes, and observes the effects of these changes based on data collected (Borko et al., 2010; Morales, 2016). PLCs help develop a culture of professionalism and continuous improvement among teachers, who are influenced to improve their teaching techniques and learners' achievement (Vescio et al., 2016). However, there are some disadvantages of these collaborative approaches to PD, which, depending on the circumstances, may make some of the methods ineffective. Conventional methods of PD do not contain such characteristics as constant, collaborative, and practical activities aimed at enhancing teaching effectiveness (Bognar et al., 2024). While single events, for example, workshops, conferences, lectures, or seminars, help raise employees' awareness, they often fail to be followed up, which is essential for change management. Also, PD that is not specific to the contextual realities of teachers could be less useful (Lee & Vongkulluksn, 2023).

Several elements and characteristics define effective PD for mathematics teachers. Research studies indicated that effective PD should comprise the following activities: workshops, mentoring, engaging, longer duration, and collaborative planning; of these, coaching is considered most beneficial because of its individualized and long-term support (Buser, 2018; Darling-Hammond et al., 2017; Desimone, 2009;

Desimone & Pak, 2017; Driscoll, 2008). Desimone (2009) stated that since the coaching processes usually contain these elements, it can positively impact teachers' instructional practices and students' academic achievement. Moreover, content-specific PD approaches are concerned with improving teachers' subject matter understanding regarding mathematics and how students learn, incorporating PCK to bridge the gap between content and methodology (Ball et al., 2008). Practical lessons and demonstrations that are used in teaching actively engage teachers and thus enable them to retain and use the knowledge in their classrooms (Loucks-Horsley et al., 2009). Furthermore, the factors that include relevance, duration, collaborative learning, and contextualization are essential in making PD coherent with teachers' previous knowledge and practice for the intended changes to be relevant and long-lasting (Desimone, 2009; Garet et al., 2001; Kazemi & Franke, 2004; Lee, 2007).

In conclusion, professional development of mathematics teachers can be defined as a dynamic process that focuses on enhancing teachers' instructional practices in class. While the traditional PD methods are informative, they are usually not sustainable and disconnected from the classrooms, meaning that the knowledge attained from them is rarely long-lasting. The more frequent and sustained forms of collaboration and professional development discussed in this section, including coaching, lesson study, action research, and PLCs, result in more substantial changes in teaching practices. Accordingly, research studies highlighted the need for effective PD programs (Darling-Hammond et al., 2017; Garet et al., 2001) and revealed that effective PD entails both the traditional and the non-traditional approaches with the characteristics of content-specific, active, coherent, long, collaborative, practice-based, and context-bound (Lee, 2007; Loucks-Horsley et al., 1996). Thus, innovative approaches to PD that imply shifts in teachers' perceptions and actions have been found to impact teacher and student performance positively. In this sense, mathematics coaching was found to be one of the most effective forms of PD according to research studies (Bengo, 2013; Buser, 2018; Desimone & Pak, 2017; Hull et al., 2009; McGatha et al., 2018; Mudzimiri, 2012; Mudzimiri et al., 2014).

Therefore, considering the examination of different PD approaches in the related literature and the nature of the present study, mathematics coaching is currently

employed as a PD approach since mathematics coaching provides a more cohesive, long-term, and tailored form of professional learning than traditional and non-traditional forms of PD. It is an integral part of the teacher's practice, is ongoing and personalized, and concentrates on the cooperative and recursive process that is critical to the development of TPACK in mathematics education (Aygün, 2016; Aygün & Işıksal-Bostan, 2019; Bengo, 2013; Campbell & Griffin, 2017; Knight, 2009). In the following section, research studies focused on mathematics coaching are presented in detail.

2.3.1. Professional Development Approach of Mathematics Coaching

Approaches to sustaining teachers' professional development are crucial for ensuring that educators stay current with best practices and strategies (Desimone, 2009). Loucks-Horsley et al. (2009) categorized professional development strategies into six categories: content, standards and research, learning and teaching, curriculum alignment and application, and professional development structure. These categories include sixteen strategies designed to support teachers' ongoing development. While these strategies can be beneficial, they may not always fully prepare teachers for today's educational landscape challenges.

To address the shortcomings of traditional professional development (PD) approaches, Joyce and Showers (1982) proposed coaching in the educational context as an alternative strategy. Unlike earlier methods, which are more general and less intensive, educational coaching is individualized and closely related to corporate coaching practices. This approach was further developed in response to research suggesting that traditional PD methods do not sufficiently prepare teachers for change or adopting new pedagogical practices (Showers & Joyce, 1996). Based on the concept of individualized professional development, mathematics coaching has emerged as a promising approach. It introduces new methods to teachers, improving their teaching ability (Bengo, 2013; Gibbons & Cobb, 2017). By coaching, teachers can reflect on their teaching, get feedback, and modify their practices, which enhances their confidence and competence and, in turn, affects students' learning (Liao et al., 2021; Larsen, 2012). The literature reveals that coaching interventions

enhance teachers' practices since the teachers who are coached are four times as likely to implement new knowledge and pedagogy as those who are not (Knight, 2007; Showers & Joyce, 1996).

Loucks-Horsley et al. (2009) categorized coaching under the professional development strategies that permit teachers to learn in pairs or alone and improve their science and mathematics teaching. This approach helps teachers think about themselves as reflective practitioners, enhance their professional knowledge and skills, and adopt new strategies (Knight, 2007; Loucks-Horsley et al., 2009; Showers, 1985). Coaching is defined as a holistic work-related learning process that aims to raise student achievement by increasing teacher effectiveness. The evolution of school-based coaching in several countries reveals that it is a continuous, effective form of professional development. Coaching has been a part of the educational system for the last two decades and is still going on with different models, such as cognitive, instructional, content-focused, and peer coaching (McGatha, 2008; Grant & Greene, 2004). Showers and Joyce (1982) developed the peer coaching model, where two teachers work together as co-workers to foster each other's learning, thus laying a foundation for the current range of coaching models. Cognitive coaching is a process that supports and encourages teachers' professional development by focusing on their thinking processes and decision-making capabilities and asking inquiry-based questions to support the process of teachers' self-improvement (Costa & Garmston, 2002). Instructional coaching has been described as the process of offering teachers guidance and feedback on particular teaching strategies with the view of enhancing the teaching and learning process. This model focuses on partnerships, targets, and ways to improve learning from the data (Knight, 2007).

Among the different coaching models, content-focused coaching is most helpful in helping teachers enhance their teaching in certain subjects (Kane & Saclarides, 2023; West, 2009). Content-focused coaching is defined as a type of coaching that aims to improve teaching practices in certain subjects, such as mathematics or literacy. This strategy incorporates planning, an observation component, feedback on the subject matter, and instructional strategies to enhance the teacher's content knowledge and student achievement (West & Staub, 2003). West and Staub (2003) have refined the

notion of content-focused coaching to mathematics teachers focusing on content and pedagogy. This model entails teachers and coaches designing, planning, and monitoring the mathematical activities and strategies for teaching and learning to improve students' performance. Based on actual lessons, the sessions enable the coaches to work with teachers directly, which impacts the teaching process (West & Staub, 2003). The model of the cyclic process of coaching in mathematics is shown in Figure 3 below.

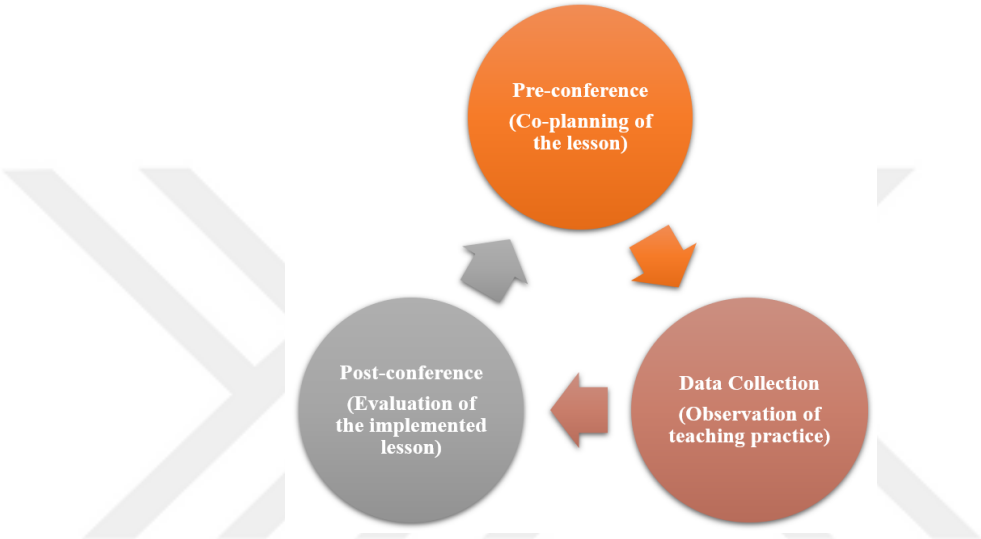


Figure 3. Mathematics coaching cycle (McGatha et al., 2018, p. 6)

Mathematics coaching is a specific type of content-based coaching that aims to enhance mathematics instruction. It is a professional partnership between a coach and a teacher in which the coach observes lessons in the classroom, offers feedback, and co-designs lessons. The purpose is to increase the teacher's content knowledge, pedagogical content knowledge, and effectiveness in implementing mathematical practices in the classroom to enhance students' understanding of mathematics (Campbell & Griffin, 2017; Gibbons & Cobb, 2017). A coaching session in content-focused coaching is typically divided into three stages. These are the co-planning of the lesson before the implementation (pre-conference), the actual teaching that takes place in the classroom (observation of the lesson), and reflecting on the conducted lesson after the lesson (post-conference). In the pre-conference phase, the coach gains insight into the teacher's approaches to lesson planning, understanding of students' thinking and beliefs about mathematics, and the teacher's understanding of

mathematics content and pedagogy (West & Staub, 2003). This stage allows the coach to work with the teacher to develop a lesson plan and identify key mathematical concepts that are essential and strategies that should be used in order to achieve the best learning outcomes. Also, the teacher and coach can sometimes collaborate in the class to determine likely misconceptions that students may have and develop strategies that may assist the students in dispelling these misconceptions (West, 2009; West & Staub, 2003).

The post-conference phase is conducted after observing the actual teaching practice, during which the teacher gets to reflect on the lesson and determine the aspect that needs to be changed (West, 2009). In this process, the coach and the teacher discuss the implementation of the lesson and take further action in the teaching-learning process, whether by elaborating on some content or reflecting on teaching strategies based on the lesson outcomes (West & Staub, 2003). The post-conference is to prepare for the next lesson by reflecting on the coach and teacher's observations and evaluations of the students and their work and classroom interactions.

Mathematics coaches are supposed to play several roles, including assisting teachers in enhancing the students' achievement in mathematics, managing the mathematics curriculum and materials, overseeing the curriculum management, and offering professional development opportunities for teachers. The coach's role may shift in the session when the focus transitions into teaching. Lesson delivery can be done solely by the teacher, the coach, or both (Gibbons & Cobb, 2017). The coach can teach the whole lesson so the teacher can focus on specific aspects of teaching, pupils' activities, or other occurrences in the class (Haneda et al., 2019). On the other hand, the coach may decide to remain passive in the lesson without interfering while at the same time supplementing the teacher as they teach the children (Gibbons & Cobb, 2016; Nugent et al., 2023). Even at this stage, the coach needs to have a good content background and experience to put forward important issues to be discussed during the post-conference phase (Knight, 2019). Besides giving individual attention to teachers, coaches also help teachers teach the students' conceptual knowledge of mathematics. Coaching enhances teachers' content knowledge and pedagogical practice by involving teachers in collaborative lesson planning, teaching, and

reflecting (Hull et al., 2009). Some responsibilities include nurturing knowledge and experience, developing learning communities, and providing meaning to assessment data to enhance teaching practices (Price-Cole, 2018).

Hence, mathematics coaching as one of the content-specific coaching strategies was employed in this study, and the coaching sessions were underpinned by five key characteristics of the focal coach's planning processes, as identified by Gibbons and Cobb (2017). These aspects consist of (1) establishing long-term teaching objectives, (2) evaluating a process of imparting knowledge, (3) classifying the current teaching practices as belonging to which stage on the development spectrum, (4) identifying the developmental plan of the particular teacher, (5) determining the activities that might be beneficial for the developmental progress of the particular teacher. Implementing these practices involves two key areas of coaching knowledge: the awareness of the teaching of ambitious mathematics and the understanding of the directions that elaborate the teacher's professional development in teaching ambitious practices. The participant teacher was observed for this research within the five dimensions in which the instructional practices were analyzed and how the coaching practices are integrated into the lesson planning done by the teacher (Gibbons & Cobb, 2017).

2.3.1.1. Research Studies on Mathematics Coaching

Despite the growing literature on coaching, many studies have been concerned with developing and maintaining coaching programs (Costa & Garmston, 2002; Desimone, 2009). Nonetheless, research focusing on mathematics coaching is relatively scarce in the literature (Kane & Saclarides, 2023; Saclarides & Harbour, 2023). Mathematics coaching as a specific field of study has gained popularity in recent years, demonstrating specific issues of the discipline and the benefits of its application in improving teaching (Bengo, 2013; Buser, 2018; Price-Cole, 2018). The following paragraphs provide insight into the general aspects of mathematics coaching, the overall success, its impact on teaching practices, the challenges faced, and the development of coaching models.

The existing studies of mathematics coaching indicate that this approach can positively affect teacher practice and, therefore, students' achievement. For instance, Price-Cole (2018) conducted a quasi-experimental study to examine the effectiveness of mathematics coaching on the teaching performances of teachers and students in middle school. The study also revealed that the teachers who participated in the coaching sessions enhanced their teaching practices to improve the student's academic performance in mathematics. This study stressed the need for sustainable professional development and the significance of school leadership in implementing coaching strategies. Likewise, Bengo (2013) identified the effect of mathematics coaching on teachers' teaching practices and students' performance using quantitative and qualitative data collection methods. The study's results suggested that coaching positively affected teachers' application of instructional strategies and their self-efficacy to teach mathematics. However, Bengo (2013) also explained several limitations, such as the time factor and the fact that the support from the school administration may not be the same, which are vital factors that define the effectiveness of the coaching program. All the mentioned studies describe the effectiveness of mathematics coaching in enhancing students' academic achievement and reveal that effective mathematics coaching should occur within the prescribed and supervised formats.

Downing (2020) conducted a quasi-experimental study to evaluate the effectiveness of different forms of professional development in improving teachers' pedagogical practices, specifically focusing on facilitating mathematical discourse in a rural U.S. elementary school. The study aimed to determine which type of professional development—workshop-based training, collaborative Instructional Team meetings, or mathematics coaching—significantly impacted teachers' ability to incorporate "Mathematics Teaching Practice: Facilitate Mathematical Discourse" into their classroom instruction. The study's findings highlighted that while all forms of professional development had some positive impact, mathematics coaching showed the most substantial growth in teachers' professional practices. Teachers who participated in the six-week coaching cycle demonstrated a notable increase in the evidence of incorporating Mathematical Discourse in their instruction compared to those who only received direct instruction or participated in collaborative training

sessions. This indicates that ongoing, job-embedded professional development, such as coaching, can enhance teachers' instructional practices more effectively than traditional forms of professional development.

While the overall usefulness of mathematics coaching has been established, the literature has also explored how it impacts teaching practices and teachers' growth. Saclarides and Harbour (2023) carried out a case study of a teacher in primary schooling to examine how an instructional coach used the coaching cycle to help the teacher change her mathematics teaching. The research examined the differentiation process, and the findings showed that the conversations between the coach and the teacher were mainly focused on content and process differentiation. At the same time, product and learning environment aspects received less attention. This study also revealed the multifaceted nature of the partnership between the coach and the teacher and the level of discussions about differentiation during the coaching meetings.

In a very similar vein to the content and context of this study, Aygün (2016) conducted qualitative research that investigated the effects of mathematics coaching on the TPACK of in-service elementary mathematics teachers while teaching polygons to 7th-grade students. The study was conducted with two teachers teaching in public schools of Türkiye, and observational, interview, and conference data was used. Her study lasted two months for each teacher, including 16 teaching hours on the 'polygons' unit. The result was that mathematics coaching helped acquire TPACK from PCK. The results showed that the teachers' TPACK improved from novice to expert levels, mainly in curriculum, assessment, learning, and teaching during the coaching process. Her study also revealed that the teacher with more years of teaching experience advanced faster along the TPACK Development Model than the less experienced teacher, and the least advanced was in the assessment aspect. The teachers offered favorable opinions regarding the outcomes of mathematics coaching, including improvements in teaching and learning and the student's achievement; however, they also mentioned some drawbacks of the process, like the stress that comes with lesson planning. These findings are consistent with the study

by Saclarides and Harbour (2023), where both studies stress the importance of coaching to improve pedagogy.

Furthermore, Kane and Saclarides (2023) have also extended the study of how coaches' participation in mathematics can enhance their knowledge and effective practices in coaching. The present study revealed that the view of coaches as engaged members of the mathematics classroom who could better understand the learners' thinking and adjust their coaching was more beneficial for both the students and the coaches. However, the study also identified that the coaches were rarely acknowledged as coaches during the collaborative activities, so the coaches could not get chances to enhance their coaching knowledge. This challenge is based on Bengo (2013), who states that there is a need to provide proper guidance on the roles and support within the coaching systems to enhance their efficiency.

However, various research works that have reviewed mathematics coaching have also pointed out the difficulties of implementing the practice. For instance, Rhue (2022) investigated the 'See it, Name it, Do it (SND)' coaching model in 5 charter schools in North Carolina concerning its impact on student performance and teacher satisfaction. Thus, the research revealed that the SND coaching model has a positive effect on students' achievement and job satisfaction; however, the process of its adoption is rather intricate and requires time and materials. Thus, it is essential to pay attention to the coaching models, which must be appropriately designed and backed up to be effective. Polly (2012) also noted that coaching enhances teachers' instructional practice since it involves receiving guidance from an expert and constant encouragement. According to teachers, coaching has positively changed teaching practices and student achievement. According to the observation made by Desimone and Garet (2015), even as the conventional forms of professional development that include workshops and lectures continue to be popular, new forms of the same, such as e-learning, are being developed. Coaching, a unique and ongoing way of improving performance, solves many problems of conventional methods because it is individualized and ongoing.

Another study by Liao et al. (2021) also explored the difficulties of coaching concerning technology incorporation. In the multiple case study of three elementary

school teachers, they concluded that authentic coaching experiences like modeling and activities effectively enhanced technology integration. Nevertheless, organizational constraints, explicitly concentrating on standardized testing, restricted the alteration of IT practices in teaching. This implies that coaching cannot be effective in such contexts without school leaders' support and the coaches' participation in planning and reflecting. Furthermore, Solarski (2021) conducted a study on a mathematics content-focused coaching model within the framework of preschool mathematics and its implications on the teachers' content knowledge, teaching practices, and confidence. It indicated an improvement in teachers' self-efficacy in teaching mathematics, although the changes in the practices did not reach a significant level within the eight-month intervention period. This is because confidence-building is a precursor to more profound changes in practice and underlines the need to pay attention to teachers' pedagogical content knowledge (PCK) in early childhood mathematics education. Further development of the coaching models is crucial to address the issues highlighted in the literature and improve the impact of mathematics coaching. Similarly, Russell et al. (2020) carried out a collective study for the enhancement of the model of mathematics coaching with 75 teachers and 32 coaches. The study results indicated that the coaches who applied the refined coaching model were more able to change the teachers' behavior. This study focused on qualitative, specific interviews. It underlined the role of concepts in enhancing teaching-related activities and further advancing the best practices for coaching models.

Moreover, Ward (2020) focused on the role of elementary mathematics coaches in in-service teachers' professional development concerning mathematics anxiety and teacher efficacy. The study identified the effects of elementary mathematics coaches positively on mathematics anxiety and teacher efficacy among in-service teachers. In the open-ended responses, co-teaching, co-planning, and curriculum support were seen as the most beneficial forms of interactions with coaches. These findings are consistent with Buser's (2018) mixed-methods study, which revealed that deep probing of mathematical objectives during pre-conferences was positively associated with effectively translating these objectives into classroom practice. These results imply that technical advancements like structured coaching cycles and collaborative

planning can contribute to the success of mathematics coaching. Each of the studies highlights the importance of having a system and working together during mathematics coaching, pointing to the fact that these are crucial for teachers' confidence and coping with problems such as mathematics anxiety. These results suggest that the success of mathematics coaching depends on the quality of the interactions between the coaches and teachers. This is improved by collaborative planning and implementing a structure for the coaching interactions.

In a mixed-methods sequential explanatory research design, Kelley (2023) investigated the effects of instructional coaching on teachers' instructional practices by focusing on the coach-teacher relationship. This research, conducted with 61 teachers from middle and high schools in a suburban district, found that instructional coaches helped teachers as they implemented new teaching methods. Through partnership and personal coaching, all teachers' needs were met and fulfilled, leading to the improvement of the best practices in teaching. Also, Kelley (2023) stressed the quality of the relationship between the teacher and the coach, noting that the more trusting relations improved the efficacy of the coaching as the coach was able to provide the teacher with more individualized support. However, Kelley (2023) also pointed out that instructional coaching had to be done in a way that would not encroach on teachers' rights in the classroom, which meant that while direction was helpful, it should not override teachers' discretion. This study also reinforced that coaching is an ongoing professional learning process with the cyclic nature of planning, implementing, and evaluating practices, resulting in improved teaching performance. Nevertheless, Kelley (2023) also noted that the support provided should be specific to the individual teacher, as underlined in the general theories of differentiated instruction. Lastly, the policy implications of the study recommended that to get the best out of instructional coaching, schools and districts must ensure that adequate time and funding are given to the coaches, besides ensuring that the coaches are well equipped through training and development to form part of the district's professional development strategy. These findings provided evidence of the importance of instructional coaching in the growth of teachers and in enhancing student achievement.

Similarly, McGatha (2008) studied the cases of two coaches to describe the different duties and roles of coaches and to provide a comprehensive understanding of teacher-coach interactions by examining each coach's level of engagement as they worked to build effective coaching relationships. McGatha (2008) explored how coaches can facilitate teachers' professional development, shedding light on the specific interactions and engagements between coaches and teachers. Consulting, cooperating, and coaching were the three support roles of a coach that McGatha (2008) identified after applying a framework from Cognitive Coaching to analyze the data. Results of her research confirmed Costa and Garmston's (2002) conclusion that support functions of coaching as coaching, consulting, and collaboration serve varied purposes and have distinct effects, with differing levels of effectiveness in promoting teachers' professional development (Costa & Garmston, 2002; McGatha, 2008). Considering the roles and effectiveness of support functions in coaching, the level of engagement of the coach in the coaching relationship of the present study is built around these three support functions: Consulting, collaboration, and coaching. Therefore, the roles and responsibilities of the coach and how the coach interacts with the teacher are formed around these support functions to enhance the teacher's professional growth (Costa & Garmston, 2002; McGatha, 2008).

Desimone (2009) defined the "core conceptual framework" of professional development, which coaches need to utilize while they design coaching cycles to advance teachers' learning. Campbell and Griffin (2017) stated the main characteristics of this framework when it is addressed in terms of mathematics coaches as follows: (1) Content-focus, addressing both content and pedagogy regarding mathematics while considering students' learning of mathematics, (2) Active learning, in which coaches and teachers involve in planning collaboratively, teaching together, observation of the classroom practice, debriefing, and shared or independent teaching demonstrations, as well as data-driven design of assessment and giving a decision, (3) Consistency in coach-teacher(s) discussions that tackle with teachers' previous points of view and beliefs regarding mathematics content, learning, and teaching considering new learning anticipations, promoting teachers' efforts to comprehend and compromise at school policy demands at levels of state, district, and local (4) Duration, by a coach's persistent efforts to identify and bring

attention to practice-related issues in teaching mathematics, and (5) Joint involvement in which a coach encourages analysis, experimentation, and reflection inside a professional community of teachers that evaluates the interpretation of student work, teaching methodologies, and mathematical meaning (Campbell & Malkus, 2011). These features demonstrate what professional development must focus on to improve teachers' knowledge (Campbell & Griffin, 2017).

Thus, in this study, the core features of the framework of Desimone (2009) were adopted as a professional development approach in designing the coaching cycles. Accordingly, the coaching cycles were planned based on Campbell and Griffin's (2017) model. The model is summarized in Figure 4 as follows;

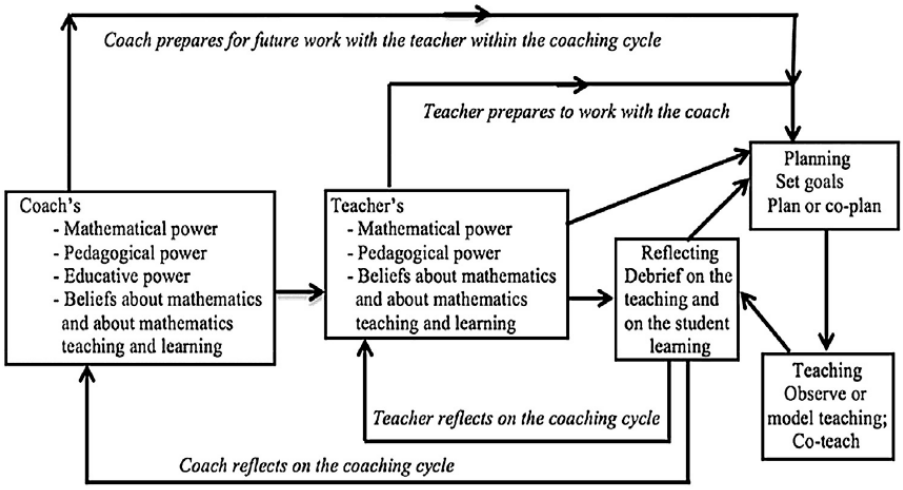


Figure 4. The co-learning process between coach and teacher in the coaching cycle is modeled by the conceptual framework (Campbell & Griffin, 2017, p. 165)

Mathematical power was defined by Jaworski (2001) as knowledge of the capable design of tasks or activities and mathematical content to get students involved in learning mathematics. Pedagogical power includes knowledge regarding developing or changing methods for teaching mathematics as well as knowledge of teaching mathematics, as pointed out in Jaworski's (2001) framework. The term educative power, which Jaworski (2001) introduced, refers to the additionary, readily available knowledge that a teacher educator requires to assist teachers in addressing mathematical and pedagogical problems. As mentioned in the framework of Jaworski (2001), coaches need to have and maintain educational power when collaborating

with teachers, in addition to having and achieving both mathematical and pedagogical power. This is crucial since coaches are called upon to improve teachers' understanding of related mathematical concepts and question and support teachers' viewpoints on teaching while promoting teachers' efforts to assign meaning to students' thinking (Moreau & Whitenack, 2013). Nonetheless, this model explicitly emphasizes that, as part of the continuous coaching cycle, both a coach and a teacher should analyze the occurrences in a lesson and consider the consequences of those events. The coach and teacher aim to discuss the insights gained from a lesson regarding students' thought processes. This dialogue enables the teacher, or both the teacher and the coach collaboratively, to design the following lessons that challenge or enhance students' comprehension, fostering their increasing mathematical sophistication.

The coach's educative power is critical throughout the debriefing processes and the subsequent lesson planning. Simultaneously, this model suggests that a coach reflects on both the conducted and intended lesson, as well as the coach's evaluation of the teacher's instructional repertoire and current knowledge, as well as on the pedagogical goals of the teacher since these goals serve to prepare the coach for future work with that teacher. As educational power is included in the coach's skill set, this coaching model suggests that the coach should have more experience in the field and more advanced knowledge than the teacher involved in the coaching. The conceptual framework that Campbell and Griffin (2017) offered to describe the co-learning process in coaching cycles also suggests that reflection might potentially enhance the learning experiences of both students and teachers. However, it is unclear if coaching is responsible for meeting the requirements of teachers solely or of both students and teachers. As seen above, this framework addresses not only the placement of the coach in schools to advance the school's instructional capacity but also the joint learning of coach and teacher as well as coaching of the teachers (Campbell & Griffin, 2017).

This study investigated teachers' TPACK development, considering the level of engagement in coaching relationships and the support function of these relations. In contrast, a middle school mathematics teacher attended content-focused mathematics

coaching as a professional development program aiming to enrich mathematics teaching with GeoGebra.

2.3.1.2. Research Studies on Teachers' Perception of Mathematics Coaching

Perception is defined as people's interpretation of experiences and how they affect their attitudes and behavior (Bandura, 1986; Pajares, 1992). In the PD context, teachers' perceptions are important since they affect their motivation to participate, their adoption of new strategies, and, therefore, the success of PD programs (Desimone & Garet, 2015; Darling-Hammond et al., 2017). This is important in the design of effective PD since literature established that positive perceptions towards PD lead to the belief that it can help enhance teaching practices (Aygün & Işıksal-Bostan, 2020; Buser, 2018; Drust, 2013; McLaughlin, 2012; Polly, 2012). In the following paragraphs, the related research on the teachers' attitudes towards mathematics coaching is discussed.

The findings of the research studies indicated that teachers' perception of mathematics coaching is primarily positive; most teachers appreciate the role of coaches in improving their teaching practices. For instance, Aygün and Işıksal-Bostan (2020) focused on the teachers involved in a mathematics coaching professional development program to examine the teachers' views. The teachers reported positive perceptions of mathematics coaching. They listed several benefits, which include improvement in teacher content knowledge and pedagogy, positive change in student achievement, support and collaboration, constructive criticism, and confidence. Additionally, the teachers noted that mathematics coaching is unique from other forms of professional development because of the process and the balance between the theoretical and the applied. According to the results of the present study, teachers suggested that mathematics coaching should be spread over the schools and employed across the country.

Similar to the study of Aygün and Işıksal-Bostan (2020), Worthington (2020) conducted a case study to assess teachers' perceptions of the efficiency of instructional coaching employed in mathematics classrooms at the secondary level.

The study participants were four secondary mathematics teachers. The participants worked with an instructional coach to develop questioning strategies and particular activities for employing in their classroom. Each participant participated in the coaching cycles by determining a goal and working to promote discourse in their classrooms. The coaching cycles were utilized by participants to decide where to facilitate discourse, discover new strategies for engaging students, discuss the likelihood of various student responses, and how to address those replies to advance the conversation. The qualitative and quantitative data collected were analyzed using a convergent approach. The research incorporated surveys comprising qualitative and quantitative questions and semi-structured interviews to evaluate how teachers perceived the changes in their practices from their initial professional development experience to their interactions with an instructional coach. Generalized findings from the four cases revealed that participants considered instructional coaching as an extension of professional development, and coaching enabled them to project what they learned in professional development into the classroom environment. As to instructional coaching, participants felt that the coaching sessions enabled them to explore teaching strategies to engage students in the conversations, plan for student discourse in their lessons, and reflect on their practices while discourse is employed.

Ketelaar et al.(2012) noted that teachers in secondary vocational education regarded coaching as applicable to their teaching practices as it focused on collaboration and PD. The teachers acknowledged the coaches who supported them in PD and helped them analyze their teaching performance. This partnership model ensured that teachers had confidence in the process and encouraged them to participate in the coaching process. Similarly, Reece and Grooms (2021) described that the teachers in their study considered mathematics instructional coaches valuable facilitators in improving teaching practices. The teachers said that the coaches played a significant role in assisting them with lesson planning, classroom management, and developing strategies to enhance student engagement in learning. Coaches were considered to have a direct and positive impact on teaching and learning processes and student achievement. Also, Warnock et al. (2022) also emphasized that coaching positively affects teacher confidence and their teaching practices. The result of the study revealed that all the teachers valued the individualized support function of the

coaching, which enabled them to resolve challenges in their classrooms. The specific coaching strategies effectively enhanced teachers' confidence and assurance in their teaching practices, improving their job satisfaction and efficacy.

Furthermore, Nicometi (2011) conducted a study to determine teachers' perceptions of the role of mathematics content coaches in changing teachers' practices and students' achievements. The study's findings showed that the teachers saw mathematics coaching as helpful in collaboration and professional growth. Teachers consider content coaches useful in improving instructional practices, classroom management, and teaching strategies. However, the teachers mentioned some difficulties, such as limited time for the coaching sessions and different perspectives on using coaching in different schools. Even though coaches believed that coaching helped the students achieve more, they could not always identify how the coaching influenced the students' performance. Accordingly, Nicometi's (2011) study also suggested that these concerns could be addressed to attain the intended objectives of mathematics coaching as envisaged by the teachers.

In a similar study conducted by Nicometi (2011), Larsen (2012) analyzed the perceptions of elementary teachers participating in a mathematics coaching program in an urban school. Larsen (2012) found that the teachers' perception of coaching was significantly positive since coaching enhanced teachers' teaching practices. The findings of the study also revealed that the participant teachers were able to employ more student-centered strategies in the teaching of mathematics through coaching, hence improving the students' learning outcomes. Another important factor that was found in relation to teachers' positive perceptions was that they established good trust and cooperation with their coaches, and this coach-teacher partnership was seen as very important for the success of cooperation in coaching. However, the teachers also expressed their concerns about the limited time spent having more in-depth coaching conversations, which they thought would have made it possible to achieve the full potential of the coaching program. Despite the fact that teachers pointed out the changes in their teaching practices in detail, they were not certain if these changes influenced the students' learning, which is relevant to a research gap. Larsen

(2012) also posited that if the time and support for coaching were to be increased, even more positive attitudes and effectiveness could be seen in coaching programs.

In a study done by Buser (2018), a mixed-method design was used to assess the three coaching moves in three different coaching cycles with three teachers. Surveys were administered at three intervals in each of the nine coaching cycles (three per teacher), and pre-conference meetings and classroom lessons were audio-recorded and transcribed. The findings of the study indicated that the teachers perceived mathematics coaching as highly positive, and they stated that the coaching cycles were beneficial in terms of professional development as they enhanced their teaching practices and content knowledge. The results also indicated that the asking of deep, specific questions about math goals during pre-conferences was correlated with the effective implementation of those goals when teaching in the classroom. Moreover, when teachers were asked to write assessing and advancing questions to be given to students in relation to the solution paths, it created chances for students to explain how they reached the solutions and for them to think further.

In a different study on teachers' perception of coaching, Drust (2013) discovered that teachers valued the feedback offered by coaches because they considered coaching as supportive and helpful in enhancing their teaching strategies. Teachers also reported that they felt more confident and used improved strategies in the classroom because of the individualized support of coaching and collaborative working setting. However, some difficulties were also identified, such as33 the variability in the implementation of the coaching approach across the schools and the limited time allocated for coaching sessions, which affected the effectiveness of the model. Similarly, McLaughlin (2012) also pointed out that teachers perceived coaching as an effective mechanism for learning and development, especially in content and teaching strategies. However, it was noted that the teachers appreciated the role of coaching in enhancing teaching and management strategies but could not easily associate the coaching with the improvement in students' performance. The teachers also indicated that there is a need to have more organized and frequent coaching sessions to enhance the benefits.

In summary, the research studies revealed that teachers perceived mathematics coaching highly positively; they acknowledged that it improves their teaching, content knowledge, and student engagement (Aygün & Işıksal-Bostan, 2020; Buser, 2018; Larsen, 2012; Worthington, 2020). The research studies also showed that the success of coaching is a result of the teacher-coach collaboration, the centering of the students, and the ability of the coaches to translate the pedagogical goals and strategies into classroom practices (Buser, 2018; Larsen, 2012). These findings support the notion of coaching as a sustainable and effective professional development strategy that can have a positive impact on teachers' teaching practices and student achievements (McLaughlin, 2012; Drust, 2013).

2.4. Summary of Literature Review

Technology usage in teaching mathematics is crucial in enhancing the students' comprehension of complex mathematical concepts (Harris & Wildman, 2019). Technologies and tools specific to mathematics, such as dynamic geometry software, help students develop their conceptual understanding by enabling them to experiment with the mathematical concepts and ideas being taught (Ball et al., 2008; Hohenwarter et al., 2009). Considering the demands of the 21st century, the application of these tools in the learning environment cannot be overlooked, given the increased application and availability of digital tools (Angeli & Valanides, 2009). However, many teachers struggle with technology integration into teaching since they do not have adequate access to resources, insufficient training, and low confidence because of the professional development they have received (Chen, 2008). The gap between the growing presence of technology in teaching and learning contexts and teachers' capability to appropriately use technology in teaching remains a challenge to effective teaching and learning (Ertmer & Ottenbreit-Leftwich, 2010). Furthermore, many teachers lack the pedagogical and technological know-how to apply subject-specific technologies. One is GeoGebra, a dynamic mathematics software developed to enhance the teaching and learning of algebra, geometry, and calculus (Hohenwarter et al., 2008). Nevertheless, most of the teachers face a lack of training on how to effectively and meaningfully integrate these technologies into their teaching (Spangenberg & De Freitas, 2019).

Shulman's concept of PCK serves as the foundation for understanding how teachers integrate content knowledge into teaching with suitable pedagogy (Shulman, 1986). Building on Shulman's idea, the TPACK framework emphasizes the need for teachers' knowledge to integrate technology into their pedagogy to enhance student learning (Koehler & Mishra, 2008; Voogt et al., 2013). TPACK is critical for teachers to navigate the complexities of technology-rich environments (Archambault & Barnett, 2010). As one of the TPACK perspectives, the transformative perspective of TPACK emphasizes integrating technology as an agent of change in teaching practices rather than simply adding to existing methods. This perspective encourages teachers to rethink their instructional strategies and adapt to the evolving educational landscape (Graham, 2011). Transformative TPACK requires continuous reflection and professional growth, which is essential for effective technology integration (Chai et al., 2013). However, many teachers still lack sufficient TPACK, which impedes their ability to effectively integrate technology into their teaching. This deficiency often results from inadequate pre-service training and limited ongoing professional development opportunities (Tondeur et al., 2017). Without a strong TPACK foundation, teachers may struggle to use technology in ways that genuinely enhance learning (Angeli & Valanides, 2009).

To this end, teachers who lack proper TPACK need specified professional development to enable them to apply technology in teaching. PD should address both technological know-how and knowledge of using technology in teaching and learning subjects (Kennedy, 2016). Effective PD has the potential to narrow the disconnect between what teachers know and what they need to know in today's classroom (Yurtseven-Avcı et al., 2020). Traditional forms of PD, including workshops, lectures, and seminars, are less effective as they are not contextual within the classroom and lack real-life application (Desimone & Pak, 2017). These forms of PD are not specific to a particular teacher, and the support offered is not continuous (Garet et al., 2001). Other forms of non-traditional PD, including coaching and collaborative learning, provide teachers with more focused and context-based support (Knight, 2019; West & Staub, 2003). Critique of traditional approaches has been that they are theoretical and do not reflect the actual classroom practice of teaching. In other words, these approaches are disconnected from the classroom and

do not negatively affect teachers' instructional practices. These methods typically rely on acquiring theoretical knowledge with little provision for the teachers to implement what they have learned in their classrooms (Desimone & Garet, 2015). Furthermore, traditional PD often adopts a 'one-size-fits-all' approach that does not consider the variations between different teachers and teachers' individual needs (Darling-Hammond et al., 2017).

Therefore, professional development should be meaningful, practical, and applicable in the classroom. Effective PD should be ongoing, done with colleagues, and directly related to classroom behavior (Desimone, 2009). Sustained PD, which includes time for teachers to think about their practice and is focused on student learning, is most beneficial to teachers (Garet et al., 2001; Guskey, 2003). Professional development that is most likely effective is content-focused, engaging, connected to other professional development activities, and participative (Desimone, 2009). It should be ongoing and offer opportunities to support the teachers' professional learning and foster collaboration and introspection (Borko et al., 2010). Efficient PD is also coherent with teachers' aspirations and their students' requirements (Darling-Hammond et al., 2017; Yurtseven-Avcı et al., 2020).

Mathematics coaching is one form of PD that represents the characteristics of effective PD, as mathematics coaching is sufficiently long, personalized, and focused on practice (Campbell & Griffin, 2017). Coaching teachers means they can enhance their TPACK in a systemic approach with the ability to provide feedback and reflection (Knight, 2019). While traditional PD is ineffective for professional growth because it is not specific to the needs of individual learners, coaching is designed for teachers and, therefore, more helpful (West & Staub, 2003). Mathematics coaching is a less generic and, therefore, more effective form of PD because it provides focused, real-time guidance within the teacher's practice (Gibbons & Cobb, 2017). Coaching also helps teachers get timely and regular feedback and time for reflection, which is crucial for changing the teaching practice (Knight, 2011). Coaching is more efficient than non-traditional approaches as it targets the multifaceted issue of integrating new technologies into mathematics teaching (Campbell & Malkus, 2010).

Considering the limitations of traditional PD approaches and the growing importance of technology in teaching, there is a strong need for effective professional development models that enhance teachers' TPACK. In this regard, mathematics coaching, a personalized and continuous intervention, has been shown to positively impact teachers' instructional practices and technology integration. Moreover, teachers generally perceive mathematics coaching favorably, highlighting its collaborative nature, immediate feedback, and the boost in confidence it provides (Aygün & Işıksal-Bostan, 2020; Buser, 2018). Therefore, this study aimed to explore how mathematics coaching affects a middle school mathematics teacher's TPACK development, providing insights into improving support for technology integration in teaching.

CHAPTER 3

METHODOLOGY

In this chapter, detailed information regarding the methodology of the current study is provided. The information was presented under ten sections respectively: (1) Research Design, (2) Context, (3) Participant Selection, (4) Intervention, (5) Data Collection Procedure, (6) Pilot Study, (7) Data Analysis, (8) Trustworthiness, (9) The Role of the Researcher, (10) Ethical Considerations, and (11) Assumptions.

The main purpose of this study was to investigate the changes in a middle school mathematics teacher's TPACK in geometry, in the learning areas of 'Angles and Measurement of Area' in particular, throughout mathematics coaching cycles. Furthermore, this study aimed to monitor the teacher's development in TPACK across five themes and corresponding descriptors within the TPACK development model, as a result of the teacher's involvement in mathematics coaching. This assessment was conducted in comparison with the teacher's TPACK levels prior to the study. This study also investigated the teacher's perceptions of mathematics coaching in developing TPACK.

To address these concerns, a qualitative research approach was employed, and the study aimed to investigate the following research questions:

1. To what extent does attending mathematics coaching influence a middle school mathematics teacher's TPACK in the learning areas of 'Angles and Measurement of Area'?

1.a. How does a middle school mathematics teacher who attends mathematics coaching make progress in the Curriculum and Assessment Theme, Learning Theme, Teaching Theme, and Access Theme within the TPACK development model?

2. What are the perspectives of a middle school mathematics teacher regarding the impact of mathematics coaching on the development of TPACK?

3.1. The Research Design of the Study

The primary objective of this study was to assess and monitor the TPACK levels of a middle school mathematics teacher at different phases: Prior to, in the course of, and following participation in mathematics coaching focused on the learning areas of 'Angles and Measurement of Area' in geometry. This study aimed to offer a thorough and detailed understanding of the progression of the teacher's TPACK in geometry. A qualitative research methodology was employed to achieve this goal, in line with the study's nature and objectives.

Creswell (2013) characterized qualitative research as a process of seeking understanding, rooted in various methodological traditions of inquiry, aiming to examine social or human challenges. Qualitative research involves placing the observer within the world as it is a situated activity. It comprises interpretive and tangible actions that bring the external world into focus. These tangible actions, manifested as material practices, have the power to transform the world into a collection of representations, encompassing memos, field notes, interviews, conversations, and photographs. Operating at this level, qualitative research embraces a naturalistic, and interpretive perspective on the world (Creswell, 2013). By seeking to interpret occurrences or make sense of these occurrences based on the meanings individuals attribute to them, qualitative research scrutinizes phenomena in their natural environments (Denzin & Lincoln, 2005). The qualitative research design provides the researcher with the opportunity to observe the processes and strategies employed by participants, understand the unfolding of these processes, and observe participants in their natural settings without imposing any intervention (Fraenkel et al., 1993; Merriam, 1988; Patton, 1990)

Creswell (2013) identified five approaches to qualitative research: case study research, narrative research, phenomenological research, grounded theory research, and ethnographic research. A case study was defined by Yin (2014) as an

investigation based on empirical data that explores a modern phenomenon within its actual context, especially when the bounds between the context and the phenomenon are not immediately apparent. In describing the case study research methodology, Yin (2014) emphasized its applicability to situations with many variables of interest compared to data points. This approach depends on multiple sources of evidence and necessitates the convergence of data in a triangulating manner. Furthermore, it leverages the previous formulation of theoretical hypotheses to direct the gathering and examination of data. The case or object of the research is the most crucial element, as can be inferred from its definition. Moreover, the primary objective of a case study is to delineate a clearly defined and bounded system wherein specific research components are encapsulated within the confines of the case (Yin, 2014). As the case constitutes a well-defined and confined system, it is suggested that researchers define it and its context carefully, and the case's boundaries are crucial in defining it (Merriam, 1998). Additionally, one of the critical elements of case studies is the analysis unit or study object. A study's unit relates to how the researcher formulated the original research question (Yin, 2014). Therefore, Creswell's (2013) and Yin's (2014) perspectives on case study served as a defining characteristic of the current study.

Based on the justifications presented above, this study adopted a single-case (holistic) qualitative research design as outlined by Yin (2003) since a confined system, or case, was thoroughly examined through comprehensive and in-depth data gathering and reporting (Creswell, 2007; Yin, 2003). In this study, the single case involved an experienced middle school mathematics teacher, and the unit of analysis was the teacher's TPACK. The boundaries for the case were defined by the teacher's 6th-grade teaching level and over a decade of teaching experience.

3.2. The Context of the Study

The context of this study covered the teaching practices of a middle school mathematics teacher working in a private middle school in Türkiye, the topics covered in the middle school mathematics curriculum published by the MoNE (2018), and the middle school students enrolled in this private middle school constituted

As for the Turkish National Education, the Ministry of National Education [MoNE] of Türkiye oversees all educational and training services and activities, including planning, programming, advancing, monitoring, and inspecting. The Turkish National Education system comprises pre-primary education (kindergarten and pre-school), primary education (primary school), elementary education (middle school), upper secondary education (high school), and higher education (undergraduate and graduate education). Compulsory schooling lasts 12 years in Türkiye. The Turkish Education System consists of four years of primary school (Grade 1 to 4), four years of middle school (lower secondary school) (Grade 5 to 8), and four years of upper secondary school (high school) (Grade 9 to 12). This education system is commonly referred to the '4+4+4' system (MoNE, 2024a).

According to the MoNE, primary and elementary education, spanning ages 6 to 13, is compulsory and cost-free in public schools, encompassing a total of eight years. Primary education consists of four-year primary schools, while elementary education involves four-year lower secondary schools (middle school), including both regular lower secondary schools and Imam Hatip middle schools (known as İmam Hatip Ortaokulu which provides both general and religious education). The overarching objective of primary and elementary education is to ensure that Turkish children acquire the knowledge, abilities, behavior, and habits that are necessary for good citizenship (MoNE, 2024a). This educational framework aims to foster alignment with national morals, preparing students for life and subsequent levels of education based on their interests, talents, and abilities (MoNE, 2018). In this study, the students whom the participating teacher taught were lower secondary school students.

In the current mathematics curriculum of primary education (Grades 5 to 8), there are five learning areas in lower secondary school mathematics: data processing and probability, geometry and measurement, algebra, and numbers and operations. The specific aims of the mathematics teaching program are as follows (MoNE, 2018, p. 9):

The student will be able to;

1. Develop mathematical literacy skills and use them effectively.

2. Comprehend and apply the concepts of mathematics in everyday situations.
3. Clearly articulate their own ideas and reasoning during the problem-solving process and identify any deficiencies in their mathematical reasoning.
4. Communicate and explain mathematical concepts in an understandable manner, use mathematical language and mathematical reasoning, and possess the ability to use the language of mathematics correctly.
5. Make sense of the links between people and objects, as well as the connections among objects, through the language and meaning of mathematics.
6. Intentionally monitor and manage their learning processes by acquiring metacognitive knowledge and abilities.
7. Use prediction and mental processing skills effectively.
8. Express concepts with different forms of representation.
9. Cultivate a positive attitude towards mathematics by engaging in learning experiences that foster confidence and a self-assured approach to solving mathematical problems.
10. Foster the traits of being systematic, meticulous, patient, and responsible.
11. Cultivate the ability to research, produce, and use information.
12. Recognize the relationship between mathematics and art and aesthetics.
13. Value mathematics by being aware that mathematics is a common value of humanity.

As for the school policy in the context of this study, the educational policy on technology is determined by the school's Directorate of Educational Technologies. In line with the mission, vision, and goals of the institution, the Directorate of Educational Technologies aims to use technology effectively and efficiently to enrich the learning experiences of students, improve the teaching methods of teachers, and support the data analysis and decision-making processes of the school administration. For teachers to have technical and technological skills in this direction, in-service trainings are regularly planned and carried out for teachers in cooperation with the school's Department of Curriculum Development and Evaluation-Assessment, with the support of professionals and academicians in the field. In this direction, the school management had a positive attitude towards all in-service trainings and

research studies supporting teachers' TPACK by evaluating the training requests coming from teachers.

To sum up, the existing elementary mathematics curriculum emphasizes teaching students the practical applications of mathematics in their daily lives, problem-solving skills, collaborative sharing of solutions and ideas, and fostering an enjoyment of learning mathematics. The primary focus of the curriculum is the principle that 'every child can learn mathematics', which encompasses many essential and modern teaching goals, various aspects of constructivism, and other teaching theories. The curriculum is built on constructivist concepts, student-centeredness, and a shift away from lecturing in favor of understanding, investigating, and conceptualizing the substance of mathematical ideas (Mullis et al., 2016).

3.3. Selection of Participant

The participant selection in this study was conducted in two stages. While the first stage was choosing a school to conduct the study, the second stage involved choosing the participating teacher from the relevant school. As the researcher of the study has also been working as a middle school mathematics teacher, he has colleagues from professional communities working at other public and private schools in the same city. One of the private middle schools where the researcher's colleagues worked was chosen for this study. Both the school administration and all the mathematics teachers at this school were encouraging and supportive of scientific research studies. Furthermore, the mathematics teachers at this school were experienced teachers (14 years of teaching on average), open to professional development programs and initiatives, and convenient for the present study. Thus, at the first sampling stage, the school was chosen using convenient sampling.

In the second stage of the participant selection, the researcher employed the purposeful sampling method to gather comprehensive data to address the research questions. The researcher visited the school and met all the mathematics teachers in the school's mathematics department to be familiar with them. The researcher also

briefed the teachers on the current study's objectives and content. After providing detailed information about the study and clarifying the questions posed by teachers concerning the study, Horzum and Öztürk's (2011) the TPACK survey (See Appendix A), which is the Turkish translation of the one developed by Schmidt et al. (2009) was administered to all volunteer teachers to assess self-perception of TPACK. Cronbach's alpha internal consistency coefficient for the Turkish version of the survey was calculated as .96 (Horzum & Öztürk, 2011). According to the survey results, the teacher with the lowest self-perception score regarding TPACK was invited to attend the study on a voluntary basis. Thus, the purposeful sampling method was also employed to provide thorough, deep, and rich data to address the present study's research questions.

The reason for selecting the teacher with the lowest score on the TPACK survey among other teachers was to develop a thorough comprehension of how the TPACK of the participating teacher develops over an extended period through conferences, classroom observations, and debriefings of the lesson implemented in the scope of mathematics coaching. Another reason was that, in comparison with inexperienced teachers, experienced teachers may be more resistant to novelties, changes, and innovations such as integrating technology into teaching practice (Hermans et al., 2008; Vannatta & Nancy, 2004). Therefore, it was crucial that the current study includes the participant who possessed these attributes.

Purposive sampling is grounded on the assumption that the researcher aims to acquire in-depth insights and understanding of the topic under study by selecting a sample that offers the most valuable information (Merriam, 2002). As such, the process of purposive sampling involves the researcher's establishment of predetermined criteria or attributes, as emphasized by Patton (1990). In this study, three specific criteria guided the selection of the participant: Being an experienced teacher, expressing a willingness to integrate technology into classroom practice, and long-term attendance at the study. These criteria served as the foundation for identifying the participant who would align with the research objectives. The participant selection in this study was thoughtfully carried out by considering both the specified criteria and the results of the TPACK survey. This purposeful selection

was aligned with the overarching goals of the study. Following the specified principles, this study's participant was an experienced middle school mathematics teacher working at a private school in Ankara, Türkiye. The selected teacher willingly and voluntarily participated in the study, aiming to enhance his professional development through mathematics coaching, specifically focusing on TPACK. The participating teacher was assigned the pseudonym Cem (he/his/him). Cem's characteristics are given in the following paragraphs.

Cem, the participant in the current study, has been teaching at the middle school (elementary) level in private education institutions for nine years. Cem graduated from the undergraduate Mathematics Department program in the Faculty of Science and Letters, which is not a teacher education program. Since alumni of the undergraduate programs of Mathematics in Türkiye are not allowed to work as teachers in educational institutions before obtaining a pedagogical formation certificate, Cem acquired this certificate after one year of training at a university's faculty of education. Cem did not take courses in pedagogy or pedagogical content knowledge during his undergraduate studies in the Mathematics Program in the Faculty of Science and Letters. According to his statements, his self-efficacy regarding TPACK was thought to be insufficient: "I improved myself in terms of pedagogy and content knowledge when I started teaching. However, I believe I have much to learn about teaching with technology effectively. What I know regarding technology combined with pedagogy is superficial." His proficiency in using technology professionally, including word processors, presentation tools, and operating systems, was sufficient. Additionally, he continuously incorporated technology into his daily routine. Even though Cem had no prior knowledge of GeoGebra, he did not have a negative attitude towards technology or teaching mathematics with technology. Instead, he was eager to learn more about integrating technology into his mathematics classes. Cem stated, "Technology use in mathematics lessons should be planned properly. I integrate technology into my classes, but I am unsure whether I do it correctly and effectively. I believe that it might be a very beneficial tool for making concepts more concrete." Cem's integration of technology into his lessons was limited to utilizing YouTube videos and digital visual materials related to the subject and engaging students in various

assessment activities from online teaching platforms such as Khan Academy. His classroom, which consisted of 24 students, was equipped with a whiteboard, a smart board, and a computer with internet access for teacher use. The learning environment in Cem's classroom was mainly teacher-centered. However, he tried to involve students in the teaching and learning process through hands-on activities and questions to help them comprehend critical concepts. The students were primarily passive listeners and note-takers in Cem's lessons.

3.4. Intervention of the Study: Mathematics Coaching

The researcher applied mathematics coaching as a professional development approach that aims to improve the teacher's understanding of the possibilities and opportunities of technology, how GeoGebra can be used effectively as a mathematics teaching and learning tool, the integration of GeoGebra into assessment and evaluation strategies, and the transfer and implementation of curriculum ideas into a technology-enriched learning environment.

The mathematics coaching process in this study comprised distinct cycles, starting with a pre-conference and concluding with a post-conference. The researcher devised This cyclic structure methodically, drawing insights from the existing literature on TPACK and mathematics coaching. The literature generally outlines three key stages in mathematics coaching: pre-conference, observation, and post-conference (McGatha et al., 2018; West & Staub, 2003). Aligning with this framework, the researcher identified and integrated these stages into the mathematics coaching process. Leveraging insights from the related literature of mathematics coaching and TPACK, specific components were introduced within each stage. For instance, Jaipal-Jamani and Figg (2015) advocated for teachers to engage in exemplary activities to enhance teachers' TPACK, leading the researcher to incorporate 'showing various exemplary technology-enriched activities' as a pre-conference component. Moreover, inspired by McGatha et al.'s (2018) recommendation that teachers reflect on their implementation during coaching, the researcher included 'reflection on practice' as a post-conference component. The detailed structure of the mathematics coaching process and its components are expounded in Table 1 as follows.

Table 1. Cyclic process of mathematics coaching

Pre-conference	Observation	Post-conference
❖ Understanding the teacher's TPACK	• Putting a new practice into effect	• Reflecting on the implemented lesson
❖ Offering technical support ❖ Illustrating high-quality technology-integrated activities and learning materials ❖ Jointly planning lessons enriched with technology	• Gathering data from classroom implementation	• Providing suggestions for the implementation • Debriefing on the implemented lesson and students' learning

Prior to the study, extensive TPACK training was designed for Cem, the participant of the study, according to the results of the TPACK survey (Schmidt et al., 2009), pre-observation, and TPACK pre-interview. The content of the training was specially planned to meet Cem's needs. This training aimed to provide training according to the TPACK components and their intersections (TK, PK, CK, TPK, TCK, PCK, and TPCK) that are insufficient in Cem's TPACK, and to support the development of all TPACK components (Mishra & Koehler, 2006). Cem started to receive this training, aiming to strengthen TPACK and TPACK components and intersections, at the beginning his participation in the first mathematics coaching In other words, the training focused on the learning domains of 'Angles and Measurement of Area' and was delivered to Cem concurrently with the coaching sessions in a way that their schedules do not overlap The training was finalized when the fourth coaching cycle was completed.

Apart from the coaching cycles (pre-conference, lesson observation, post-conference), the researcher provided Cem with 5 hours of TPACK training per week for two weeks for his development in technology, pedagogy, and content knowledge in addition to the intersections of these knowledge domains regarding the learning areas of 'Angles and Measurement of Area' (instructional design, misconceptions and their solutions , effective teaching methods, course-specific technologies, technology-based assessment and evaluation tools and so on). In this 2-week training lasting 10 hours, Cem was provided with detailed information about what technology integration is. The explanations were delivered through lectures on the approaches

that explain an individual's motivation to use technology, such as the Technology Acceptance Model (Davis et al., 1989), and how tools become integral parts of educational practices through their use, adoption, and adaptation by teachers and students as explained in the Theory of Instrumentalization (Rabardel, 1995). In addition, detailed information was provided on the theoretical framework of TPACK, TPACK components, and the interaction and intersection of these components (Mishra & Koehler, 2006).

In coaching sessions, Cem's deficiencies in various knowledge domains outlined in the TPACK framework (TK, PK, CK, TPK, PCK, TCK, TPCK) were addressed. The coach identified these gaps based on Cem's expressions, thoughts, and reflections before conferences and during lesson observations. During the pre-conference phase of each coaching cycle, intensified training targeted the deficient TPACK component. Consequently, subsequent coaching cycles reduced training intensity for the improved component while increasing focus on other deficient knowledge domains. This approach aimed to comprehensively develop Cem's understanding of technological (T), pedagogical (P), and content (C) knowledge and their intersections, thus fostering holistic TPACK development. For instance, in MCC 2, Cem's TPACK was identified as deficient, leading to intensified training during the lesson planning phase of the pre-conference in the MCC. The training emphasized understanding the pedagogical benefits and limitations of various technological tools in line with suitable pedagogical methods, theories, and frameworks (Davis et al., 1989; Mishra & Koehler, 2006; Rabardel, 1995). Consequently, Cem learned to select technology that aligns well with the content being taught and the instructional strategies being employed. To provide another illustration, when Cem was tasked with teaching the area of a triangle and the concept of height within a triangle during the MCC 3, he exhibited deficiencies in PK (Mishra & Koehler, 2006), evident in his lesson plan and statements during the pre-conference. After the coach identified Cem's deficiencies relevant to the teaching outlined in his lesson plan, he intensified training specifically on PK. Through this approach, the coach aimed to enlighten Cem about PK, which encompasses his understanding of diverse teaching practices, strategies, and methods to enhance student learning, illustrated through case studies, varied units and topics, and examples of various teaching techniques.

A significant part of this 2-week training was devoted to GeoGebra, a technology specific to mathematics. Cem was taught how to use basic tools of GeoGebra, the GeoGebra Lesson Resources website (https://www.geogebra.org/materials), how to create and use an account on this website, how to assign GeoGebra homework, how to download GeoGebra activities and how to create GeoGebra activities or assessment tasks. Conceptual and procedural understanding, effective teaching methods in mathematics, and the role of technology in teaching methods were other aspects addressed in the training provided to Cem. In addition, multiple representations (daily life situations, pictures, graphs, tables, verbal expressions, symbolic representations, etc.) and good examples of these representations, which are of great importance in mathematics teaching, were shared with Cem. Deductive and inductive strategies play a crucial role in students' comprehension of geometry topics, and the best examples of these strategies were also shared with Cem in this training.

In addition, the learning areas of 'Angles and Measurement of Area,' which are taught within the scope of the present study, were discussed with Cem in detail from the perspective of content knowledge. Specific and common misconceptions were addressed in light of the related studies in the literature, and recommendations for eliminating these misconceptions were shared with Cem. Examples of misconceptions and ways to minimize them were also discussed with Cem. In addition, effective assessment and evaluation techniques in mathematics and good examples of these techniques were shared. In keeping with the nature of this study, Cem also discussed technology-based assessment and evaluation methods, techniques, and tools appropriate to the content and teaching methodology. The use of digital assessment tools (e.g., GeoGebra assessment worksheet, GeoGebra homework, Nearpod, Teachermade, Kahoot, Quizizz, Google Forms, and so on) in and out of class, and good examples of the use of these platforms were shown to Cem within the scope of the training.

3.4.1. Pre-conference

In the initial stage of each mathematics coaching cycle, individual pre-conferences were held with Cem, spanning approximately 90 minutes each. These pre-

conferences, centered around lesson planning, served as a platform for the researcher as the coach to grasp Cem's TPACK and instructional planning tendencies. Before each pre-conference, Cem formulated mathematics lessons with a corresponding lesson plan, specifically focusing on a geometry topic enhanced with GeoGebra. These plans were then shared with the coach. Engaging in mathematical discussions during the pre-conference, the meticulously prepared lesson plans underwent collaborative review and enhancement with the guidance of the coach. Additionally, reflections, discussions, and issues addressed in the post-conference of a previously implemented lesson were considered in the pre-conference of the following lesson. In these pre-conferences, the coach shared mathematical, pedagogical, and educative power and insights, combining beliefs about mathematics and teaching and learning mathematics with technology. The coach's expertise enriched Cem's existing knowledge and abilities in mathematics, pedagogy, and education, merging both parties' capacities. Collaboratively, they set goals for future work and planned lessons. Moreover, the coach supported addressing deficiencies in each TPACK component (Technology, Pedagogy, and Content) and their intersections (TPK, TCK, PCK, TPCK) throughout the coaching cycles based on observed deficiencies.

During the first pre-conference (pre-conference 1) session of the Mathematics Coaching Cycle 1 (MCC 1), which focused on co-planning the lesson for teaching 'Constructing an angle congruent to a given angle', the coach suggested integrating GeoGebra to help students visualize and construct angles accurately. Using GeoGebra, students could manipulate the rays and better understand the properties of angles through interactive activities. Innovative teaching approaches, such as inductive strategies, were emphasized to encourage students to derive the properties of angles through exploration and observation.

The pre-conference 2 session of the MCC 2 focused on exploring the properties of adjacent, complementary, supplementary, opposite angles, and solving related problems. The coach recommended using GeoGebra to create dynamic angle models, allowing students to explore and manipulate the angles to observe how their properties change. This helped in making abstract concepts more tangible. The coach also emphasized using collaborative group work to solve related problems, fostering

deeper understanding through peer discussions. Deductive strategies were also integrated, guiding students from general rules to specific instances. The coach shared multiple representations of angle relationships using GeoGebra to align concepts with students' existing mental schemas.

During the pre-conference 3 session of the MCC 3, which focused on constructing the area relation of a triangle and solving related problems, the coach introduced GeoGebra activities where students could draw and manipulate triangles to observe how the height affects the area. Using GeoGebra, students practiced drawing heights in different types of triangles and understood the geometric properties through visual aids. The coach highlighted the importance of connecting the area of triangles to the more familiar area of rectangles to reinforce understanding. The coach also addressed common misconceptions, such as confusing the height with the side length, by providing clear visual examples and interactive exercises. Co-planning three hours of instruction, focusing on drawing heights in triangles, including obtuse triangles, on dotted or squared paper, discussed the most effective ways to teach a triangle's area relation.

During the pre-conference 4 session of the MCC 4, which focused on forming the area relation of a parallelogram and solving related problems, the coach guided Cem to use GeoGebra to demonstrate how the area of a parallelogram can be derived from the area of a rectangle. Students were encouraged to explore the properties of parallelograms through GeoGebra, manipulating the shapes to see how the area remains constant when the base and height are unchanged. The coach shared exemplary GeoGebra-enriched teaching practices, showing how to effectively integrate the software into lessons. Additionally, the coach provided online assessment practices using GeoGebra, allowing students to complete interactive quizzes and receive immediate feedback.

Throughout these pre-conferences, the coach aimed to foster innovative teaching approaches supported by technology. This included employing multiple representations to align concepts with existing mental schemas and using technology to transition between representations. Deductive and inductive strategies were

incorporated to enhance the teaching of geometric concepts. The coach also addressed common misconceptions and learning difficulties by providing literature articles and discussing overcoming these challenges with effective teaching strategies.

Cem was empowered to accept, reject, or propose alternatives to the coach's suggestions, fostering collaborative adjustments to the recommendations in the pre-conference sessions. Beyond addressing curricular concerns, these sessions also involved instructional discussions on suitable and effective teaching strategies. Furthermore, the coach shared common misconceptions and learning difficulties related to the topic through literature articles with Cem. These sessions aimed to motivate Cem to adopt the role of a guide of exploration in facilitating student learning through technology rather than a director. The focus was on fostering a discourse environment among students and between Cem and his students. Cem was guided on creating a GeoGebra-enhanced environment where students took responsibility for their learning. Additionally, the researcher advised integrating activities for an active learning environment, emphasizing self-exploration and Cem serving as a guide. The lesson plan was then restructured based on these discussions, providing a shared focus on technology integration for observation during mathematics coaching. Technical assistance was also offered by the coach when needed, including searching for ideas on technology integration into the teaching of the topic and exemplary GeoGebra activities on the internet. For instance, the coach taught Cem how to use the website 'https://www.geogebra.org/materials'. Pre-conferencing not only offered a glimpse into Cem's thinking processes and necessities but also provided the coach with the opportunity to offer feedback during the post-conference, drawing from the actual teaching practice and sharing the lesson plan for the implemented lesson.

In summary, during the pre-conference phase of the mathematics coaching cycles, the lesson planning with Cem was centered around several key aspects: Leveraging technology for conceptual learning (Niess, 2005, 2005a, 2006, 2007, 2011), enabling students to fully utilize technology in the lesson (Hughes, 2005), employing multiple representations to align concepts with existing mental schemas and using technology in transitioning between representations (Kaput, 1992), incorporating deductive and

inductive strategies in teaching the subject (Polly, 2011), integrating technology-based assessment and evaluation practices (Reeves, 2000), utilizing technology across all areas of mathematics (Cuban, Kirkpatrick, & Peck, 2001), employing technology as a tool for teaching and learning (Drijvers et al., 2010), leveraging technology to foster mathematical thinking (Sherman, 2014), using technology to provide students with active participation opportunities in lessons (Goos et al., 2003), and fostering self-management skills in their learning (Zimmerman, 2002), as well as using technology to deepen mathematical understanding (Hennessey et al., 2005).

3.4.2. Observation

Observing Cem's teaching practice constituted another essential component of the coaching process. In each mathematics coaching cycle, the coach allocated two to four hours to observe Cem's lessons. During these observation sessions, Cem implemented a GeoGebra-enhanced lesson, which had been the central focus of the pre-conference. The coach diligently recorded notes on classroom dynamics, intending to delve into them during the subsequent post-conference discussion. Observation of the actual teaching practice of the co-designed lesson proved to be valuable data during the subsequent post-conference, facilitating reflections on the integration of technology as a tool of learning and teaching. In addition, the coach observed and recorded students' work on GeoGebra, how they used GeoGebra and students' post-use expressions, in-class discussions on the topic, students' inferences, questions they asked, and their answers to the questions, and provided insights that guided the next steps in the coaching process. The classroom observation of Cem's teaching practice offered crucial evidence about his level in the TPACK Development Model, specifically gauging Cem's utilization of GeoGebra as either a solely demonstration tool or a dynamic tool in teaching. Consequently, the observation of teaching practice functioned as evidence of Cem's TPACK level and progress in the TPACK Development Model.

3.4.3. Post-conference

Following the observation of actual teaching practice, a comprehensive post-conference session took place in which the coach and Cem engaged to delve into the

intricacies of the lesson, particularly emphasizing the incorporation of technology. In this post-conference, the coach actively prompted Cem to verbalize his insights into the lesson, nurturing his professional growth. Cem participated in reflective discussions on his performance, encompassing technology, pedagogy, and content. Within these post-conference dialogues, Cem and the coach collaboratively explored avenues for enhancing the lesson. Establishing an open environment where Cem could articulate issues, concerns, and critiques regarding the observed lesson yielded valuable perspectives on what held significance for Cem, shedding light on his TPACK. Moreover, during the post-conference, the coach provided constructive feedback and recommendations to elevate the TPACK of Cem and enhance his instructional methodologies.

The coach shared insights from observing the actual teaching practice and the notes taken during the session, providing constructive feedback. To illustrate, in one instance of the mathematics coaching cycle's post-conference, the coach advised Cem to adopt a more student-centered approach and encouraged Cem to allow students to explore concepts independently and give them the necessary time to do so. Following the feedback, the coach and Cem agreed to approach GeoGebra with a new perspective, emphasizing growth. In summary, the post-conference was pivotal in evaluating the learning outcomes, shaping future instruction, and modifying upcoming lesson plans. Its primary aim was to perfect future lessons in terms of TPACK based on the reflections of both Cem and the coach.

By making recommendations to Cem on issues, such as deficiencies observed during actual teaching, unachieved instructional objectives, level of students' participation in the lesson, the roles of Cem and his students, and possible misconceptions, effective teaching solutions are presented for future lesson planning and implementation. To illustrate, in terms of addressing deficiencies in actual teaching, the coach suggested incorporating more student-centered activities, such as group work or hands-on activities using GeoGebra, to explore the area relation of triangles. This approach would encourage students to investigate and discover the concepts independently. In the next lesson, Cem planned activities where students used GeoGebra to manipulate and visualize triangles, helping them understand the area

formula through exploration. If there were unachieved instructional objectives, the coach recommended breaking down the problem-solving process into smaller, more manageable steps and providing scaffolded practice. Additionally, the coach suggested using technology to create interactive problem-solving exercises that guide students step-by-step. Following this advice, Cem developed a series of GeoGebra applets that allowed students to solve more straightforward problems first and gradually move to more complex ones, ensuring they understood each step before progressing. When student participation was minimal, with only a few students answering questions and engaging in discussions, the coach advised Cem to use think-pair-share activities and technology tools like interactive polling to increase student engagement. This approach would allow all students to participate and share their ideas. Considering these recommendations, Cem incorporated think-pair-share activities where students discussed their solutions with a partner before sharing them with the class. He also used an interactive polling tool to gather responses from all students, allowing him to gauge their understanding and adjust his teaching accordingly.

As another example, when the lesson was predominantly teacher-led, with Cem doing most of the talking and demonstrating, the coach suggested shifting the classroom dynamics by allowing students to take on more active roles, such as presenting their findings or leading a part of the lesson using technology. Accordingly, Cem assigned small groups of students to present their problem-solving methods using GeoGebra. This increased student engagement and allowed them to articulate their understanding and receive feedback from their peers and the teacher. As for addressing possible misconceptions, some students held misconceptions about the properties of triangles, which hindered their understanding of the area formula. The coach recommended using technology to create dynamic visualizations that address these misconceptions. For example, showing how the area of a triangle remains constant even when its shape changes, as long as the base and height are unchanged. Consequently, Cem used GeoGebra to demonstrate how manipulating the angles of a triangle affects its shape but not its area. This visual representation helped students grasp the concept more concretely and corrected their misconceptions. Following these improvements in teaching the area of triangles,

Cem and the coach planned to approach the next topic, the area relation of parallelograms, with similar strategies. This included using GeoGebra to explore and solve problems related to the area of parallelograms, ensuring that the successful techniques applied in teaching triangles would also enhance student understanding and engagement in the upcoming lessons.

3.5. The Data Gathering Procedure

The researcher, as the coach, structured the intervention of this study into three phases to monitor the changes and progress in Cem's TPACK: (1) Prior to mathematics coaching, (2) In the course of mathematics coaching, and (3) Following mathematics coaching. In other words, the data collection process of the current research encompassed three stages. Firstly, prior to mathematics coaching, observation of teaching practice (Pre-observation) and TPACK interview (pre-interview) were conducted to determine Cem's TPACK level. A two-hour classroom observation was administered to verify the extent to which the pre-interview about TPACK and Cem's responses in the pre-interview were put into practice in teaching practice. These activities aimed to determine Cem's TPACK levels according to the TPACK Development Model prior to the commencement of mathematics coaching. Secondly, in the course of mathematics coaching, three cyclic processes of pre-conference, observation, and post-conference were conducted as part of mathematics coaching.

The mathematics coaching cycles were performed four times in this study and designated as Mathematics Coaching Cycle 1 (MCC1) through Mathematics Coaching Cycle 4 (MCC4). In the last phase of the study, following mathematics coaching, the observation of teaching practice (post-observation) and TPACK interview (post-interview) to determine Cem's final TPACK level according to the TPACK Development Model, and Mathematics Coaching Interview to examine Cem's perceptions on mathematics coaching for enhancing TPACK were conducted following the completion of the study. Post-observation was done to verify the extent to which the post-interview about TPACK and Cem's responses in the post-interview were put into practice in teaching practice. These activities aimed to evaluate the

impact and changes in Cem's TPACK after the intervention of mathematics coaching. This comprehensive research design allowed for a thorough examination of Cem's development in TPACK throughout different stages of the coaching process.

The researcher systematically monitored and evaluated Cem's TPACK levels throughout multiple coaching cycles to track the development of TPACK. As previously outlined, each cycle of mathematics coaching consisted of three stages: Lesson co-planning by 'pre-conference,' observing the actual teaching practice of the planned lesson by 'classroom observation,' and stage of evaluation of the implemented lesson by 'post-conference.' After completing the mathematics coaching cycles, the researcher implemented the following steps: (a) Post-Mathematics Coaching: No pre- and post-conference sessions were conducted with Cem after four mathematics coaching cycles. Cem independently designed and implemented a lesson without the coach's support. Cem was observed for three hours during classroom practice, and he was conducted post-interviews on TPACK after the implemented lesson of three hours, (b) Determination of the final TPACK level: The researcher assessed Cem's final TPACK level based on the post-observation and post-interview on TPACK. Also, a mathematics coaching interview was conducted to comprehend Cem's perceptions regarding the role of mathematics coaching in developing TPACK. The interpretation of the TPACK development of Cem was framed according to the themes and related descriptors of the TPACK Development Model developed by Niess, Ronau, et al. (2009).

Before commencing the study, Cem and the researcher held a preliminary meeting to establish the hours and objectives for the lessons. The chosen unit, 'Angles and Measurement of Area' aligns with the sixth-grade level of the national mathematics curriculum of Türkiye. As outlined in the national curriculum (MoNE, 2018), this unit encompasses eight objectives and is recommended to be taught over 25 lesson hours. However, for the aims of this study, which specifically focused on Cem's TPACK in geometry, two objectives related to units of Measurement of Area, land survey measurements, and conversions were excluded. Consequently, the study concentrated on six learning objectives pertaining to the investigated topic. For the

learning areas of ‘Angles and Measurement of Area’, the observation of lessons spanned 17 hours, encompassing pre-observation, observation 1 through observation 4, and post-observation. Pre-observation occurred before the commencement of mathematics coaching. Observations 1 to 4 were conducted as integral components of coaching cycles throughout the mathematics coaching process. Post-observation took place after the conclusion of four cycles of mathematics coaching. During the post-observation, Cem planned and implemented the lessons without the coach’s support. Hours of observations and the learning objectives regarding the learning areas of ‘Angles and Measurement of Area’ are outlined in Table 2.

Table 2. Hours of observations and the objectives regarding the learning areas of ‘Angles and Measurement of Area’

Process	Objectives	Hours
Pre-observation	- To know that an angle is formed by two rays with the same starting point and shows it with a symbol	2
MCC1	- To construct a congruent angle to an angle	2
MCC2	- To discover the properties of adjacent, complementary, supplementary, and opposite angles, and solve related problems	4
MCC3	- To form the area relation(formula) of a triangle and solves related problems.	3
MCC4	- To construct the formula of a parallelogram’s area and solve related problems	3
Post-observation	- To solve problems related to the area of triangle, rectangle, and parallelogram	3

(MCC: Mathematics Coaching Cycle)

To obtain a comprehensive understanding of Cem’s development in TPACK in geometry via mathematics coaching, the researcher (acting as the coach) employed various data collection methods, including interviews, observations, and field notes. The data collection process involved multiple types of interviews, namely pre-interviews, pre-conferences, post-conferences, post-interviews, and a specific interview focused on the perceptions of mathematics coaching for improving TPACK. Observations were conducted before, during, and following mathematics

coaching sessions, allowing for a close examination of how the designed lessons were implemented. Detailed field notes were taken during each classroom observation to complement the observational data. The data collection period spanned from April 2023 to May 2023 and involved working with Cem, a middle school mathematics teacher, in 6th-grade mathematics classrooms. The timesheet of the study detailing the data gathering from Cem is given in Table 3. The entire study process is illustrated in Figure 5, providing a visual representation of the stages and data gathering tools involved in examining Cem's TPACK development through mathematics coaching.

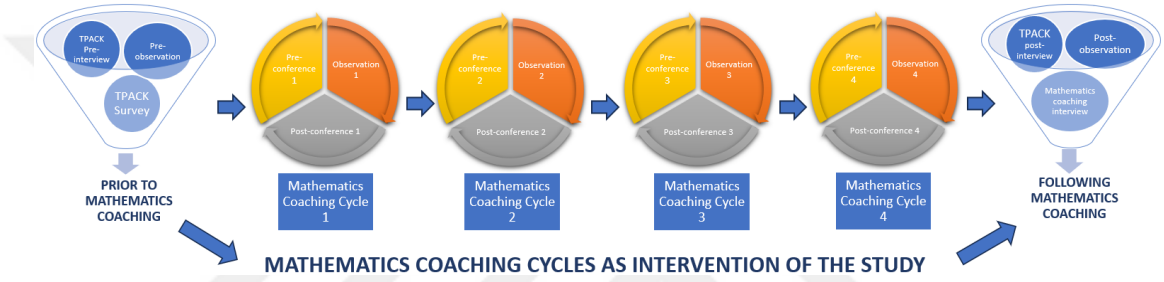


Figure 5. The flowchart of the study

3.5.1. The Data Gathering Tools

Various types of data were gathered before, during, and following the Mathematics Coaching Cycle (MCC) to address the research questions of this study. The data gathering process comprised the following steps: 1) Conducting a pre-interview with Cem on TPACK prior to the commencement of the study, 2) Observing the actual teaching practices of Cem to determine his TPACK level prior to the commencement of the study, 3) Conducting pre-conferences individually with Cem in the course of MCC, 4) Observing the actual teaching practices of Cem in the course of MCC, 5) Conducting post-conferences individually with Cem in the course of MCC, 6) Conducting post-interview with Cem on TPACK following the MCC, and 7) Performing mathematics coaching interview with Cem on his perceptions regarding the role of coaching in developing TPACK following the MCC. All the interviews employed in this study were developed as semi-structured interviews.

Table 3. Timesheet of data collection from Cem

Learning Objective and the type of data gathering	Pre-Conference	Observation	Post-Conference
(BEFORE MATHEMATICS COACHING) **PRE-INTERVIEW** (25 April 2023)		**PRE-OBSERVATION**	
M.6.3.1.1 Knows that an angle is formed by two rays with the same starting point and shows it with a symbol.	**No pre-conference**	1. lesson 26 April 2023 2. lesson 27 April 2023	**No post-conference**
(MCC1) M.6.3.1.2. Constructs an angle congruent to an angle.	1 May 2023	1. lesson 2 May 2023 2. lesson 3 May 2023	3 May 2023
(MCC2) M.6.3.1.3. Discovers the properties of adjacent, complementary, supplementary, and opposite angles; solves related problems.	3 May 2023	1. lesson 4 May 2023 2. lesson 4 May 2023 3. lesson 8 May 2023 4. lesson 9 May 2023	9 May 2023
(MCC3) M.6.3.2.1. Forms the area relation(formula) of a triangle and solves related problems.	9 May 2023	1. lesson 11 May 2023 2. lesson 11 May 2023 3. lesson 16 May 2023	16 May 2023
(MCC4) M.6.3.2.2. Forms the area relation(formula) of a parallelogram and solves related problems.	17 May 2023	1. lesson 18 May 2023 2. lesson 18 May 2023 3. lesson 23 May 2023	23 May 2023
(AFTER MATHEMATICS COACHING) **POST-INTERVIEW** (2 June 2023)		**POST-OBSERVATION**	
M.6.3.2.5. Solves problems related to the area of triangle, rectangle, and parallelogram.	**No pre-conference**	1. lesson 24 May 2023 2. lesson 25 May 2023 3. lesson 1 June 2023	**No post-conference**

3.5.1.1. The Data Gathering Tools Prior to Mathematics Coaching

Prior to the commencement of mathematics coaching cycles, the data were gathered via an interview on TPACK (pre-interview) and observation of the classroom practice (pre-observation). These activities were conducted to determine the TPACK of the participating teacher prior to the study, utilizing the TPACK development model as a framework.

3.5.1.1.1. Pre-interview Prior to Mathematics Coaching

To explore and assess TPACK levels of Cem concerning geometry according to Niess, Ronau, et al.'s (2009) TPACK development model, a pre-interview on TPACK was conducted prior to initiating mathematics coaching. The pre-interview was conducted using a semi-structured interview protocol, centering around the first research question (See Appendix B). These online interviews, lasting approximately 90 minutes each, were video recorded with Cem's consent. The choice of semi-structured interviews for this study allowed flexibility, enabling Cem to elaborate on or seek clarification for questions as needed. The TPACK pre-interview protocol that the researcher developed the TPACK interview protocol drawing upon the major themes and descriptors from the TPACK Development Model (Niess, Ronau, et al., 2009) employed open-ended questions to gauge Cem's understanding of technology integration in geometry, aligned with the TPACK Development Model. Cem's responses to the interview protocol of TPACK furnished data to ascertain his TPACK proficiency before engaging in mathematics coaching. Table 4 presents these themes and their associated descriptors (Niess, Ronau, et al., 2009, p. 11).

Table 4. Major themes and their descriptors in the TPACK Development Model

Themes	**Descriptors**
Curriculum and Assessment	• Curriculum, the treatment of the subject matter • Assessment, assessing the students' understanding
Learning	• Focus on subject matter (i.e., learning of mathematics topics) • Demonstration of conceptions of how students learn (i.e., development of students' thinking skills)

Table 4. (continued)

Teaching	• Focus on subject matter (i.e., learning of mathematics topics) • Instructional approaches • Classroom environment • Professional development
Access	• Usage (whether or not students are allowed to use technology) • Barriers (how teachers address barriers to technology integration) • Availability (how technology makes higher levels and more mathematics available for investigation for greater numbers of more and more diverse students.

The curriculum and assessment theme of the TPACK Development Model comprises two key descriptors: (1) *Curriculum descriptor* and (2) *Assessment descriptor*. The descriptor of the curriculum emphasizes the handling of mathematical contents, delving into how technology, curriculum, and materials used for the objectives of the curriculum are incorporated into the learning and teaching of mathematics. On the other hand, the descriptor of assessment centers on evaluating students' understanding by technology (Niess, Ronau, et al., 2009). Below are examples of questions from the TPACK interview that pertain to the curriculum and assessment theme:

- ❖ To what degree do you follow the national curriculum and textbook when teaching ‘Angles and Measurement of Area’ with technology? Are there modifications you make, and if so, what is the rationale behind these adjustments?
- ❖ What specific technological tools do you employ when teaching ‘Angles and Measurement of Area’? Could you provide insights into the reasons behind your choice of these tools and elaborate on their significance in your teaching approach?
- ❖ How do you integrate technology into the assessment of students' learning? Could you describe the methods and processes you use to incorporate technology into the assessment?
- ❖ In the assessment and evaluation process, what role do you attribute to technology? How does technology contribute to the overall assessment and

evaluation of students' understanding in the context of ‘Angles and Measurement of Area’?

Mathematics learning and the conception of student thinking descriptors are two essential descriptors covered in the TPACK Development Model Learning Theme. This theme delves into whether teachers effectively integrate technology as a learning tool into mathematics teaching and the improvement in thinking skills of learners through technological means. This theme also concentrates on subject matter (comprehension of mathematics topics) and demonstrating insight into the understanding of students' learning (i.e., development in thinking skills of students) (Niess, Ronau, et al., 2009). Below are questions related to this theme in the TPACK interview:

- In what ways does the incorporation of technology in teaching ‘Angles and Measurement of Area’ impact students' comprehension of the subject?
- How do you employ technology to foster students' conceptual understanding of "Angles and Measurement of Area’? Could you provide an illustrative example of using technology to deepen students' comprehension of the subject?
- In what ways do you utilize technology to address and rectify students' misconceptions or difficulties in grasping specific concepts related to ‘Angles and Measurement of Area’, such as drawing congruent angles, recognizing angle types, or understanding the calculation of the area of a triangle or parallelogram? Could you offer detailed examples to illustrate this process?

The teaching theme of the TPACK Development Model comprises four integral descriptors: (1) Mathematics learning descriptor, (2) Instructional descriptor, (3) Environment descriptor, and (4) Professional development descriptor. This theme delves into teacher’s perspectives on technology, their dispositions toward professional development, their understanding of instructional strategies incorporating technology, and the creation of learning environments that utilize technology as a tool for learning in individual and collaborative learning settings (Niess, Ronau, et al., 2009). Sample questions associated with this theme in the TPACK interview are given as follows:

- How do you use technology to introduce concepts students have never encountered? What is your rationale behind this approach, and could you provide an example to illustrate?
- When teaching the subject, do you think you employ inductive or deductive instructional strategies? Can you elaborate on the reasons behind your choice and offer an example illustrating your approach?
- What role do students play in your lessons, and how do you create a learning environment that ensures active participation? Could you provide specific strategies or examples that facilitate and encourage students' engagement in the learning process?
- What specific content or objectives do you envision for technology-related education? What purposes or goals do you hope this education will serve, and how do you envision its impact on teaching and learning?

The Access theme is another theme of the TPACK Development model, encompassing the descriptors of (1) *Usage,* (2) *Barrier*, and (3) *Availability*, which examines whether students are allowed to use the technology, the possible barriers that teachers may face in technology integration and how to deal with these barriers when they are encountered, and how technology is used to access more advanced mathematics for student exploration (Niess, Ronau, et al., 2009). Some questions from the TPACK interview associated with this theme are given as follows:

- What potential difficulties might arise when integrating technology into the teaching of 'Angles and Measurement of Area'?
- How do you proactively address and prepare for potential challenges students may face when using technology in 'Angles and Measurement of Area' lessons?
- In your teaching of 'Angles and Measurement of Area', do you leverage technology to incorporate multiple representations of mathematical concepts? If so, how?
- How do you employ technology to facilitate smooth transitions between different mathematical representations during your lessons on 'Angles and Measurement of Area'?

3.5.1.1.2. Pre-observation Prior to Mathematics Coaching

Observations of classroom practices prior to the beginning of coaching, named 'Pre-observations,' were administered to assess the teacher's levels of TPACK, considering the major themes and descriptors of the TPACK development model. Creswell (2013) mentioned that a researcher's involvement in observations can range from being a complete participant to a complete observer. To comprehensively analyze the data gathered, the researcher was in the role of a non-participant observer while observing a mathematics teacher within an authentic classroom setting. To put it differently, the researcher functioned as a nonparticipant observer, refraining from active participation in lessons to prevent influencing the teacher's behaviors while implementing technology-enhanced activities. This data collection method provided valuable evidence for addressing this study's first research question and sub-question.

In the pre-observation of Cem teaching the concept of angles, students were expected to learn that an angle is formed by two rays with the same starting point and to show an angle with a symbol. These pre-observations spanned two lesson hours, and the classroom practices were video recorded with Cem's consent. Video recording of pre-observations was employed to increase the study's trustworthiness through data triangulation. Considering the major themes and their descriptors in the TPACK Development Model (Niess, Ronau, et al., 2009), the researcher developed an observation protocol (See Appendix D) to employ during all the observations of actual teaching practices. This observation protocol encompassed three sections: (1) Basic information about the lesson, (2) Assessment of Cem's TPACK based on the performed lesson according to the TPACK Development model, and (3) Field notes for Cem's technology integration to highlight crucial indicators regarding his TPACK level. The first section included details such as the date of the lesson, teacher information, the researcher's place as an observer in the classroom, learning objectives, classroom size, seating plan, and technological facilities. The second section involved evaluating TPACK using the development model of TPACK. The themes and descriptors of the model were filled based on observed behaviors or similar actions during teaching practice. The third section consisted of field notes placed under the evaluation form for each theme and related descriptors. The field

note was used as a written record of the observations, experiences, and insights gained during the classroom observation.

During the observation of Cem's lesson on angles, the researcher aimed to gather data addressing the following questions: (1) Whether Cem effectively used Smartboard and dynamic geometry software to illustrate how angles are formed by two rays with a common starting point and used technology to visually represent angles to help students understand the abstract concept more concretely (TK), (2) Whether Cem employed a variety of teaching strategies, including direct instruction and guided practice, to explain the concept of angles or not. Whether Cem asked probing questions to check for understanding and encouraged student participation or not (PK), (3) Whether Cem demonstrated a solid understanding of the mathematical concept of angles, providing clear and accurate explanations and representations or not (CK). Pre-observations were conducted prior to coaching to understand how Cem performed in the actual teaching practice of co-planned lessons and integrated technology. These pre-observations helped to determine Cem's TPACK levels prior to the mathematics coaching cycles. As mentioned previously, the observation protocol was employed for classroom observations during the mathematics coaching cycles, completed by the researcher after each observation.

To ensure a comprehensive and reliable assessment of Cem's TPACK levels, a second coder responsible for analyzing video recordings of observed lessons also filled out the same observation form. This dual coding approach added robustness to the evaluation process. In the 'field notes' section of the observation protocol, the researcher documented notes highlighting both strengths and weaknesses observed in the classroom practices. These notes aimed to provide insights for assessing Cem's TPACK level. Following Merriam's (1988) recommendation, the researcher documented the field notes to provide a narrative depiction of the participant, course flow, and the learning environment. This documentation included direct quotations or summaries of students' statements and actions. Additionally, the researcher's personal thoughts, feelings, intuitions, perceptions, and views on the challenges faced during the classroom observations were recorded in the field notes. Cem's classroom practice was captured through these field notes, offering complementary

information to data collected from other data collection instruments, such as semi-structured interviews and surveys, as Bogdan and Biklen (2007) suggested.

3.5.1.2. The Data Gathering Tools in the Course of Mathematics Coaching

The researcher gathered data by way of pre-conferences (pre-conferences 1 to 4), observations of actual teaching practices (observations 1 to 4), and post-conferences after the lesson implementation (post-conferences 1 to 4) conducted during four mathematics coaching cycles. Pre-conferences were held online, and each spanned around 90 minutes. Classroom observations spanned 2+4+3+3 lesson hours and were videotaped. Post-conferences were held online, and each spanned around 45 minutes. These data collection methods aimed to continuously monitor Cem's TPACK levels and changes in his levels and TPACK development throughout the coaching process.

3.5.1.2.1. Pre-conferences in the course of Mathematics Coaching

As part of the mathematics coaching process, the researcher administered four pre-conferences. Cem collaboratively designed mathematics lessons in these sessions through content-specific discussions with the coach. These pre-conferences focused on lesson planning, providing the researcher, as a coach, with insights into the TPACK of Cem and position Cem in the TPACK Development Model. For this purpose, the researcher developed a protocol for the pre-conference. The pre-conference protocol was developed to align with the themes and descriptors stated in the TPACK Development Model, customized to suit the specific context of the present study (See Appendix E).

The pre-conference protocol comprised two main sections: (1) *Review of the lesson plan* and (2) *Enhancing the lesson plan*. In the first section of the pre-conference protocol, Cem was asked questions to elicit his practices in designing technology-enriched lessons and Cem’s understanding of technology inclusion. In this way, Cem’s deficiencies in TPACK were identified, and necessary support was provided to him to increase his Technological Pedagogical Content Knowledge for future lessons and lesson implementation. Sample questions from the pre-conference

protocol that the researcher developed concerning the four major themes of the TPACK Development Model are given in Table 5 below.

Table 5. Sample questions from the reviewing the lesson section of the pre-conference protocol

Curriculum and Assessment Theme
❖ To what extent did you adhere to the textbook and national curriculum when preparing the lesson plan? If you made changes, what were they? Why did you make changes? ❖ Which technological tools will you use in the lesson? Why did you choose these tools? ❖ Did you plan to use technology to assess students' learning? If so, can you explain how you will do this? What will the role of the technology you will use be in the measurement and evaluation process?
Learning Theme
❖ To what extent do you think technology will contribute to the conceptual learning of this subject? ❖ How will the technology you use in the lesson deepen student learning? Can you give an example? ❖ Have you planned to use technology to eliminate students' misconceptions or to help them understand concepts they have difficulty understanding? If so, can you explain how you planned it?
Teaching Theme
❖ Will you make use of inductive or deductive strategies in the lesson? If yes, how did you plan to use these strategies? What will the function of technology be when using inductive or deductive strategies? ❖ As a teacher, what will your role be in the lesson? How will the teaching process progress in your lesson? Could you elaborate on how you make use of technology? ❖ What is the role of the student in the lesson? How will you ensure active student participation in the lesson?
Access Theme
❖ Will your students use technology in this lesson? If yes, for what purpose will they use it? Can you explain? ❖ What precautions did you take before the lesson to cope with students' difficulties in using technology? ❖ Did you plan to utilize technology in the transition between representations? If so, how will you do this?

After the lesson plan review section, the researcher allowed Cem to progress in the TPACK Development Model by asking questions to restructure and improve the lesson plan to enhance Cem's technology integration in mathematics lessons. In

addition to asking questions, Cem's various suggestions and good instructional ideas were shared. Sample questions from this section are outlined in Table 6 below.

Table 6. Sample questions from the improving the lesson section of the pre-conference protocol

Highlighting the strong aspects of the teacher's lesson plan and guiding improvement
❖ (referring to an activity from the teacher's lesson plan) It was well thought out to include such an activity in the lesson. (Some good examples are shown) What do you think about incorporating an activity like this?
Discussing activities from the lesson plan that may have a limited impact on student's comprehension of the relevant concept
❖ Do you believe the technology-enriched activity you integrated into the lesson plan will enhance your student's comprehension of the topic? What alternative activities could be considered? What do you think about integrating one of these activities? ……(Good examples are shown.)……Would it be advantageous for the students?
Presenting exemplary GeoGebra activities and engaging in discussions.
❖ (Good examples are shown) Do you believe that this activity has the potential to enhance students' conceptual understanding?
Offering advice on TPACK
❖ Do you think that integrating this …(good examples are shown)… GeoGebra activity into the lesson with the teaching method you will use for teaching the subject can deepen students' understanding? ❖ Providing constructive feedback and suggestions for refining the lesson plan to enhance technological integration and pedagogical effectiveness.

3.5.1.2.2. Observation in the course of Mathematics Coaching

The researcher engaged in observations using observation protocol for Cem as a component of the mathematics coaching cycles (MCC1 to MCC4). Across Observation 1 to Observation 4, the researcher closely monitored how Cem implemented the lessons co-designed during the coaching process. During these observations, the researcher took field notes on issues such as the learning environment, Cem's behaviors, instructional decisions, obstacles encountered, difficulties experienced, classroom discussions, questions, and answers to evaluate Cem's TPACK levels consistently.

3.5.1.2.3. Post-conference Protocol

The post-conference protocol was applied to create a setting where Cem and the coach could ponder the implemented lesson and engage in discussions from a TPACK perspective (See Appendix G). The researcher tailored the post-conference protocol according to the themes and descriptors stated in the TPACK Development Model (Niess, Ronau, et al. 2009). Initially, the researcher posed questions to elicit Cem's thoughts on the strong sides and drawbacks of the observed teaching implementation, particularly concerning the inclusion of technology. Subsequently, Cem was provided with instances, anecdotes, and footage from the classroom to facilitate a thorough analysis. Additionally, the coach contributed suggestions for advancing Cem's TPACK. Furthermore, the issues discussed, emphasized, and addressed in the post-conference of the previously performed lesson provided Cem with the knowledge to plan the next lesson for effective and TPACK-strong teaching. Furthermore, the pre-conference of the next lesson reminded Cem of the crucial points discussed in the post-conference of the previous lesson. This contributed to Cem's continuous and constructive improvement in the deficiencies in all TPACK components separately. Sample questions asked to Cem during the post-conference are given in Table 7 below.

Table 7. Sample questions from the post-conference protocol

Questions about the entire lesson
• What are your thoughts about your technology use in the lesson you implemented today? What went well? What could be improved? How can it be improved? What are your suggestions? • Did you teach the lesson as you planned? Was there anything you did not specify in the lesson plan but experienced in practice? Why?
Curriculum and Assessment Theme
• Was your lesson in line with the textbook and national curriculum? Was your use of technology as recommended in the curriculum? Did you teach the lesson considering the recommendations in the curriculum (e.g., learning that should not be mentioned)? • [Showing a video extract from the lesson] ..... What did you aim to measure here, and how did you measure it? Could you have done it more effectively? Could it be more effective to use technology in the measurement and evaluation process as .....? What do you think about this? (suggestions are made)

Table 7. (continued)

Learning Theme
• To what extent do you think that the technology you used in the lesson you implemented today contributed to the students' conceptual learning? Did you receive any feedback (comments, answers, etc.) from the students that they discovered the concept? Can you explain by giving an example from a part of the lesson? • Did you use technology to eliminate possible difficulties and misconceptions? If yes, how did you use it?
Teaching Theme
• To what extent do you think your teaching method in this course was effective in students' learning? Why? • Can you describe the learning environment in the lesson? Would it have been more effective if the learning environment had been organized differently?
Access Theme
• Did you have any difficulty in integrating technology into the lesson? If there was a difficulty (by showing footage from the lesson) ..... do you think you had difficulties in this section? If yes, do you think you were able to overcome it? Would it have been more effective to bring a solution in the form of ..... (suggestions are made)? • (By showing footage from the lesson) ..... What was the purpose of using technology here? To what extent do you think its use here contributed to students' conceptual learning?

3.5.1.3. The Data Gathering Tools Following Mathematics Coaching

The researcher gathered data through various means, including post-observation sessions, post-interview, and mathematics coaching interview on Cem's perceptions of the role of mathematics coaching in developing TPACK conducted following the mathematics coaching sessions. These data gathering methods were employed to assess the TPACK levels of Cem and to uncover his perspectives on mathematics coaching.

3.5.1.3.1. Post-Observation Following Mathematics Coaching

Cem designed and implemented the lessons covering the last learning objective of the unit, which was taught in three lesson hours. The learning objective focused on 'solving problems related to the area of a triangle, rectangle, and parallelogram'. After completing four coaching cycles under the guidance of the coach, the researcher conducted post-observations to assess Cem's final TPACK level

following his attendance at the program of mathematics coaching. The field notes were taken, and the observation protocol (See Appendix D) was utilized during the post-observations. Notably, during the planning and enactment phases of the observed lesson, the researcher did not coach Cem in instructional design using GeoGebra. In brief, Cem received no support in instructional planning and implementation of the lesson, allowing the observations after mathematics coaching (post-observation) to reveal Cem's level of TPACK that he reached without the coach's support, independently in other words. The post-observation session lasted three hours and was video recorded.

3.5.1.3.2. Post-interview Following Mathematics Coaching

A post-interview on TPACK with Cem was conducted after he completed the actual teaching practice of the last learning objective. The TPACK interview protocol (See Appendix B) used for the pre-interview was also employed for the post-interview. The researcher aimed to gather comprehensive evidence to determine Cem's final TPACK level. Consequently, the post-interview provided additional data to support evidence of Cem's development in TPACK and assisted the researcher in determining Cem's final TPACK level following mathematics coaching. The post-interview session lasted approximately 90 minutes and was recorded on video.

3.5.1.3.3. Mathematics Coaching Interview Following Mathematics Coaching

The interview of mathematics coaching (See Appendix I) involved the researcher posing open-ended questions to Cem after the post-interview and post-observation. The aim of conducting this interview was to collect extensive data on Cem's perceptions regarding the mathematics coaching program for the development of TPACK. This study's second research question was addressed using mathematics coaching interview data. The researcher sought insights into Cem's perspectives on the benefits of a coaching program and any challenges or drawbacks he perceived. The interview of mathematics coaching lasted around 30 minutes and was recorded on video.

3.6. The Pilot Study

Yin (2009) stated that performing a pilot case study is the last step in preparing for the data gathering. Following the same procedures and utilizing the same data collection tools, a pilot case study was carried out with another teacher at the same private middle school, serving as a crucial step in refining the research approach. The pilot case study involved an experienced mathematics teacher, Melis (she/her), with 21 years of teaching experience. Teaching the concept of slope to 8th-grade students, Melis voluntarily participated in the pilot study and demonstrated openness to innovations and professional development. She possessed sufficient knowledge and proficiency in using technology, specifically GeoGebra and Desmos, for mathematics teaching. Melis's classroom, equipped with a smart board, whiteboard, and a computer for the teacher's use, mirrored the physical learning settings of the main study. The only differences were the grade level, topics taught, and the number of mathematics coaching cycles (one coaching cycle).

For the analysis of Melis's change in TPACK prior to, in the course of, and following coaching, various tools for data collection were employed, including pre-interview, pre-observation, pre-conference, observation of classroom practice, post-conference, post-observation, post-interview, and mathematics coaching interview. The pilot case study facilitated a thorough evaluation of the suitability of these tools. The researcher, acting as a coach, gained valuable insights into supporting a teacher for TPACK development during a coaching cycle. The researcher also gained valuable insights into the role and responsibilities of the coach, the roles and effectiveness of support functions in coaching (consulting, collaboration, and coaching), and the level of engagement of the coach in a coaching relationship. Moreover, the pilot case study informed the researcher about how the coach interacts with the teacher around these support functions to enhance the teacher's professional growth during the main study (Costa & Garmston, 2002; McGatha et al., 2008). Following the pilot study, necessary adjustments and improvements were made to the data collection tools, procedures, and the functioning of the coaching cycles. Also, the interview questions, interview protocol, coaching content, and the level of support to be provided to the teacher were revised. The content of the discussions to

be held pre- and post-conference in the main study was refined based on the experiences of the pilot study. Parts deemed insufficient in contributing to the teacher's TPACK development were removed. The pilot study refined the data collection process and aided in coordinating time and identifying the necessary data sources for the main study. The pilot study was crucial in enhancing the main research study's overall quality, trustworthiness, and effectiveness.

3.7. Data Analysis

Qualitative research data gathering, and analysis are concurrent and iterative processes characterized by reflection and ongoing engagement (Merriam, 1988). As Patton (2002) noted, the initial stages of data analysis often commence simultaneously with the data collection phase. The data analysis was described as the systematic process of seeking and organizing the transcriptions of interviews, field notes, and any collected materials to enhance comprehension and present findings to the reader (Bogdan & Biklen, 1982). This approach highlights the dynamic and interconnected nature of qualitative research, where the continuous interplay between collecting data and deriving meaning from it contributes to a comprehensive and nuanced understanding of the research phenomenon (Bogdan & Biklen, 2007; Miles & Huberman, 1994).

To generate a comprehensive depiction of Cem's development in TPACK related to the learning areas of 'Angles and Measurement of Area' as geometry concepts, this study employed semi-structured interviews, field notes, pre-conferences, document analysis of lesson plans of Cem, classroom observations, and post-conferences for data analysis. All audio recordings from the interviews and conferences and video footage from classroom observations underwent transcription. This meticulous process ensured translating spoken or visual data into written form. Subsequently, a thorough database for this case study was created by methodically organizing and integrating all the data acquired on Cem. This approach facilitated the structured data analysis, holistically examining Cem's experiences and practices within the study context.

To address the first research question and its sub-question, 'To what extent does attending mathematics coaching influence a middle school mathematics teacher's

TPACK in the learning areas of ‘Angles and Measurement of Area?’, various sources of data were analyzed prior to, in the course of, and following mathematics coaching. These data sources included transcriptions of pre/post-conferences, pre/post–interviews, recordings of videos, observations, and field notes from these observations. Cem’s advancement in TPACK was monitored and evaluated using deductive analysis, according to the development model of TPACK.

The deductive analysis is deemed suitable when data analysis adheres to an existing framework, requiring the pre-selection of themes and codes based on prior literature, theories, or the specifics of the research question (Patton, 2002). In the scope of data analysis of this study, the initial step involved transcribing videotaped interviews and observations, with subsequent review by the researcher to ensure accuracy. Then, relevant quotes were highlighted in transcriptions. The TPACK levels of development (Recognizing, accepting, adapting, exploring, advancing) and major themes (Curriculum and assessment, learning, teaching, access) were noted using the TPACK Development Model (See Appendix K), which included 11 descriptors and examples regarding Cem’s behavior clarifying the TPACK levels.

To determine the TPACK levels of Cem prior to mathematics coaching, transcripts of the pre-observation and pre-interview and video recordings were analyzed. Analysis of pre-conferences, observations, and post-conferences provided the researcher with identifying the TPACK levels of Cem during the respective coaching cycle. Post-interview transcripts and video recordings of post-observation regarding teaching practice were analyzed to assess Cem's final TPACK levels following mathematics coaching. While the descriptors in the TPACK Development Model (excluding the professional development descriptor of teaching theme) were searched and analyzed in all the data collected throughout the stages of mathematics coaching, the level of the theme of professional development of teaching was determined through the relevant questions in the TPACK interview conducted prior to and following mathematics coaching. This comprehensive approach allowed for a thorough understanding of the influence of mathematics coaching on TPACK of Cem in geometry.

Furthermore, during the observed lessons, when more than one behavior specified with different TPACK level descriptors was observed in any theme, the TPACK level of Cem was determined as the level to which the higher-level behavior was assigned. For example, the TPACK level of Cem who exhibited a behavior specified with a descriptor at both 'recognizing level' and 'accepting level' or a behavior equivalent to this behavior in the access theme was determined as 'accepting level' by accepting that he/she exhibited this behavior because he/she could access to a higher level. To provide a detailed understanding, here is an example definition of *the usage descriptor* within the Access Theme of the sequential TPACK Development Model (Niess, Ronau, et al., 2009):

Recognizing Level: Allows students to utilize technology 'solely' once they have mastered specific concepts.
Example: Beliefs about how students should learn mathematics present a challenge to the utilization of technology tools in mathematical exploration.
Accepting Level: Students employ technology in restricted ways during standard instructional sessions.
Example: Student engagements with technology are confined to brief and tightly controlled situations.
Adapting Level: Allows students to utilize technology in specially crafted units.
Example: Access to and utilization of technology is provided for exploring new topics, typically with the teacher's demonstration.
Exploring Level: Allows students to utilize technology to explore particular mathematical topics.
Example: Access to and encouragement for using technology is consistently available for mathematical exploration during most classroom periods.
Advancing Level: Allows students to use technology in every facet of the mathematics class.
Example: Technology is an opportunity to challenge preconceptions about the mathematical concepts students can master.

As can be seen from the example given above, thorough examples and explanations were furnished within the model of TPACK development for each TPACK level

concerning *the usage descriptor* of the Access theme. Similarly, examples of teachers' behavior regarding the levels, themes, and descriptors were clearly stated in the TPACK development model for curriculum and assessment, teaching, and learning themes. As an illustrative example, in the case of the use of GeoGebra as a tool that facilitates students' exploration of concepts through multiple representations, the researcher evaluated whether Cem only allowed students to use GeoGebra after introducing a concept and determined that these students reached mastery, which indicates a practice at *the recognizing level* as mentioned above. Additionally, a second specialist in mathematics education was involved in coding all the data gathered. In cases where inconsistencies or disagreements arose during the process of data coding, the second coder (an independent researcher of mathematics education with a Ph.D. degree) and the researcher of this study engaged in discussions, ultimately reaching a consensus for inter-coder reliability by the conclusion of the coding process (Miles & Huberman, 1994). This meticulous approach ensured a thorough assessment of Cem's TPACK levels within the Access theme, specifically concerning *the usage descriptor* and integrating tools such as GeoGebra into the teaching.

'What are the perspectives of a middle school mathematics teacher regarding the impact of mathematics coaching on the development of TPACK?' was the second research question of this study. By addressing this research question through data analysis, the researcher aimed to explore Cem's perceptions of mathematics coaching regarding the role and benefits of coaching for TPACK development. For this purpose, the transcriptions of semi-structured interviews and video recordings of interviews were analyzed using an inductive process of open coding. The interview transcripts underwent analysis in a systematic sequence involving data organization, category and theme generation, pattern identification, data coding, verification of emergent understandings, exploration of alternative explanations, and report writing (Marshall & Rossman, 2014).

3.8. Trustworthiness of the Study

Validity and reliability are crucial considerations for researchers throughout the study's research design, data analysis, and assessment of its overall quality (Patton,

2002). In qualitative research, validity pertains to the accuracy of findings perceived by the researcher, participants, or readers (Creswell & Miller, 2000). Trustworthiness, by Creswell's (2015) description, revolves around validating findings and encompasses a set of strategies designed to enhance the accuracy and credibility of the research outcomes. Lincoln and Guba (1985) argued in favor of alternative terms to characterize the concepts typically referred to as internal validity, external validity, reliability, and objectivity. They asserted that these alternative terminologies are better suited to capture the essence of qualitative research. Therefore, the three primary criteria put forth by Lincoln and Guba (1985) to assure trustworthiness comprise (1) Credibility, (2) Transferability, (3) Objectivity (Confirmability), and (4) Dependability. The strategies employed to promote this study's trustworthiness within these criteria are outlined in the following paragraphs.

The first criterion, Credibility, synonymous with internal validity, aims to determine if a study effectively measures or tests its intended objectives (Shenton, 2004). Several strategies recommended in the literature were adopted to bolster the credibility of this study. These strategies included triangulation, ensuring the participant's sincerity and honesty, conducting frequent debriefing sessions, and subjecting the study to peer scrutiny by other mathematics education researchers (Shenton, 2004). Triangulation was employed by comparing transcriptions of interviews with Cem with his lesson plans and classroom observations, providing a multi-perspective approach to address issues from different angles. This method evaluated the consistency between Cem's actions and statements and uncovered routines that might not be apparent in a single lesson, thereby enhancing the study's credibility (Denzin, 1978; Patton, 1999). Furthermore, to ensure sincerity and honesty, Cem was encouraged to express his genuine thoughts and experiences regarding the use of technology in his mathematics teaching (Lincoln & Guba, 1985). Debriefing sessions were conducted regularly to discuss the findings and reflections, allowing Cem to clarify any misunderstandings and contribute additional insights (Shenton, 2004). The study was also subjected to peer scrutiny by other mathematics education researchers, who provided feedback on the methodology, findings, and interpretations (Lincoln & Guba, 1985). Encouraging honesty among informants was another method implemented to enhance credibility. The participant

was informed of his right to decline participation, ensuring that only willing participants might contribute to the study, resulting in more reliable and comprehensive data. The participant of this study, Cem, was chosen among teachers who volunteered to participate in the present study. Additionally, the researcher emphasized independence from any institution or organization during nonparticipant observation to create an environment in which participant feel more comfortable while performing actual teaching activities, aligning with Shenton's (2004) suggestions. This approach aimed to enhance the credibility of the data gathered and this study's overall trustworthiness. Also, Cem and the researcher met frequently to fulfill the principle of conducting frequent debriefing sessions. This way, the researcher informed the participant about the study's progress. To fulfill the principle of subjecting the research study to peer scrutiny, the researcher included his mathematics education researcher friends in the data analysis process.

Creswell (2013) asserted that prolonged engagement and persistent observation require establishing trust with participants, understanding the culture, and verifying information to identify potential distortions introduced by the researcher or informants. Therefore, the observations were conducted in environments deemed trustworthy and comfortable. Furthermore, the researcher established close connections with Cem's mathematics classrooms before the study, aiming to foster trust and create a comfortable atmosphere for Cem and his students. Cem took part in the study for a total of 1.5 months. As for member check as another important procedure for credibility, the researcher engaged in discussions with Cem to ensure a thorough understanding of his responses throughout the pre-conferences, interviews, and post-conferences. Furthermore, the study's transcript was shared with Cem, allowing him to contribute to his representation as the teacher in the research.

The second criterion, Transferability of findings, or Generalizability, is another criterion Lincoln and Guba (1985) advocated to enhance the study's trustworthiness. While generalizability remains a contentious concern in qualitative research, the researchers offered some strategies for generalizing findings over the years. As one of these qualitative researchers, Merriam (1995) suggested strategies such as "thick description, multi-site designs, sampling within and modal comparisons" (Merriam,

1995, p. 59) could be employed to bolster the criteria of transferability. Moreover, Merriam (1995) suggested that generalizability should not be the primary concern of the researcher but instead of the 'consumer of the research' (p. 58). Consequently, this study adopted the thick description strategy, which aligned with the transferability criterion. Thick description involves presenting readers with comprehensive details regarding the phenomenon under investigation, enabling them to assess the similarity between the conducted study and their research regarding the transferability of findings. In this study, detailed case vignettes of a middle school mathematics teacher and comprehensive information about the study's context are presented to the readers, facilitating an informed decision regarding the transferability of the findings. This approach aimed to enhance the applicability and relevance of the research to similar contexts or situations.

The third criterion, Objectivity (Confirmability), as another crucial issue for the trustworthiness of the study, refers to the process of demonstrating that findings and data interpretations are derived directly from the collected data and are not a product of the researcher's imagination (Tobin & Begley, 2004). Objectivity also refers to the effort to ensure that the findings and conclusions drawn from the research are not unduly influenced by the researcher's personal biases, values, or preconceived notions" (Creswell, 2013; Patton, 2015). Lincoln and Guba (1985) recommended triangulation, an audit trail, and maintaining a reflective journal as methods to ensure the confirmability of qualitative research. An audit trail serves as tangible evidence that the researcher did not selectively find information to align with preconceived notions, considering both the process and the resulting data (Bowen, 2009). To establish an audit trail in this study, the research process involved collecting and preserving the data in a raw state, recording interviews, field notes taken during observations, and documents as lesson plans, allowing for crosschecking. Furthermore, the researcher incorporated direct quotations in the study to enhance confirmability. These practices served as a means of directly presenting participant's voices and perspectives, contributing to the credibility and authenticity of the research findings.

The fourth criterion, dependability, assesses the extent to which the findings can be replicated when the study is done over with the same context (Merriam, 1995;

Shenton, 2004). To enhance the dependability of the data, Baxter and Jack (2008) recommended involving different researchers in the coding process to achieve consensus on generated codes and categories. Following completing the initial coding phase, a second mathematics education researcher, focusing on a similar subject, independently coded the same data set in this study. Inter-coder reliability was determined using the percent agreement method, comparing the coded data by the two observers (coders) and calculating the agreement by dividing the number of agreed-upon codes by the number of agreements in total added to disagreements. The two researchers collaboratively developed and refined the final codebook through regular meetings and extensive discussions. The faculty member overseeing the study as advisor contributed to the revision process during these meetings. This iterative approach, involving multiple meetings and discussions, ensured the examination of inter-rater reliability through the development of the codebooks. According to Miles and Huberman's (1994) guidelines and formula, an inter-coder reliability percentage of nearly 90% is deemed acceptable. The inter-reliability between the two coders in this study was calculated to be 96%, indicating a high level of agreement.

3.9. The Role of the Researcher

Mathematics coaches play a crucial role in enhancing instructional practices and student outcomes by providing targeted support to teachers (Bengo, 2013). Based on related literature, several vital characteristics define an effective mathematics coach. These characteristics were revealed in the related literature as expertise in mathematics content (Knight, 2007; McGatha et al., 2018), pedagogical skills (Campbell & Malkus, 2011; Gibbons & Cobb, 2017), technological proficiency (McGatha, 2009), coaching and mentoring abilities (Costa & Garmston, 2002; Knight, 2011; Mangin & Dunsmore, 2015), professional development leadership (Desimone & Pak, 2017; Loucks-Horsley, Love et al., 2009), collaborative mindset (Campbell & Malkus, 2011), commitment to continuous improvement (McGatha et al., 2018), supportive and encouraging (Ehrich et al., 2004), and minimizing bias and ensuring validity (Campbell & Malkus, 2011; McGatha et al., 2018).

In the context of this study, the researcher exemplified the essential characteristics of an effective mathematics coach, as identified in the relevant literature. The researcher's qualifications and experiences align closely with these traits, highlighting their suitability for the coaching role. This alignment is particularly significant in enhancing teachers' instructional practices and TPACK proficiency. The researcher holds a bachelor's degree in elementary education, specializing in mathematics, and a master's degree in technology integration with GeoGebra, completed in 2014. This solid foundation in mathematics content is crucial for a mathematics coach, as it enables the researcher to provide accurate and insightful guidance to teachers, helping them develop effective instructional strategies and address student misconceptions (Knight, 2007). Moreover, with 11 years of experience as a middle school mathematics teacher, teaching grades 5 through 8, the researcher has honed pedagogical skills specific to mathematics instruction. This extensive teaching experience equips the researcher with the ability to model effective teaching practices and support Cem in implementing strategies that promote conceptual understanding and procedural fluency (Campbell & Malkus, 2011; Gibbons & Cobb, 2017). Furthermore, the researcher's significant knowledge and experience in educational technologies, including 13 years of using GeoGebra, positions him well in this characteristic. The researcher's proficiency extends to various field-specific technologies such as Cabri 3D, Geometer's Sketchpad, Geometry Expressions, and other subject-specific technologies. This technological proficiency allowed the researcher to assist Cem in integrating technology into their mathematics instruction effectively, enhancing the learning experience (Niess, 2011). Moreover, the researcher attended certification program of 'Coaching and Mentoring' from continuing education center of a prestigious university in Türkiye before conducting this study. Therefore, throughout the coaching process, the researcher demonstrated strong coaching and mentoring skills, engaging in listening, guiding, and encouraging Cem. By establishing a favorable, close, constructive, and supportive relationship with Cem, the researcher ensured a comfortable environment for professional growth. This supportive relationship is essential for providing constructive feedback and fostering reflective practice among teachers (Costa & Garmston, 2002; Mangin & Dunsmore, 2015; Knight, 2011).

In this study, the researcher acted as the coach, reflecting his mathematical, pedagogical, and educational expertise. By carrying out planning, reflection, and debriefing processes in close collaboration with Cem, the researcher took on a leadership role in professional development. This included designing and implementing professional development programs tailored to Cem's needs and promoting best practices in mathematics instruction (Desimone & Pak, 2017; Loucks-Horsley, Love, et al., 2009). Besides, the researcher fostered a collaborative environment by working closely with Cem in planning and conducting debriefing sessions. This approach promoted a collaborative mindset, essential for creating a supportive community of practice within the school, allowing Cem to share experiences, discuss challenges, and explore best practices.

In addition, the researcher encouraged Cem to examine instructional methods critically, reflect on their effectiveness, and make necessary adjustments. By modeling and promoting a commitment to continuous improvement, the researcher helped Cem develop a growth mindset and a commitment to ongoing professional development (Schwille, 2008). Additionally, the researcher exhibited a supportive and encouraging demeanor by providing personalized feedback based on classroom observations and engaging in detailed discussions about integrating technology into lessons. This approach helped build Cem's confidence and motivation to improve instructional practices (Ehrich et al., 2004). Moreover, to minimize potential bias and uphold internal validity, the researcher engaged in preliminary interactions with Cem and observed his lessons before the study commenced (Campbell & Malkus, 2011; McGatha et al., 2018). This proactive approach helped the researcher become acquainted with Cem's teaching context, ensuring that subsequent coaching interventions were relevant and contextually appropriate (Gallucci et al., 2010).

In summary, the researcher's extensive background in mathematics education, pedagogical skills, technological proficiency, and commitment to continuous improvement align seamlessly with the key characteristics of an effective mathematics coach. This alignment underscores the researcher's capability to provide effective coaching and support to enhance the instructional practices of mathematics teachers.

3.10. Ethical Considerations

The current study adhered to the ethical procedures established by the university, as outlined by the Ethics Committee of Human Research. To obtain confirmation for the study concerning ethical considerations, an application form of ethics, along with data collection tools, a study summary, and consent forms (participant information form and voluntary participation form), was submitted to the Ethics Committee of Human Research. Once the Ethics Committee of the University granted approval based on ethical principles, an application was then submitted to the Republic of Türkiye Ministry of National Education (MoNE) to acquire authorization for conducting the study at the school affiliated with MoNE, involving a thorough examination of the application process.

Upon receiving approval from MoNE, the participant teacher was informed that his participation in the study was based on volunteering. Before the research, the participant signed a consent form outlining the study's purpose and granting permission for audio and video recording. Participant information was confidential, and a pseudonym was used when reporting results. The researcher kept video recordings secure, and all the data were exclusively accessed for data analysis and interpretation of the results. The participant was ensured that the video recordings of his teaching practices would remain confidential and would not be shared with any third parties under any circumstances. Additionally, the participant was informed that the study results would be shared with them upon completion of the research.

3.11. Assumptions of the Study

The present study is based on several assumptions. Firstly, it was assumed that the participant teacher provided truthful and accurate information during the data collection process. To mitigate potential distortion in responses, the participant was encouraged to share his genuine thoughts without hesitation before each interview session. Emphasis was placed on the significance of his authentic ideas, highlighting his contribution to educational literature. Questions were formulated and presented to participant in a friendly manner, aligning with Yin's (2014) criteria for a well-

qualified interview. Additionally, it was assumed that the teaching practices observed throughout the study accurately reflected the teacher's genuine instructional behaviors and routines, providing an authentic representation of his classroom practices.

CHAPTER 4

FINDINGS

The primary aim of this case study was to investigate the changes in TPACK of a middle school mathematics teacher who participated in mathematics coaching focused on geometry. The data collected were analyzed using the TPACK Development Model proposed by Niess, Ronau, et al. (2009). The changes and development in the teacher's TPACK were examined across two learning areas of geometry taught at the 6th-grade level of the mathematics curriculum: 'Angles' and 'Measurement of Area'. Mathematics Coaching Cycles (MCCs) 1 and 2 were conducted within the 'Angles' learning area, while MCCs 3 and 4 focused on the 'Measurement of Area'.

This chapter is organized into three main sections. The first two sections address the first research question by presenting the findings on the changes and development in the teacher's TPACK. The first section describes the changes and development in the teacher's TPACK in the learning area of 'Angles' through his participation in MCC 1 and 2. The second section describes the learning area of 'Measurement of Area' through his involvement in MCC 3 and 4. The third section of this chapter addresses the second research question by exploring the teacher's perceptions of mathematics coaching for the development of TPACK.

4.1. The TPACK Development of Cem in the Learning Area of 'Angles'

This section explains how Cem combined GeoGebra, Geometry, and Pedagogy into his teaching practices in the learning area of 'Angles'. This section emphasizes Cem's TPACK level before coaching and his TPACK development during coaching. Cem's teaching performance was assessed based on the four themes (Curriculum and

Assessment, Learning, Teaching, and Access Theme) and the associated descriptors (*curriculum, assessment, mathematics learning-learning theme, conception of student thinking, mathematics learning-teaching theme, instructional, environment, professional development, usage, barrier,* and *availability*) outlined in the TPACK development model (Niess, Ronau, et al., 2009).

4.1.1. The Curriculum and Assessment Theme of the TPACK Development Model

The theme of curriculum and assessment includes two descriptors, one focusing on curriculum and the other on assessment. *The curriculum descriptor* centers on how subject matter is approached and integrating teaching and learning processes with curriculum, curricular materials, and technology. *The assessment descriptor* highlights teachers' decision-making processes when assessing students' learning and their employment of technology in the assessment process (Niess, Ronau, et al., 2009). In the upcoming sections, Cem's TPACK was scrutinized concerning the curriculum and assessment theme within the TPACK Development Model.

4.1.1.1. TPACK Level of Cem Regarding Curriculum and Assessment Theme Before Mathematics Coaching

Cem's TPACK level was assessed by observing his lessons for two hours before the mathematics coaching process's weekly pre-conference/observation/post-conference phases. Before mathematics coaching, he taught the learning objective '*Understanding that an angle is formed by two rays with a common starting point and representing it symbolically*' within the learning area of angles. All quotes and dialogues in the upcoming paragraphs were sourced from the TPACK pre-interview and the pre-observation lesson conducted before the initiation of mathematics coaching.

Cem's actual teaching practice before mathematics coaching indicated *the recognizing level* for *the curriculum* and *assessment descriptor*s in the curriculum and assessment theme. In the following paragraphs, this lesson is explained in detail.

Cem began the lesson by asking students if they had brought the materials he requested earlier and checking them. He started the lesson with a presentation and asked the students, ‘Have you ever wondered how the symbols for numbers were found?’ He received answers from the students. Without answering the question, Cem continued and said they would answer it later in the lesson. He then played a video from YouTube of a ski jumper's world record attempt. Before starting the video, he asked the students how far they thought the skier could jump and gathered their responses. He asked why they thought they were watching this video. When the skier was in the air, he asked what shape the skier's skis formed with his feet. He received answers from the students. Combining the students' answers with his explanations, Cem then asked where else angles could be seen in daily life. Students came to the board and drew examples of angles from daily life. Cem then projected examples like scissors, a compass, and the hands of a clock to demonstrate angles. Returning to the question he posed at the beginning of the lesson, Cem explained the relationship between the symbols for numbers and angles using mathematics history. Moving on to angles, Cem asked students to define an angle. Receiving various responses, he asked one student to develop another student's answer and corrected the responses. Cem then provided the mathematical definition of an angle without using any technology while discovering the concept of an angle. Projecting a section from the students' textbook on the board, he discussed concepts like the definition of an angle, the rays forming an angle, the vertex of an angle, and the symbolic representation of an angle. He drew the angle symbol in the direction the rays opened and asked students to compare the sizes of these angles. This allowed him to address a common misconception about representing the angle between two rays. He asked how the measure of an angle would change as the rays extended, explaining and correcting related misconceptions. He then asked several students to symbolically represent the angle projected on the screen and explained it further. He discussed how using a single letter to represent an angle would not be suitable when there were three rays with the same starting point and explained the difference between a triangle symbol and an angle symbol. Students then wrote the projected section in their notebooks. Cem continued by discussing an angle's interior and exterior regions, projecting the section about angle regions on the screen, asking students to note them, and ending the lesson. He did not measure students' learning with any

questions except those he asked and did not use technology for conceptual learning or assessment.

In the second lesson, Cem started by discussing the day's objectives, telling students they would use protractors. Recalling the previous lesson, he said they would do a group activity using what they learned before. He continued by asking about the unit of angle measurement and what angles are measured with. Correcting or adding to the student's responses, he transitioned to learning angle measurement and angle measuring tools. After receiving the answers to the questions "degree" and "protractor" from the students, he said they would measure angles with a protractor later in the lesson. Projecting a section from the student's book on the screen, he discussed the symbolic representation of an angle, the unit of angle measurement (degree), and the symbolic representation of an angle's measure. He then asked students to note the projected section. Cem opened a Khan Academy activity about comparing angles, which was not about measuring angles but deciding which type of angle (acute, obtuse, right, or straight) a given angle was based on previous knowledge. After explaining the questions in the Khan Academy activity, he asked different students to solve each, allowing seven students to solve the questions. After completing this section, he asked students, “What would happen if there were no angles?” and received their responses. He asked, “What would our houses look like without angles?” and asked students to form groups of 4-5 and draw houses without angles. Students formed groups and started working while Cem moved among the groups, providing feedback and checking their work. After completing the activity, he called a representative from each group to present their drawings. Students shared their drawings with their peers one by one. At the end of the lesson, Cem distributed exit cards aiming to measure what was learned in the two lessons. He checked the students' responses on the exit cards, provided feedback, and ended the lesson. Cem replied as follows when asked why he uses technology in the classroom during the TPACK pre-interview conducted before the study:

> “I have been actively using technology in teaching for the last 4-5 years, and I can say that students learn abstract concepts much better when visualizing them. Therefore, I think technology is beneficial for students to make sense of the subject via visualization of the concepts through technology. This is why I use technology to visualize the concepts in my classroom.”

Even though Cem stated the abovementioned statements during the TPACK pre-interview, he did not use GeoGebra in actual teaching practice to teach the concept of angle, which is formed by two rays with the same starting points. Instead, Cem utilized a presentation on PowerPoint, including visuals of angles in real-life context throughout the lesson and a ski long jump video on the topic at the beginning of the lesson. Cem asked students questions about the video and the visuals to make students discover the concept. During the actual teaching practice, he displayed angles using technology to represent angles visually. Cem perceived these visuals as contributing to understanding the concept of angles but needed clarification about how they could aid students in grasping the fundamental concept. However, he did not use GeoGebra or other subject-specific technology to enhance students' understanding of the topic. In brief, it was clear that Cem's knowledge of technology integration in education was limited to presenting a PowerPoint presentation and various visuals attached to this presentation to the students and having the students watch videos related to the subject.

Considering Cem's teaching practice before mathematics coaching, it was concluded that Cem's pre-observation aligned with his pre-interview statements. Therefore, Cem's teaching practice was consistent with the indicator of *the recognizing level* of *the curriculum descriptor* of the Curriculum and Assessment Theme: "Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum." (Niess, Ronau, et al., 2009, p. 20). Niess, Ronau, et al. (2009) argued that *the recognizing level* is characterized by teachers who have both the ability and knowledge to use technologies of type graphing calculators. Niess, Ronau, et al. (2009) stated that these teachers can also see where particular technologies fit into or relate to topics covered in their curriculums. Nevertheless, teachers at this level rarely think about using technology in teaching, and most are not willing to make it part of their practice or develop and try out new teaching approaches that might allow them to work for grades specific to this level. The features of Cem's teaching practice in the pre-observation phase were consistent with the level of recognizing. This consistency was evident from the fact that Cem used a YouTube video of a ski athlete's world record attempt to introduce the concept of angle and to show his students an example

of the concept in an everyday context, thus using technology to present a mathematical idea visually and contextually. Cem's use of technology in this way provided evidence that he utilized technological tools to display curriculum-related content. Furthermore, Cem's use of technology was more of a static tool to visualize the concept rather than a deep integration for teaching and learning purposes. Furthermore, Cem's association of the angle concept with everyday objects such as scissors, compasses, and clock hands showed his understanding that using familiar objects can help students make sense of mathematical concepts. However, Cem did not make transformative use of technology to clarify and concretize his verbal explanations. Although Cem's content knowledge was demonstrated by his explanations of the historical context of angles and number symbols, technological tools did not support these explanations. Throughout the lessons, Cem gave the mathematical definition of angle and used traditional methods (verbal explanations) rather than technology to eliminate the related misconceptions. Cem explained concepts without using technology for deep cognitive engagement and projected various chapters from the textbook and questions on the board. Cem's instructional approach reflected his awareness of the benefits of technology but his lack of knowledge and reluctance to incorporate it into his teaching practices consistently. In addition, Cem preferred the direct teaching method instead of using technology to assess students' learning or conceptual learning. The fact that Cem did not use technology to assess learning and tasks supporting higher-order cognitive thinking, consistent with *the recognizing level* of the TPACK development model, revealed his reluctance to fully adopt technology as a teaching tool due to his lack of self-confidence and knowledge.

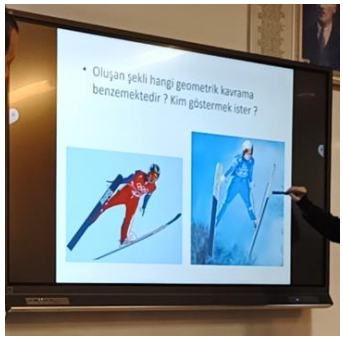


Figure 6. Cem guiding students through real-life examples to explain the concept of angles.

When Cem was asked during the TPACK pre-interview whether he uses technology during the assessment and evaluation process or not, he replied:

> "In the assessment and evaluation part, I want to see the answers given by the students, solution steps, etc., concretely. I believe the technology for assessing students' learning distracts students from the main mathematical ideas. In other words, I prefer written material more. Also, I do not think I am good at using technology in the assessment and evaluation process during the lesson. Therefore, I prefer traditional assessment and evaluation methods such as observation during the lesson, question and answer, and exit cards at the end of the lesson."

It was clearly understood from Cem's pre-interview statements that he preferred traditional assessment methods: "In the assessment and evaluation part of the lesson, I want to see students' answers, solution steps, etc. concretely. Using technology to assess students' learning distracts students from the main mathematical ideas". These statements proved that Cem resisted using technology for assessment because he believed it interfered with determining students' understanding of mathematics. Furthermore, Cem supported his opinion by emphasizing that his lessons used traditional assessment methods such as observation, question-answer, and exit cards. Cem's use of technology for assessment did not directly involve students in the technological assessment process. It was limited to entering exam results into the school grading system and looking at the classes' learning percentages and grade point averages from this system. In short, Cem did not use technology for instructional purposes or to assess student learning. Cem's assessment of students' learning in the classroom involved opening a Khan Academy activity with multiple-choice questions and Cem marking the answers himself on the website. In summary, Cem preferred traditional methods to integrate technology into the assessment process in his teaching practice before mathematics coaching.

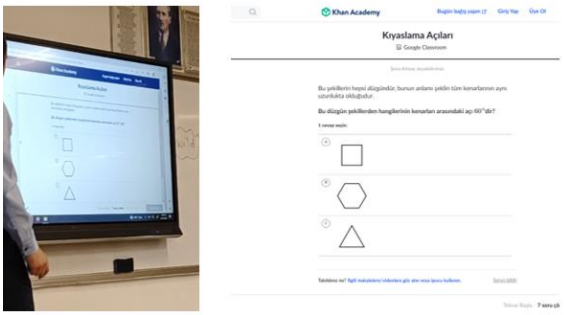


Figure 7. Cem using the smartboard to display a Khan Academy question

In light of Cem's teaching practice before mathematics coaching, it can be said that Cem's reluctance to embrace technology in assessments fully and his preference for traditional methods, both in his statements during the pre-interview and pre-observation, demonstrated his alignment with the descriptor of *the recognizing level* concerning *the assessment descriptor* of Curriculum and Assessment Theme as follows: "Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics." (Niess, Ronau, et al., 2009, p. 20). Niess, Ronau, et al. (2009) stated that teachers at *the recognizing level* of *the assessment descriptor* were resistant to using technology to assess student's learning because they think technology interferes with determining students' understanding of mathematics. Cem's statements in the pre-interview and his teaching practice reflected these characteristics. During the TPACK pre-interview, Cem acknowledged the potential benefits of tools such as Nearpod and Kahoot. However, he did not integrate Nearpod or Kahoot into his assessment methods in his teaching practice before mathematics coaching. Instead, he showed resistance and reluctance to fully adopt technological tools in assessment practices, preferring written materials and traditional methods.

4.1.1.2. TPACK Level of Cem Regarding Curriculum and Assessment Theme in the MCC 1

To determine Cem's TPACK level regarding the Curriculum and Assessment Theme at the Mathematics Coaching Cycle 1 (MCC 1), which focused on the learning objective '*Constructing congruent angle to a given angle*,' his teaching practices were observed for two hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the MCC 1 constituted the sources of all the quotations and dialogues in the following paragraphs. Cem's actual teaching practice in MCC 1 during mathematics coaching indicated *the adapting level* of TPACK for both *the curriculum* and *assessment descriptor*s of the Curriculum and Assessment Theme. In the following paragraphs, MCC 1 lessons are described in detail.

Cem started the lesson by checking the homework he had given in the previous lesson. Then, he projected the questions from the last lesson on the exit card about

the concept of angles and represented angles with symbols on the smart board screen. Then, he answered the questions together with the students. He gave feedback to different students by allowing them to answer the questions on the exit card. After that, Cem opened the "What did we learn?" page in the presentation and checked how well the topic learned in the previous lesson was understood with another question about representing angles with symbols. In this way, he reminded the students of what they had learned in the last lesson and moved on to the types of angles. Cem controlled the flow of the lesson through the PowerPoint presentation and followed the parts that he needed to mention in the lesson according to the order of the presentation. Cem briefly mentioned the content of today's lesson and drew attention to the subject with questions such as "How many types of angles do you know? How do you decide which angle it is? How do you measure angles?". Before getting answers to these three questions, Cem stated that he realized that the students already had some knowledge about the types of angles and received answers from different students about different types of angles. The students gave examples of various types of angles with the measures of the angles (such as 83 degrees is an acute angle). He verified and elaborated the answers from the students.

He continued the lesson with the question, "How would you measure angles if we did not have protractors in our lives?" and received answers from the students about drawing or measuring angles without protractors. In response to the student's answers, he tried to make them realize the necessity of protractors by asking them different questions related to their answers. Then, he projected the types of angles on the screen via the presentation and had the students write them down. Cem then briefly explained the protractor, the miter, the unit of angle measurement, and the symbolic representation of the measure of the angle using the page he projected on the presentation. After this section, he opened a video from Khan Academy (named "How to use a protractor") about measuring an angle using a protractor. He asked the students to watch it carefully. Cem paused the video occasionally and provided detailed explanations about the part mentioned in the video. He then asked the students to create an angle with the help of a ruler and measure this angle. He drew an angle with a large protractor on the whiteboard and showed how to measure it as an example. Meanwhile, the students drew an angle with a ruler in their notebooks

and continued to measure it with the protractor. After drawing the angle on the board, Cem gave information about which side of the protractor should be used to measure the angle, which was a common confusion among students when measuring with the protractor. Cem then walked around to check the students' work and gave them feedback. After this, Cem opened the GeoGebra activity, “Measuring angles with a protractor.” He said he would explain the activity and let the students explore it. He said that they would measure the interior angles of the triangle in the activity and check whether the sum of the interior angles is 180 degrees. Cem measured one angle of the triangle using the dynamic feature of GeoGebra. He called two students to the board to measure the other two angles. The first student tried to measure on the smart board. Then Cem said that he could make a more accurate measurement using the computer mouse, and the student measured the angle using the computer mouse. The next student first placed the protractor on the angle incorrectly. Then, after Cem helped and guided the student, the student measured the angle correctly. When Cem measured the angles, he did not use the magnification feature with the mouse wheel so that the angle written on the protractor on GeoGebra could be seen better. He wrote the three angles on the whiteboard and then showed the students that the sum of the measurements of the three angles was 180 degrees. In this way, Cem used GeoGebra as a verification tool.

From here, he opened the other GeoGebra activity, “Measuring Angles Using Protractor,” and explained the purpose of this activity. Then he said, "I want a student to come and explore the activity". A student came to the board and said, "We have an angle and a protractor. What do you think we are expected to do here?” The student answered, "The angle should be measured," and started to engage in the GeoGebra activity. The student tried to measure the angle by placing the protractor on the angle, measured the angle, and told Cem the answer, but the student's result was wrong. Cem wrote the student's answer in the relevant box. After receiving "the result is incorrect" feedback from the activity, he asked the student to measure the angle again and guided him in doing so. Cem realized that the student needed to place the corner of the angle in the right place and used the angle measure written on the wrong side of the protractor. Cem then guided the student with questions and explanations and verified the student's second measurement. Cem constantly

communicated with the student while he was using the activity. In this way, he assessed the student's learning on the topic using GeoGebra and gave feedback to the student. After that, another student came to the board and measured another angle correctly. Cem then used the dynamic feature of GeoGebra to change the angle again and had different students measure different angle measures, but not all students were involved in this process. He ended the lesson without giving homework. Apart from that, he did not assess students' learning using technology.

He started the second lesson by ‘drawing congruent angles’ by reminding the students of the previous lesson and the activities (Khan Academy, GeoGebra) (measuring angles, drawing congruent angles with a protractor). He mentioned the aims and content of today's lesson. Cem stated that in this lesson, they will draw an angle equal to an angle of a given size using virtual angle measurements. Then, he opened the Khan Academy and asked a student to do the activity on the board. He asked various questions to the students on the board (What are you going to do? What are we asked to do?). The student responded to Cem's questions by saying how many degrees of an angle he had to draw. He explained how he placed the protractor and an adjustable angle model on a draggable protractor and drew an angle of 10 degrees. In doing so, the student took advantage of the dynamic properties of the protractor and the angle model (moving the rays of the angle to change its degree, dragging and rotating the protractor to position it correctly, etc.). After seeing that the result was correct by pressing the "check" button in the activity, Cem called another student to the board to do the next activity. Cem helped and guided the student when the student had difficulty placing the protractor in the correct position. With explanations and questions, the student could correctly set the angle equal to the given angle on the protractor using the angle model. Cem then called five other students to the board. While the students were working on the activity on the blackboard, Cem explained the concept and the activity to the other students and contributed to their conceptual learning.

Then, Cem moved on to the activity Congruent angles - GeoGebra. Before opening the activity, he asked questions such as “What is an equal angle? What do you understand when you say equal angle? How do you know that two angles are equal?”

Cem asked a student who answered, “We decide by measuring with an angle gauge,” to elaborate on his answer. In this way, other students deepened their learning about the concept. Cem received different answers from other students and gave feedback to them. Then he answered the question by saying we could decide by measuring with a protractor and opened the activity Congruent angles - GeoGebra. Explaining the activity briefly, Cem aimed to make the students realize that the three congruent angles given in the activity were set to the same measure when he dynamically changed the measures of the angles in the activity from the arms of the angle. Cem used GeoGebra as a dynamic teaching tool instead of a demonstration tool. From there, Cem switched to a context from daily life, the game of billiards, and showed the students a video of the performance of a famous Turkish billiards player.

Here, Cem used technology to transition between representations. After briefly explaining how billiards is played and the rules, he asked the students, “Why did we watch this video? What could it have to do with our topic?” he asked. He received answers from the students. Cem tried to make the students realize that the angle of arrival and departure of the billiard ball from the band after the shot was equal. He then asked the students about the relationship between the angles of the ball's arrival and departure from the band and gave feedback on the student's answers. After receiving the answer "congruent angles" from the students, he projected a visual of the congruent angles made by the balls on the billiard table in the presentation. After the students realized that the tape's arrival and departure angles were congruent, Cem projected the definition of congruent angles on the board. After that, he mentioned that this situation also occurs in the reflection of light or sound and associated interdisciplinary concepts. After that, he asked the students to write down the definition of congruent angles in their notebooks and then draw two congruent angles in their notebooks with the help of a protractor. Cem gave the students some time, and during this time, he explained more to the class about drawing congruent angles and told the students what they were expected to do. He walked among the students, checked their work, and gave feedback. Cem then shared the home task with the students and gave explanations on how to do the task. The home task he gave the students was an interactive application called ‘MathsPad’, which dynamically contains geometric construction tools. Cem told the students that he expected them to

use geometric construction tools (compasses, rulers, etc.) to create rays, draw angles, measure angles, draw congruent angles, etc., in this online application and to send these works to him via e-mail. Then, he allowed two students to work on the "MathsPad" worksheet on the smart board as an example of how to do it for the whole class. Then, he ended the lesson by giving homework from the textbook. Also, he did not use any other technology-supported tool to assess students' learning.

Cem was enthusiastic about incorporating technology into the lesson during the pre-conference for the jointly planned session. Based on his remarks, it was evident that he encountered no challenges in pinpointing subjects within his curriculum suitable for integrating technology as an educational aid. Essentially, he concurred with the coach's recommendations regarding subject-specific technology-supported activities. Nevertheless, he needed to explore how to employ topics from his curriculum to integrate technology into the learning process seamlessly. Additionally, he needed to actively seek ideas and strategies to utilize technology more interactively to enhance students' understanding and application of mathematical concepts. In MCC 1, Cem's instructional approach with his participation in mathematics coaching, was oriented towards fostering students' conceptual understanding and encouraging a more discovery-based learning of the curriculum. This shift demonstrated that the discussions, suggestions, examples, and lesson planning during the pre-conference session enhanced Cem's TPACK level within *the curriculum descriptor*. The coach influenced Cem's technology, mathematics, pedagogy, and education perspective. Cem's understanding and use of the technological tools in the instructional context were influenced by the coach's belief about mathematics and its teaching and learning.

Cem's goal was to enable the students to explore the basic principles by applying them in actual teaching situations with the help of technology. In the pre-conference session where this lesson was planned, he showed much interest in developing activities in which students not only observe the use of technology in explaining mathematical concepts but also participate in activities that would require them to use technology to solve problems that are related to the mathematical concepts the students have learned. Cem ensured that the basic skills of constructing an angle with

a given measure and identifying the measure of an angle that has been built or given were well captured in the lesson. Cem mobilized the students for this purpose into activities in which they had to perform similar tasks with conventional means tools such as pencil and paper and with the use of technological and dynamic tools such as Khan Academy, GeoGebra, and MathsPad. Thus, he went beyond using GeoGebra as a tool whose primary aim is to represent mathematical objects and utilized both the verification and dynamic possibilities of the program. Cem showed tremendous understanding of how the use of technology is beneficial in the teaching and learning of mathematics curriculum as he was able to use a billiards game video in the actual lesson. Specifically, in the second lesson of MCC 1, in the following GeoGebra activity, Cem had expected the students to understand that all three angles are angles with the same measure by sliding the slider. He intended to help the students start thinking about congruent angles by asking, 'Why do all three angles change simultaneously and by the same amount when I move this slider?'. In this case, he employed GeoGebra as a dynamic teaching tool, not a demonstration tool.

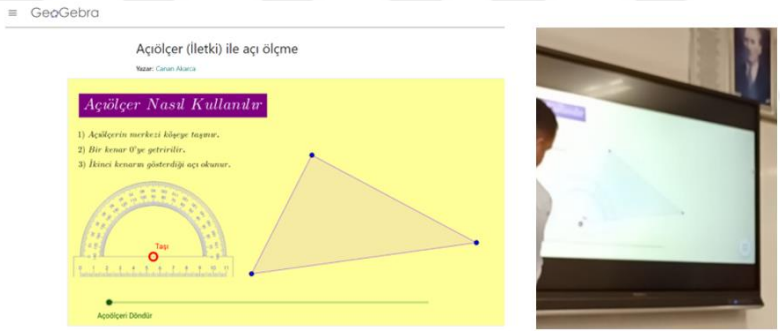


Figure 8. Cem demonstrating angle measurement using GeoGebra Protractor Activity 1

Cem used visuals from real life concerning the key concept to explain it to the students, making the subject more real. Also, his strategy was to create both visuals in dynamic geometry software where the two angles could be constructed and measured. The GeoGebra activities used in the lesson to let the students discover congruent angles also indicate his awareness of how appropriate technologies may be used to enhance student learning. It was observed that even though Cem was exposed to several GeoGebra activities in the pre-conference, he chose activities that were congruent with his teaching beliefs and strategies. This decision-making process

demonstrates his thinking in integrating technology, which stresses an individualized approach to ensure that the technology will improve the learning process of his students.

When Cem was asked about whether the GeoGebra activities he selected for the implementation would enhance the students' conceptual understanding of the topic in the pre-conference of MCC 1, he responded as follows:

> "I think these activities will help students deepen their understanding and explore the concept through visualization, manipulation, verifications, and geometric constructions. For example, students will watch the Khan Academy video and learn about using a protractor. Then, they will measure the interior angles of a triangle with a digital protractor in GeoGebra. Hence, they will realize that the sum of the interior angle measures is 180 degrees. Here, the fact that GeoGebra verifies the sum of the angle measurements will show the students that their measurement using GeoGebra is correct. In the next GeoGebra activity, only a protractor and an angle exist. The student will find the given angle's measurement independently on the board; I will only guide them. Then, there is the Khan Academy angle measurement activity. As students use these independently and check their results, they will realize what they are doing right and wrong. I think they will easily discover the concept visually by measuring and drawing angles in different measures."

When Cem was asked post-conference 1 what he thought about using technology in the lesson and what could be improved, he answered the following:

> "First of all, I must say that the student's interest in the lesson and the subject increased maybe five times. I can say this since even the most timid students in the class asked questions and wanted to come to the board and use the GeoGebra activity. After the lesson, I witnessed how the dynamics of the lesson, the class, and the quality of teaching changed when technologies such as GeoGebra and other digital tools specific to mathematics lessons were used. Everything went as I planned. I was also very pleased with the positive student feedback."

Although Cem initially recognized the potential benefits of technology, he needed to integrate technology into his curriculum meaningfully and effectively. In MCC 1, he clearly understood the benefits of technology and incorporated technology into the curriculum to effectively enhance teaching and learning. Therefore, based on Cem's statements during the pre-conference of MCC 1 and his actual teaching practices, it

was evident that he was at *the adapting level* for *the curriculum descriptor* within the Curriculum and Assessment theme of the TPACK Development Model of Niess et al. (2009). Cem's actions and reflections aligned with the indicator of *the adapting level* in *curriculum descriptor*: "Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum." (Niess, Ronau, et al., 2009, p. 20). Niess, Ronau, et al. (2009) stated that teachers at *the adapting level* of *curriculum descriptor* engage in activities that led to adopting or rejecting technology in teaching and comprehending the benefits of integrating suitable technologies. Cem's active incorporation of GeoGebra and Khan Academy in his lessons showed his willingness to try new ideas and assess their effectiveness in enhancing student learning. Cem used technology to reinforce concepts that had been previously taught. For example, Cem used GeoGebra to involve students in measuring and verifying angles instead of just showing the angles statically. Thus, he improved and strengthened students' understanding through practical application. Teachers at *the adapting level* in *the curriculum descriptor* use technology effectively to guide students toward intended learning outcomes and manage classroom activities accordingly. Cem's structured use of a PowerPoint presentation to control the lesson flow, guided activities with GeoGebra, and interactive questioning demonstrated his ability to manage the technology-enhanced lesson efficiently (Niess, Ronau, et al., 2009). This progress of Cem corresponds to progress in the TPACK dimension of the TPACK framework. Cem's ability to integrate technology in a meaningful and planned way to support and enhance the mathematics curriculum reflects his improvement and development in using technology as an educational tool. Cem's progress points to his developing competence in blending technology with pedagogy to facilitate deeper understanding and engagement among his students, thus demonstrating that his TCK has developed to move towards TPACK.

To assist Cem in progressing from *the recognizing level* to *the adapting level* for *the curriculum descriptor* of the Curriculum and Assessment Theme of Niess et al.'s (2009) TPACK Development Model, the coach employed the following support functions in the pre-conferences of MCC 1. The coach achieved this by showing Cem examples of how technology can be effectively utilized in mathematics

teaching and learning and the positive results from the students' achievement when technology is included in the teaching process. The coach and Cem set up lessons in which content-appropriate technology would be embedded, the intended mathematics concepts were noted, and activities that involved higher-order thinking were incorporated. The coach discussed pedagogical approaches and the use of lesson-specific technologies with Cem. In addition, the coach emphasized how technology can support inquiry-based and collaborative learning and provided hands-on training on tools such as GeoGebra, MathsPad, and Khan Academy. During this process, the coach tried to ensure that Cem was comfortable using these tools and provided additional resources, such as lesson plans and interactive activities. Cem was encouraged to engage in reflective practice, and the coach encouraged Cem to identify areas for technology development and keep a reflective journal. The coach modeled technology integration during lesson planning and provided Cem with a practical and detailed description of the tools to create engaging and effective lessons and assess students' understanding in real-time. During the pre-conference, the coach provided Cem with continuous feedback and support. In addition, Cem and the coach set progressive goals, starting with small changes and gradually increasing the use of technology. These strategies helped Cem move from *the recognizing level* to *the adapting level* by clearly understanding the benefits of using technology and integrating technology into the curriculum. Cem's progress reflected his development in blending technology with pedagogy to increase student engagement and understanding, thus improving his TPACK.

As for *the assessment descriptor* of Curriculum and Assessment Theme in MCC 1, instead of assessing students' learning after the end of the lesson, Cem allowed different students to use technology throughout the lesson as he taught the concept and gave them feedback after using the technology. In addition, students drew and measured angles and two congruent angles using paper, a pencil, and a protractor, a geometric construction tool, and did the same with digital learning tools such as GeoGebra, MathsPad, and Khan Academy. He checked the students' work in the notebook and gave them feedback. Cem gave feedback to the students on their work and learning outcomes whenever they used the technology on the smart board. He gave feedback to students who needed help working on the smart board with guiding

questions and explanations. Cem also assessed students' learning by asking questions about what was expected to be done in the activity projected on the smartboard through GeoGebra or other digital tools, as well as projected visuals or videos. Cem allowed the use of GeoGebra, MathsPad, and Khan Academy in the assessment but designed it to focus on collecting students' conceptual understanding of congruent angles and their procedural understanding.

While Cem recognized the appropriateness of incorporating technology into assessments, his perspective could have been more robust, primarily considering its use while teaching the concept. Despite this acknowledgment, he should have proactively explored the diverse possibilities of technology-based assessment items and questions, including technology-active, inactive, neutral, or passive elements. During the pre-conference discussions, it was inferred from Cem's statements that he grasped the importance of tailoring assessment questions to different aspects of understanding, distinguishing between conceptual and procedural understandings. Although he did not actively explore various types of technology-based assessment items, his recognition of the need for differentiated questions in the presence of technology suggests an understanding of technology's nuanced role in assessing students' learning. Instead of using multiple-choice assessment items, Cem posed open-ended questions for large group discussion, measured students' learning in GeoGebra activities, and gave them feedback accordingly. Instead of using technology-supported assessment tools at the end of the lesson, Cem used them throughout the lesson. This was evident throughout the lesson in Cem's use of Khan Academy, angle measurement in GeoGebra, angle construction, angle measurement, and angle congruence in MathsPad.

Cem permitted using GeoGebra, MathsPad, and Khan Academy in assessments. Yet, he tailored the assessment design to emphasize procedural understanding and conceptual comprehension of congruent angles among students. In doing so, Cem recognized the value of assessing not just the mechanical application of knowledge but also delving into students' more profound understanding of the underlying concepts related to congruent angles. This approach aligns with a comprehensive evaluation strategy beyond mere procedural knowledge and considers the students'

conceptual grasp of the mathematical principles involved. Thus, Cem's actual teaching practice was more advanced than how he taught the content before mathematics coaching regarding his technology use in assessment. Cem did not measure the learning of all students with a technological tool. Instead, he only allowed students who volunteered to participate in this process. Cem used the following GeoGebra activity in the figure to assess students' learning.

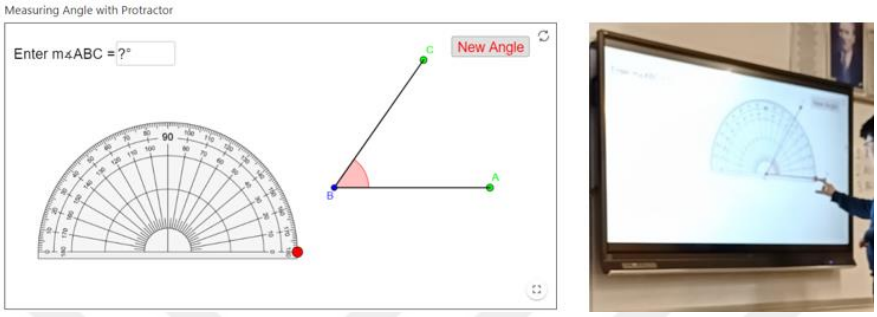


Figure 9. Cem measuring an angle using the protractor activity 2 in GeoGebra

Cem was hesitant to fully integrate technology into the assessment, even though the coach offered several technology-based assessment practices at the pre-conference of MCC 1, including GeoGebra assignments, Kahoot, etc. Instead, he preferred having some students use GeoGebra to construct congruent angles, draw the angle and find its measure, ask other students some exploratory questions, and check their work in a notebook during the lesson. However, Cem suggested incorporating the assessment item of Khan Academy into the lesson plan himself. In addition, during the pre-conference of MCC 1, he said as follows:

> "I don't think I have adequate knowledge to assess students' learning with technology fully. Also, students don't have technological tools. I have no other way but to use the smart board. So, I think the best way for me to incorporate technology into the assessment and evaluation process would be to provide students with a learning environment where they can engage in the activity and give them feedback on their misconceptions and mislearning. Of course, to assess the learning of such a topic, I will use different questions or assessment types such as geometric construction questions instead of multiple-choice questions because it is a geometry topic."

When Cem was asked in post-conference 1 to what extent he used technology to assess student's learning in the lesson, he replied as follows:

> "In this lesson, where I integrated GeoGebra, the type of questions I asked students changed. Open-ended questions that could result in multiple guesses and answers helped students go deeper into the topic and concept. I observed that the answers I received to these questions contributed more to students' conceptual understanding and mathematical reasoning. Doing this with paper and pencil type questions is much more difficult."

Cem's statements during the pre-conference of MCC 1 and his actual teaching practices aligned with the indicator: "Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings)." (Niess, Ronau, et al., 2009, p. 20). Hence, it was evident that he was at *the adapting level* for *the assessment descriptor* within the Curriculum and Assessment Theme. Niess, Ronau, et al. (2009) pointed out that teachers at *the adapting level* of *the assessment descriptor* undertake activities that result in the decision towards integrating or non-integrating technology in assessments. Cem used GeoGebra and Khan Academy to check students' knowledge and testifies that he is ready to use technology in class effectively. Cem provided feedback during technology-based activities, demonstrating his understanding that it is crucial to provide feedback while working, a characteristic of *the adapting level*.

Teachers at this level develop assessments to find conceptual and procedural knowledge through technology. Cem's systematic use of digital tools in the form of assessment showed that he could effectively coordinate the use of such tools. Cem's use of technology in teaching mathematics in a way that helps in assessment indicates that he had developed the use of technology in education. The following are some of his statements and practices during MCC 1 that gave evidence of this shift. At the onset, he appreciated the role of technology, yet he did not incorporate it into his assessment. However, in MCC 1, he had a good idea of the advantages of technology in assessment and, therefore, applied it generously to improve the assessment of student learning. This indicates his TPK development as he gradually learned to integrate technology into teaching and learning to achieve better understanding and active student participation (Mishra & Koehler, 2006).

The coach supported Cem in many ways during the pre-conference of MCC 1 to help Cem progress from *recognizing* to *adapting level* in *assessment descriptor*. Coach

introduced Cem to the concept of technology-based assessment, provided examples of effective technology integration, and introduced him to assessment tools such as GeoGebra assessment worksheet, GeoGebra homework, Nearpod, Teachermade, Kahoot, Quizizz, and Google Forms. Coach showed Cem how these tools can be used to enhance both conceptual and procedural understanding. In this way, he helped Cem to understand the benefits of incorporating technology into his assessments using different question types and assessment items. In collaboration, the coach and Cem planned lessons that included technology-based assessments aligned with the learning objectives and content. The coach provided Cem with hands-on training on specific educational technologies such as GeoGebra assessment worksheets, GeoGebra homework, Nearpod, Teachermade, Kahoot, Quizizz, and Google Forms. In addition, to support the implementation, Cem was provided with resources such as teaching videos, manuals, teacher material-sharing platforms, and sample activities. Emphasizing the importance of immediate feedback, the coach discussed strategies for giving real-time feedback with Cem that can be used when students are engaged with digital tools.

Cem was encouraged to evaluate current teaching practices critically and think reflectively about identifying areas for technological improvement. Specific and achievable goals for integrating technology into students' learning were also set. The coach frequently emphasized that Cem should integrate the assessment tools he learned in the lesson and test their effectiveness. The coach modeled how to incorporate technology into assessment practices effectively and demonstrated how to use assessment results (quiz or exam grade) to adapt and change teaching. The coach provided Cem with continuous support and feedback as he used the assessment tools he had learned before the conference and scheduled follow-up meetings to discuss his strengths and challenges. In this way, the coach helped Cem to improve his approach. These comprehensive support functions supported Cem's progress in the TPACK Development Model by enabling him to improve his competence in using technology in the assessment phase of the lesson. This progress contributed to Cem's ability to blend technology with pedagogy to assess student learning more effectively and in detail.

4.1.1.3. TPACK Level of Cem Regarding Curriculum and Assessment Theme in the MCC 2

To determine Cem's TPACK level regarding the Curriculum and Assessment Theme during the MCC 2, which focused on the learning objective '*Explores the properties of complementary, supplementary, adjacent, opposite angles, and solves related problems*,' his teaching practices were observed for four hours. The quotations and dialogues in the subsequent paragraphs were sourced from the pre-conferences, post-conferences, and mathematics lessons conducted within the MCC 2.

Cem's actual teaching practice in the MCC 2 during mathematics coaching indicated *the adapting level* of TPACK both for *the curriculum descriptor* and *the assessment descriptor* of the Curriculum and Assessment Theme. In the following paragraphs, MCC 2 lessons are described in detail.

Cem began the lesson after checking homework and asking students about what they had learned in the previous class. Controlling the lesson flow with a smart board presentation, Cem received responses from students such as 'Measuring angles, drawing congruent angles, and symbolizing angles' to his questions. After briefly summarizing these concepts, he moved on to a new topic. After informing students about the current lesson, he grabbed their attention with questions like, "What types of angles do we see in twin villas? What can you say about angles from the ball hitting the band on a billiard table?" These questions aimed to draw attention to the topic and assess students' prior knowledge. Using drawings on the board to make questions more straightforward, Cem explained the answers by illustrating the situation specified in the questions as students provided their answers. Emphasizing the concept of adjacent angles between twin villas and angles formed on a billiard table, Cem highlighted the common vertex to students. He then used a ballerina visual to discuss angles formed in the body (between arms and legs) during various movements. Later, he projected other relevant images from daily life, such as junctions, traffic signs, artworks, and airplane designs, onto the board, emphasizing the topic's use and importance. Continuing with angles between the hour and minute hands on a clock and angles formed in a mace, Cem asked students which angles

among those formed could be adjacent. As students provided answers, Cem deepened the discussion by opening the section on the definition of adjacent angles on the next page and asking students to take notes in their notebooks. Cem then introduced the 'Let's Explore Adjacent Angles' section, opened the 'GeoGebra - Adjacent Angles' activity, and invited a student to the board. Briefing the student on the activity, Cem had the student explore the relationship between angles by dragging the blue dots and using GeoGebra's dynamic features. While doing so, Cem asked exploratory questions like "How many angles are there? What is the relationship between the angles? What is the sum of the angle measures?". Cem allowed different students to demonstrate adjacent and non-adjacent angles with an example; the student identified the angles with a common vertex, saying that 'these angles are adjacent because there is a situation as stated in the definition.' Referring to another example in the presentation, Cem asked another student to come to the board to label adjacent and non-adjacent angles correctly. Following the students' solution performance, Cem used this example to address a typical misconception that many students have about angles formed between four rays with the same starting point. He explained this possible misconception in detail. When the lesson was over, Cem did not employ any technological device to evaluate students' learning. During the class, he assessed students' achievement through several means, including quizzing and questioning while the students were implementing the activities and large group discussions.

Cem started the second lesson by giving the students a problem concerning complementary angles. He allowed a student to work on the problem on the board, and the student solved it using the angle relationships. Cem assisted the student by posing questions when he was stuck and helping him apply what he knew to solve the problem. When analyzing the student's solution and ideas, Cem offered the following remarks. Cem then re-approached the problem by appealing to the relationship between angles and their measures and requested the students to write down what was being discussed. Starting the "GeoGebra – Complementary Angles" activity, Cem said, "What do complementary angles mean?" He explained the concept and proceeded with the GeoGebra activity by adjusting the angles' corners and measures using sliders. He did not allow any students to use the activity while in

class, but he used it for himself. After that, he adjusted the angles in the GeoGebra, made the angles more than 90 degrees, and let the students define complementary angles. Cem continued to pose questions such as, “What are the complementary angles of a 1-degree angle? Which angle has complementary angles equal to itself?” to test the concept’s understanding. Cem turned to the slide that defined complementary angles and told the students to write it down. Cem then moved to an example where he asked students to draw an angle with the help of a protractor and then draw another angle complementary to the previous one in measure. The students were then required to cut out the drawn angles and stick them in their exercise books. Finally, Cem requested the students to observe and state their opinions concerning the angle produced by the summation of these angles. At the same time, Cem offered more elaborations to the entire class regarding the idea of complementary angles besides the anticipated outcomes of the activity. He provided feedback to the students one-on-one by going through their work. Cem then opened the “GeoGebra – Adjacent Complementary Angles” activity and asked questions to let students focus on the angle relationship. He mentioned that the sum of the measures of adjacent complementary angles, which is 90 degrees, is the same regardless of the angles’ measures in GeoGebra. Cem did not allow any students to use the activity but instead used GeoGebra as a static tool for his demonstration. Next, he launched the “GeoGebra – Adjacent Complementary Angles 2” to illustrate to the students that although the angles change with the movement of the common vertex in this activity, the total degree of the angles is always 90. Thus, after the activity, Cem went to the presentation page containing the definition of adjacent complementary angles and told the students to write it down. When students were taking notes, Cem also asked questions and elaborated on the concept to ensure everybody understood it. In the end, Cem provided students with a problem that concerned complementary angles and called a student to the board to work on the problem with a connection between angle measures. Once the student was able to solve the problem posed by Cem, the lesson was concluded. Cem did not give homework, and for the final assessment of all students, he did not use any technology at the end of the lesson. In the class, he assessed the students’ comprehension by asking questions, questioning while students were using the activities, and having large group discussions.

Cem continued teaching the third lesson by posing questions about the concepts covered in the prior lesson, including adjacent angles, complementary angles, and adjacent complementary angles. Then, he briefly explained what he had learned from the previous lesson according to the feedback given by students and then introduced the theme of the present lesson, which was complementary angles. Cem then opened the GeoGebra – Complementary Angles 2 activity, giving information about the activity to the students and starting its use with the following question: What does it mean for two angles to be complementary? He requested the student to notice any change that would occur when he adjusted the sliders in the activity. The student indicated that one of the angles had to decrease as the other increased. The student then put one of the angles to 0 degrees and noted the other angle to be 180 degrees, thus deducing that their sum equals 180 degrees. Cem then requested another student to explain complementary angles in the student's own words. This student gave a similar answer. Finally, Cem defined the topic, stating that if the sum of two angles equals 180 degrees, the angles are known as complementary angles, and then proceeded with the explanation. At this stage, none of the students incorporated any technology. Following the question, Cem asked: "According to what do we classify angles as adjacent supplementary angles or non-adjacent supplementary angles?" Some of the answers Cem got from the students included "they should have a common ray" and "they should have a common vertex". These answers indicated that the students had a good grasp of the fact that if two angles are complementary, then their neighbors are also complementary and could apply this to any angle.

Cem then opened the GeoGebra - Complementary Angles activity. At this stage, Cem applied GeoGebra as a demonstration tool and did not permit the students to work on the activity. He first gave a brief background of the activity, then adjusted the sliders and asked the student to compare the two given angles, which are adjacent complementary angles and non-adjacent complementary angles. In this way, Cem used this question with the help of the angles and measured the level of students' comprehension of the condition for adjacent complementary angles. Another student said, "In the left angle, the blue and purple angles are added; in the right angle, the angles are divided into two." Later on, Cem explained the case of the adjacent complementary angles with an example. He then took a big protractor and made an

acute angle on the whiteboard. Permitting the student to complete the adjacent complementary angle, the student was later permitted to allow another student to draw the complementary angle to the initial angle drawn. Then, Cem invited the students to the GeoGebra activity "Adjacent Complementary Angles" and moved the sliders to change the position of the angles. In this way, he wanted the angles to be connected and simultaneously divided at a certain point to help the students differentiate between adjacent and non-adjacent complementary angles. GeoGebra was employed by Cem only as a demonstration tool. Cem went further and opened the GeoGebra - Adjacent Complementary & Complementary Angles activity, used the dynamic features to change the angles, and asked the students questions. Two students answered that the total measures of the angles did not change, the first being 90 degrees and the second 180 degrees. Cem then gave the students a problem concerning adjacent complementary angles and let a student solve it on the board. The student is right in solving the problem by relating the angle measures. Then, Cem logged into the "Khan Academy – Complementary and Supplementary Angles" and had the students do some fill-in-the-blank questions regarding complementary and supplementary angles. Four students were allowed to finish the solutions and put the answers in the required spaces. Once all the students finished their work, Cem described the solutions to the class in general. Cem proceeded with another Khan Academy activity; seven students were given problems to solve on the board and gave feedback on their solutions.

Cem then opened the activity, "GeoGebra – Opposite Angles," where GeoGebra's dynamic feature was applied to manipulate the angles by dragging them from their vertices or rays. In doing this, he posed some questions to the students to help them understand the opposite angles and develop a generalization of their measures. Cem posed questions such as "What are opposite angles? What can be said about their measures? How is the term opposite angles defined?" After this, Cem provided an activity where he had an example of intersecting rays, and different students were given a chance to find the opposite angles. Cem then briefly explained the concept from his understanding to ensure that the students understood it correctly. Cem then moved on to the GeoGebra activity on complementary and supplementary angles and angles around a point where the students were told what the activity was about and

asked to name some complementary angles around a point. When the students answered the question, he challenged them to name other adjacent supplementary angles. After their answers, he asked them to give him some angles opposite the ones they had listed. Cem supplemented the answers provided by the students with feedback and elaboration to improve students' comprehension. Last, Cem opened the GeoGebra – Opposite Angles 2 activity to show the students, using the dynamic features, that the measures of complementary angles are equal. He did not employ technology to assess the learning of all students at the end of the lesson.

Cem began the fourth lesson by explaining what the students would be doing in the class for that lesson. Then, he asked the students what they could recall about the prior lessons' topics, including complementary angles, supplementary angles, adjacent angles, adjacent supplementary angles, and opposite angles. Cem collected the students' comments and checked their understanding before moving on to the next activity. He then gave his remarks. Then, Cem navigated to the online multiple-choice quiz titled "Khan Academy – Complementary, Supplementary, and Opposite Angles" to check on the previous concepts. As for each question, he provided a chance for a student who wanted to answer to do that. Cem gave feedback to the students' answers and described the situations when they had challenges. He followed this process with seven students, allowing them to ask questions and explain concepts to further their understanding. Then, Cem started the GeoGebra – Complementary Angles activity, telling the students this would reinforce the previous day's lessons. Cem kept on questioning the students while they were doing the activity. He then opened the GeoGebra – Supplementary Angles activity and introduced an activity on GeoGebra that included all angles in summarizing the whole topic section. For this activity, it is possible to let the students actively learn and compare the different types of angles. Cem wanted to improve the students' comprehension; thus, he invited two students to come and work on GeoGebra to look at these angles. For the rest of the lesson, Cem opened different exercises from the textbook and let other students answer the questions. He allowed six other students to come and work on the board, solving and discussing the questions. Toward the last five minutes of the class, Cem clicked on the "GeoGebra Homework" page and guided the students on how to do the assignment before closing the class.

The approach in which Cem taught the topic in MCC 2 was similar to MCC 1, as it was the level at which he adhered to the curriculum and the materials he used to plan and deliver the lesson. Cem did not alter the national curriculum used to teach the subject and approached the subject as provided in the curriculum. Cem's consciousness of the benefits of appropriate technologies that can be used in the teaching and learning of Mathematics was evident in his pre-conference comments and the MCC 2. In detail, Cem identified the main ideas that the students investigated through GeoGebra. He devised lesson plans that presented mathematical ideas to the students using technology and activities where students used GeoGebra to check or deepen the concepts. Once the students could recognize different angles, they were allowed to use GeoGebra to check if two angles were equal or if there was a relationship between them. However, Cem did not examine ways to incorporate technology into his current curriculum to improve learning. He also did not actively look for ideas or strategies to integrate technology more extensively in developing the mathematics content taught to students. Furthermore, Cem did not revise his previous mathematics lessons to include technology or generate his ideas.

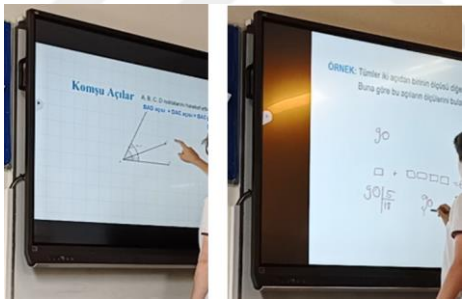


Figure 10. Students working on a GeoGebra activity and textbook problems in MCC 2 lesson

When Cem was asked whether the GeoGebra activities he chose to use in the classroom would contribute to students' conceptual understanding of the topic, he stated as follows in the pre-conference of MCC 2:

> "I think it will contribute to students' conceptual learning because, with the GeoGebra activities I will use, we will have the opportunity to examine many angles, many different situations, and the relationship between angles with different angles and measures. For example, they can observe the equality of two angles by changing the arms of the angle instantaneously. I will do this

> by using it as a dynamic tool and having the students use it. This way, they will learn the subject by experiencing it, away from memorization. This is how I planned to use technology as a teaching and learning tool for teaching the curriculum."

In addition, when Cem was asked at the pre-conference of MCC 2 what criteria he considered when choosing the technology to be used in the course and why he chose this technology, he replied as follows:

> "When selecting the technology, I pay attention to the level of the students, that is, that the technological tool I choose is a tool that students can use easily and is understandable. Apart from that, it is also crucial that it is suitable for the subject I teach. In this sense, GeoGebra is a very suitable tool for geometry subjects. I can say that I mainly consider these."

Even though Cem improved in terms of the treatment of the subject matter, he did not seek ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning according to the observation of his teaching practice in MCC 2 and statements in the pre-conference of MCC 2. Cem did not explore using topics in his curriculum to incorporate technology as a learning tool. Instead, Cem integrated technology in line with the coach's recommendations. Thus, regarding *the curriculum descriptor* of the TPACK development model, Cem could not make enough progress to a more advanced level and remained at the same level of MCC 1.

Therefore, the statements of Cem at the pre-conference of MCC 2 and his actual teaching practice during MCC 2 provided evidence to place him at *the adapting level* of TPACK for *the curriculum descriptor* of the Curriculum and Assessment Theme as this evidence aligned with the indicator of this level: "Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum." (Niess, Ronau, et al., 2009, p. 20). Niess, Ronau, et al. (2009) stated that these teachers also developed technology-supported activities for students to verify and reinforce these mathematical concepts. These teachers at *the adapting level* of *the curriculum descriptor* target the core topics that students investigate with technology and design lessons to demonstrate mathematical concepts with technology. Cem's teaching practice in MCC 2 reflects these

characteristics. Cem's integration of various GeoGebra activities aimed at mathematical explorations to teach different types of angles, properties of angles, and relationships between different angles demonstrated his skills and knowledge in incorporating technology in a meaningful way. His teaching approach showed that he targeted key topics students explored with technology. He demonstrated mathematical concepts with technology and activities for students to use technology to verify or reinforce these concepts. For example, he used GeoGebra to dynamically illustrate the properties of angles dynamically, thus reinforcing students' understanding through interactive visualizations. Students could explore mathematical ideas about different angles in a dynamic learning environment. Cem's structured approach to using digital tools in his lessons demonstrates his ability to manage and apply technology effectively.

To support the development of Cem in TPACK, the coach applied various strategies focusing on *the curriculum descriptor* during the pre-conference of MCC 2. The topics of the pre-conference session were complementary, supplementary, adjacent, opposite angles, and solving related problems. The coach introduced Cem to examples of successful technology integration specific to angles and then showed Cem how tools such as GeoGebra can visualize and manipulate angles dynamically. In this way, the coach supported Cem to gain a deeper insight into the different angles and their properties. The coach and Cem identified key concepts, designed activities that integrated technology to reinforce these concepts, and planned the lesson together. Cem suggested using interactive activities where students could explore the relationships between different types of angles by adjusting measurements and observing the results. The coach suggested integrating various pedagogical approaches in the lesson, such as inquiry-based and collaborative learning, which can be used harmoniously with technology to enhance student engagement and learning. The coach emphasized the effectiveness of these methods and encouraged Cem to use applications where students can explore angle relationships through guided discovery using technology and digital tools for problem-solving. The importance of making changes in the curriculum and integrating technology into the curriculum through examples; the coach gave Cem hands-on training on creating interactive angle activities using GeoGebra. The coach

provided Cem with resources such as interactive problem-solving exercises, GeoGebra activities, and instructional videos on angles prepared by different teachers. This helped Cem feel comfortable with technology and understand its potential benefits. The coach encouraged Cem to identify areas where technology could enhance the curriculum and reflect on the strengths and weaknesses of his implemented lessons.

As for *the assessment descriptor* of Curriculum and Assessment Theme in the MCC 2, how he assessed students' learning was similar to MCC 1. Cem asked students questions about concepts throughout the lesson. The manner of questioning varied, sometimes involving large group discussions and others consisting of individual questions. In addition, he had students use protractors to draw angles and asked them to determine the complements and supplements of these angles. In another part of the lesson, Cem conducted activities to reinforce concepts learned from the textbook, involving students in problem-solving situations. Furthermore, while working with students on GeoGebra activities, Cem asked them questions, provided feedback, and assessed students' learning through the GeoGebra homework assignment at the end of the lesson.

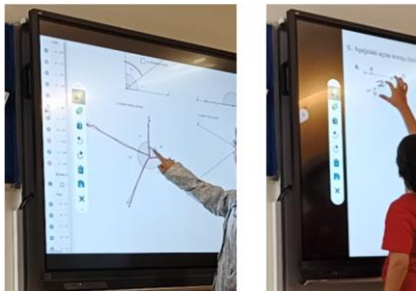


Figure 11. Students while working on GeoGebra and exercises from the textbook

To detail the GeoGebra homework, in the collaborative integration of the GeoGebra homework task by the coach and the teacher, the objective was for students to create constructions in a dynamic geometry environment, test their geometric ideas and assumptions, and respond to open-ended questions about the concepts. After completing the teaching of the subject, students were assigned a task accessible at https://www.geogebra.org/classroom. Cem provided detailed instructions to the students on accessing the assignment and shared the necessary information. Students

who accessed the assignment followed the instructions and submitted their work as a GeoGebra worksheet to their teacher. As students completed the tasks in the study, their names and works were displayed on the page of the corresponding task in Cem's GeoGebra account.

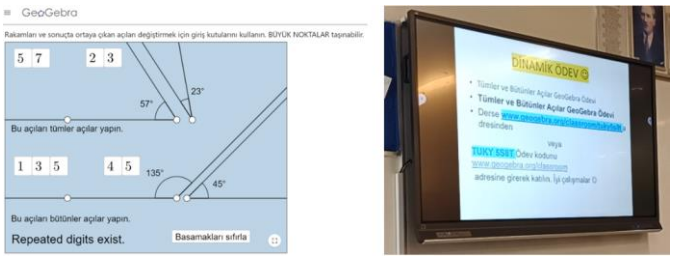


Figure 12. Screenshots of GeoGebra homework assignment on complementary and supplementary angles

Based on his statements in the pre-conference of MCC 2, it was inferred that Cem acknowledged the need to pose different assessment items or questions when technology is used during assessments, distinguishing between conceptual and procedural understandings. During the MCC 2 pre-conference, when asked about the role of technology in the assessment and evaluation process of the lesson, Cem responded as follows:

> "The role of technology will essentially be to expedite and streamline the assessment of students' learning. It enables us to analyze numerous examples swiftly and provide prompt feedback on various solutions and answers from different students. Of course, we need to carefully select the appropriate assessment tools and items for this purpose. I intend to do this in this lesson while the students are working on GeoGebra and through online platforms such as Khan Academy. Additionally, I will assign GeoGebra homework with detailed instructions and individually examine each student's work, offering feedback. This will be something I am attempting for the first time in my professional career."

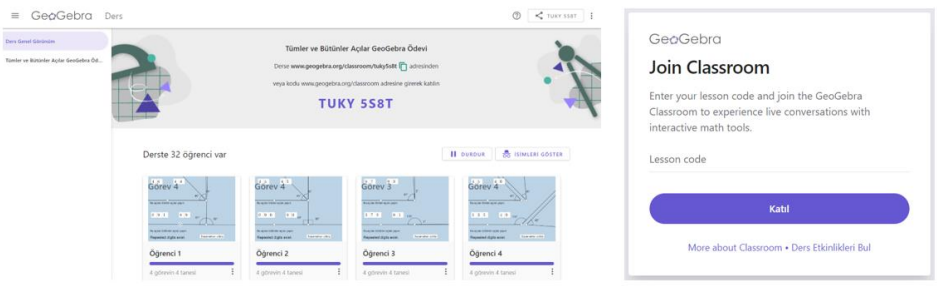


Figure 13. Steps to access the GeoGebra homework assignment on complementary and supplementary angles

In pre-conference 2, the coach implemented targeted strategies to help Cem enhance his TPACK based on *the assessment descriptor* by focusing on complementary, supplementary, adjacent, and opposite angles and solving related problems. The coach introduced Cem to GeoGebra as a powerful tool for assessing students' understanding of angles. Coach demonstrated how GeoGebra could create dynamic, interactive activities in which students can manipulate and explore angles in various ways. This allowed Cem to see how technology could provide immediate feedback and a deeper understanding of geometric concepts. The coach and Cem collaboratively planned lessons incorporating GeoGebra for teaching and assessment. Using GeoGebra, they designed activities for students to construct and measure complementary, supplementary, adjacent, and opposing angles. Adding GeoGebra to homework assignments would require students to apply their knowledge and think critically about the relationships between angles. The coach suggested that students integrate GeoGebra into their homework assignments. They discussed how Cem could create GeoGebra worksheets that students could complete online, allowing them to explore angle relationships at their own pace. Using the GeoGebra, the coach demonstrated how to set up these assignments, collect student responses, and provide feedback. By doing this, students received timely feedback on their work and continued engaging with the material outside the classroom. It was highlighted that GeoGebra could assess Cem's understanding in real-time during lessons. They demonstrated that Cem could assess students' understanding interactively through GeoGebra activities projected on the smart board. By having students come up to the board and manipulate angles, Cem could immediately see their thinking processes and provide instant feedback, guiding students through any misconceptions they had.

The coach encouraged Cem to reflect on assessment practices and consider how technology could be integrated more effectively to enhance student learning. Moving beyond paper-and-pencil assessments, they discussed different types of assessment items and types in a dynamic learning environment, such as questions that can be asked when students are using GeoGebra activities and types of feedback on students' inferences after using GeoGebra. The coach emphasized the importance of assessing both procedural and conceptual understanding. The coach shared examples

and practices with Cem on how GeoGebra and different technological tools can be used to achieve this. They set specific and achievable goals together for integrating GeoGebra into assessment practices. Small planning steps, such as adding a few GeoGebra activities to each lesson, were followed by a gradual increase in the complexity, variety, and number of these activities. The coach supported Cem to gain competence and confidence in integrating technology into assessment by transferring his expertise and involving him in hands-on activities. The strategies used by the coach before the conference supported the development of his TPACK in terms of *the assessment descriptor*. Having experience using GeoGebra as an assessment tool, giving online homework assignments, and learning the importance of real-time feedback, Cem was able to see the benefits of integrating technology into assessment practices. This progress reflected Cem's growing competence in blending technology with pedagogy.

When Cem was asked in post-conference 2 to what extent he used technology to assess students' learning in the lesson, he responded as follows:

> "After the MCC 2 lesson, I realized I could assess students' learning more easily and better using technology. The questions I asked in this lesson led students to explore more concepts. In addition, the GeoGebra online homework also aroused excitement in the students. Thanks to this assignment, I could monitor their learning using technology and give them feedback. By the evening of the day after the lesson, they had all completed the GeoGebra homework. This greatly motivated me to use technology to assess students' learning."

Even though Cem demonstrated improvement in terms of *the assessment descriptor* of TPACK by incorporating different assessment items in the actual teaching practice of MCC 2, he needed to explore the integration of topics in his curriculum to integrate technology as a tool for learning. Moreover, he did not actively seek ideas and strategies for implementing technology in a more integral role in developing the mathematics that students are learning. Additionally, he developed his ideas about leveraging technology to enhance the current curriculum, initiate modifications to pre-existing activities, or create new activities for the current curriculum. For example, the coach offered Cem a GeoGebra homework assignment. While Cem did not originate the idea of assigning GeoGebra homework, he was willing to integrate

such an assessment type into his lesson. Consequently, despite these improvements, Cem needed to progress to the next level when considering the behaviors outlined in the level indicators specified by Niess, Ronau, et al. (2009). Thus, the statements of Cem at pre-conference of MCC 2 and actual teaching practice at MCC 2 could provide evidence to place him at *the adapting level* of TPACK for *assessment descriptor* of the Curriculum and Assessment Theme since this evidence aligned with the following indicator of *the adapting level*: "Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings)." (Niess, Ronau, et al., 2009, p. 20). Niess, Ronau, et al. (2009) stated that teachers at *the adapting level* of *the assessment descriptor* recognize the need to ask different questions when assessing students' learning with technology. These teachers develop assessments that simultaneously support conceptual and procedural understandings through technology (Niess, Ronau, et al., 2009). Cem's teaching practices at MCC 2 reflected the characteristics of teachers at this level. Cem effectively assessed students' conceptual and procedural understanding of angles using GeoGebra, GeoGebra assignments, and Khan Academy. He engaged his students interactively by including differentiated items in the assessment. Cem enabled students to explore complementary, supplementary, adjacent, and opposite angles in the dynamic learning environment provided by GeoGebra activities and other tools. This revealed that the assessment of learning went beyond procedural tasks and was in line with the level of adapting. Cem asked students to demonstrate their conceptual understanding through GeoGebra interactive tasks and focused on assessing students' ability to use geometric concepts in a technology-supported learning environment. Cem assessed students' learning using real-life applications, manipulations in a dynamic learning environment, and open-ended questions. All these demonstrated Cem's ability to manage and apply technology effectively. For example, Cem allowed students to manipulate angles, recognize the relationship between different angles, and use GeoGebra to solve problems involving these angles, demonstrating that the assessments served a well-rounded understanding. In addition, Cem provided real-time feedback to the students during the technology-based activities, enabling them to analyze their misconceptions and eliminate their learning errors.

4.1.1.4. The Progress of Cem in TPACK Regarding Curriculum and Assessment Theme Through Mathematics Coaching in the Learning Area of 'Angles'

The figure 14 presents Cem's TPACK levels for *curriculum* and *assessment descriptor*s of Curriculum and Assessment Theme based on his teaching practices in technology-enriched lessons during mathematics coaching and before coaching (pre-observation lesson). Cem progressed in curriculum and *assessment descriptor*s according to the TPACK development model compared to before mathematics coaching. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

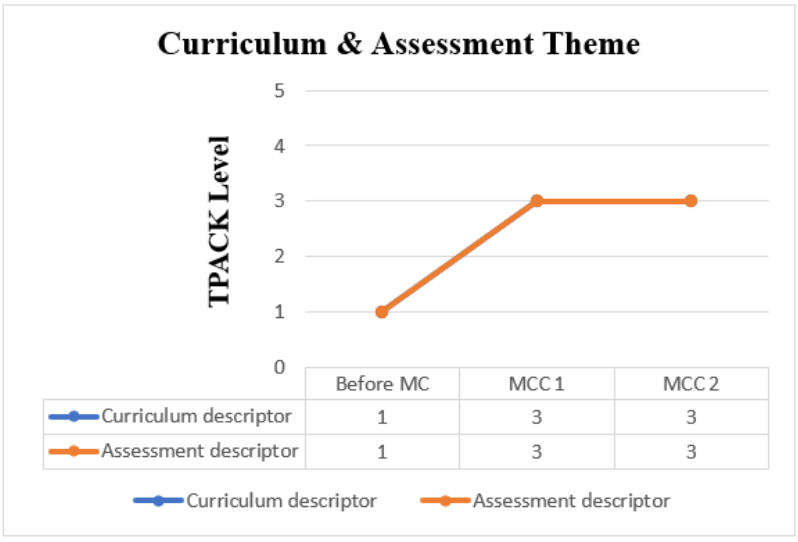


Figure 14. The progress of Cem of the Curriculum and Assessment Theme in the learning area of Angles

The curriculum descriptor pertains to understanding curriculum content and materials that incorporate technology into mathematics teaching and learning. Before mathematics coaching, Cem's level in the TPACK development model's *curriculum descriptor* was at *the recognizing level* (level 1). Cem acknowledged the potential usefulness of technology in illustrating mathematical concepts aligned with the curriculum but did not utilize subject-specific technology during the lesson. Before mathematics coaching, Cem's technology integration into the curriculum was limited. He used a video presentation and had students answer multiple-choice questions

online. However, aside from these instances, Cem primarily relied on real-life examples to visually represent angle concepts. While he perceived these visuals as aiding in understanding the concept, he expressed uncertainty about their effectiveness in facilitating student learning. Technology was primarily employed as a presentation tool for teaching angle-related concepts, with no further adaptation or modification of the curriculum to incorporate technological tools.

Cem's levels in *the curriculum descriptor* of the TPACK Development model at MCC 1 and MCC 2 indicated *the adapting level* (level 3). Through his involvement in the mathematics coaching cycles, Cem demonstrated advancement in *the curriculum descriptor*. He understood the advantages of integrating suitable technologies as aids for teaching and learning mathematics curriculum. Cem actively engaged students in exploring key topics using GeoGebra. He designed lessons to illustrate geometric concepts using technology and devised activities where students utilized technology to confirm and strengthen their understanding of angle-related concepts. Once students had acquired the skills to construct congruent angles, identify various types of angles, and establish connections between different angle types based on their properties, Cem tasked them with employing GeoGebra to validate relationships among diverse angle types.

As for *the assessment descriptor*, this descriptor is related to assessing the students' understanding of technology. Cem's level in *the assessment descriptor* of the TPACK Development model before mathematics coaching indicated *the recognizing level* (level 1). Cem opposed using technology in the assessment, stating that technology interfered with determining students' understanding of mathematics. Instead, he stated that if technology is to be used, it should focus on teaching concepts, and his teaching practice was consistent with this idea. Therefore, Cem did not allow technology to assess students' understanding of angles. Through his involvement in the mathematics coaching cycles, Cem progressed in *the assessment descriptor*. Cem's levels in *the assessment descriptor* of the TPACK Development model at MCC 1 and MCC 2 indicated *the adapting level* (level 3). Cem recognized the importance of adjusting assessment approaches when technology is permitted, ensuring that questions and assessment items are tailored to deepen students'

comprehension of concepts and procedures. He began integrating interactive digital tools such as MathsPad, Khan Academy, and GeoGebra into assessments. Still, he structured the assessments to emphasize the exploration of students' conceptual grasp of angles alongside their procedural proficiency.

4.1.2. The Learning Theme of the TPACK Development Model

The learning theme encompasses two descriptors: Mathematics learning and the conception of student thinking. Mathematics learning focuses on the teacher's perspective on student learning and the role of technology as a learning tool. *The conception of student thinking descriptor* within the learning theme concentrates on how students' thinking skills evolve through the utilization of technology (Niess, Ronau, et al., 2009). In the upcoming sections, Cem's TPACK is scrutinized within the framework of the learning theme of the TPACK Development Model, utilizing descriptors related to mathematics learning and the conception of student thinking.

4.1.2.1. TPACK Level of Cem Regarding Learning Theme Before Mathematics Coaching

Two hours of Cem's lessons were observed before he participated in mathematics coaching to determine his TPACK level regarding the Learning Theme while teaching the learning objective '*Understanding that an angle is formed by two rays with a common starting point and representing it symbolically.*' The pre-TPACK interviews with Cem and the pre-observation lessons before mathematics coaching are the source of all quotes and dialogue in the following paragraphs.

Cem's actual teaching practice before mathematics coaching indicated *the accepting level* of TPACK for both *the mathematics learning descriptor* and *conception of student thinking descriptor* of the Learning Theme. When Cem was asked about the role of technology in students' learning during pre-interview, he said:

> "With technology, the subject becomes more understandable by the students. I try to use technology for this purpose, especially since geometry is more

> abstract and visual than mathematics. However, students need to focus on me at the beginning of the subject or when I am explaining key concepts. When the technology is on, they can sometimes move away from the subject's focus. First, I try to make them fully grasp what they need to learn. Then, I use technology to visualize the concept with technology as a reinforcement tool to make it clear in their minds."

The statements of Cem during the pre-interview and his actual teaching practice provided evidence that Cem restricted the use of technology by students, especially during the introduction and development of key concepts. Thus, Cem's actual teaching practice at pre-observation and statements at pre-interview aligned with the following indicator of *the accepting level* for *the mathematics learning descriptor* of the Learning Theme: "Has concerns about students' attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities." (Niess, Ronau, et al., 2009, p. 21). *The accepting level* in *the mathematics learning descriptor* of the Learning Theme is characterized by teachers forming favorable or unfavorable attitudes toward teaching and learning content topics with technology. Teachers at this level may try out activities with their students, but technology is not a consistent thought when planning lessons. They practice integrating technology but often have concerns about its potential to distract students from learning appropriate mathematical content (Niess, Ronau, et al., 2009). Cem's teaching practice before mathematics coaching reflected the characteristics of the teachers at this level. In his teaching practice, Cem engaged students by posing questions and exploring the concept verbally and through traditional methods without involving technology in the students' tasks. He provided detailed verbal explanations and used the board to demonstrate concepts, ensuring students grasped the basics before introducing technological tools. Cem mentioned using technology as a reinforcement tool after students have understood the fundamental concepts. This approach reflected his selective use of technology, indicating a reluctance to rely on it for initial learning and discovery. It is consistent with *the accepting level* of TPACK, where technology is not yet an integral part of the teaching process. Cem's statements and actions show that he has formed specific attitudes about using technology in his teaching practice. While he acknowledged its usefulness in visualizing concepts, he also expressed significant concerns about its potential to distract students from the primary learning objectives.

When Cem was asked whether he uses technology to help students discover a concept, he answered as follows:

> "Even though I show the angle in a visualized way, I find it more important for students to draw it and touch the pencil and paper. Let's say the student draws the angle and then verifies the result by measuring it with technology. In this way, when the student draws or participates in an activity using technology, I may feel like they have not fully learned, as if something is missing. Maybe this is not the case, but I think this is the case in my current teaching practice. Therefore, I feel that when the student tries to verify the accuracy of what the student does with technology, the student may not be able to develop the main idea of the subject thoroughly. Thus, I want them to see them draw without using technology.

Cem's actual teaching practice at pre-observation and statements at pre-interview aligned with the following indicator of *the accepting level* of TPACK for *the conception of student thinking descriptor* of the Learning Theme: "Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics." (Niess, Ronau, et al., 2009, p. 21). *The accepting level* for *the conception of student thinking descriptor* is characterized by teachers who develop attitudes towards using technology in teaching. Teachers at this level are typically concerned that technology may interfere with students' appropriate mathematical thinking skills development. Moreover, even when these teachers have tried to use technology, they still determine its role in promoting deep understanding and cognitive engagement in students (Niess, Ronau, et al., 2009).

Cem's teaching practice aligned with what he outlined in the pre-interview. Cem believed that students could learn how to draw an angle and then draw another angle equal to this angle only by using paper and pencil, which was how he taught. Consistent with this belief, he showed the students how to use a physical protractor on the whiteboard and then asked them to measure angles and draw congruent angles using the protractors he asked them to bring before the lesson. Cem presented the concept of drawing congruent angles and explained it verbally in detail. He succeeded in attracting students' interest by asking questions to explore the concept, but he did not utilize technology while performing various tasks related to drawing congruent angles.

4.1.2.2. TPACK Level of Cem Regarding Learning Theme in the MCC 1

To determine Cem's TPACK level of the Learning Themes during the first mathematics coaching cycle, which focused on the learning objective '*Constructing congruent angle to a given angle,*' his teaching practices were observed for two hours. The quotations and dialogues in the upcoming paragraphs were derived from the pre-conferences, post-conferences, and mathematics lessons conducted as part of Mathematics Coaching Cycle 1 (MCC 1). During mathematics coaching, Cem's actual teaching practice in MCC 1 indicated *the adapting level* of TPACK for *the mathematics learning descriptor* and *conception of student thinking descriptor* of the Learning Theme.

Cem's actual teaching practice was more advanced regarding *the mathematics learning descriptor* than his teaching practice before the mathematics coaching. Cem started exploring, experimenting, and implementing GeoGebra, MathsPad, Khan Academy, and subject-related visuals and videos as a tool for mathematics learning. In the MCC 1 lesson, Cem prioritized showing how students learned concepts and supporting the development of students' thinking skills more than before the math coaching. In the pre-mathematics coaching lesson, Cem did not use technology as a learning tool except for projecting daily life visuals related to the topic (highlighting the angles of a ski athlete's body during his performance), projecting the multiple-choice question on the board on the online learning platform and marking the answer he received from the students, and projecting a few definitions for the students to note down. Still, he did not use technology as a static demonstration tool in this lesson. Instead, he used technology as a dynamic learning and verification tool to assess students' learning, introduce and explore the concept, and make them connect.

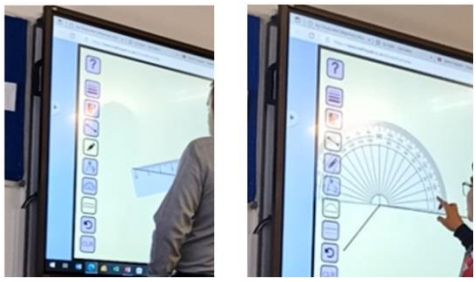


Figure 15. Cem and a student using MathsPad for constructing congruent angles in MCC 1 lesson

Cem started exploring, experimenting, and practicing integrating technologies as mathematics learning tools in his MCC 1 course. In doing so, he allowed students to explore the topic using technology. Moreover, when Cem was asked at the MCC 1 pre-conference what he would use these GeoGebra activities and other technological materials for, he said:

> "This is the first time I will integrate technology in such a planned way. We chose the activities together; we discussed how useful they would be for students, and we thought about all of them. In our current meeting, I see that technology is more than just using it as a static visual tool. I mean, the thing is to teach this concept with technology in a way that supports students' learning and develops their thinking skills. I aim to do this, but how I and the students use it in the lesson will give me beneficial feedback."

Cem's statements at the pre-conference and his actual teaching practice at MCC 1 aligned with the following indicator: "Begins to explore, experiment and practice integrating technologies as mathematics learning tools." (Niess, Ronau, et al., 2009). Therefore, these provided evidence to place him at *the adapting level* of TPACK for *the mathematics learning descriptor* of the Learning Theme. Niess, Ronau, et al. (2009) stated that teachers at *the adapting level* of *the mathematics learning descriptor* of the Learning Theme integrate technologies as tools for learning mathematics, explore different technologies, and begin to experiment and apply these technologies. Teachers at this level meaningfully and systematically incorporate technology into their teaching to facilitate students' exploration and understanding of mathematical concepts by providing opportunities for students to have hands-on experiences and engage in interactive learning environments. Cem's teaching practice in MCC 1 reflected the features of teachers at *the adapting level* of *the mathematics learning descriptor*. In the MCC 1 lesson, Cem began to explore and experiment with integrating course-specific technologies such as GeoGebra, MathsPad, and Khan Academy into his teaching, consistent with the characteristics of *adapting level* teachers. Cem used GeoGebra and MathsPad to dynamically show angles, measure different angles, and draw congruent ones. He actively involved his students in exploring mathematical concepts using technology and allowed students to interact with the technology and observe real-time changes. As Cem started incorporating technology to enhance his teaching practices, this exploration and

experimentation with new digital tools indicated his *adapting level*. For example, students interacted with technology through the GeoGebra activity to measure the angles of a triangle and dynamically verify their sums. This interaction contributed to students' exploration of the concept and their practical understanding of mathematical principles. Cem's approach aligned with *the adapting level* characteristic, which is defined as having students explore mathematical concepts using technology. By having his students watch Khan Academy videos on how to use a protractor and then reinforcing this with hands-on activities using a physical protractor and GeoGebra, Cem effectively used technology for systematic, planned, and instructional purposes to support and deepen student learning. Using technology for such purposes is one of the characteristics of teachers at *the adapting level*. Cem used technology to present concepts, guide exploration, and provide feedback to students. He walked around the classroom checking students' work and providing instant feedback to help them correct their mistakes and better understand the concepts. Cem's interactive and supportive use of technology in this way reflected his level of adapting.

When Cem was asked in post-conference 2 to what extent he thought that the use of technology in the lesson contributed to students' conceptual learning, he replied as follows:

> "Students were already familiar with the concept of angle. However, creating an angle with rays starting from the same point, learning to use the protractor in a dynamic learning environment, making various manipulations, measuring the angle they created with digital tools, and drawing an angle equal to this angle in a dynamic learning environment made a big difference in their conceptual learning."

Compared with the TPACK level in the pre-mathematics coaching, it was concluded that Cem progressed from *the accepting level* (level 2) to *the adapting level* (level 3) regarding *the mathematics learning descriptor* in MCC 1. The elements of the cooperating process and the coach's role were presented as crucial in supporting Cem's development from *the accepting level* toward *the adapting level* in *the mathematics learning descriptor* of the TPACK development model. Cem constructed knowledge through group and guided training sessions to develop his

CK and PCK. Before engaging in the MCC 1, Cem identified new technological tools and strategies to implement into his practice. The coaching process proved helpful in offering Cem chances to practice how integrating technologies like GeoGebra, MathsPad, and Khan Academy could be done. Thus, with the help of close cooperation with the coach, Cem was able to organize and enact technology-supported activities that would foster students' engagement and learning of mathematical concepts. The coach was always present, giving feedback and providing support to Cem to make him able to think about his teaching and practice and change if needed. This guided the experimentation, and the focus on the intentional use of technology helped to bring Cem to *the adapting level*, where, in addition to using technologies for presenting ideas, he started using them in activities that would actively involve the students. Thus, Cem's lessons became more demonstrative and informative; the students appeared to grasp the issues better and behave more positively.

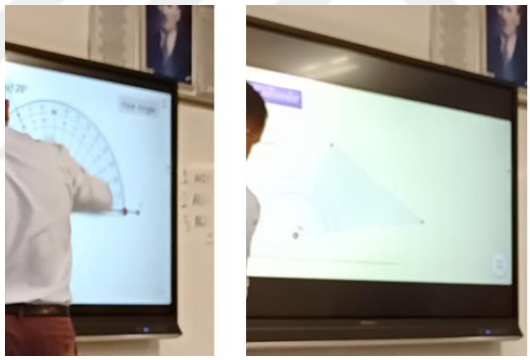


Figure 16. Cem while measuring angles using a protractor on GeoGebra

Concerning *the conception of student thinking descriptor* of the Learning Theme, Cem's actual teaching practices exhibited a higher level of advancement compared to how he taught, addressed concepts, eliminated potential misconceptions, and applied a constructivist approach to help students comprehend the concept before mathematics coaching, specifically in terms of *the conception of student thinking descriptor*. Cem did not use GeoGebra, MathsPad, and Khan Academy as static demonstration tools. Instead, both Cem and his students actively utilized GeoGebra's dynamic features. Nevertheless, despite students employing technology for conceptual understanding, the assessment of students' thinking primarily occurred

without using technology. The technologies selected by Cem, their integration into the lesson, and his statements in the pre-conference reflected an evolving understanding of how students learn, contributing to the development of students' thinking skills. All these observations indicated that Cem has progressed to *the advancing level* in *the conception of student thinking descriptor* within the Learning Theme.

When Cem was asked at the MCC 1 pre-conference how the technology he would use in teaching would deepen students' learning, he replied as follows;

> "Students have ideas about the concept of angle, but it needs to be taught concretely to associate their mental schemas. I planned to do this by having students use pencils, paper, rulers, and protractors, making inferences from the video, and allowing them to test their conjectures and ideas through technology dynamically. We discussed how they think and how and how they might think in which situation. With all this in mind, I want to develop students' ability to think about concepts and support them in conceptual relationships."

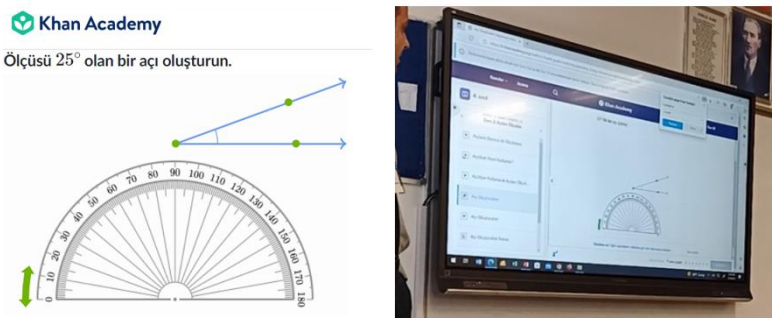


Figure 17. The interactive Khan Academy activity on measuring and constructing angles Cem used

The statements of Cem at the pre-conference of MCC 1 and actual teaching practice at MCC 1 aligned with the following indicator: "Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning." (Niess, Ronau, et al. 2009, p. 21). Therefore, these provided evidence to place him at *the adapting level* of TPACK for *the conception of student thinking descriptor* of the Learning Theme. *The adapting level* in the TPACK development model for *the conception of student thinking descriptor* of the Learning Theme is defined by how

teachers develop adequate mathematical thinking when technology is incorporated into learning. At this level, teachers purposefully and knowledgeable use technologies to enhance students' learning and investigate mathematical concepts, stressing practical and socially interactive approaches (Niess, Ronau, et al., 2009). Factors evident in Cem's teaching practice aligned with teachers at *the adapting level*. In the context of the MCC 1 lesson, Cem started to work on the identified mathematical thinking skills by using technology as a medium. For example, he used GeoGebra and made angle measurements as students viewed the application and saw actual changes. Utilizing this form of technology allowed students to have a better grasp of the lessons that were being taught and be able to interact more with the geometric concepts. Overall, Cem's integration of technology was purposeful and tied to the lesson's learning objectives in that he intentionally used it to teach mathematical ideas. In addition, when teaching how to use a protractor, Cem also employed videos from Khan Academy to supplement a hands-on approach with physical protractors and GeoGebra to incorporate technology that would further enhance the learning process. This fits *the adapting level*, where the teacher intentionally integrates technology into teaching and learning practices. In the GeoGebra activity, the students used their instruments to measure angles in a triangle and then check out the sum of angles dynamically. This engagement ensured the students got a first-hand experience of the technology and how it could be used to expound on the concept. They appreciated how the technology embodied the various mathematical concepts. This engaging and discovery-based type of learning is characteristic of *the adapting level*, which involves using technology to learn mathematics. Cem employed technology through presentations and discussions while also using it to give feedback and help students learn. He moved around the classroom, checked students' work, provided feedback, and explained corrections on the spot so students could revise their work. This kind of technology integration of Cem for learning and assessment and students' thinking process can be referred to as *the adapting level*, where the teachers utilize the technology to support student learning.

Compared with the TPACK level before mathematics coaching, it was observed that Cem progressed from *the accepting level* (level 2) to *the adapting level* (level 3)

regarding *the conception of student thinking descriptor* in MCC 1. Coaching played a crucial role in Cem's progression. The coach presented Cem with various technological tools, including GeoGebra, MathsPad, and Khan Academy. Cem was introduced to these tools and was then able to use them in guided sessions to improve his teaching practices. The coaching sessions helped him identify examples and models of how these technologies can be used in teaching and learning mathematics to enable the students to discover more about the subject. During the coaching, Cem had joint planning sessions with his coach. They identified and chose the technological tools that could be effectively used for the students' learning. Thus, Cem reconsidered his teaching methods and created a more explicit plan for using technology in the classroom. Cem received feedback from the coach and suggestions for improvement that would assist him in fine-tuning his strategies and increasing his self-efficacy in integrating technology into learning. Coaching stressed the need to ensure that learning was done with the students and enhance their mathematical thinking abilities. The coach suggested Cem develop activities in which students can learn mathematical concepts using gadgets. Realizing this shift helped Cem to appreciate the importance of using technology to get the student's active participation and foster more profound levels of thinking. The coach was also physically present in Cem's lessons and could assist him in real time. The coach followed and watched how Cem was teaching and provided him with suggestions as to how he could do it differently. This immediate support allowed Cem to test the new approaches and understand how technology could be better applied to the students' enhanced mathematical thinking. Thus, thanks to coaching, Cem enhanced his Technological Pedagogical Knowledge (TPK) and learned how to use technology in teaching. He also improved his pedagogical content knowledge (PCK) by learning different tactics for effectively conveying mathematics concepts to the students. These simultaneous enhancements enabled Cem to incorporate technology into his teaching as a supplement and a crucial element of the teaching process that enhances students' learning and concept formation.

4.1.2.3. TPACK Level of Cem Regarding Learning Theme in the MCC 2

To determine Cem's TPACK level regarding the Learning Themes during the second mathematics coaching cycle, which focused on the learning objective '*Explores the*

properties of complementary, supplementary, adjacent, opposite angles, and solves related problems,' his teaching practices were observed for four hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 2 (MCC 2) constituted the sources of all the quotations and dialogues in the following paragraphs.Cem's actual teaching practice in the MCC 2 during mathematics coaching indicated *the adapting level* of TPACK for *the mathematics learning descriptor* while *the exploring level* of TPACK for *the conception of student thinking descriptor* of the Learning Theme.

Cem's instructional approach in MCC 2 closely mirrored his methodology in MCC 1 regarding *the mathematics learning descriptor*. In MCC 2, Cem explored, experimented, and implemented GeoGebra as a mathematics learning tool. Notably, in the MCC 2 lesson, Cem emphasized illustrating how students grasp concepts and enhance the development of their thinking skills more than in MCC 1. This is evident in Cem's choice to have students employ GeoGebra to explore various activities related to adjacent, supplementary, complementary, and opposite angles. He also used more exploratory questions, adopting a constructivist approach to guide students toward the concepts rather than providing direct answers or explanations. Essentially, alongside the integration of technology, Cem facilitated his students' exploration of the topic with technology. In summary, Cem initiated the exploration, experimentation, and application of technologies as tools for mathematics learning. Students delved into various mathematics topics (e.g., sub-topics of angles: Adjacent angles, supplementary angles, adjacent supplementary angles, complementary angles, adjacent complementary angles, opposite angles). This indicates that students under Cem's guidance did not use technology to explore many topics from different learning areas. In other words, the students did not extend their exploration beyond the specific topic at hand using technology.

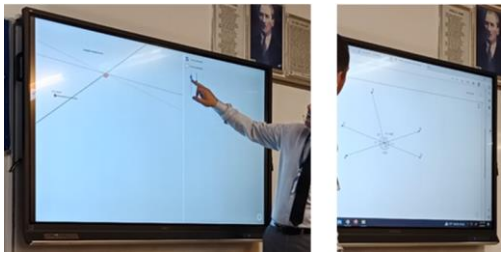


Figure 18. Cem while using the opposite angles activity of GeoGebra

Moreover, when questioned during the MCC 2 pre-conference about the purpose of employing GeoGebra activities and other technological materials, Cem stated:

> "The main reason for using these technological resources is to visualize the concept and contribute to students' conceptual learning. I planned to do this by practicing the technology in the classroom. I will try to use technology as a learning tool to guide students' learning and as a teaching tool at the center of my demonstration. In the 2nd coaching cycle, I think I will do what and how I will do more appropriately by considering my previous experiences."

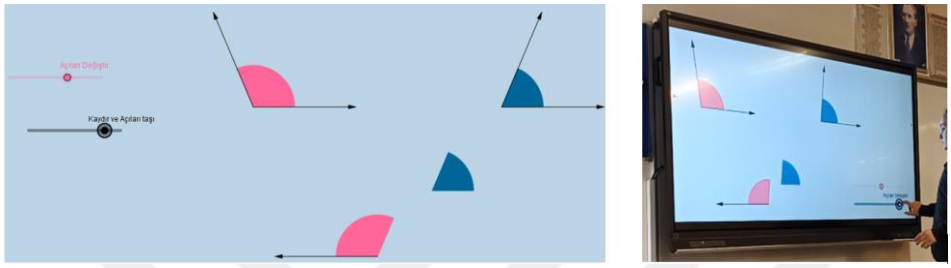


Figure 19. A student while exploring adjacent complementary angles using GeoGebra

Cem's statements were aligned with the following indicator: "Begins to explore, experiment and practice integrating technologies as mathematics learning tools." (Niess, Ronau, et al., 2009, p. 21). Thus, Cem's statements at the pre-conference of MCC 2 and actual teaching practice at MCC 2 could provide evidence to place him at *the adapting level* of TPACK for *mathematics learning descriptor* of the Learning Theme. *The adapting level* of *the mathematics learning descriptor* is characterized by teachers who start to search, test, and apply the integration of technologies as tools in mathematics learning. At this level, teachers intentionally use technology to enhance student's learning processes and discover mathematical concepts since students can manipulate objects and interact with the content. In accordance with these, Cem first started to use technologies like GeoGebra and further showed the properties of complementary, supplementary, adjacent supplementary, adjacent complementary, and opposite angles using GeoGebra activities in the MCC 2 lesson. These are the characteristics of *the adapting level* where teachers begin to use technology in their teaching practices. Regarding the purposeful integration of technology in the classroom, Cem's intentional use of technology in teaching particular mathematical concepts can be noted. He employed GeoGebra to

demonstrate that complementary angles sum up to 90 degrees and to depict the angles' relationships. Thus, Cem advertised how he planned his lesson and how technology can be included to enhance learning. This exemplifies *the adapting level* clearly because all these elements are purposefully integrated. While Cem did not let the students change the features of the GeoGebra activities directly, he encouraged them to investigate mathematical concepts by interacting with the program. For instance, he required students to describe complementary angles in their own words following a demonstration using GeoGebra. He also put questions to test students' comprehension of angle relationships. This approach ensured that the students could discover and develop their concepts, corresponding to *the adapting level* feature whereby students manipulate mathematical concepts using technology. Cem incorporated technology into the lesson to introduce and discuss concepts, provide feedback, and guide the students. He stopped and asked questions, assessed students' answers, and provided feedback during GeoGebra activities. This application of technology in assessment and learning is at *the adapting level*, whereby Cem employed technology to help develop students' thinking and comprehension.

As for *the conception of student thinking descriptor*, in the actual teaching practice of MCC 2, Cem demonstrated his commitment to guiding students in understanding mathematical concepts through meticulous planning, implementation, and reflective practices. He strategically incorporated technology-enriched activities, ensuring they align with the curriculum and contribute to meaningful student engagement. Cem's approach involved introducing technological tools and carefully evaluating the impact of these activities on student learning outcomes in mathematics and their attitudes toward the subject. By actively managing these technology-enriched activities, Cem aimed to direct and enhance student engagement, fostering a sense of self-direction in their learning. However, despite these positive practices, there were certain areas where Cem's approach fell short. Technology still needs to be fully integrated into the mathematics curriculum, as he perceived it more as an additional component than an integral part of the instructional process. The missed opportunity to engage students in high-level thinking activities, such as project-based learning and complex problem-solving, highlighted a potential area for further development.

However, Cem did not leverage technology to its full potential in developing advanced levels of understanding among students regarding mathematical concepts. This aspect represents an area where future efforts could be directed to harness the full benefits of technology in enhancing students' mathematical comprehension and problem-solving skills. Also, the observations made during the actual teaching practice of MCC 2 revealed that Cem's students not only actively engaged with the GeoGebra activities but also demonstrated a high level of enjoyment in the learning process. The enthusiasm and participation were notably strong among the students. When Cem inquired whether they would be interested in more GeoGebra activities, all students expressed a strong desire to continue working with GeoGebra across all subjects, positively impacting their learning experiences. These developments serve as compelling evidence that Cem has advanced to a higher level in *the conception of student thinking descriptor* of the Learning Theme. The students' enthusiasm and eagerness to integrate GeoGebra into their learning process suggest that Cem's instructional approach successfully fostered a positive and engaging environment, aligning to enhance students' thinking skills and attitudes toward mathematics.

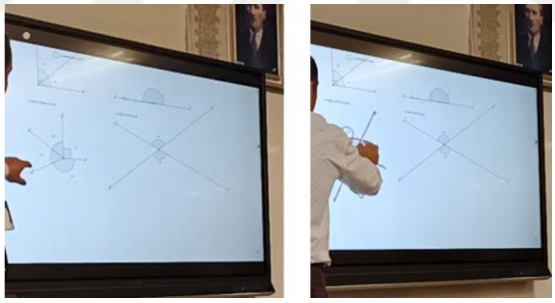


Figure 20. Cem while using GeoGebra to address a common misconception in opposite angles

Cem's teaching approach demonstrated a keen awareness of students' comprehension levels, leading him to tailor his instruction to address specific areas of difficulty. When responding to student questions, he strategically probed to pinpoint the aspect that the student found challenging. For instance, when a student struggled to grasp the concept of opposite angles, Cem initiated the discussion by asking about the essential condition for two angles to be considered opposite. Ensuring the student understood the necessary conditions, Cem emphasized the linearity of the rays forming opposite angles. Subsequently, the student corrected their misunderstanding

of the concept and applied this newfound understanding to another example. This instructional strategy showcased Cem's ability to diagnose learning errors, provide targeted support, and facilitate meaningful student reflection.

When questioned about the role of technology in students' learning for the planned lesson, Cem responded in pre-conference 2 as follows:

> "The main aim is to lead students to discover the concept. We planned to use GeoGebra activities as a tool for this. I aimed to deepen students' understanding through the questions I would ask students and the feedback I would provide them while I was using these activities and while students were using them. So, I can say that this will be the main role of GeoGebra activities."

The coaching process, mainly during pre-conference 2, was pivotal in helping Cem move from the adapting state in the conception of student thinking description to the exploring state. The coach's role was crucial in ensuring that Cem embraced the following changes in teaching practice, including continued professional development, training in the use of technology, student-centered approach, feedback, and creativity. Thus, Cem incorporated technology to foster active student participation and comprehension, which showed a substantial improvement in technology application to improve the teaching and learning process. Furthermore, the analysis of Cem's progression in *the conception of student thinking descriptor* showed improvement in the aspects of TPACK. His technological literacy became better and, thus, contributed to positive changes in his teaching and the way he presented mathematical information. These improvements in the TK, PK, TPK, TCK, PCK, and TPACK benchmarks indicate that he has enhanced his capacity to incorporate technology into his teaching practice to foster student learning.

When Cem was asked in post-conference 2 how he used technology to eliminate possible difficulties and misconceptions about the topic, he responded as follows;

> "For example, there is a common misconception about the subject. The intersecting rays must intersect linearly for the measures of opposite angles to be equal. Sometimes, the image of intersecting rays can be misleading for students. This is a learning challenge for many students. By utilizing the dynamic features of GeoGebra, students discovered that the equality of

> opposite angles is only valid when linearity is ensured and that the measures of the angles are not equal when linearity is not ensured."

Cem's statements during the pre-conference of MCC 2 and his enhanced actual teaching practice in MCC 2 aligned with the following indicator: "Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding." (Niess, Ronau, et al., 2009, p. 21). Thus, these served as evidence to position him at *the exploring level* of TPACK for *the conception of student thinking descriptor* of the Learning Theme. *The exploring level* in the TPACK development model for *the conception of student thinking descriptor* of the Learning Theme is characterized by the teachers' intentional effort to teach and learn content topics using technology. At this level, teachers develop activities to fit into their curriculum, seek various strategies to teach the content, and are ready to model new ways of viewing mathematical ideas using technology as a medium. They enable students to learn and apply technology to solve real-life situations (Niess, Ronau, et al., 2009).

Cem used GeoGebra in the MCC 2 lesson and tried implementing it in the teaching practice. He applied GeoGebra activities to display the characteristics of angles, specifically complementary, supplementary, adjacent complementary, adjacent supplementary, and opposite angles. Using technology in his lesson design and delivery is a characteristic of *the exploring level* in which the teachers use technology as an integrated part of their teaching to improve student learning. Cem revised and created activities that are congruent with his curriculum plans. For instance, Cem planned activities to teach students that the opposite angles are equal. He employed GeoGebra to adjust the intersection of two lines and display how the opposite angles are always equal in measure, even when the lines are moved around. Cem presented problems regarding opposite angles to the students, for instance, finding the measures of the unknown angles when one angle is given. This activity assisted students in seeing and comprehending the characteristics of opposite angles and how to use this knowledge to solve related problems. Such alignment and restructuring of the activities to incorporate technology are characteristic of *the exploring level* in which the teachers try to search for and integrate new techniques of teaching the content.

Cem provided the students with an opportunity to discover mathematical concepts through technology. When directing the students to work on GeoGebra, he prompted them to define complementary angles in their own words after demonstrating it on GeoGebra. He asked questions such as, "What are the complementary angles of a 1-degree angle?" and "Which angle has complementary angles equal to itself?" to address their comprehension. Likewise, he employed GeoGebra to teach supplementary and opposite angles; he asked the students to watch and explain the characteristics of these angles. This is because enhancing students' exploration and conception of concepts through technology fits the characteristics of teachers at *the exploring level*, where students employ technology to explore and tackle problems. Cem showed how mathematics can be approached and taught differently with the help of technology. For instance, he was able to engage the students with GeoGebra to demonstrate the angle properties where the sum of adjacent supplementary angles remains constant at 180 degrees even when the angles' measures are altered. He also showed on GeoGebra how opposite angles are equal by moving the vertex. This is another crucial aspect of *the exploring level*, characterized by the teachers' propensity to employ technology to help students better understand mathematical concepts. During the MCC 2 pre-conference, Cem stated that he would like to include technology in a planned and purposeful way, thus showing his openness to searching and trying new approaches to teaching. Hence, Cem implemented GeoGebra activities by applying dynamic visuals to teaching angles and using technological tools in the classroom. This is characteristic of *the exploring level* due to using an investigative approach. Compared with the TPACK level in MCC 1, it was noted that Cem's level shifted from *the adapting level* (level 3) to *the exploring level* (level 4) on the conception of the student thinking descriptor in MCC 2.

4.1.2.4. The Progress of Cem in TPACK Regarding Learning Theme Through Mathematics Coaching in the Learning Area of 'Angles'

The figure 21 presents Cem's TPACK levels for *mathematics learning* and *conception of student thinking descriptors* of Learning Theme based on his teaching practices in technology-enriched lessons during mathematics coaching and before coaching (pre-observation lesson). Cem progressed in *mathematics learning*

*descriptor*s and *the conception of student thinking descriptor*s according to the TPACK development model compared to before mathematics coaching. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

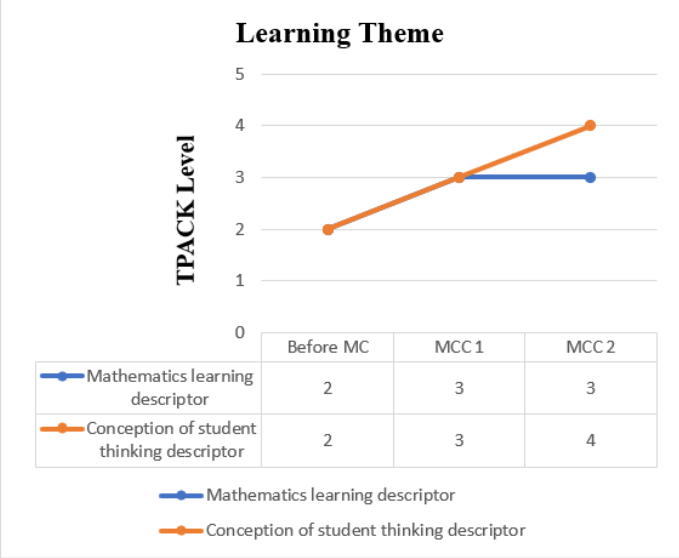


Figure 21. The progress of Cem of the Learning Theme in the learning area of Angles

The mathematics learning descriptor pertains to the concentration on the subject matter, which involves students' learning of mathematical topics. This descriptor also focuses on the teacher's viewpoint regarding teaching mathematics using technology and their methods for incorporating technology into students' learning. Essentially, it concerns the teacher's stance and actions regarding the fusion of technology into mathematics education.

Before mathematics coaching, Cem's level in the TPACK development model's *mathematics learning descriptor* was at *the accepting level* (level 2). Cem was concerned that students might become more preoccupied with technology rather than grasping mathematical concepts before mathematics coaching. As a result, he limited students' access to technology, particularly during the introduction and in-depth exploration of crucial concepts. He believed students would better concentrate on learning if the teacher-controlled technology usage. Cem demonstrated advancement

in *the mathematics learning descriptor* through his involvement in the mathematics coaching cycles. Cem's levels in *the mathematics learning descriptor* of the TPACK Development model at MCC 1 and MCC 2 indicated *the adapting level* (level 3). Cem started delving into, experimenting with, and applying the integration of technologies like GeoGebra and Khan Academy as learning and teaching tools for mathematics learning. Students actively explored geometric concepts related to angles through the utilization of GeoGebra.

As for *the conception of student thinking descriptor*, this descriptor is related to the demonstration of conceptions of how students learn and develop their thinking skills. Cem's level in the TPACK development model's *conception of student thinking descriptor* before mathematics coaching was at *the accepting level* (level 2). Before attending mathematics coaching, Cem was concerned that students might not cultivate suitable mathematical reasoning abilities when technology was solely used as a mathematical exploration verification tool. Consequently, Cem often structured activities involving repeated technology without use, employing traditional tools like pen, paper, protractor, and ruler. This approach ensured that students genuinely grasped the concepts within the angle learning domain. Through his involvement in the mathematics coaching cycles, Cem progressed in *the conception of student thinking descriptor*. Cem's levels in *the conception of student thinking descriptor* of the TPACK Development model at MCC 1 indicated *adapting level* (level 3). Cem started fostering the development of students' mathematical reasoning skills by incorporating GeoGebra and other technological tools as aids for learning in MCC 2 teaching practice. While students primarily utilized technology to grasp most concepts, the assessment of student thinking largely remained independent of technology. Cem's levels in *the conception of student thinking descriptor* of the TPACK Development model at MCC 2 indicated *the exploring level* (level 4). Cem meticulously devised, executed, and reflected upon teaching strategies, prioritizing the guidance of students toward conceptual understanding. He implemented GeoGebra activities and evaluated their effectiveness in enhancing students' mathematical understanding and attitudes toward the subject. Cem adeptly orchestrated GeoGebra activities to foster student engagement and promote self-directed learning in mathematics.

4.1.3. Teaching Theme of TPACK Development Model

The teaching theme revolves around teachers' perspectives on technology, competence in employing instructional strategies that integrate technology, and capacity to establish mathematical environments utilizing technology to facilitate individual and group-based mathematical learning and creativity. This theme includes three descriptors: Mathematical learning, instructional, and environment (Niess, Ronau, et al., 2009). In the subsequent sections, the TPACK of Cem is examined within the context of the teaching theme of the TPACK Development Model.

4.1.3.1. TPACK Level of Cem Regarding Teaching Theme Before Mathematics Coaching

Two hours of Cem's lessons were observed before he participated in mathematics coaching to determine his TPACK level in the Teaching Theme while teaching the learning objective '*Understanding that an angle is formed by two rays with a common starting point and representing it symbolically.*' The pre-TPACK interviews with the teacher and the pre-observation lessons before mathematics coaching are the source of all quotes and dialogue in the following paragraphs.

Cem's actual teaching practice before mathematics coaching indicated *the accepting level* of TPACK for the *mathematics learning* and *professional development descriptors*, while *the recognizing level* of TPACK for instructional and *environment descriptor*s of TPACK of the Teaching Theme.

When Cem was asked whether he uses technology in teaching a concept that students have not learned before, he said:

> "Of course I use it. For example, I show the students a video about the subject at the beginning of the lesson. This creates an awareness in students. They try to discover the concept in this way."

This statement was consistent with Cem's actual teaching practice because Cem used technology in the lesson only as a visual demonstration tool where he played videos

and displayed pictures instead of making students discover the concept of angle. Cem continued referring to students' use of technology as follows:

> "I provide them with an environment where they can benefit from technology with research tasks I give them before the lesson. They do research using technology and come to the next lesson, and it contributes to their learning."

Cem asked questions to understand students' prior learning and their misconceptions. He gave information about common student mistakes. However, he did not use technology to take precautions against misconceptions about the concept of angle. During the teaching, Cem posed the question, "How would our houses be if there were no angles?" to the students as a group work question. He asked the students to form groups of 4-5 and draw houses without angles. The students formed groups and started to work. During this time, Cem went around the groups to check their work and provide feedback. Then, after the work was completed, he called one representative from each group to the board to introduce their drawings. Students took turns to share their drawings with their peers.



Figure 22. Group work of Cem's students on the concept of angles

In addition, Cem, referring to his use of technology, said at TPACK pre-interview:

> "I often used technological tools such as Nearpod, Kahoot, etc., during the pandemic. In the current process, I try to use such tools whenever possible. I can give homework questions that they can complete online with Nearpod. I can generally express it in this way."

Both in his interview statements and in the actual teaching practice, it was clear that Cem did not use technology as a visual presentation tool in the lesson to improve students' learning and support conceptual learning. He also stated in the TPACK pre-

interview that he gave the students tasks that required them to use technology before the lesson (outside the classroom teaching). When Cem was asked which teaching method and technique provides effective learning with technology integration, he replied as follows during the pre-interview of TPACK;

> "I often use the question and answer and large group discussion method. Within my role in the lesson, I use technology to present a visual or video that arouses curiosity. Sometimes, I stop the video and ask questions about that part. Sometimes, when the video is over, I ask why we watched the video and what they understand from this video. Students can discuss among themselves why they watched it. When we come to the end of the video, they know the topic or its relation to it. It is all for them to discover.

Although Cem stated in the interview that he frequently integrated technology into the teaching of the subject, the use of technology in the lesson was limited to opening presentations on the computer and projecting the image of the computer on the smart board screen. Cem only used technology as an internet-connected tool to show a video to students and to open a study on his website to measure students' perception of measure value related to angles. In other words, Cem did not use technology to support students' conceptual understanding or develop a concept. Instead, he used technology for basic or repetitive tasks in the classroom, such as getting answers from the students for the work on Khan Academy.

Cem's statements and his teaching practice before mathematics coaching aligned with the following indicator: "Uses technology activities at the end of units, for "days off," or for activities peripheral to classroom instruction." (Niess, Ronau, et al., 2009, p. 22). Therefore, these could provide evidence to place Cem at *the accepting level* of TPACK for *the mathematics learning descriptor* of the Teaching Theme. *The accepting level* in the TPACK development model for *the mathematics learning descriptor* is defined by teachers adopting attitudes to the use of technology but mainly using conventional approaches. They may use technology at times, but the incorporation of technology into teaching is not frequent (Niess, Ronau, et al., 2009). Cem's use of materials and instruction was straightforward and primarily low-tech; for example, he displayed questions and portions of the textbook to write on and showed a YouTube video. However, the above-illustrated cases show that teachers

were sensitive to the possibilities of technology, although its application was confined to improving existing teaching and learning models. Cem's lessons were primarily didactic, as he controlled most activities and offered explanations. For instance, he elaborated on angles and how they are depicted mathematically without incorporating technology to let the students discover or learn through interactions. Cem used the Khan Academy activity, where he managed the technology and guided the students' actions instead of having the students work with the tools independently. This conforms with *the accepting level* signifying the institution's resistance to completely involving technology in the students' learning process.

Cem explained the symbolic representation of the angle, the degree, which is the unit of angle measure, and the symbolic representation of the measure of the angle. Then, he opened a study on Khan Academy to compare angles. This exercise was not about measuring angles but about deciding which angle is appropriate for a given angle according to the types of angles. Students answered the questions using their previous knowledge of acute, obtuse, and right angles. These could provide evidence that Cem thought that the students had a sufficient understanding of the concepts related to angles after his verbal explanations on the concept of angles, knowledge summary, and related key learning. Then, he wanted to measure this with multiple choice questions he opened on the Khan Academy page as a technological tool and provide feedback based on student answers. In other words, Cem did not teach the concept of angle via technology but attempted to enhance the understanding of the concept of angle that he taught without technology by the Khan Academy page.

Cem's statements in the TPACK pre-interview and teaching practice before mathematics coaching aligned with the following indicator: "Does not use technology to develop mathematical concepts." (Niess, Ronau, et al., 2009, p. 22). Therefore, these could provide evidence to place Cem at *the recognizing level* of TPACK for *the instructional descriptor* of the Teaching Theme. Niess, Ronau, et al. (2009), in their TPACK development model for *the instructional descriptor*, identified *the recognizing level* as one in which teachers are aware of the potential benefits of technology, but this is not applied constantly in their practice. Cem also ensured that he availed of a YouTube video and projected pictures that showed him

that he was aware of the emerging technologies. However, these tools were not infused into the instructional practices; instead, they made the students aware of specific learning tools, and interest was generated at the onset. Again, in both lessons, Cem was in control of the use of technology and adopted an approach that involved using technology to illustrate his lectures rather than putting in place a student-centered learning environment characterized by an enhanced use of the awards. This shoved an understanding of the many advantages of adopting technology in the teaching and learning process but failed to harness them to impact the process's instructiveness optimally.

As for *the environment descriptor*, Cem's statements at pre-interview and teaching practice before mathematics coaching aligned with the following indicator: "Uses technology to reinforce concepts taught without technology." (Niess, Ronau, et al., 2009, p. 22). Therefore, these could provide evidence to place Cem at *the recognizing level* of TPACK for of the Teaching Theme. In the TPACK development model, *the recognizing level* of *the environment descriptor* is defined as teachers who have ideas about how technology can enhance the learning environment. At the same time, they do not actively use them to design a stimulating learning environment constantly (Niess, Ronau, et al., 2009). Sometimes, technology integration in Cem's lessons was missing or not applied in a way that improved the teaching-learning process. The approaches to teaching and learning included the conventional ones, with technology being incidental and not the mainstream. Cem questioned students to determine their existing knowledge and any misconceptions they had; however, he did not appropriately capitalize on the use of technology to alleviate these misconceptions. This suggests a mixed understanding of how technology could improve the learning environment, but this was not fully achieved.

When Cem was asked what he thought about the professional development activities in pre-conference 1 and whether there was training he would like to attend, he said

> "Technological tools attract students' attention, and they enjoy seeing such applications in the lesson. Therefore, I take care to be intertwined with technology. I try to get all the training I can get since I am not sure I use technology in the lesson exactly as it should be. In the professional training I

> want to receive, I am interested in the technological materials used in international educational institutions. These schools use digital tools, technological education techniques, websites, game sites, etc."

Cem's statements aligned with the following indicator: "Recognizes the need to participate in technology related PD." (Niess, Ronau, et al., 2009, p. 22). Hence, his statements provided evidence to place him at *the accepting level* of TPACK for *the professional development descriptor* of the Teaching Theme. Niess, Ronau, et al. (2009) stated that teachers at *the accepting level* for the *professional development descriptor* understand the values of professional development for themselves and start to build positive attitudes to using technology as a part of teachers' professional learning process. Still, they are not yet involved in constantly improving their uses of technology in the teaching and learning process. As Cem said, "Before the lessons, I give them tasks that they can complete with the help of technology. They use technology for research before the next lesson, which enhances their learning." Despite this, it shows a relatively low level of integration because technology is employed more as an additional tool helping with assignments than an essential tool for teaching. During the pre-interview, Cem said technology helps educate students when a concept is presented visually. Thus, despite recognizing the advantages, his approach to teaching remained relatively shallow, touching on the use of technology in the context of professional learning and development only occasionally.

4.1.3.2. TPACK Level of Cem Regarding Teaching Theme in the MCC 1

To determine Cem's TPACK level regarding the Teaching Theme at the first mathematics coaching cycle, which focused on the learning objective '*Constructing congruent angle to a given angle,*' his teaching practices were observed for two hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 1 (MCC 1) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in MCC 1 indicated *the adapting level* of TPACK for *the mathematics learning descriptor* and *instructional descriptor* while *the accepting level* of TPACK for *the environment descriptor* of the Teaching Theme.

Cem incorporated technology into the instructional process to enhance, reinforce, and support students' comprehension of mathematical concepts. Furthermore, students actively employed technology as a learning aid to review and solidify their understanding of concepts previously taught by the teacher. Before introducing the concept of congruent angles, Cem sought to ensure that students had mastered foundational topics, such as constructing and measuring angles. To achieve this, he had students construct and measure angles within a dynamic learning environment using technology and traditional materials like paper, pencil, and protractor. This approach demonstrated that students actively utilized technology to revisit and strengthen their grasp of mathematical concepts introduced by Cem in earlier lessons. Students interacted with educational technology as an integral part of their learning or as supplementary materials provided by Cem.

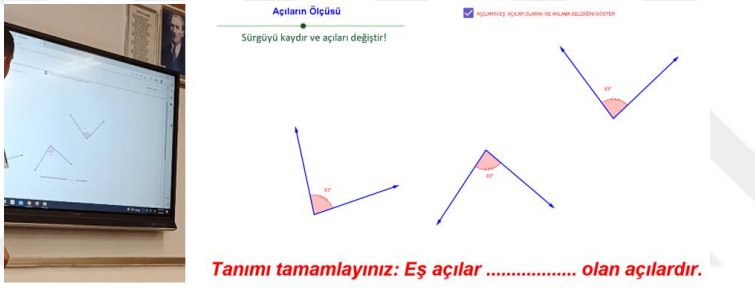


Figure 23. Congruent angles activity of GeoGebra Cem used

Cem's teaching practices provided evidence supporting the placement of him at *the adapting level* of the TPACK for *mathematics learning descriptor* of the Teaching Theme. This alignment is observed in Cem's actual teaching practice at MCC 1, reflecting the indicator: "Uses technology to enhance or reinforce mathematics ideas that students have learned previously." (Niess, Ronau, et al., 2009, p. 22). *The adapting level* of the TPACK development model for *the mathematics learning descriptor* was characterized by teachers who started considering the possibilities of using technology to support their mathematics learning. At this level, teachers use technology to support students' learning and investigations of mathematical concepts and ideas through manipulation and interaction (Niess, Ronau, et al., 2009). Cem's teaching practice aligned with the features of teachers at *the adapting level*. Cem applied GeoGebra to visually and interactively explain and calculate angles and their

measures so that students could see the changes on the spot. He employed activities from GeoGebra, such as the “Measuring angles with a protractor” and “Measuring Angles Using Protractor,” to ensure that students knew how to measure angles well. Cem asked students to come and stand in front of the board to work with the GeoGebra activities, in which he demonstrated the process and corrected them. This was because the students could get an animation of how the angle measurement works, thus falling under *the adapting level* where technology is used to supplement prior learning of mathematics concepts. Cem got students to watch Khan Academy videos that teach how to use a protractor and followed this up with practical activities where he showed them how to do it. This was a good integration of technology in teaching and learning. This purposeful integration is an aspect that relates to *the adapting level*.

Furthermore, Cem’s utilization of GeoGebra and Khan Academy to supplement and consolidate students’ learning of mathematical concepts depicted better ICT literacy (TCK). He effectively used these tools to help the students have a clear picture of angles and to help them understand the concept. He employed GeoGebra to show the angle measurements and made the concepts more practical through the activities. These instances indicate that Cem's proficiency in *the mathematics learning descriptor* progressed to a higher TPACK level, transitioning from *the accepting* to *adapting level*. However, Cem did not communicate his insights regarding the lesson's progression and outcomes with his colleagues. He omitted details about whether the learning objectives were met, the effectiveness of the technological tools he selected, and the results of implementing technology-based lessons in the classroom. This absence of information prevented Cem from being classified at *the exploring level* in the TPACK development model for *the mathematics learning descriptor*.

In the pre-conference of MCC 1, the coach contributed to Cem’s progression from *the accepting* to *the adapting level* of *the mathematics learning descriptor in* Teaching Theme. Several strategies and support from the coach were evident in this progression of Cem’s skill. Both the coach and Cem discussed the objectives of the planned changes in the mathematics teaching/learning process regarding technology

integration. They also stressed using technology in the classroom to present information to the students and support what they have already learned. Additionally, the coach explained some efficient strategies and practices for using technology in mathematics teaching and learning to Cem. They analyzed examples of practices and lessons they had watched in which technology was employed to enhance and consolidate students' learning of mathematical ideas. These examples helped Cem get specific ideas about how to apply technology to support the topics he had taught concerning mathematics. This exposure made Cem introduce activities using GeoGebra and Khan Academy to help the students understand angles. Thus, the coach gave constant feedback and engaged in reflective dialogue regarding his teaching strategies with Cem. They analyzed the content of lesson plans that Cem prepared and some of the possibilities of integrating technology into teaching and learning to improve students' experience. Cem's feedback and reflection allowed him to understand his weaknesses and how he can modify his teaching methods. He started integrating technology into teaching and learning to enhance students' knowledge, thus depicting a shift to the adapting stage. Therefore, the coach assisted Cem in progressing from *the accepting level* to *the adapting level* in *the mathematics learning descriptor*. The enhancement of technology applications to strengthen the concepts previously learned in MCC 1 was evident in Cem's case.

When Cem was asked in post-conference 1 to what extent he thought that the use of technology in the lesson contributed to students' learning how to draw an angle congruent to a given angle, he answered as follows;

> "Students drew and measured the angle using the protractor in their notebooks and did the same with digital tools. Later on, they liked drawing with a digital tool like MathsPad more, where they drew more angles, measured these angles, and drew angles equal to these angles. Therefore, I think using technology as a geometric construction and verification tool contributed to their conceptual learning.

As the coach, the researcher proposed various GeoGebra activities and other technological tools to Cem. Despite the recommendation to select and incorporate the activities that would contribute most to students' learning, Cem used only three of the suggested GeoGebra activities independently. Additionally, he proposed

integrating a Khan Academy activity and a billiards game video into the lesson. In the pre-conference discussions, the coach had advised Cem to present the billiard table example as a problem situation, suggesting that students explore the concept through a problem sentence relevant to the scenario. If Cem had followed this approach using technology, it could have fostered the development of students' higher-order thinking skills within a problem-based context. However, Cem chose to analyze footage from the billiards game video within a real-life context and incorporate the GeoGebra activities selected for the MCC 1 lesson.

During the pre-conference of MCC 1 within mathematics coaching, the coach extensively shared insights and experiences with Cem on various topics, including the integration of GeoGebra, accessing GeoGebra-supported mathematical content, utilizing technological teaching materials, correctly integrating technology with teaching methodology, effective technology-supported teaching practices, and planning for technology-supported assessment and evaluation. Cem's subsequent performance in the lesson showed that he was keen on implementing the knowledge he gained during the coaching process, which served as a form of professional development. Consequently, Cem tried to adapt his mathematics lessons to incorporate the knowledge and technologies acquired during the pre-conference and teacher training.

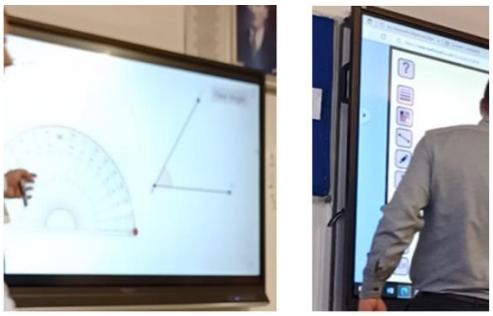


Figure 24. Cem using GeoGebra and MathsPad in teaching of constructing congruent angles

Given Cem's familiarity with his students' learning characteristics, prior learning, academic achievements, and other aspects of the student profile, he tailored technology-based lessons to address his students' specific needs. To put it more

precisely, Cem realized that before introducing the concept of congruent angles and how to form two congruent angles, his students needed to master the skills of angle construction and measurement. Cem realized that his students lacked the preliminary learning to construct an angle and measure a given angle using a protractor, so he planned his instruction according to the needs of his students.

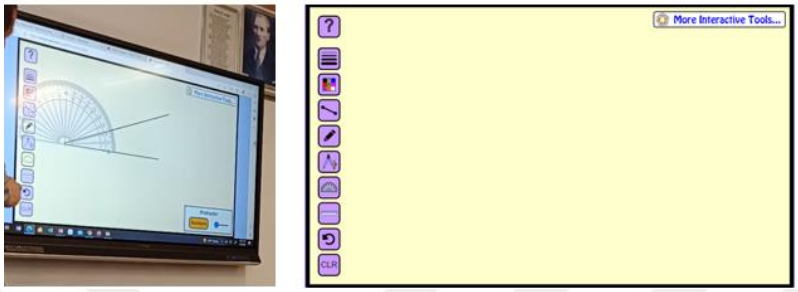


Figure 25. Cem using MathsPad for constructing congruent angles

When Cem was asked in post-conference 1 to what extent he thought the teaching method he used in the lesson was effective in students' learning, he answered as follows;

> "I used the discovery learning method in the lesson. Due to the nature of the teaching method, students had to discover the concept themselves. I tried to put them in situations where they could explore the concept. I asked various questions, and they had the opportunity to try their mathematical guesses and ideas. Here, GeoGebra had the role of a tool that helped with the exploration.

When Cem's teaching practices in MCC 1 were compared with his teaching performance before the mathematics coaching, notable improvements were observed in his teaching methods and techniques. The way he integrated technology into these methods and the overall alignment between the technology used and the chosen teaching methods throughout the lesson demonstrated a more supportive and constructive approach to students' conceptual learning and exploration of the concept. This positive change indicated that the discussions, suggestions, examples, and lesson plans from the pre-conference sessions significantly contributed to Cem's TPACK level in *the instructional descriptor*. The coach effectively conveyed his mathematical, pedagogical, and educational expertise and beliefs about mathematics and its teaching and learning to Cem.

Therefore, these aspects provided evidence to place Cem at *the adapting level* of TPACK for *the instructional descriptor* of the Teaching Theme as his actual teaching practices at MCC 1 aligned with the following indicator: "Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes." (Niess, Ronau, et al., 2009, p. 22). *The adapting level* in the TPACK development model for *the instructional descriptor* is defined as teachers who engage in the simplest form of professional development activities with technologies and attempt to transform lessons for their mathematics classes. At this level, teachers use technology more purposefully and consistently (Niess, Ronau, et al., 2009). Cem's teaching practice aligned with the teachers' characteristics at *the adapting level*. Cem worked systematically with lesson plans aligned with technological tools, as in the case of professional development. He used PowerPoint in his teachings to orchestrate the lessons in a way that addressed all the crucial points well. Cem changed his teaching approaches to include technology-based activities that benefited the students. For example, he used GeoGebra to estimate angles and Khan Academy videos to introduce the use of protractors. He also urged the students to try these tools, thus allowing them to practice and learn through experiences. Cem understood the students' learning characteristics, prior knowledge, and academic performance; therefore, he integrated technology-based lessons to meet their learning needs. For instance, understanding that his students had to be trained in constructing and measuring angles before being introduced to the concept of congruent angles, Cem arranged his instruction as such. Moreover, Cem's detailed lesson plans embedded with technology and modified professional development activities for classroom application showed an improvement in TPK. This is in line with the use of technology in teaching and learning, which was evident in his classroom strategies (TPK) (Mishra & Koehler, 2006). His teaching practice included Khan Academy videos and GeoGebra activities, during which the students were required to go through technological manipulations in GeoGebra. Furthermore, Cem incorporated technology-based activities into the lessons to enhance students' learning, improving his professed pedagogical knowledge in teaching mathematical content. He changed the lessons and applied technology to explain and enforce the concepts in math, such as the angle symbols and measurements.

As for *the environment descriptor* of the Teaching Theme, Cem structured his classroom learning environment around individual student work, with students engaging in exploratory questions during large group discussions or individualized queries. Group work and peer teaching were absent in his approach. Cem conducted the lesson using a presentation, tightly controlling the flow, technology use, activities, visuals, and time management. Compared to previous lessons, his instructional approach in MCC 1 demonstrated a significantly more organized and structured environment. In contrast to the prior lesson, Cem's teaching in MCC 1 was highly controlled and methodically structured, dictating the order of concepts and the use of technology by both the teacher and students. While aiming for students to recognize key learning through the GeoGebra activity, Cem did not facilitate activities to discover concepts independently without teacher guidance. Technology followed a precise sequence of steps, and Cem utilized GeoGebra's dynamic features systematically for specific purposes, such as constructing and measuring angles or verifying congruent angles. Cem did not employ GeoGebra, Khan Academy, and MathsPad as demonstration or static tools but rather as dynamic tools integrated throughout the lesson in a systematic and controlled manner. The emphasis was on using technology to develop or enhance specific skills, such as constructing and measuring angles, rather than exploring or experimenting with its broader capabilities—this targeted and goal-oriented use of technology aligned with Cem's intention to enhance students' understanding.

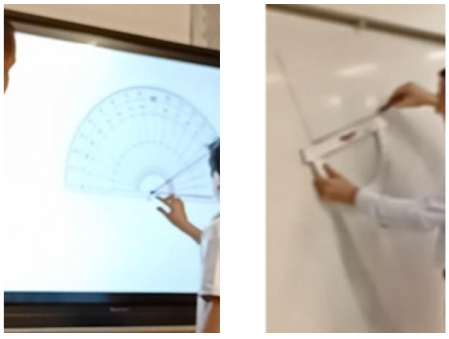


Figure 26. Cem guiding a student on GeoGebra while constructing an angle using a protractor

When Cem was asked about the learning environment in the lesson in post-conference 1, he answered as follows;

> "Since it was the first course, I integrated GeoGebra and other technologies into my teaching so intensively that I was usually in control. So, I cannot say that my course was completely student-centered. However, I wanted to be cautious because it was the first time I integrated course-specific technologies such as GeoGebra and MathsPad as an integrated part of my course. Therefore, I kept control of technology use. I allowed using technology to the students who wanted to participate in the lesson and whom I thought would have no problem using the technology."

These practices provided evidence showing that Cem transitioned to a higher TPACK level for *the environment descriptor*, indicating development in TPACK. Therefore, these aspects could be considered evidence for placing Cem at *the accepting level* of TPACK for *the environment descriptor* of the Teaching Theme as his teaching practices at MCC 1 aligned with the following indicator: "Tightly manages and orchestrates instruction using technology." (Niess, Ronau, et al., 2009, p. 22). Niess, Ronau, et al. (2009) stated that *the accepting level* in the TPACK development model for *the environment descriptor* is described by teachers who coordinate and direct the use of technology to instruct students. Yet, they may still be building their confidence to design learning environments that fully integrate technology. Cem paid much attention to the proper use of technology in the class. He operated on GeoGebra and MathsPad applications and explained how to measure and recognize congruent angles to the students. This approach helped achieve the appropriate use of technology in teaching and learning but also depicted a regulated and ordered system. While Cem fed back to the students and evaluated student learning through technology, including when guiding students through GeoGebra and working on MathsPad, he did not utilize technology to assess each student's learning. This implies that there is building confidence in using technology for instructional management, but the need for a more fully integrated, technology-enhanced environment is demonstrated.

The coach discussed with Cem the importance of creating a technology-rich learning environment and provided strategies for managing and orchestrating technology use in the classroom. This guidance helped Cem understand how to manage and use technology to enhance the learning environment effectively. This moves from recognizing technology's potential to actively managing its use indicates *the accepting level*. The coach emphasized strategies for managing technology-enhanced

instruction, including providing feedback and assessing student learning using technological tools. Cem implemented these strategies by tightly managing the GeoGebra activities and providing structured feedback. This approach ensured that technology was used effectively to support learning, reflecting *the accepting level* where teachers manage and orchestrate technology use. The coach worked with Cem to build his confidence and proficiency in using technology. They practiced using various tools and discussed how to handle potential challenges. This practice and support helped Cem feel more confident using technology in his classroom, enabling him to create a more integrated and engaging learning environment. This progression from recognizing to *accepting level* reflects an increased confidence and proficiency in using technology. To sum up, through collaborative planning, practical demonstrations, and focused feedback during pre-conference 1, the coaching process helped Cem progress significantly in his TPACK development. He moved from the *accepting* to *adapting level* in the mathematics learning descriptor, from recognizing to adapting level in the instructional descriptor, and from recognizing to *accepting level* in *the environment descriptor*. These advancements demonstrate the effectiveness of targeted coaching in enhancing teachers' integration of technology into their teaching practices.

Cem’s enhancement in the use of technology-integrated teaching and learning in giving feedback and evaluating the extent of student learning through technological materials led to significant growth in technological pedagogical knowledge (TPK). GeoGebra was well-controlled; thus, the teacher gave guidance and constructive criticism to the students during the lesson. Moreover, enhancing the subject’s self-efficacy and mastery of multiple technological tools to enhance their classroom application demonstrated growth in TK. He thus used tools such as GeoGebra and Khan Academy, among others, to create a technology-enhanced learning environment and regulate their application.

4.1.3.3. TPACK Level of Cem Regarding Teaching Theme in the MCC 2

To determine Cem's TPACK level of the Teaching Theme at the second mathematics coaching cycle, which focused on the learning objective ‘*Explores the properties of*

complementary, supplementary, adjacent, opposite angles, and solves related problems,' his teaching practices were observed for four hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 2 (MCC 2) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in MCC 2 indicated *the exploring level* of TPACK for *the instructional descriptor* while *adapting the level of TPACK for both* mathematics learning and *the environment descriptor* of the Teaching Theme.

Cem's MCC 2 lesson was similar to MCC 1 in terms of *the mathematics learning descriptor* of the Teaching Theme in how it deals with the subject and integrates technology into teaching. In the actual teaching practice of MCC 2, Cem effectively integrated technology into the instructional process to enhance, reinforce, and support students' comprehension of mathematical concepts. Furthermore, students actively employed technology as a learning aid to review and solidify their understanding of concepts previously taught by the teacher. Cem introduced the concepts to the students from simple to complex, facilitating connections between these concepts. For example, students who had previously learned about angles and angle measurement were encouraged to explore this concept in association with other types of angles. After understanding the necessary and sufficient conditions for two angles to be adjacent angles, students confirmed their ideas and the definition of the concept in both the dynamic learning environment and their work using pencil, paper, and a protractor.

Similarly, Cem introduced the concept of adjacent angles before moving to adjacent supplementary angles. Subsequently, students learned about adjacent angles, allowing them to discover a new concept formed by combining two previously introduced concepts. This approach demonstrated that students actively utilized technology to revisit and strengthen their grasp of mathematical concepts introduced by Cem in earlier lessons. Students interacted with educational technology as either an integral part of their learning or as supplementary materials provided by Cem.

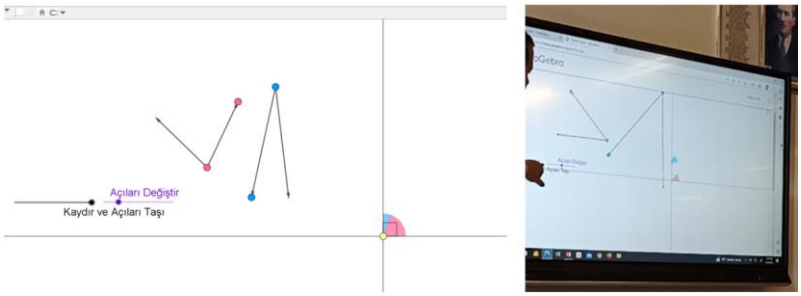


Figure 27. Adjacent supplementary angles activity of GeoGebra Cem used in the MCC 2 lesson

In the second pre-conference session, the coach stressed the use of technology to build and consolidate the mathematics knowledge already learned by the students. The coach shared with the facilitator the possible ways in which students' learning of these concepts can be enhanced with the proper and regular application of technology. The coach then gave more examples of effectively incorporating technology in teaching mathematics. This included introducing new GeoGebra activities that could supplement the students' learning of angles and other geometrical concepts. The coach also showed how these activities could assist students in manipulating mathematical concepts and ideas in their minds. The coach and Cem collaborated on enhancing the lesson plans so that technology can still be integrated into the process. They concentrated on creating tasks in which students employed GeoGebra to elicit and check their mathematical conceptions, which was consistent with the strategy they had used in the earlier unit. The coach regularly gave Cem feedback on his lesson plans and teaching strategies during the coaching process. At the end of every lesson, they evaluated the strategies' success and the aspects that need improvement. This ongoing reflection enabled Cem to continue enhancing the use of technology to support students' learning.

Cem's knowledge of students' understanding of mathematics, thinking, and learning with technology improved in the MCC 2 course. However, Cem did not involve students in activities with high-level thinking skills, such as projects or problem-solving tasks, utilizing technology as a learning tool. Cem employed 13 distinct GeoGebra activities and three different Khan Academy online assessment activities at various points in the lesson, allowing students to engage with them. He utilized

these technological resources as learning tools, placing students at the center, and as demonstration tools, utilizing dynamic features under his control. Cem employed these activities to stimulate student exploration of the concept and to reinforce conceptual learning. Although he presented various problems to the students during the lesson, these problems lacked a contextual basis. These problems were not introduced to the students when technology was a learning tool. Despite being open to integrating suggested GeoGebra activities into the lesson to enhance student learning, Cem, during the MCC 2 pre-conference, opted to use activities solely to support discovery-based conceptual learning and student engagement. This was despite the coach's suggestion to present various problem situations to the students in context and leverage technology as a student tool to enhance high-level thinking skills. If Cem had followed this approach using technology, it could have fostered the development of students' higher-order thinking skills within a problem-based or project-based context.



Figure 28. Real-life examples used in the MCC 2 lesson

Also, Cem did not communicate his insights with colleagues regarding the lessons' progression and outcomes. He omitted details about whether the learning objectives were met, the effectiveness of the technological tools selected, and the results of implementing technology-based lessons in the classroom. This lack of information prevented Cem from being placed at *the exploring level* in the TPACK development model for *the mathematics learning descriptor*. When Cem was asked at the MCC 2 pre-conference how he thought technology would contribute to students' learning of this topic, he replied as follows;

> "I make sure that the GeoGebra activities or other technological tools I integrate into my teaching are compatible with the characteristics of my students, the subject matter, and my teaching method. The activities we choose will help students explore the concept. Of course, it will also help me

> prevent possible misconceptions and allow students to test their predictions. In other words, I think technology will contribute to the learning of the subject in this way."

When Cem was asked at post-conference 2 how the use of technology in the lesson affected the lesson, he replied as follows;

> "Students learned concepts they encountered for the first time in this lesson. GeoGebra had a very positive effect on teaching these concepts and the students' conceptual learning. Without GeoGebra, they would have no choice but to memorize the rules. For example, "the measures of opposite angles are equal. It is enough for me to know that this is so." Nevertheless, GeoGebra activities led them to learn by exploring the concept, and they examined in detail why these angles are equal and made their inferences."

Therefore, all these instructional choices and decisions of Cem and his approach to integrating technology in the MCC 2 lesson provided evidence supporting the placement of Cem at *the adapting level* of the TPACK for *the mathematics learning descriptor* of Teaching Theme. This alignment was evident in Cem's actual teaching practice at MCC 2 and his statements at the MCC 2 pre-conference concerning the following indicator of *the adapting level* of the Teaching Theme: "Uses technology to enhance or reinforce mathematics ideas that students have learned previously." (Niess, Ronau, et al., 2009, p. 22). Niess, Ronau, et al. (2009) asserted that teachers at *the adapting level* integrate technology to teach or re-teach concepts that students have previously learned. MCC 2 also showed this feature in Cem's practice. Cem showed how students can be helped to understand angle properties and revise their learning through GeoGebra. Cem applied GeoGebra in his instructions to visually explain the adjacent, complementary, and supplementary angles. For example, he employed GeoGebra activities to demonstrate the angles and asked the students to identify and define such relationships in angles, thus emphasizing prior learned knowledge. He used GeoGebra, for instance, to make the students recall some concepts they were already familiar with. For instance, he employed GeoGebra to explain the characteristics of angles to enhance the student's ability to picture and grasp the mathematical concepts introduced in previous classes. His incorporation of digital tools in consolidating and revising mathematic concepts depicts *the adapting level*. Cem first collected homework, and then, to help the students remember the

previous information, he asked them to recall the methods of measuring angles and drawing equal angles. He then employed GeoGebra to model adjacent angles to build the student's knowledge of the subject. Cem deployed GeoGebra in the teaching of complementary angles, whereby, using GeoGebra, the students could see the change in angle measures. Cem used GeoGebra to revisit and strengthen the concept of supplementary angles and demonstrate it to the students efficiently.

Cem's teaching practices in MCC 2 were more advanced than those of MCC 1; his teaching practices evidenced this. In this case, Cem went further in applying, investigating, and applying GeoGebra in teaching Mathematics in MCC 2. Most significantly, he paid more attention to displaying student learning of concepts and developing their thinking ability, which went beyond the level of instruction presented in MCC 1. In MCC 2, with a clear plan, Cem incorporated GeoGebra in various activities, including teaching angles: adjacent, supplementary, complementary, and opposite angles. In this aspect, he differed from MCC 1 in encouraging students to ask questions and carry out activities that promoted constructivism. Instead of directly answering students' questions, Cem guided students to find the solution through specific applications and made the class more student-oriented and based on questions and investigation.

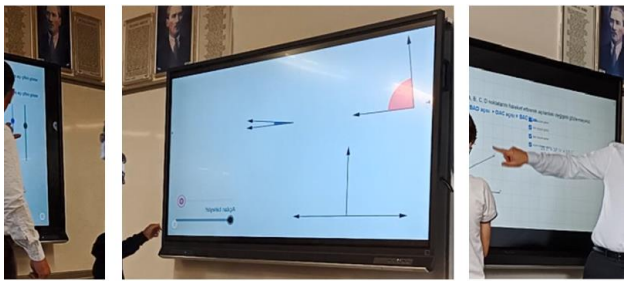


Figure 29. Cem while guiding his students during GeoGebra use in the MCC 2 lesson

In addition, Cem's planning, implementation, and self-assessment of the teaching and learning processes were more purposeful in MCC 2. He integrated activities that are technology-supported not only in measuring the students' Math proficiency but also in managing the students and their learning. Therefore, the application and integration of technology in teaching and learning were more effective in MCC 2

than in MCC 1 through Cem's teaching methods. Thus, Cem assisted students in mathematical investigations by technology and stood more as a guide than a director.

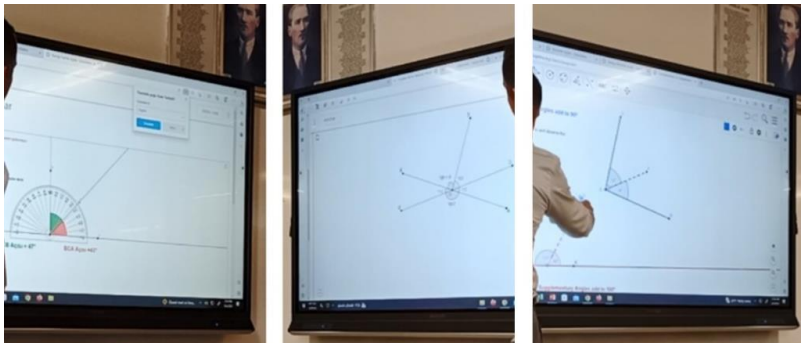


Figure 30. Cem while using dynamic properties of GeoGebra in the MCC 2 lesson

At post-conference 2, when Cem was asked to what extent he thought the teaching method he used in the lesson was effective in students' learning, he gave the following answer;

> "Except for the part of the lesson where the students took notes in their notebooks, the lesson was based on the Discovery Based Learning teaching method enriched with GeoGebra activities. I think this teaching method worked harmoniously with GeoGebra and achieved its purpose. It was a much more successful course in achieving the learning objectives than MCC 1. I guided the students in discovering the information without giving it to them. Everything, from the lesson's introduction to the assessment of students' learning at the end, was as planned."



Figure 31. Cem while utilizing multiple representations in the MCC 2 lesson

Cem's teaching practices in MCC 2 provided evidence to place him at *the exploring level* of TPACK for *the instructional descriptor* within the Teaching Theme as Cem's

teaching practice at MCC 2 aligned with *the exploring level* indicator: "Engages students in explorations of mathematics with technology where the teacher is in the role of a guide rather than the director of the exploration." (Niess, Ronau, et al., 2009, p. 23). Niess, Ronau, et al. (2009) noted that the teachers at *the exploring level* facilitate students' mathematical activities using technology while the teachers' roles are to assist rather than lead. In this regard, Cem's practice reflected this characteristic well. He engaged students in learning activities that exposed them to technology as a tool for learning mathematical concepts by taking more of a facilitator role rather than a teacher's role. For example, Cem prepared activities where students could change angles using GeoGebra and find some relations by themselves. He organized the activities so that students could explore and work on problems on their own while he was there to assist them if necessary. Cem gave the students GeoGebra to do activities and discover such concepts as complementary and supplementary angles. He facilitated activities whereby he posed questions and got the students to manipulate angles and observe relationships; this made it a student-centered class. Cem supported this process by asking the students to work with GeoGebra on the smart board, demonstrating the steps of the activities, and offering suggestions and help. He helped the students reason out and discover more about the angles through the problems and concepts he presented. Cem helped students to find adjacent supplementary angles and vertical angles in GeoGebra, giving questions, allowing students to change the angles, and giving feedback if necessary. Also, Cem employed GeoGebra to assist students in a detailed lesson that covered all the angles' categories. He directed the students to explain terms such as adjacent, complementary, and supplementary angles and made them learn independently. In these activities, Cem also played the role of the teacher as he allowed the students to explore freely, with him being there to direct them and answer their questions. All these observations indicated that Cem progressed from *adapting* to *the exploring level* of the TPACK development model for *the instructional descriptor*. Furthermore, Cem's proficiency in involving students in the investigations of mathematics with the use of technology and where he was more of a facilitator than a conductor showed that he had enhanced his Technological Pedagogical Knowledge (TPK). This dimension was about realizing how the teaching and learning processes could be different when certain technologies were adopted in specific manners. Cem

was able to develop his skills further in creating more engaging and active learning activities that involve the students through technology, particularly GeoGebra in TPK.

In the pre-conference of MCC 2, the coach shared and discussed many ideas with Cem regarding different issues. These issues were incorporating GeoGebra, GeoGebra-supported math content, the use of technological teaching materials, the proper integration of technology in teaching, the use of technology-enhanced teaching strategies, and the integration of technology for assessment and evaluation. The positive enhancements and changes indicated that the discussions, suggestions, examples, and lesson plans from the pre-conference sessions significantly improved Cem's TPACK level in *the instructional descriptor*. In short, the coach successfully shared his mathematical, pedagogical, and educational knowledge and views on mathematics and its teaching and learning with Cem. During the pre-conference 2, the coach suggested that Cem alter the conventional teacher-centered approach to a student-centered one. The coach suggested creating activities to make the students more curious about the given subjects using technology. The coach also emphasized being more of a guide and less of a driver regarding the student's learning and development of mathematical concepts. To this end, the coach, in collaboration with Cem, planned GeoGebra activities for the students to perform. These activities were intended to assist the student in controlling the variables and observing the effects of changes on the variables as they occur in the real world, thus enhancing the student's understanding of the content. Cem's coach helped him establish the goals for forming conjectures and their testing and sharing with the students. Throughout all the stages of the coaching process, the coach was comforting and providing advice to Cem. They discussed the lessons' steps, activities, the possible difficulties, and how to address them. The researchers' encouragement to Cem put him at ease in doing the exploratory activities and managing the class. He presented the students with mathematics with technology and let them solve it with him, guiding them through the process. This is seen in how he employed GeoGebra to allow students' explorations during MCC 2. These strategies helped Cem move from the adapting to *the exploring level* in *the instructional descriptor*.

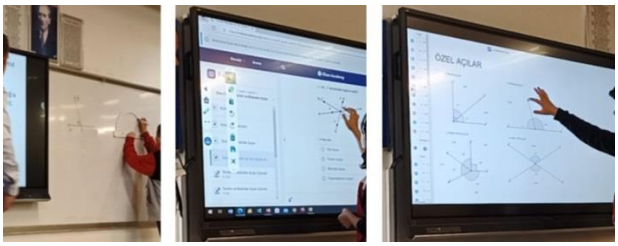


Figure 32. Screenshots from the learning environment of Cem's MCC 2 lesson

As for *the environment descriptor*, there were notable advancements in Cem's instructional approach in MCC 2 compared to his practices in MCC 1. The learning environment, encompassing the roles of both students and teachers, organizational methods, handling of subject matter and concepts, and the purpose of integrating technology, demonstrated a more advanced and constructivist nature. In MCC 1, Cem predominantly employed technology in a deductive and teacher-directed manner to maintain control over the learning activities. However, in MCC 2, he transitioned towards teaching methods that enabled students to explore topics using technology throughout the lesson. Cem organized the classroom environment around individual student work, facilitating exploratory questions during large group discussions or through personalized questioning. Cem used digital tools and physical materials to teach the concept (such as measuring the angle using the GeoGebra protractor and drawing an angle and its complementary angle on the whiteboard with the protractor).

When Cem was asked at the MCC 2 pre-conference what his role in the course would be, he replied as follows:

> "I planned to act as a guide, providing students with various technology-based activities and clear directions. While students are busy with GeoGebra or Khan Academy activities, I will guide them to explore the concept with questions and explanations."

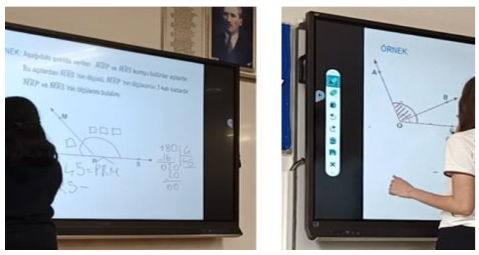


Figure 33. Cem's students while sharing their mathematical thinking and problem solutions in MCC 2

In MCC 2, Cem maintained his instructional approach by not incorporating group work but introducing peer teaching into the instruction. In addition, Cem allowed different students to share their various solutions to a problem with the class. Shifting from a general to a specific strategy, he employed technology differently than MCC 1. While the emphasis in MCC 1 was on developing particular skills like drawing congruent angles and measuring angles using technology, MCC 2 saw Cem using technology to guide students in autonomously discovering concepts such as adjacent, complementary, adjacent complementary, supplementary, adjacent supplementary, and opposite angles, without explicit guidance as long as students faced no difficulties. Furthermore, Cem cultivated a classroom environment encouraging students to express their mathematical ideas openly. Leveraging peer teaching, he had one student explain a topic, concept, problem, or situation to another student, promoting the sharing of answers and fostering collaborative learning.

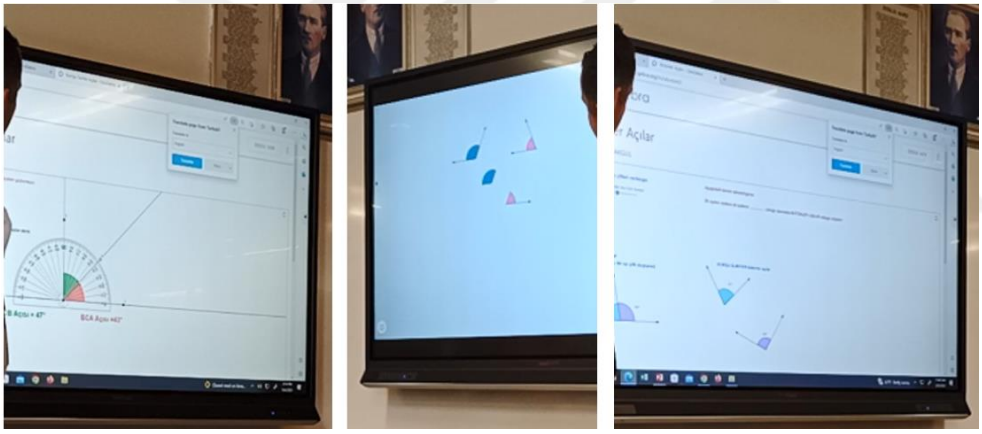


Figure 34. Cem while using dynamic properties of GeoGebra in the MCC 2 lesson

When Cem was asked in post-conference 2 what the roles of the students and himself as a teacher were in the lesson and how technology had an impact on these roles, he responded as follows;

> "During the lesson, I acted as a facilitator and guide, stepping back from being the primary source of knowledge. This allowed me to support student inquiry and learning. The students became active explorers, using GeoGebra to investigate and understand opposite angles. They engaged directly with the technology and reflected on how it helped them learn, deepening their understanding and connecting their discoveries to broader mathematical generalizations.

Cem's statements and teaching practices during MCC 2 provided clear evidence of his placement at *the adapting level* for *the environment descriptor* within the Teaching Theme of the TPACK Development Model by Niess, Ronau, et al. (2009). His primarily deductive and teacher-directed use of technology to maintain control over the learning process reflected the critical features of teachers at this level. These practices demonstrated his competence in using technology to support structured and guided learning, enhancing his TPACK. Therefore, these observations supported placing him at *the adapting level* of TPACK for *the environment descriptor* of the Teaching Theme. Cem's teaching practice at MCC 2 aligned with the indicator: "Instructional strategies with technologies are primarily deductive, teacher-directed to maintain control of how the activity progresses." (Niess, Ronau, et al., 2009, p. 22). Niess, Ronau, et al. (2009) asserted that teachers at *the adapting level* for *the environment descriptor* of the Teaching Theme mainly employ instructional strategies with deductive and expository technologies. This approach is employed to ascertain that the activity's learning objectives are achieved as planned since the flow of the activity is controlled. Teachers at this level also use technology in teaching but in a manner that guides students' interaction with a particular learning sequence. In this respect, Cem's practice was characterized by these features. Although he used the computer as part of the lesson, his approaches were mainly deductive and teacher-led to ensure control over the flow of the activity. For instance, he applied GeoGebra to explain ideas and guide the student's usage of the tool to achieve learning goals. Cem managed the lesson through the smart board and specifically guided the GeoGebra activity on adjacent angles to ensure the students got the idea he wanted them to grasp. He also gave directions and information systematically while managing the overall learning activities. Cem utilized GeoGebra to teach complementary angles; he controlled the tool, and the students answered the questions. This allowed for the creation of the deductive approach, which ensured that students copied him and understood the main ideas through technology. Cem also used GeoGebra to show supplementary angles to the students; however, he guided the students through the process carefully. Thus, by controlling the development of the lesson and the student's activities, he kept the class effective for learning. Moreover, Cem's structured, teacher-directed use of technology to maintain control over the learning environment illustrated his TPK development. This

dimension includes understanding how to effectively manage a classroom environment incorporating technology (Mishra & Koehler, 2006). By balancing technology integration and teacher control, Cem improved his ability to use technology to enhance pedagogical strategies and classroom management.

During the pre-conference of MCC 2, the coach stressed the need for a management strategy in the class while incorporating technology in the teaching process in the pre-conference of MCC 2. They shared ideas on allowing the students to be creative while ensuring they achieve the set goals without compromising class discipline. In pre-conference 2, the coach successfully modeled teacher-led strategies for implementing technology. It depicted how to present the activity, start the students, and slowly let them manage the technology independently. The coach and Cem focused on the deductive teaching strategies integrated with technology. They concentrated on preparing the learning activities through which Cem could control the learning process, and the students were provided with a specific route to take to learn the major concepts. This was comprised of lectures, which were in a step-by-step format, demonstrations, and guided exercises done using GeoGebra. These strategies assisted Cem in strengthening his position at *the adapting level* in *the environment descriptor*. His technological techniques were mainly deductive and teacher-centered to keep the flow of the activity under his control. Thus, following this approach, Cem could incorporate technology in the classroom without the student straying off the topic. In conclusion, it can be noted that the coach improved Cem's instructional and *environment descriptor*s during pre-conference 2 and the coaching process. Thus, the coach supported exploration, designed interactive activities, provided continuous support, emphasized structured guidance, demonstrated teacher-directed techniques, and developed deductive instructional strategies to enhance Cem’s TPACK. Thus, Cem managed to involve students in the meaningful use of technology in mathematics while keeping the overall discipline of the class. All these showed that discussions, suggestions, examples, and shared teaching practices and lesson plans significantly contributed to Cem's TPACK level in *the environment descriptor*. In other words, the coach could reflect his mathematical, pedagogical, and educative power with his beliefs about mathematics and mathematics teaching and learning to Cem. Moreover, these were considered

evidence that Cem moved to a higher TPACK level for *the environment descriptor* and showed development in TPACK.

4.1.3.4. The Progress of Cem in TPACK Regarding Teaching Theme Through Mathematics Coaching in the Learning Area of 'Angles'

The figure 35 presents Cem's TPACK levels for *mathematics learning*, *instructional*, and *environment descriptor*s of Teaching Theme based on his teaching practices in technology-enriched lessons during mathematics coaching and before mathematics coaching (pre-observation lesson). Cem progressed in *mathematics learning*, *instructional*, and *environment descriptor*s according to the TPACK development model compared to before mathematics coaching. Cem's TPACK level in *the professional development descriptor* of the Teaching Theme was determined from the TPACK interview conducted before and after mathematics coaching. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

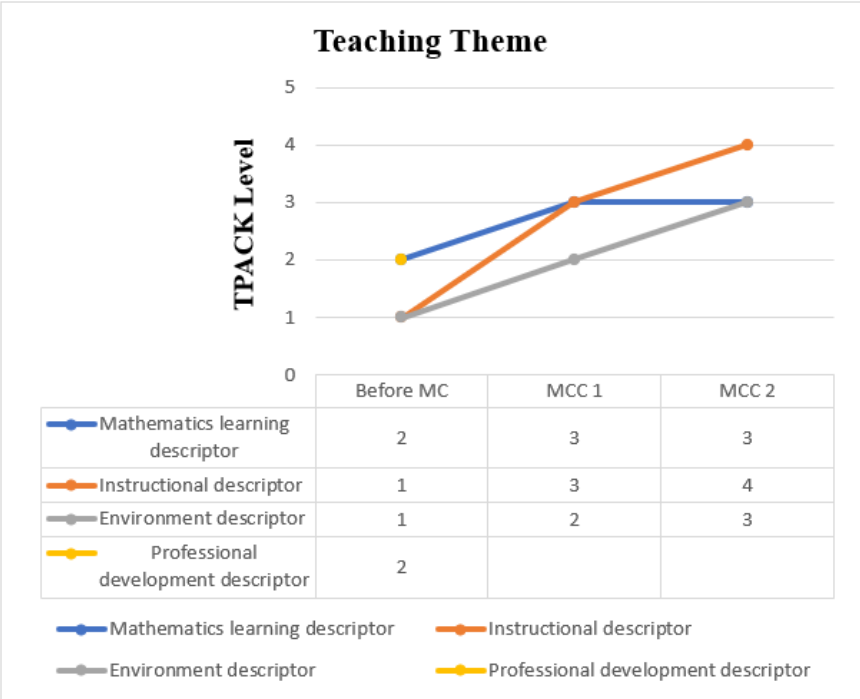


Figure 35. The progress of Cem of the Teaching Theme in the learning area of Angles

Mathematics learning descriptor focuses on how technology facilitates mathematics learning, such as reinforcing concepts, enhancing understanding, or engaging students in high-level thinking activities (Niess et al., 2009). Before mathematics coaching, Cem's level in the TPACK development model's *Mathematics learning descriptor* was at *the accepting level* (level 2). In the teaching practice before mathematics coaching, Cem utilized technology to a limited extent. He employed it primarily to introduce the concept of angles at the outset of the lesson and to underscore their relevance in everyday contexts. Cem accomplished this by presenting a video on the topic and initiating a study on angle comparison via Khan Academy. However, the activity did not entail angle measurement but rather involved discerning the appropriate angle type for given scenarios based on angle classifications. Cem refrained from incorporating technology-supported activities for concepts demanding advanced technological skills. Cem's use of technology remained confined to its role as a static demonstration tool—primarily for playing videos or projecting visuals—within the classroom. Cem progressed in *mathematics learning descriptor*s to more advanced levels through his involvement in the mathematics coaching cycles. Cem's levels in *mathematics learning descriptor* of the TPACK Development model at MCC 1 and MCC 2 indicated *adapting level* (level 3). Cem employed technology to strengthen and solidify geometrical concepts previously covered in class. Consequently, students utilized GeoGebra, MathsPad, and Khan Academy to reinforce their understanding of angles. This included activities such as measuring given angles, constructing angles with specific measures, and identifying various angle relationships, such as congruent, adjacent, opposite, supplementary, and complementary angles.

Instructional descriptor assesses the teacher's instructional strategies when integrating technology into mathematics teaching, ranging from direct instruction to more student-centered approaches (Niess et al., 2009). Through his involvement in the mathematics coaching cycles, Cem demonstrated advancement in *instructional descriptor*. Before mathematics coaching, Cem's level in the TPACK development model's *instructional descriptor* was at *the recognizing level* (level 1). Cem did not use technology to develop geometrical or mathematical concepts. Instead, Cem used technology in class for menial activities such as demonstrating angles via real-life

situations and classifying angles via multiple choice study on Khan Academy (acute angle, obtuse angle, and so on). Cem progressed in *the instructional descriptor* to more advanced levels through his involvement in the mathematics coaching cycles. Cem's level in the TPACK development model's *instructional descriptor* at MCC 1 indicated *the adapting level* (level 3). During the TPACK training session centered on integrating GeoGebra, Cem underwent training with a coach. Following this training, he mimicked basic professional development activities by incorporating various technologies into his mathematics classes. He initiated technology-based lessons tailored to the specific needs of his students. Cem's level in the TPACK development model's *instructional descriptor* at MCC 2 indicated *exploring level* (level 4). Cem facilitated student exploration of angle concepts using technology, assuming the role of a guide rather than a director in the exploration process. Students actively used technology to delve into angle concepts, with Cem providing guidance and support throughout the exploration activities. Cem used teaching strategies such as discovery teaching, peer teaching, large group discussion, and deductive and inductive teaching strategies integrated with GeoGebra.

Environment descriptor examines how the classroom environment is managed when technology is incorporated, including the level of control exerted by the teacher and the degree of student engagement and self-direction (Niess et al., 2009). Before mathematics coaching, Cem's level in the TPACK development model's *environment descriptor* was at *the recognizing level* (level 1). Cem employed technology to solidify concepts introduced without its use. Specifically, he concentrated on angle concepts, guiding students through activities like constructing angles with specified measures, establishing congruent angles, and investigating various types of angles, such as adjacent and opposite angles, using technology. Once students demonstrated proficiency with angles, Cem summarized their understanding, providing examples using MathsPad or engaging in activities on GeoGebra. Through his involvement in the mathematics coaching cycles, Cem demonstrated advancement in *Environment descriptor*. Cem's level in the TPACK development model's *Environment descriptor* at MCC 1 indicated *the accepting level* (level 2). Cem tightly controlled the lesson using GeoGebra and other technologies, guiding students through a structured presentation and closely managing the flow of activities, visuals, and time. While the

objective was for students to grasp key concepts via GeoGebra, Cem did not facilitate independent discovery activities; instead, they followed a precise sequence of steps and utilized GeoGebra's dynamic features for specific purposes, such as angle construction and measurement. In summary, Cem's teaching approach in MCC 1 was highly structured and controlled, with predetermined concepts and technology usage directed by teachers and students. Cem's level in the TPACK development model's *environment descriptor* at MCC 2 indicated *the adapting level* (level 3). Cem began to adopt instructional approaches that allowed students to explore mainly with GeoGebra for most of the lesson. Cem's strategies for teaching with GeoGebra, MathsPad, and Khan Academy exercises in MCC 2 were primarily deductive and teacher-directed to control how the activities progressed. However, students played a more active role in the MCC 2 course than the MCC 1 course and were more involved in the learning process based on GeoGebra activities.

4.1.4. Access Theme of TPACK Development Model

The access theme includes three descriptors: Usage, barrier, and availability. *The usage descriptor* evaluates whether the teacher allows students to use GeoGebra or any other technology. The barriers descriptor emphasizes how the teacher deals with challenges related to technology integration. Additionally, *the availability descriptor* examines how technology offers access to higher levels of mathematics and increases opportunities for student exploration (Niess, Ronau, et al., 2009). In the subsequent sections, the TPACK of Cem is analyzed within the framework of the access theme of the TPACK Development Model.

4.1.4.1. TPACK Level of Cem Regarding Access Theme Before Mathematics Coaching

To determine Cem's TPACK level in the access theme, his lessons, which focused on the learning objective '*Understanding that an angle is formed by two rays with a common starting point and representing it symbolically,*' were observed for two hours before participating in mathematics coaching. The pre-TPACK interviews with the teacher and the pre-observation lessons before mathematics coaching are the source of all quotes and dialogue in the following paragraphs.

Before mathematics coaching, Cem's actual teaching practice indicated *the recognizing level* for usage, barrier, and *availability descriptor*s of TPACK of the Access Theme.

When Cem was asked in the TPACK pre-interview whether the students would use technology in the lesson and, if so, how and when they would use it, he answered as follows;

> "Students will use it in classroom activities. They will also use technology throughout the topic with technology-based research tasks outside the lesson. So, there will be tasks where students will use technology. For example, I tell them to investigate what GeoGebra is, but of course, especially some materials or technological tools may be things that students are not very familiar with. Therefore, I show, explain, and guide them occasionally."

Although Cem answered the question above in the TPACK pre-interview, the students never used the technology in the lesson during the teaching practice before mathematics coaching. The students did not have the opportunity to test their ideas and support their conceptual learning by facing a domain-specific technology. They were not presented with a technology-based research task before the lesson, as Cem mentioned in the TPACK pre-interview. Cem considered the students' use of technology in the context of the research they would do on the internet to come to class prepared for the topic. Instead, the students only followed the teacher's instructions, and students were passive listeners when the teacher used a computer, presentation tool, smart board, projected computer screen onto smartboard, etc. If the students had learned and mastered the topic and key concepts, they would probably have had their students explain their research task with a technological presentation tool. Also, Cem prioritized his verbal explanations, the visuals he projected on the screen, the presentation of the lesson, and the drawings he made on the board. Thus, Cem's teaching in the actual teaching practice was inconsistent with what he stated in the pre-interview of TPACK. He received the answer from a student, tapped the screen, and switched to the next question in the Khan Academy online learning platform study by using the technology himself. Therefore, it is believed that Cem believes that teaching students the concept of angles using paper and pencil by direct

teaching and a student-centered approach would be more effective, and the students' access to technology is not essential for their learning.

Cem's statements in pre-interview and actual teaching practice were aligned with the following indicator: "Permits students to use technology 'only' after mastering certain concepts." (Niess, Ronau, et al., 2009, p. 23). Hence, these could provide evidence to place him at *the recognizing level* of TPACK for *the usage descriptor* of the Access Theme. Niess, Ronau, et al. (2009) stated that teachers at *the recognizing level* in *usage descriptor* permit students to use technology 'only' after mastering certain concepts. Beliefs about how students need to learn mathematics challenge mathematical exploration with technology tools. Cem's statements and teaching practices align with *the recognizing level* of *the usage descriptor*. He allowed the use of technology only after ensuring that students had mastered the basic concepts through traditional methods. Cem started with a presentation and video introducing angles but did not use GeoGebra to explain the mathematical definition or facilitate discovery. This showed his belief that students must first understand concepts without technological interference. The Khan Academy activity was introduced only after students had learned about angles and their measurements. This reinforced that Cem viewed technology as a reinforcement tool, not a primary instructional method. Cem's approach reflects the idea in the teacher's comment that students need to understand the underlying mathematical concepts before using technology like GeoGebra for problem-solving.

When Cem was asked in the TPACK pre-interview about the difficulties that might be encountered in technology integration in the lesson and whether he had any concerns about this issue, he said:

> "Managing teaching with technology is a difficult thing. Both classroom management and the execution of the activity must go simultaneously. For this reason, getting students up to the board individually and involving them in an activity prepared with a technological tool can sometimes disrupt the classroom organization. For this reason, I keep control of the use of technology in my hands. For example, I take the answer choice or answer to a question projected on the screen from the student and mark it myself."

Cem's statements at pre-interview and actual teaching practice aligned with the indicator of *the recognizing level*: "Resists consideration of changes in the content taught although it becomes accessible to more students through technology." (Niess, Ronau, et al., 2009, p. 23). Hence, these could provide evidence to place him at *the recognizing level* of TPACK for *barrier descriptor* of the Access Theme. Niess, Ronau, et al. (2009) stated that teachers at *the recognizing level* in *barrier descriptor* resist considering changes in the content taught. However, it becomes accessible to more students through technology. In the views of these teachers, students' access to technology should be available for them 'after' they have learned the given concepts using paper and pencil procedures and only for rote activities. Similarly, Cem's approach resisted changing content delivery methods despite technology making the content more accessible. After initial teaching, he preferred traditional methods and incorporated technology only as a supplementary tool. Cem relied on verbal explanations, board work, and textbook projections to teach angles without integrating GeoGebra for initial explanations. This reflected his resistance to using technology for primary content delivery. This aligns with the characteristics of the teachers at *the recognizing level*, which emphasizes understanding the concepts first before using technology for enhanced learning (Niess, Ronau, et al., 2009). Although Cem used Khan Academy, it was only after the main instruction was delivered through traditional methods. Technology was used mainly to reinforce concepts rather than to teach them initially.

During the actual teaching practice, Cem did not utilize the affordances of technology to adjust the angle by dragging and measuring it quickly on GeoGebra. However, what he stated during the pre-interview could prove that he knows that technology may facilitate calculations in problems that necessitate long calculations. When Cem was asked about the multiple representations that can be used in teaching the topic and the role of technology in multiple representations in TPACK pre-interview, he answered as follows:

> "I think effective learning will be achieved after showing the concept with more than one representation and having the student establish a relationship. For example, I give an example from daily life in the presentation; then, I show how that angle can be expressed with a symbol. I show the angle as a

shape by drawing it. I can show it in a presentation in more than one form. Alternatively, I can go from more abstract to more concrete with dynamic software. Of course, some more complex or straightforward situations can be shown very easily with technology. For example, in some problems, when the base and side lengths are given in decimal notation, it is much easier to calculate the area with technology."

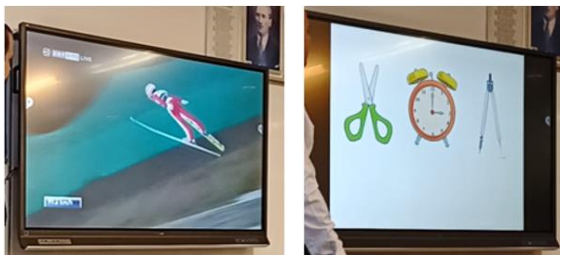


Figure 36. Cem while using multiple representations of the concept of angle

Cem's actual teaching practice before mathematics coaching aligned with the following indicator: "Notices that authentic problems are more likely to involve 'unfriendly numbers' and may be more easily solved if students had calculators." (Niess, Ronau, et al., 2009, p. 23). Thus, these could provide evidence to place him at *the recognizing level* of TPACK for *the availability descriptor* of the Access Theme. Niess, Ronau, et al. (2009) indicated that teachers at *the recognizing level* in *the availability descriptor* realize that real-life problems are more likely to contain 'unfriendly numbers' and may be solved more easily if the students are provided with technology like the calculator. In other words, *the recognizing level* teachers know that in the real world, many problems present difficult or unusual numbers that can be called 'unfriendly' and cannot be solved easily with paper and pencil. It is understood that some of these problems can be solved with the help of tools, such as calculators or software like GeoGebra. However, these teachers may still not readily apply these tools to their teaching even after they have been acknowledged. Instead, they may use them occasionally or after the students try to solve the problems the traditional way. Cem understood the role of technology in solving problems or, in this case, aiding in students' learning process, but he failed to incorporate it into his introductory class. He said that the idea of angle can be explained better with technology, but he only uses it once the students have understood the basics. Cem introduced angles without using technology but knew technology could help somehow. This agrees with his notion that technology is good but not necessarily for

the core teaching and learning process. The Khan Academy activity was presented to the students as an additional activity to perform and not as an extension of the lesson. This is the same as giving the students technology-based assignments as penalized tasks, which depicts the speaker's conservative attitude towards technology. Cem also pointed out that tools such as GeoGebra can process more extensive data and real-world problems, but they should be employed once the students understand the fundamentals.

4.1.4.2. TPACK Level of Cem Regarding Access Theme in the MCC 1

To determine Cem's TPACK level of the Access Theme at the first mathematics coaching cycle, which focused on the learning objective '*Constructing congruent angle to a given angle,*' his teaching practices were observed for two hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 1 (MCC 1) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in MCC 1 indicated *the adapting level* for usage and *barrier descriptor* while *the accepting level* for *availability descriptor* of TPACK of the Access Theme.

Cem ensured that technology was accessible and functional for students to grasp the concept through his demonstrations in the actual teaching practice of MCC 1. Cem's approach to making technology accessible for his students was primarily centered on enhancing their understanding of angles, constructing different angles, utilizing a protractor, and measuring the measure of a given angle using the protractor. Notably, students did not independently use technology to explore the concept without direct support and guidance from the teacher. In this context, Cem employed a teacher-centered methodology focusing on "teacher demonstration." The unit focused on "Angles and Area Measurement," and its inherent topics were conducive to visualizing concepts using technological capabilities. Recognizing this, Cem devised a tailored plan for this unit, allowing students to use technology in a specially designed framework and test their predictions. To achieve this, Cem leveraged the

dynamic features and affordances, such as manipulation and dragging, provided by GeoGebra, MathsPad, and Khan Academy. These tools were strategically employed to enhance conceptual and procedural understanding of the targeted mathematical concept.

In the TPACK pre-interview conducted before the mathematics coaching, when asked about whether students would use technology during the lesson, Cem responded as follows:

> "Certainly, students need hands-on experience with technology. If I were the sole user, I would not capture the level of interest and motivation I desire from them. Consequently, following the introduction and the explanation of the activity's purpose, I will allow students to approach the board and interact with the technology. They can attempt to find answers, test mathematical hypotheses, and verify their solutions. This approach is intended to foster active participation in the lesson and promote a more meaningful learning experience."

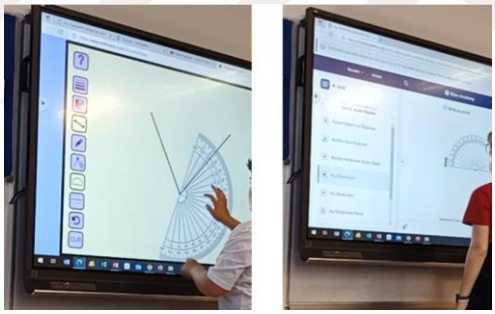


Figure 37. Students while working on MathsPad and Khan Academy

Cem's approach to incorporating technology and permitting its use among his students diverged significantly from the prior lesson. In the class preceding the math coaching session, neither Cem nor his students employed any technology tailored to the math curriculum. In the lesson before the mathematics coaching, Cem refrained from utilizing technological tools conducive to teaching and enhancing conceptual understanding, except for displaying a video on the subject (featuring a ski athlete) and a presentation on the smart board, where he directed the lesson's progression. However, Cem's technology integration in the subsequent lesson began with a "teacher demonstration" method, allowing students to utilize the technology

themselves. Consequently, he evaluated the students' comprehension while enabling them to grasp the concept through practical application. In Cem's MCC 1 lesson, preceding the introduction of technology for the students, he instructed them to construct an angle with a ruler in their notebooks and measure a given angle using a protractor. Cem then expounded on the concepts using his explanations and presentation. Ensuring the students were prepared to apply the concepts, he introduced technology only after this initial phase. This approach enabled students to test their hypotheses, learn through trial and error, identify misconceptions or mislearning, verify their solutions, and delve into the concept. Furthermore, Cem provided guidance and assistance, furnishing feedback on learning challenges or difficulties in utilizing the technology. Thus, Cem aimed to facilitate conceptual learning by empowering students to monitor their educational progress.

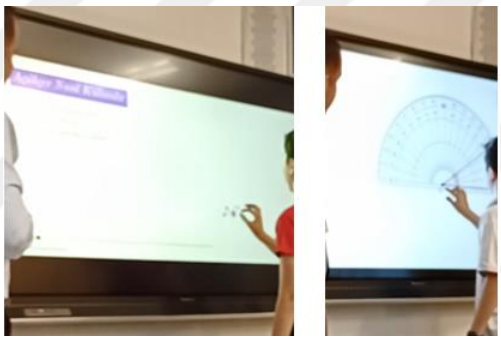


Figure 38. Students while working on GeoGebra for measuring angles

When Cem was asked in post-conference 1 how and for what purpose students use technology in class, he replied as follows;

> "Students started to use technology for the first time in this lesson. They measured various angles using a digital protractor in GeoGebra. We examined in great detail which of the angles written on the outside and inside of the protractor they should use. Apart from that, students constructed angles on MathsPad and determined the measurements of these angles. Then, they drew another angle with this measure. The topic of this lesson was to construct a congruent angle to a given angle. I think they did everything required by this outcome."

Cem's actual teaching practice of MCC 1 aligned with the following indicator: "Permits students to use technology in specifically designed units." (Niess, Ronau, et

al., 2009, p. 24). Thus, it was evident that he transitioned to a higher TPACK level in *the usage descriptor*, placing him at *the adapting level* within the Access Theme. Niess, Ronau, et al. (2009) defined teachers at *the adapting level* in *the usage descriptor* as activities that result in adopting or not adopting the technological tool in the teaching practice. Cem's statements and teaching practices are consistent with *the adapting level* of *the usage descriptor*. He meaningfully used technology to boost the teaching and learning process. Thus, Cem employed GeoGebra to illustrate the concept of angles and their characteristics and students' engagement through visualization. He let the students use the smart board and the computer mouse to measure angles, thus proving that he is ready and willing to use the available technology in class. Cem also incorporated GeoGebra and Khan Academy activities to capture the student's attention in the class and help them use technology to draw and measure angles. This shows his strategy of integrating technological tools to support teaching and learning, which correlates with *the adapting level*. Moreover, incorporating technology in Cem's teaching practices, as guided by the pre-conference inputs, signifies a positive evolution in his TPACK, demonstrating an enhanced ability to integrate technology effectively into his teaching approach. Cem's progress in using technology in the teaching and learning process showed that he had gained better knowledge and application of the technological tools in teaching. This progression indicated an enhancement of his Technological Knowledge (TK) as he progressed from identifying the affordances of technology to integrating them into his teaching plan.

The pre-conference discussions, suggestions, examples, and lesson plans collectively played a significant role in elevating Cem's TPACK level, particularly in *the usage descriptor*. Essentially, the coach effectively guided Cem in reflecting on his proficiency in mathematics, pedagogy, and education, aligning them with his beliefs about teaching and learning mathematics. This alignment was evident in Cem's actual teaching practices during MCC 1, showcasing a development in his TPACK. In pre-conference 2, the coach stressed the value of technology use in teaching, that is, using it often and not occasionally to support students' learning and increase their participation. Besides the supplementary tools, the coach shared with Cem ways and examples of how he could integrate technology into his teaching daily. The coach

showed how it is possible to effectively incorporate technology tools into the class, especially GeoGebra and Khan Academy, and how they can be used in various activities. The coach supported Cem in designing lessons that incorporate technology use in both demonstration and students' activities. This contained the development of activities that required the students to use technology to solve mathematical problems with minimal guidance. The coach observed Cem's classes and gave him feedback on how he could improve and enhance his teaching methods and ensure the effective use of technology in the classroom.

Regarding *the barrier descriptor* of the Access Theme, in the actual teaching practice of MCC 1, Cem utilized technology to enhance his mathematics lessons, teach the concepts, and provide students with a new perspective on approaching mathematics. No obstacles, unexpected events, or situations were encountered while Cem was using the technology. Despite Cem's lack of concerns about access and management issues related to integrating technology in the classroom, he did not assess students' understanding of using a technological tool and whether all students learned the concepts with technology throughout the entire lesson. In other words, Technology was not heavily incorporated into assessments of all students' understanding. During the pre-conference of MCC 1, Cem's statements did not demonstrate that he recognized challenges for teaching mathematics with technologies or explored strategies and ideas to minimize the impact of those challenges. Additionally, Cem did not actively seek ways to acquire technology for classroom use or initiate strategies to address potential technology management challenges. The absence of extensive technology integration in assessments suggests an area where Cem's engagement with technology could be further enhanced. Furthermore, proactively seeking solutions and developing methods for technology management issues could contribute to a more comprehensive and effective technology integration into his teaching practices.

When Cem was asked whether he or his students had any difficulties in using technology in post-conference 1, he replied as follows;

> "I did not experience any particular difficulties, but for the first time, the transformation from the direct teaching method, where I explain and then the

students take notes, to such a different teaching approach and classroom environment created some stress for me. The students had to wait for me to check the presentation on the computer and open GeoGebra activities or other digital tools from the presentation. Interestingly, because this GeoGebra enriched lesson was unusual for my students, they listened much more attentively and disciplined than usual. So, I was much more relaxed in classroom management compared to my previous classes."

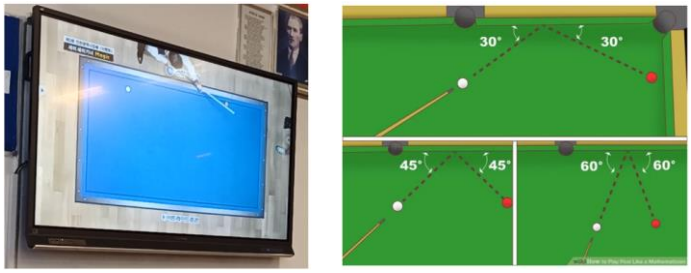


Figure 39. Cem while examining the congruent angles through multiple representations

The coach assisted Cem in dealing with possible obstacles to technology use and enabled him to find ways out and feel at ease. They shared the problems, like controlling the students' usage of devices in class and equality of access and generated solutions for those problems. Cem was concerned about how to use technology in the classroom, and the coach offered ideas on maintaining order while encouraging the students' use of technology. The coach supported Cem in recognizing and applying resources that are available for him to use, such as the technology at school and free online resources. As for the training, the coach advised Cem to attend the workshops and other training activities related to the use of technologies.

Hence, the observed practices served as evidence to position Cem at *the adapting level* of TPACK for *the barrier descriptor* of the Access Theme. This alignment is particularly evident in Cem's teaching practices during MCC 1, where he demonstrated consistency with the indicator of *the adapting level* of TPACK for *the barrier descriptor* within the Access Theme as follows: "Uses technology as a tool to enhance mathematics lessons to provide students a new way to approach mathematics" (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) identified that teachers at *the adapting level* in *the barrier descriptor* acknowledge

the difficulties in teaching with technology but try to find ways to overcome them. These teachers integrate technology into mathematics teaching to offer the students a new form of learning in mathematics. The lesson plan Cem developed exemplified a positive attitude toward technology use and professional development efforts in identifying ways to use GeoGebra and similar tools to support students' learning. Cem had the students check the sum of interior angles in a triangle using GeoGebra, which gave them a way to learn interactively. He assisted the students in accessing and using the technology and solutions to their problems. Cem also employed GeoGebra to teach students about equal angles: he adjusted the angles and showed how they are connected to real life, such as billiards. This demonstrates his attempt to use technology to enhance the teaching and learning of mathematical concepts. This means that Cem appropriately integrated technology as an assistance in teaching mathematics to the students to provide them with a better and different way of learning mathematics. Cem's analysis of the barriers to technology use and his problem-solving skills in dealing with such barriers also revealed an enhancement in his Technological Knowledge (TK). He shifted from identifying the barriers to modifying his practices to minimize their impact, demonstrating a clearer understanding of the management and implementation of technology.

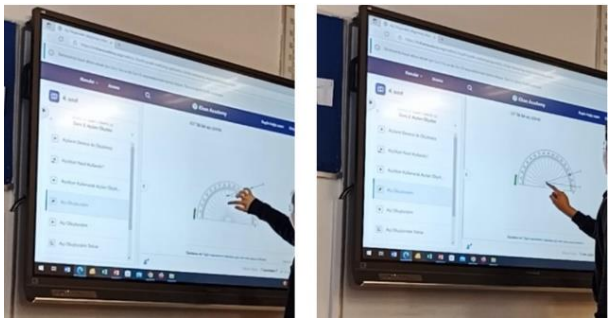


Figure 40. Cem's students, while exploring different examples of angles on Khan Academy

In implementing MCC 1, Cem utilized GeoGebra not solely as a static demonstration tool but as a dynamic instrument to concretize the concept of congruent angles. He permitted students to actively engage with GeoGebra, MathsPad, and Khan Academy, empowering them to construct angles, measure these angles, and determine their congruence. Cem opted for these dynamic learning tools due to their

capacity to facilitate a more significant number of examples, such as manipulating sliders to change the angle measures, thereby providing students with an enriched exploration experience. Incorporating dynamic geometry software played a crucial role in the students' exploration of mathematical conjectures, formation of connections, and development of generalizations.

GeoGebra's interactive and dynamic features enhanced their decision-making processes, contributing to improved analytical skills and a deeper understanding of the subject matter. Furthermore, Cem expanded the learning experience by linking real-life contexts to mathematical concepts, exemplified by the billiards game illustration. Employing diverse representations—verbal explanations, drawings, symbolic expressions, and real-life examples—Cem aimed to solidify students' comprehension by bridging abstract mathematical concepts with tangible, real-world scenarios. Considering these practices, it is evident that the pre-conference discussions, suggestions, examples, and lesson plans significantly contributed to Cem's TPACK level in *the usage descriptor*. This alignment indicates a progression to a higher TPACK level for *the availability descriptor*, showcasing development in TPACK.

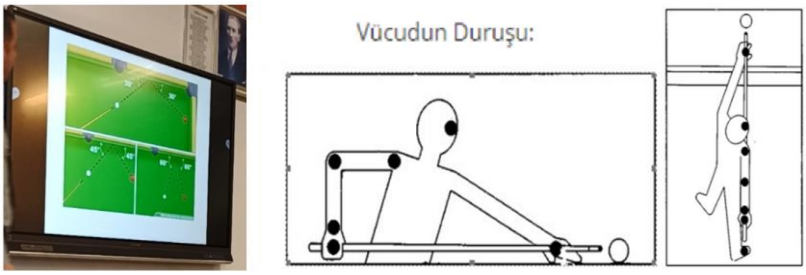


Figure 41. Cem’s representation of the concept from real-life contexts

When Cem was asked in post-conference 1 whether he utilized technology in terms of multiple representations in the course, he replied as follows:

> "In the lesson, I used the examples we discussed in the pre-conference to teach the concept of congruent angles to the students. However, I did not use GeoGebra for multiple representations. I verbally told the students about the reflection of light and sound on a surface. I also showed a video of a billiard ball hitting a billiard table's tape with the same incidence and departure angle.

> I tried to make students understand a congruent angle by using visuals of this situation. Students then had the opportunity to observe and analyze this situation in GeoGebra and MathsPad."

Consequently, all these provided substantial evidence to position Cem at *the accepting level* of TPACK for *the availability descriptor* within the Access Theme. The indicator supports this alignment: "Calculators permit a greater number of examples to be explored by students." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) pointed out that teachers at *the accepting level* in *the availability descriptor* have positive or negative perceptions concerning the incorporation of technology in the teaching and learning process and are aware of the possibility of using it but are not always consistent in doing so. Cem applied tools like Khan Academy and GeoGebra for particular activities but did not apply them in the lesson. He did not rely on technology as a sole teaching/learning tool but used it to support teaching and learning. Even though Cem employed GeoGebra and Khan Academy activities to complement the lesson, he still employed conventional strategies, and technology was not interwoven into each learning activity. This is consistent with *the accepting level* of the questionnaire, portraying him as a cautious person when it comes to using technological resources. Moreover, Cem's improvement in identifying and using technological resources pointed to an improvement in his Technological Knowledge (TK). His progression to *the accepting level* revealed that he became more positive about using technology and began to incorporate it into his teaching more so than before.

During the pre-conference 1, the coach wanted to ensure that the technology was easily accessible and mostly used in the classroom. They assisted Cem and helped him learn about the various technologies the school provided and the possibilities of acquiring other useful tools for teaching. The coach pointed out that the school's resources, such as devices and applications like GeoGebra and Khan Academy, are available. The coach supported Cem in planning the lessons where these resources could be integrated most effectively so that technology could become a common tool used in students' learning process. The coach transferred his technological literacy to Cem and encouraged him to search for new technologies and applications of current tools to increase learning achievements and students' motivation.

4.1.4.3. TPACK Level of Cem Regarding Access Theme in the MCC 2

To determine Cem's TPACK level of the Access Theme at the second mathematics coaching cycle, which focused on the learning objective '*Explores the properties of complementary, supplementary, adjacent, opposite angles, and solves related problems*,' his teaching practices were observed for four hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 2 (MCC 2) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in MCC 2 indicated *the exploring level* for usage and *barrier descriptor* while *the adapting level* for *availability descriptor* of TPACK of the Access Theme.

In Cem's teaching practice during MCC 1, his approach to incorporating technology in the classroom was less advanced compared to his later practices in MCC 2. Specifically, *the usage descriptor* of the Access Theme refers to the permission granted to students to use technology to explore specific mathematical topics. In MCC 1, Cem's use of technology was more teacher-centered, suggesting that the teacher played a more dominant role in using technology within the classroom. As Cem transitioned to MCC 2, a noticeable shift occurred in his teaching approach, particularly regarding technology usage. In this later phase, Cem's teaching practice became more advanced, reflecting an increased emphasis on student engagement and exploration. In MCC 2, students were not only permitted to use technology for mathematical exploration but access and encouragement for such exploration were prevalent throughout most of the teaching time.

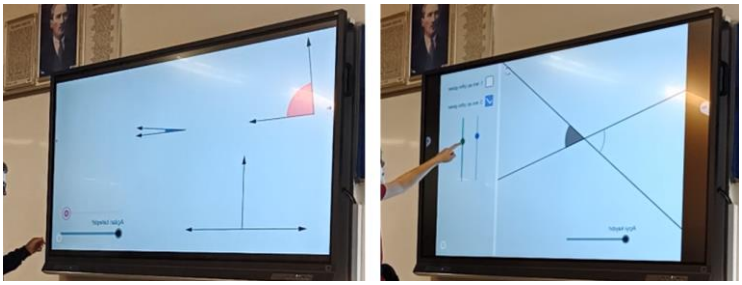


Figure 42. Students while discovering complementary and opposite angles on GeoGebra in MCC 2

Unlike MCC 1 which focused on the learning objective '*Constructing congruent angle to a given angle*,' where technology might have been employed by the teacher primarily to convey information, in MCC 2, Cem's use of technology became more student-centered and discovery-oriented. The students were not only passive listeners who memorized knowledge because they engaged GeoGebra in a way that involved dragging, manipulating, sliding, and observing the concepts of complementary, supplementary, adjacent, and opposite angles—the motivation for providing technology changed from being focused on the teacher to being focused on the student. The focus was to equip the students to understand the concepts being taught, enhance their knowledge, and relate it to other topics learned. In conclusion, it was clear that Cem's lesson in MCC 2 showed improved and more considerate use of technological tools as he engaged the students in discovering complementary, supplementary, adjacent, and opposite angles.

The coach was important in this process since he supported the transition to *the exploring level* of *the usage descriptor* through modeling, co-planning of the lesson, and encouragement. The various indicators indicated that Cem's TPACK level regarding *the usage descriptor* was improved during the pre-conference discussions, recommendations, examples, and lesson plans. In other words, the coach elicited his mathematical, pedagogical, and educative authority from Cem by matching his beliefs about mathematics and mathematics teaching and learning with Cem's beliefs. In the pre-conferences of MCC 2, the coach demonstrated how to incorporate technology into teaching strategies to assist the students in using it to obtain more information. For example, the coach showed Cem how to use GeoGebra to learn more about the angles with the help of practical tasks related to complementary, supplementary, adjacent, and opposite angles. Cem's coach supported him in creating the lesson plans, which involved technology for the students and the applications available, for example, GeoGebra and Khan Academy for teaching angles. This planning entailed integrating student activities aimed at using technology to enhance the students' understanding of geometric concepts. The coach also recommended that Cem allow students to interact with the technology more and offered ideas on guiding and managing these interactions. Cem was assured that technology would be integrated into teaching and learning from this support. The

strategies described above were observable in Cem's teaching performance in MCC 2. As advised by his coach, he made sure that he involved the students in using technology in the search for some mathematical terms, which agrees with *the exploring level* where teachers encourage students to use technology in mathematics learning. Cem allowed the students to use GeoGebra to determine what adjacent angles are and how they function. He told students to modify the angle values and observe the relations between them to demonstrate that he permitted the use of technology in some parts of the class and ensured that it was well encouraged. Cem asked the students to solve some activities on GeoGebra to introduce them to complementary angles. This practice showed that he was at *the exploring level* because he introduced technology the students could use to investigate and learn mathematical concepts. Cem integrated technology in class, and the students could solve problems from supplementary angles using GeoGebra. This was so as he used technology in teaching almost throughout the lesson, a characteristic of teachers at *the exploring level*.

This approach created the opportunity for the students not only to have access to the technologies but also to use them when exploring mathematics. In all the teaching sessions, Cem ensured that the use of technology was more student-focused and discovery-based. The students were allowed to use technology to identify mathematical aspects, including complementary, supplementary, adjacent, and opposite angles. Cem's teaching practices in MCC 2 demonstrated a shift from a teacher-centered to a more collaborative and interactive classroom dynamic, where technology became a valuable instrument for students' mathematical exploration. Students, empowered by the available technology, actively explored mathematical concepts. They used digital resources to deepen their understanding of complementary, supplementary, adjacent, and opposite angles. The encouraged use of technology during most of the teaching time allowed students to investigate these mathematical topics independently, make meaningful connections between concepts, and develop a more profound grasp of the subject matter.

When Cem was asked what the students used the technology for in post-conference 2, he replied as follows:

> "The students used many activities related to complementary, supplementary, adjacent, and opposite angles. They came to the blackboard and used the smart board to move various sliders or points through which the angles pass, and they practiced both the properties and definitions of these types of angles. They also explored the equality of measures of opposite angles using GeoGebra's dynamic features as a learning tool."

These were considered evidence that Cem moved to a higher TPACK level for *the usage descriptor* and showed development in TPACK. Therefore, all these could provide evidence to place him at *the exploring level* of TPACK for *the usage descriptor* of the Access Theme since Cem's actual teaching practice at MCC 2 was aligned with the following indicator: "Permits students to use technology for exploring specific mathematical topics." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) indicated that teachers at *the exploring level* let the students use technology to explore specific mathematical areas and insisted that technology should be made possible and recommended for mathematics learning during most class periods. These features characterized Cem's practice during MCC 2. He specifically challenged students to apply technology to discover certain aspects of mathematics, which is consonant with *the exploring level* that requires teachers to stimulate students' interest in using technology for mathematics learning. Cem let the students work with GeoGebra and find out what adjacent angles are for themselves. He allowed the students to adjust the angles and watch the relationships between the angles. He showed that he was willing to allow the use of technology for specific topics and made sure that it was accessible and recommended to the students during class. Cem then demonstrated complementary angles using GeoGebra to get the students to define the concept and its practical use. This practice depicts that he embraced *the exploring level* since he provided technology to the students so that they could discover mathematical concepts. Cem applied technology in the lesson and made the students use GeoGebra to analyze supplementary angles and their characteristics. This was evident in this case because he allowed the use of technology in the class most of the time, which is one of the characteristics of teachers at *the exploring level*. Moreover, Cem's progression in *the usage descriptor* from the recognizing to *exploring level* reflects significant growth in Technological Knowledge (TK) and Technological Pedagogical Knowledge (TPK) (Mishra & Koehler, 2006).

Concerning *the barrier descriptor* of the Access Theme, Cem's approach to addressing and minimizing obstacles to technology integration in the actual teaching practice of MCC 2 was more advanced than how he did in MCC 1 teaching practice. When Cem was asked in pre-conference 2 about the possible challenges he might face in the use of technology during the lesson and the precautions he had taken before the lesson to overcome these challenges, he responded as follows:

> "We discussed the possible challenges during the training and pre-conference 1. During the coaching, I learned many things I did not know or lacked in selecting appropriate technological materials, time and classroom management, making technological infrastructure and facilities ready for use, and integrating technology into the lesson. Considering these, I will go to the classroom before the lesson and check that the hardware, software, and internet connection are working smoothly. I prepared problems to write on the whiteboard in case of power outages, etc., and prepared worksheets for students to use in group work. I took precautions in this way.

The coach supported the development of Cem to *the exploring level* for *the barrier descriptor* through concerns, resources, and professional development. Before the conferences, the coach introduced possible challenges that may hinder the integration of technology and ideas to address them. The coach answered Cem's questions regarding controlling technology in the learning environment and gave accurate suggestions. The coach helped Cem to recognize the existing resources and to find more tools that could be helpful for teaching. This involved the use of accessible internet sources and equipment that is supplied by the school, especially when it comes to the angles. The coach convinced Cem to engage in professional development geared towards technology to enhance learning, thus boosting the latter's confidence and competence. The following features were also evident in Cem's practice during MCC 2. He tried to find ways to integrate technology even if there were barriers, which shows that he belongs to *the exploring level* wherein teachers are motivated to find ways to lessen the effects of challenges. Following the coach's recommendations, Cem identified and dealt with technological issues and guided the students through finding adjacent angles using GeoGebra. Cem also made plans to make GeoGebra accessible to all students; he regulated the classroom atmosphere to enable the tool to discover complementary angles. Cem also strived to use technology lessons such as GeoGebra concerning supplementary angles even

after encountering challenges, proving that he was keen on providing the best tools for students to solve problems. In the pre-conference 2, Cem stated as follows:

> "I examined lesson plans prepared by different teachers for different grade levels and for different subjects and shared them with other teachers on various websites. I analyzed the flow, content, teaching method, and technologies selected for all these in these lesson plans. This, in my opinion, positively affected my technology integration into the lesson. I will also utilize suggested time and classroom management strategies when implementing a technology-supported lesson."

As Cem mentioned in pre-conference 2, he went to the classroom before the lesson started, checked the PC, and launched GeoGebra activities both on PC and online. He took all necessary precautions in case of any technical problems (electricity, hardware problems, and so on). Cem's actions and teaching approach in MCC 2 demonstrate his awareness of the challenges of teaching mathematics using technology. He actively sought strategies and ideas to mitigate these challenges. Additionally, Cem embraced the coach's suggestion from pre-conference 2 to incorporate a GeoGebra homework assignment into assessments. Throughout the lesson, Cem extensively utilized Khan Academy and GeoGebra activities to assess students' conceptual understanding. This underscores the pervasive use of technology in assessments. Furthermore, Cem proactively sought ways to acquire technology for classroom use and developed methods to address technology management issues. When Cem was asked whether he or his students had any difficulties in using the technology in post-conference 2, he replied as follows:

> "We had no difficulties except a few times when we accidentally enlarged the screen on the GeoGebra worksheet. We had already discussed with you in the pre-conference before I implemented the lesson that such a situation could happen. There is a refresh button in the upper right corner of the GeoGebra activity worksheet. Pressing this button brings the activity back to its original state. This way, we solved it easily. After MCC 1, the students eliminated our amateurishness in opening activities, sliding sliders, activating and deactivating checkboxes, etc. Therefore, my students and I used this lesson much more comfortably."

Therefore, all these could provide evidence to place him at *the exploring level* of TPACK for *barrier descriptor* of the Access Theme since Cem's actual teaching

practice at MCC 2 was aligned with the indicator of: “Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges.” (Niess, Ronau, et al., 2009, p. 24). As Niess, Ronau, et al. (2009) pointed out, the teachers at *the exploring level* understand the problems that may arise in integrating technology into teaching mathematics but seek ways and means of lessening the effects of such issues. These teachers also looked for ways to get hold of technology for teaching and learning purposes and started to develop ways to manage the technology used in the classroom. The characteristics of the post-MCC 1 period were also observed in Cem’s practice during the MCC 2. He searched for ways to use technology even where there could be obstacles, which shows that he is at *the exploring level* where teachers are driven by the desire to find strategies and ideas to help overcome their challenges. Cem employed GeoGebra to let the students investigate adjacent angles and deal with technical issues. This identified his attempts to reduce the effects of barriers by seeking ways to do so. Cem came up with measures to ensure that every student could interact with the technology; for instance, he arranged activities to help students master how to use GeoGebra and, at the same time, control the class. This practice showed his enterprise in seeking and managing technology, one of the characteristics of teachers at the exploring stage. Nevertheless, Cem kept looking for ways to integrate technology even in the given situation, showing his concern for using technology in teaching and developing the students’ problem-solving skills. This can be placed at *the exploring level* because he identified problems and devised new ways of solving them. Furthermore, Cem’s progression in *the barrier descriptor* from the recognizing to *exploring level* showcases his growth in Technological Knowledge (TK) and Technological Pedagogical Knowledge (TPK) (Mishra & Koehler, 2006).

During pre-conference 2, the coach helped Cem to progress to *the adapting level* for *the availability descriptor* by showing them how to use technology, develop lessons, and provide further help. The coach showed how, with dynamic geometry software such as GeoGebra, one could teach mathematics differently and in a way that would help the students establish connections that would otherwise be very hard to make. This was especially important when introducing concepts of complementary,

supplementary, adjacent, and opposite angles. The coach helped Cem design the lessons, incorporating GeoGebra and other tools to enhance students' understanding and show them the relationships between the concepts. The coach continuously observed and gave feedback to Cem. He guided Cem on how he could further improve the use of the technology in a way that would passively improve the understanding of the students on mathematical concepts.

The above strategies characterized Cem's practice in MCC 2. He was able to teach concepts in mathematics differently and employed such tools as GeoGebra to enable students to establish relations that could not be easily made without technology. As the coach advised, Cem used the GeoGebra application to teach adjacent angles, and students saw the real-time changes in angles and their adjacent relations. Cem used GeoGebra to confirm complementary angles and presented another way of developing the concept using technology. Thus, Cem helped the students visualize supplementary angles and properties of supplementary angles using the dynamic learning environment of GeoGebra.

In the MCC 1 lesson, Cem allowed students to explore more examples, particularly involving the measurement or construction of congruent angles using a digital protractor. Students predominantly used GeoGebra for this exploration. In contrast to the MCC 1 approach, Cem altered his teaching strategy by incorporating GeoGebra as a technological tool, providing access to previously challenging connections, especially in terms of relationships and connections in the MCC 2 lesson. Students utilized the dynamic features of GeoGebra to investigate and establish connections between different angle types. Integrating GeoGebra into the lesson allowed students to learn conceptually and make connections between concepts. This approach was not present in the MCC 1 lesson. For instance, Cem guided his students to employ GeoGebra to understand the relationship between opposite and supplementary angles in the actual teaching practice of MCC 2. Through this method, students better understood the equality in the measures of opposite angles. Cem aimed to leverage technology to make higher levels of mathematics accessible for investigation, catering to a larger and more diverse student population.

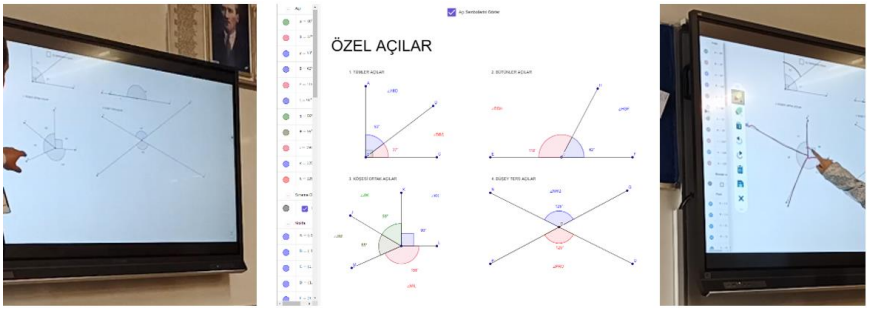


Figure 43. Cem and his student while associating the concepts in the MCC 2 lesson

In post-conference 2, when Cem was asked whether he used technology to move between representations, he responded as follows;

> "I allowed students to use GeoGebra to investigate different types of angles and make connections between them. At the beginning of the lesson, I used various visuals representing different angles. From these visuals, we transitioned to a dynamic learning environment. I tried to use multiple representations in this way.

Therefore, all these could provide evidence to place him at *the adapting level* of TPACK for *availability descriptor* of the Access Theme since Cem's actual teaching practice at MCC 2 was aligned with the indicator of: "Concepts are taught differently since technology provides access to connections formerly out of reach." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) claimed that teachers at *the adapting level* use different methods to teach concepts since technology offers connections between previously unavailable concepts. This is a way in which these teachers can use dynamic geometry software to explore and relate mathematical ideas. MCC 2 also suggested similar features of Cem's practice. He incorporated technology in teaching mathematics and demonstrated how technology can make concepts challenging to make otherwise easier. This showed that he is at *the adapting level* because teachers are using dynamic geometry software to assist the students' activities and learning. Cem employed GeoGebra to explain the idea of adjacent angles; he utilized the application so that students could observe angles' relationships, making it easier for them to grasp the concept. This practice represented the aspect of teaching concepts differently with the use of technology, which is typical of *the adapting level*. These included using GeoGebra to teach complementary angles, and Cem used it to aid students in discovering the concept

through the application. This was at *the adapting level*, making the students link up using the dynamic software. Cem had GeoGebra to help the students discover supplementary angles, their properties, and their relationships. This shows how he was employing the help of technology to ensure that the students could get to mathematical relationships that they could otherwise not easily get to, another characteristic of *the adapting level*. Moreover, Cem's progression in the availability of descriptors from the recognizing to *the adapting level* demonstrates his improvement in technological content knowledge (TCK) and pedagogical knowledge (TPK). All these showed that the pre-conference discussions, suggestions, examples, and lesson plans contributed to Cem's TPACK level in *the availability descriptor*. Put differently, the coach reflected his mathematical, pedagogical, and educative power with his beliefs about mathematics and mathematics teaching and learning to Cem during pre-conference 2.

4.1.4.4. The Progress of Cem in TPACK Regarding Access Theme Through Mathematics Coaching in the Learning Area of 'Angles'

The figure 44 presents Cem's TPACK levels for *usage*, *barrier*, and *availability descriptor*s of Access Theme based on his teaching practices in technology-enriched lessons during mathematics coaching and before coaching (pre-observation lesson). Cem progressed in *usage*, *barrier*, and *availability descriptor*s according to the TPACK development model compared to before mathematics coaching.

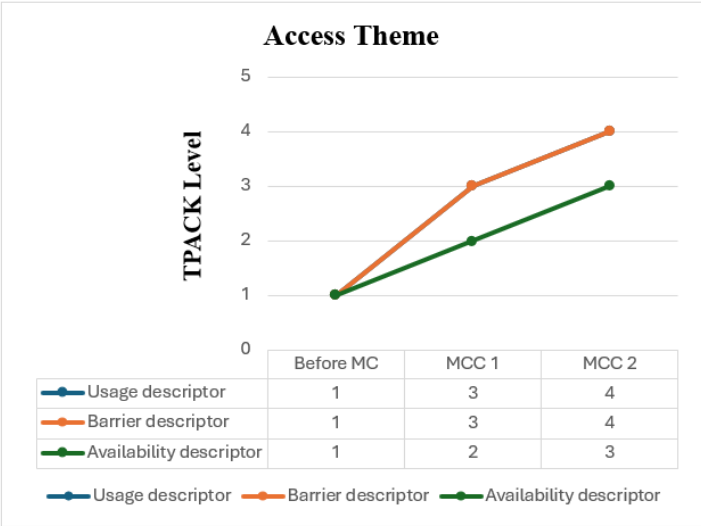


Figure 44. The progress of Cem of the Access Theme in the learning area of Angles

The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

Usage descriptor revolves around whether students should have permission to utilize technology as part of their learning process within the classroom. Before mathematics coaching, Cem's level in the TPACK development model's *Usage descriptor* was at *the recognizing level* (level 1). In the teaching practice before the mathematics coaching, Cem did not allow students to use technology to explore, reinforce, or strengthen angle concepts. Cem allowed students to use technology only after he thought they had a complete understanding of certain concepts, and this use was not aimed at conceptual learning or concept exploration but only at marking the correct option in a multiple-choice task. Cem did not allow students to make mathematical discoveries with technology tools due to his beliefs about how students should learn mathematics.

Cem progressed in *usage descriptor* to more advanced levels through his involvement in the mathematics coaching cycles. Cem's level in the TPACK development model's *Usage descriptor* at MCC 1 indicated *the adapting level* (level 3). Cem began to allow students to use technologies such as GeoGebra, MathsPad, and Khan Academy in a specially designed unit of angles. Students were allowed to utilize these tools, often under Cem's guidance, to delve into angle-related concepts. The student's engagement with technology primarily involved adjusting sliders, altering angles, manipulating angle components, observing resultant changes, and drawing conclusions based on their observations. Cem's level in the TPACK development model's *usage descriptor* at MCC 2 indicated *exploring level* (level 4). Cem permitted students to use technology to explore supplementary, complementary, and adjacent and opposite angles. Access to and use of technology was available for students, and mathematical exploration was encouraged during most class times. The primary focus was empowering students to autonomously investigate concepts, nurturing a more profound comprehension, and facilitating connections with prior knowledge. In essence, Cem's instructional approach in MCC 2 showcased a refined and student-centered technology integration, promoting active exploration and

participation among students as they delved into the complexities of complementary, supplementary, adjacent, and opposite angles.

The barrier descriptor of Access Theme focuses on the difficulties teachers face in ensuring that technology is smoothly incorporated into teaching activities and how the teachers handle barriers to technology integration. Before the mathematics coaching, Cem's level in *the barrier descriptor* was at level two: *the accepting level*. Before the mathematics coaching, Cem was worried about the use of technology in the classroom and how to manage it because he never planned a lesson focused on technology integration systematically and effectively. Hence, Cem was unaware of when and how students would apply the technology, how he would handle a situation where a student or even he could not incorporate the technology, and what other students would do while one student was using the technology. The students applied the technology to the simple learning tasks (solving the multiple-choice questions in the online application of Khan Academy). Cem's level in *the barrier descriptor* moved from novice to proficient through his participation in the mathematics coaching cycles. Cem's level in the TPACK development model's *Barrier descriptor* at MCC 1 was at *the adapting level* (level 3). Cem integrated technology into his mathematics classrooms to help the students approach the subject differently, like learning by exploring and testing the students' ideas and conclusions, examining and working through many examples, and making inferences. However, the concepts of angles introduced to the students through technology were not evaluated with technology. Cem's level of the TPACK development model's *barrier descriptor* at MCC 2 was at *the exploring level* (level 4). Cem was conscious of the problems using math technologies such as GeoGebra but was always looking for ways to solve them. He actively integrated technology into assessments, sought ways of obtaining technology for use in the classroom, and began developing solutions to manage technology.

The availability descriptor explains how technology enhances the possibilities for many students to learn and practice mathematics. By using online tools, computer programs, and interactive models, students from various populations can be introduced to the higher levels of mathematics and tools that were once unavailable.

Before mathematics coaching, Cem was at *the recognizing level* (level 1) in the TPACK development model's *Availability descriptor*. This is another example of technology helping solve real-life problems because Cem understood the concept. Nevertheless, he failed to incorporate technology in teaching advanced mathematical concepts to the students. In other words, he did not use a single technological tool to extend the walls of mathematical learning for students of all types. Cem's development in *the availability descriptor* for the mathematics coaching cycles was noted to have improved to a higher level. Cem's level in the TPACK development model's *Availability descriptor* at MCC 1 was at *the accepting level* (Level 2). Cem, therefore, introduced GeoGebra and MathsPad into his classroom to allow the students to look at more examples. The dynamic feature of GeoGebra made it easier for students to manipulate objects virtually to learn more about mathematical concepts. For example, they could have been working with geometric shapes, changing the values of some parameters, and observing the impact on the connections between different angles. Using GeoGebra allowed students to participate in hands-on activities that helped them learn the dynamic aspect of mathematical relations and geometry. Cem's level for the TPACK development model's *Availability descriptor* at MCC 2 pointed to *the adapting level* (level 3). Cem changed his teaching strategies after adopting subject-specific technologies such as GeoGebra, through which he got a link previously beyond his reach. They used dynamic geometry software to investigate and relate types of angles, as shown below. They operated the software by changing the sliders, changing the angles, changing the angle components, watching the effects of the changes, and coming up with conclusions.

4.2. The TPACK Development of Cem in Learning Area of 'Measurement of Area.'

This section explains how Cem integrated/implemented GeoGebra, Geometry and Pedagogy in teaching the learning area of 'Measurement of Area'. This section shows Cem's TPACK growth as a middle school mathematics teacher during and after mathematics coaching. Cem's performance was evaluated based on the four themes (Curriculum and Assessment, Learning, Teaching, and Access Theme) and

related descriptors (curriculum, assessment, learning-mathematics learning, conception of students' thinking, teaching-mathematics learning, instruction, environment, professional development, use, barrier, and availability) of the TPACK development model (Niess, Ronau, et al., 2009).

After completing the instruction on the "angles" learning domain and its coaching cycles (MCC 1 and MCC 2), the TPACK pre-interview conducted with the teacher before the study served as a post-test for the ‘Angles’ learning domain. This approach allowed for monitoring TPACK changes in the ‘Angles’ learning domain following MCC 1 and MCC 2. This pre-interview also functioned as a baseline assessment before the coaching cycles (MCC 3 and MCC 4) focused on ‘Measurement of Area’ began. Consequently, it was evident that the teacher's TPACK proficiency in the "Measurement of Area" domain was initially low before MCC 3 and MCC 4. Essentially, this methodology ensured that any observed changes in the teacher's TPACK level within the ‘Measurement of Area’ domain were attributable to the impact of mathematics coaching rather than the teacher's existing knowledge and experience.

4.2.1. The Curriculum and Assessment Theme

The theme of curriculum and assessment comprises two descriptors: One for curriculum and the other for assessment. *The curriculum descriptor* centers on how subject matter is approached and integrating teaching and learning processes with curriculum, curricular materials, and technology. *The assessment descriptor* highlights teachers’ decision-making processes when assessing students' comprehension and their employment of technology in the assessment process. In the following sections, Cem's TPACK is examined in relation to the curriculum and assessment theme of the TPACK Development Model.

4.2.1.1. TPACK Level of Cem Regarding Curriculum and Assessment Theme in the MCC 3

To determine Cem's TPACK level in the third mathematics coaching cycle, which focused on the learning objective ‘*Establishes the area relation of a triangle and*

solves related problems,' his teaching practices were observed for three hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 3 (MCC 3) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in the MCC 3 during mathematics coaching indicated *the exploring level* of TPACK both for *the curriculum descriptor* and *the assessment descriptor* of the Curriculum and Assessment Theme. His proactive approach to integrating technology into his curriculum and exploring various technology-based assessment methods reflected significant growth in his TPACK.

Cem commenced the initial MCC 3 lesson on the objective "Establishes the area relation of a triangle and solves related problems" by refreshing the students' memory on concepts covered in the preceding lesson (such as opposite angles, complementary angles, etc.). Following the students' recap of these concepts, he offered feedback and further elucidation before delving into the topic of triangle height, outlining the lesson's objectives to the class, and introducing the subject matter with inquiries like "What defines the height of a triangle? How is it constructed? Under what conditions do three heights exist within a triangle? When does a height coincide with a side of a triangle? When are heights depicted outside the triangle?" Cem initiated with queries to gauge the students' prior understanding. Those wishing to respond to Cem's questions elaborated on their solutions on the whiteboard. At this juncture, Cem posed additional thought-provoking questions to the students, deferring the correct answers for later in the session. Subsequently, Cem illustrated real-life examples of triangle heights (such as the leaning tower of Pisa, a tent, the sail of a sailboat, a house roof, etc.) and prompted the students with inquiries like "How can triangle height be constructed? When do three heights exist within a triangle? When does a height coincide with a side of a triangle? When are heights depicted outside the triangle?" Following this, Cem utilized GeoGebra activities to facilitate student exploration of triangle heights, their placement in various triangle types, and construction methods. Through GeoGebra activities, students investigated triangle heights in different contexts, later applying their findings to worksheets projected by Cem. Cem allowed other students to present

their answers on the board. In the final stage of the lesson, Cem assigned various triangle height exercises from the textbook and elaborated on the "GeoGebra - Height in a Triangle" homework assignment in detail. He also shared the instruction video regarding this GeoGebra assignment, which the coach recorded and uploaded to YouTube.

Cem began the second lesson, about the area relation of a triangle, by explaining the GeoGebra task given to the students at the end of the first lesson. The task included questions such as "How can we determine the area of a triangle? What significance does calculating triangle area hold? How many methods exist for calculating triangle area? Is there a connection between calculating triangle area and computing rectangle area?". Cem began the process by examining the students' preconceptions regarding this topic and providing feedback on their predictions and reasons. He then exposed practical examples like Egyptian Pyramids, triangles in bridges with different sections, house roofs, and even the Louvre Museum to enable the students to see the application of the concept. Then, Cem got students involved in GeoGebra activities to determine the relationship between the rectangles and triangles' area formulas. While the students were performing the activities, Cem assisted them, asked questions, and urged them to express their ideas. Finally, Cem gave the directions and the explanations, guiding the students to GeoGebra activities that dealt with the relationship between area and height in a triangle. Thus, students engaged in real-life activities that involved the comparison of triangles with the same bases but different heights and observed the changes in the height or base length. Then, Cem presented the students various problems that include different base and height values of triangles to determine the area and to explain the area relationship in their own words. Cem then gave the students several exercises from the textbook about triangle area and then explained the GeoGebra – Area Calculation in a Triangle assignment in detail. He also provided the link to the instructional video for this GeoGebra task, which the coach created and posted on YouTube. Towards the end of the 2nd lesson, Cem gave exit cards to the students and checked the students' level of understanding. This assessment had four questions, each taking 3 to 4 minutes to answer. Cem looked at the papers of the students who wanted to show him their results and gave them feedback, and the students then exited the class.

Cem began the third lesson by sharing with the students and elaborating on the outcomes of the ‘area calculation GeoGebra homework task’ given in the previous lesson. He then solved the area relation of the triangle on the smart board and reviewed how to calculate the area of the triangles that use distinct base sizes and their corresponding height. Cem allowed students who wanted to solve the area to come to the board, and he explained the calculation to them in detail in case they did not understand it. After that, on the smart board, he wrote exercises from the textbook, which are triangle height construction and area calculation, so different students could try them. Using several Khan Academy online activities, Cem additionally checked on students’ learning and allowed several diverse students to answer questions.

During the whole lesson with the target students, Cem strived to build the students’ conceptions by providing constant feedback and further investigative questions to the target students when they were working on the text and the Khan Academy problem-solving tasks. He posed a problem to solve wherein different lengths of the base and height of the triangle must be computed for its area and asked one student to solve the problem. When students provided solutions, Cem also engaged all the class members in a discussion, further clarifying problem-solving strategies. Furthermore, he handed out the worksheets and homework assignments about triangle area calculations in the different parts of the textbook. Cem then applied a quiz on WordWall called ‘Quiz Show, ' its content was to calculate the triangle area. Students took turns on the quiz; each tried to answer the questions as much as possible before time ran out. Thus, with the final action of forming a WordWall, Cem closed the lesson.

Before practicing the implementation of MCC 3, Cem focused his research on the use of technology as a medium of learning within the areas provided by the national curriculum. He could discuss his innovative curriculum ideas with the coach before the conference. Based on his lesson plan analysis, it is apparent that he is keen on information and pedagogy regarding adopting technology as a more pivotal component in students’ mathematics learning process. Growth was noted in Cem’s introduction of technology into the curriculum, as evidenced by his improvement

from the methods seen in MCC 2 and his interactions during the pre-meeting sessions. He tried to find an activity on the GeoGebra resource website where a triangle's area relationship can be displayed more visually and through the dynamic graphic. Cem paid scrupulous attention to the outcome descriptions and necessities and suggested approaches in the national curriculum guide on mathematics, which sponsors educators as they plan their lessons.

When Cem could find a suitable and effective activity for the lesson, he introduced it to the coach during the pre-conference of MCC 3. Explaining the area relation of a triangle via the area of a rectangle not only adhered to the suggestions from the textbook and the curriculum but also embraced Cem's preferred method. Also, instead of highlighting several specific heights, Cem used GeoGebra's versatility to analyze the triangle area based on the given base with different heights. This new strategy helped students demonstrate many different heights, which, in turn, helped broaden their understanding.

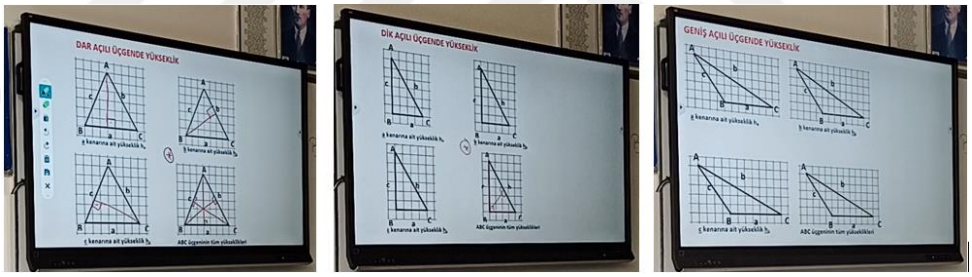


Figure 45. Exercises on constructing heights in different types of triangles from the student textbook in the MCC 3 lesson

These remarks indicated that Cem altered the previous mathematics lessons to include the use of technology and developed the ideas that helped him enhance the conventional lessons to incorporate technology. Also, in the lesson plan modified at pre-conference 3 and in Cem's teaching design in the actual classroom teaching of MCC 3, it was noted that he incorporated technology as a part of the integrated teaching of the subject and not as an add-on to the lesson. Therefore, he began to change the previously performed activities or create new ones that would fit the new curriculum. All these facts point to the fact that Cem progressed to *the advancing level* of *the curriculum descriptor* within the Curriculum and Assessment theme.

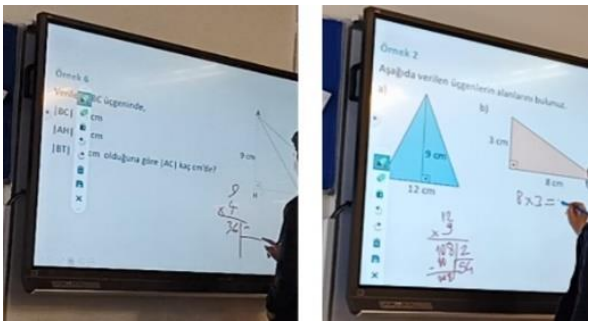


Figure 46. Students, while working on exercises from the textbook in the MCC 3 lesson

Furthermore, when Cem was asked in pre-conference 3 to what extent he benefited from the middle school mathematics curriculum while developing the lesson content, he replied as follows:

> "When planning my teaching, I stayed within the limits specified in the curriculum. I saw that adapting these activities to digital tools was possible during the coaching process. Therefore, I started to search for appropriate GeoGebra activities by examining the teacher's guide textbooks and curricula published in previous years. In other words, I stuck to the national curriculum but integrated technology into it."

The coach met with Cem to develop lessons that would demand the use of technology to a higher degree during pre-conference 3. They developed plans to use GeoGebra to solve problems on triangle heights and areas; hence, GeoGebra is used as a supportive tool and not just an extra tool. The coach told Cem that using technology to verify or refute hypotheses was insufficient. They also highlighted integrating technology into the teaching and learning process. The coach demonstrated how the application can be applied in teaching mathematics, and not only that but also as an enhancement of learning. Some of the new methods of teaching that the coach introduced and demonstrated include using real-life objects/instances and dynamic geometry software in teaching mathematics. This made Cem realize the importance of technology, and they came up with new and improved teaching and learning methods. The coach suggested sources and illustrations of how technology can incorporate geometry into Cem's learning process. They shared the lesson plans, the activities, and even videos that depict how the enhancement of technology can enhance the learning and teaching of

mathematics. The coach often discussed Cem's lesson plans and teaching approach. The coach suggested that Cem consider the teaching process and the possibilities of utilizing technological tools to enhance student learning. Thus, this reflective practice helped in the formation of Cem's own beliefs about the role of technology in the curriculum. As the coach advised and suggested, Cem used GeoGebra activities to have students investigate triangle heights and solve the problems on worksheets according to their solutions.

When Cem was asked in post-conference 3 what he thought about using technology in class, he replied as follows;

> "In the lesson, I taught the height of a triangle and the area relation of a triangle with GeoGebra. Then, I tried to use technology as a learning tool to solve problems related to these. In the first cycles of coaching, I perceived technology only as a tool that enriches the lesson. In this lesson, I think I integrated GeoGebra into the lesson more integratively because it was based on GeoGebra activities and exploring concepts from the beginning."

Cem's statements in pre-conference 3 and actual teaching practice of MCC 3 aligned with the following indicator: "Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning." (Niess, Ronau, et al., 2009, p. 20). Therefore, all these could provide evidence to place him at *the exploring level* of TPACK for *Curriculum descriptor* of the Curriculum and Assessment Theme. Niess, Ronau, et al. (2009) noted that at *the exploring level* for *Curriculum descriptor*, teachers examine the appropriateness of the topics in their curriculum for integrating technology as a tool and look for ideas and strategies on how to expand the use of technology in teaching and learning of mathematics. These teachers also modify their past mathematics lessons to integrate technology and develop ideas on using technology in mathematics curriculums (Niess, Ronau, et al., 2009). The practices of Cem in the MCC 3 also carried these characteristics. He sought out and incorporated technology into his instruction and showed creativity in his planning for integrating technology into the teaching of math concepts. Cem employed GeoGebra activities to help students investigate the location of heights in different types of triangles and

the construction of heights. Using real-life examples, Cem was able to give the students new approaches to learning the height of a triangle with the help of dynamic tools such as GeoGebra. Cem got the engaged students to work on GeoGebra activities to establish the connection between the rectangles and triangle area, particularly the height or base length changes. This approach revealed his attempt to incorporate technology into his teaching plan meaningfully. Cem employed GeoGebra and Khan Academy activities to gauge and improve students' comprehension of triangle area determination. Some strategies he used to incorporate interactions and competition into the WordWall quiz include designing the quiz so that students would have to solve mathematical problems and puzzles as they participated.

When compared with the TPACK level of *curriculum descriptor* in MCC 2, it was concluded that Cem progressed from *adapting level* (level 3) to *exploring level* (level 4) in terms of *curriculum descriptor* in MCC 3. Cem's movement from the adapting to *the exploring level* in *the curriculum descriptor* of the Curriculum and Assessment Theme compared to MCC 2 relates to the improvement in the TPK dimension of the TPACK framework. This development indicated that Cem was gradually learning to connect theoretical knowledge on pedagogical matters with the practical application of technology in teaching and learning. Thus, by reviewing and employing new methods, including GeoGebra for triangle heights and area relations, real-life examples, WordWall quizzes, and Khan Academy assignments, Cem depicted the growth of his professional knowledge of integrating technology into teaching. This strategy not only improved the students' learning but also matched the essential characteristics of teachers at *the exploring level* for both the curriculum and assessment.

As for *the assessment descriptor* in MCC 3, unlike the assessment approach in MCC 2 teaching practice, where assessment and evaluation are conducted without the use of technology, Cem, in MCC 3 teaching practice, not only understood the need for different questions/items during the assessment of students' learning when technology is allowed but also went beyond this to incorporate such elements into his lessons. In other words, in the teaching practice of MCC 3, Cem integrated

technology and assessed students' learning using various types of measurement items such as GeoGebra triangle height construction and triangle area calculation homework assignment, multiple choice and fill-in-the-blank exercises from Khan Academy online platform, multiple choice exercises through the online platform of quiz show (WordWall), height construction and area calculation exercises from textbook.

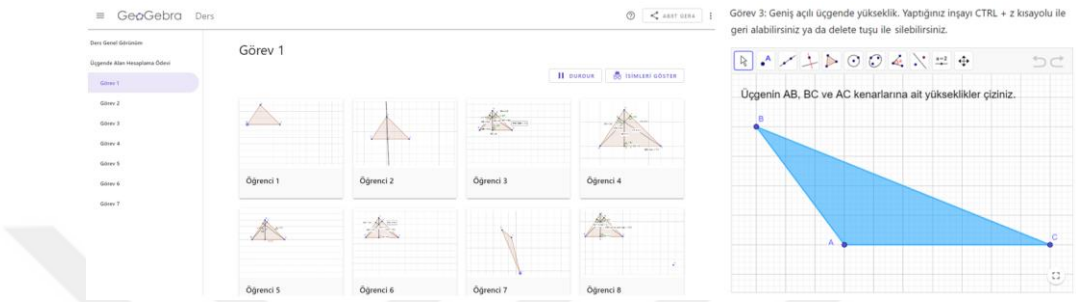


Figure 47. GeoGebra dynamic worksheet (homework assignment) Cem used in MCC 3

Cem tasked students with a GeoGebra home assignment to gauge their comprehension of triangle height and area calculations outside regular class hours. To access the assignment, students utilized an assigned code on GeoGebra Classroom, inputting their names and classes upon completion. Their submissions, including geometric constructions and area calculations, were viewable on Cem's GeoGebra page. Cem provided personalized feedback to each student regarding their solutions and calculations. Through this approach, Cem implemented assessments requiring students to demonstrate their grasp of mathematical concepts using technology beyond traditional pen-and-paper methods.

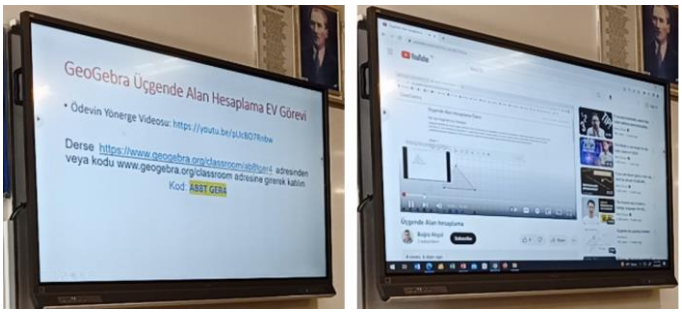


Figure 48. A screenshot from the MCC 3 lesson about accessing the GeoGebra assignment and the instruction video

Before the MCC 3 lesson, Cem extensively explored the integration of various technology-driven assessment formats and queries. He collaboratively designed assessments with the coach that required students to demonstrate their comprehension of mathematical concepts using suitable technology beyond traditional pen-and-paper tasks. Moreover, he asked students different questions, such as “What is the height in a triangle? How can we construct it? Why do we need it? When are heights shown outside the triangle? Is there a relationship between calculating the area of a triangle and calculating the area of a rectangle? How many different ways can you calculate the area of a triangle?” to enhance students’ conceptual understanding and provide students with feedback.

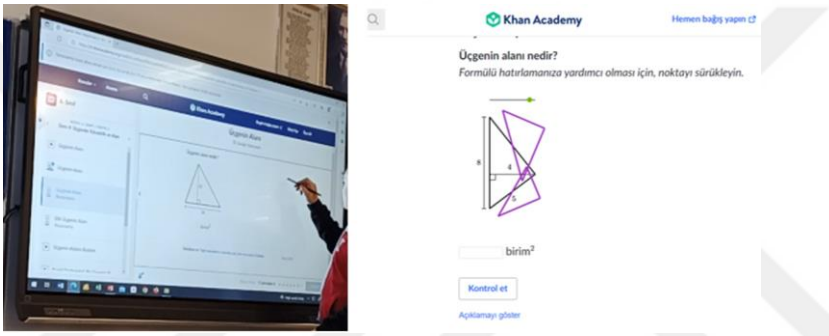


Figure 49. A student, while working on Khan Academy in MCC 3 lesson

When Cem was questioned during the pre-conference of MCC 3 about his strategy for leveraging technology in assessing students' learning, he outlined the following approach:

> “I conducted various research to assess students' learning in different ways. As a result, I plan to employ GeoGebra activities to facilitate their understanding of the area formula of triangles by posing a series of questions tailored to the subject matter and my teaching methodology. Additionally, I have devised a mini contest featuring alternating turns for students to engage in multiple-choice and online competition formats, which students typically enjoy. Furthermore, I intend to incorporate online fill-in-the-blank, multiple-choice area calculation exercises and multiple-choice questions available at Khan Academy. I will also utilize textbook-based questions and construction activities projected onto the board, where students can actively participate by sketching geometric figures with pencils. This dual approach assesses students' learning in contexts where technology is utilized actively and passively. Throughout the lesson, I will pose open-ended, short-answer, and conceptual relationship questions individually and as part of a broader class

discussion. In summary, this outlines my approach to integrating technology into the assessment and evaluation process."

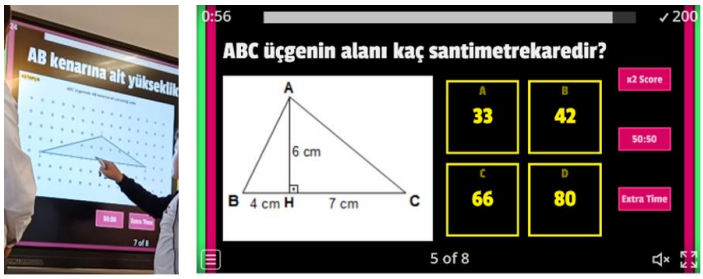


Figure 50. A student while performing an online quiz on WordWall in the MCC 3 lesson

When Cem was asked to what extent he used technology to assess students' learning in the post-conference lesson 3, he replied as follows:

> "I used different types of technology-based assessment items and questions in the lesson. In this way, I expected students to show their understanding of mathematical ideas and concepts using GeoGebra that extended beyond paper and pencil type questions."

At the pre-conference of MCC 3, the coach suggested that Cem consider various technology-based assessments other than the conventional ones he has been using. The coach helped Cem search for different assessment items that can be used with the help of various technological tools for assessing students' learning. The coach gave instances to support how various tools such as GeoGebra, Khan Academy, and WordWall could be used. Before the MCC 3 conference, the coach and Cem worked on designing new assessment options, which included technology in assessment. This collaboration involved providing more engaging tasks, enabling the students to showcase their knowledge rather than simply writing down their answers on paper. For instance, they defined activities where GeoGebra would be employed in developing an understanding of the heights of triangles and the relationship between the areas of triangles, as well as the WordWall quiz in calculating them. The coach offered ways to utilize technology best to give prompt and precise feedback to the participants. With such tools as GeoGebra and Khan Academy, Cem could quickly know the understanding level of the students and provide them with feedback that would enable them to correct their mistakes. He stressed the role of feedback in the

learning process and showed how using technology can improve this aspect. The coach proposed the idea of employing multimedia items, like videos and quizzes, to design assessment tasks that are interesting and informative. The use of instructional videos recorded by the coach and shared with the students presented a case of how more extended learning and assessment can be offered to Cem. Consequently, Cem employed GeoGebra activities to check the students' understanding of triangle heights. Students were allowed to share their findings on the whiteboard, which made the assessment interactive. Cem got the engaged students to use GeoGebra to investigate the area of triangles. He set a GeoGebra assignment with a video tutorial to follow; this way, the students can explore and learn the concepts independently. Cem brought a WordWall quiz related to triangle area calculation as an engaging and competitive assessment method. In addition, he applied Khan Academy activities to check the student's comprehension and incorporate them into his assessment plan. The pre-conference discussions, suggestions, examples, and lesson plans contributed to Cem's TPACK level in *the assessment descriptor*. In other words, the coach could reflect his mathematical, pedagogical, and educative power with his beliefs about mathematics and mathematics teaching and learning to Cem. To sum up, through this coaching process, Cem moved beyond solely understanding the benefits of technology in assessment to actively investigating and implementing various technology-based assessment methods. This shift allowed Cem to design assessments that provided more profound insights into students' conceptual and procedural understanding, ultimately improving the quality and effectiveness of his teaching and assessment practices.

Cem's alignment with the indicator "Actively investigates the use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive)." (Niess, Ronau, et al., 2009, p. 21) further demonstrated his development in TPACK. Specifically, Cem's statements in pre-conference 3 and his teaching practice in MCC 3 showcased his engagement with technology-enhanced assessment strategies. This evidence placed him at *the exploring level* of TPACK for *the assessment descriptor* of the Curriculum and Assessment Theme. Niess, Ronau, et al. (2009) noted that teachers in *the exploring level* for *the assessment descriptor* seek for various technology-based assessment items and

questions (active, inactive, neutral or passive). Teachers at this level also create tests and examinations that demand students to demonstrate their knowledge of mathematical concepts through technology that goes beyond the conventional use of paper and pen. Cem's practices in MCC 3 illustrated these features as well. He became conversant and creatively incorporated different technological assessment tools to enable the students to show their understanding. Cem used GeoGebra activities and worksheets to evaluate the student's knowledge of the heights of triangles.

By requiring the students to present their answers on the board and elaborate on the answers, he made the assessment not only paper- and pencil-based. Cem employed GeoGebra to study the link between area and height of a triangle through the spreadsheet view of GeoGebra and set the GeoGebra – Area Calculation in a Triangle as homework. He also submitted an instructional video the coach took, integrating other media into the evaluation procedure. Cem brought in a WordWall quiz in which students answered questions about the triangle area following the quiz format. He also applied Khan Academy online exercises to check the students' progress and incorporated multiple technology-based assessments into his teaching.

Moreover, comparing Cem's TPACK level in MCC 2 and MCC 3 revealed his progression from *the adapting level* (level 3) to *the exploring level* (level 4) regarding *the assessment descriptor*. To sum up, Cem's level at MCC 2 was higher than his level at the Curriculum and Assessment theme of MCC 3. At the same time, he moved from the adapting to *exploring level* in *the assessment descriptor*, which showed improvement in the Technological Pedagogical Knowledge (TPK) component of the TPACK framework. Improvement in Cem's TPK was observed during the study when Cem enriched his teaching tools and practices with technology-based assessment items, including GeoGebra and Khan Academy, to check students' comprehension of mathematical concepts. Furthermore, he developed assessments that extended beyond the conventional approaches, incorporating fun activities such as WordWall quizzes to gauge the student's knowledge and comprehension and helping the students to employ technology as a tool to express their understanding of mathematical concepts, thus making the technology an essential part of the assessment process.

4.2.1.2. TPACK Level of Cem Regarding Curriculum and Assessment Theme in the MCC 4

To determine Cem's TPACK level in the fourth mathematics coaching cycle, which focused on the learning objective '*Establishes the area relation of a parallelogram and solves related problems*,' his teaching practices were observed for three hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 4 (MCC 4) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in the MCC 4 demonstrated *the advancing level* of TPACK for *the curriculum descriptor* while *the exploring level* of TPACK for *the assessment descriptor* of the Curriculum and Assessment Theme according to the TPACK Development Model.

Cem started the first lesson of the MCC 4 teaching practice by recalling the information learned in the previous lesson. He asked students questions about area and height in a triangle and gave feedback on their answers. After checking the students' prior learning, he told them about the objective of today's lesson. Then, he projected a problem about area calculation in a parallelogram on the screen. He asked the students to guess the solution to the problem and listened to their suggestions. After this part, he told the students they could solve such issues after learning this topic. Then, he had the students define parallelogram and gave feedback to the students' definitions. Projecting visuals on the screen with examples of parallelograms from daily life situations, Cem asked, "What is height? How do we determine the height of a parallelogram? Why do we need height in a parallelogram? What does it mean for the height of a parallelogram to be in the inner and outer regions of the parallelogram?" and listened to the answers of different students. To answer the question, "What is the height of a parallelogram, and how can we draw it?" he allowed students to show their mathematical estimations and height drawings on the board.

He encouraged the students to share their answers with the class and gave feedback to the students. After the students shared their ideas about height, Cem tried to make

them intuit the concept's meaning by drawing height on the board and drawing what height is in a parallelogram. He then opened the GeoGebra - Height on a parallelogram activity and explained the purpose and instructions of the activity to the students. Cem allowed many students to use GeoGebra's dynamic features to transform the activity's parallelograms into parallelograms of different sizes and examine the heights of the bases. While the students were engaged in the activity, he continuously gave feedback and asked them conceptual questions. Cem had the students explore the concept of height in the GeoGebra parallelogram height drawing activity. Then, he opened a worksheet with dotted paper and had the students practice drawing heights. At the end of the lesson, he explained the GeoGebra - Drawing height on a parallelogram homework task to the students and ended the lesson by giving homework for drawing various heights from the book. At the end of the lesson, he distributed exit cards to evaluate the students' learning. After completing this 5-question assessment on drawing the height of a parallelogram, the students left the classroom. Cem gave feedback on the results of the exit cards brought by the students.

Cem started the second lesson of the MCC 4 teaching practice by giving feedback to the students about the GeoGebra - Drawing the height of a parallelogram homework task. Cem then asked the students how they calculated the area of a rectangle and triangle. He aimed to emphasize the importance of base and height in finding the area. Cem then asked if the area of these shapes could be used to find the area of a parallelogram and listened to the students' guesses and mathematical ideas. He then allowed one student to do the shapes inside a parallelogram activity in GeoGebra. The student filled the parallelogram with two triangles and one rectangle by dragging the shapes inside the parallelogram. The student shared the geometric situations he noticed with Cem and his classmates. Moving on to another GeoGebra activity, Cem opened an activity called "cut-part-move the parallelogram" to find the area of a parallelogram by using the area of a rectangle. He allowed one of his students to work on this activity and listened to the student's conclusions. Then, another student worked on this activity and shared his findings with Cem and his classmates. The students used expressions to show that they realized that the area of a parallelogram

is the same as the area of a rectangle by moving the triangle on the parallelogram to the other side.

After the students, Cem used this activity himself and supported the students' learning with various explanations. After this part, Cem projected a parallelogram where the area could be calculated by counting the unit squares inside the parallelogram. After allowing one student to do so, Cem then projected another parallelogram whose area could not be calculated by counting the unit squares and made the students realize that a relation was needed to calculate the area of the parallelogram. Cem opened another GeoGebra activity and allowed one student to work on the activity. In this activity, he made the students realize that the area of a parallelogram can be calculated by counting the unit squares, moving the triangle in the parallelogram, and using the area of the rectangle. The student, who could change the amount of fill transparency with the help of the slider in the parallelogram activity, shared his conclusions with his classmates. Cem gave feedback to the students and made additional explanations to the class. To have students explore the area relation of a parallelogram using the area of a triangle, Cem opened another GeoGebra activity and allowed one student to work on the activity. The student made changes such as moving the sliders and corners, changing the triangle size, etc. The student then used a slider to realize that a parallelogram consists of 2 congruent triangles due to the addition of the triangle's partner after the rotation transformation and shared this with Cem and his classmates.

Cem opened another similar GeoGebra activity and allowed another student to listen to the student's observations and conclusions after working on GeoGebra. Cem then opened another activity, proving that the area of a parallelogram with different bases and heights is the same. Cem allowed a student to use GeoGebra to discover this. The student pointed out that the areas are calculated similarly, but the base and height lengths differ. Cem then used the activity himself and gave detailed explanations. Cem then asked the students to write down the area relation of a parallelogram in their own words. At the end of the lesson, Cem explained the GeoGebra - Parallelogram Area calculation GeoGebra homework to the students and evaluated their learning by distributing the exit card related to calculating the area of

a parallelogram. After giving feedback to the students' solutions on the exit card, he ended the lesson by assigning various area and height drawing exercises from the textbook.

In the last lesson of the MCC 4 teaching practice, Cem permitted a student to solve the area problem, which he shared with his students at the beginning of the first lesson. Then, Cem had the students practice constructing the height and calculating the area of a parallelogram and solve problems related to them. He projected the exercises and problems from the textbook on the screen and allowed the students to answer these questions. The students first practiced drawing height and area calculation questions and then continued to solve different questions through the WordWall Quiz Show online learning platform. Cem allowed students to solve in groups of 2 students with peer teaching. In this way, students decided on the solution together, deepening their understanding and confirming what they had learned about the subject through collaborative learning. Cem evaluated the learning of all students with the Quiz Show. Cem gave feedback to the student groups whose solutions were wrong and ended the lesson by distributing the exit cards to the students.

As for *the curriculum descriptor* of the Curriculum and Assessment Theme, Cem enhanced the curriculum by integrating GeoGebra as a valuable tool for both teaching and learning. He adjusted the curriculum to incorporate GeoGebra, allowing students to delve into the concept of parallelogram area. Recognizing GeoGebra's dynamic capabilities, Cem modified existing activities within the curriculum to better align with its interactive nature. Furthermore, he drew inspiration from the existing curriculum and developed innovative approaches, leveraging GeoGebra to deepen students' understanding of the subject matter.



Figure 51. Parallelogram activities of GeoGebra from MCC 4 lesson

In MCC 4, Cem investigated the use of topics of height in triangles and area relation of triangles in their curriculum for including technology as a tool for learning and sought ideas and strategies for implementing technology in a more integral role for the development of mathematics that students are learning. He adapted his previous mathematics lessons to include technology. Moreover, he developed his ideas about using technology to enhance the current curriculum; thus, he began altering preexisting activities or creating new activities for the current curriculum. In addition to what Cem did in MCC 3, Cem recognized the imperative of continually innovating his curriculum to effectively integrate technology as a tool for teaching and learning in MCC 4. For instance, he devised inventive methods leveraging technology to enrich his students' mathematical cognition. One approach involved employing a virtual cutting activity to dissect a triangle from one side of a parallelogram and relocating it to the opposite side, thereby investigating the correlation between the areas of rectangles and parallelograms. In another instance, he utilized the knowledge of triangle area acquired in previous lessons, orchestrating a virtual folding exercise to prompt students to uncover the equivalence between the combined areas of two congruent triangles and that of a parallelogram. By harnessing technology as an educational aid, Cem adapted and advanced the curriculum, incorporating activities that enabled students to delve into the area relationships inherent in parallelograms through various triangles or parallelograms.

Furthermore, at the pre-conference of MCC 4, Cem mentioned the changes and improvements that could be made to the curriculum as follows;

> "The activities specified in the curriculum are not integrated with technology. Therefore, it is crucial to transfer these activities to a dynamic learning environment using technology as a teaching and learning tool. In the curriculum, it is recommended that we utilize information technologies while teaching the subject, but many teachers do not know how to do this. It is easier for teachers to use what is written in the curriculum as it is. We can develop more effective teaching materials and opportunities by developing or modifying the existing curriculum activities. In short, to integrate technology into teaching most effectively, it is necessary to make various modifications and improvements in the curriculum."

Moreover, when Cem was asked about the way he integrated technology into the curriculum in post-conference 4, he gave the following answer;

> "In the curriculum, it is suggested that we include works on dotted or squared paper to make students discover the height and area of a parallelogram. We should use the area relation of a rectangle to make them discover the area relation of a parallelogram, and we should also emphasize that squares and rectangles are special cases of parallelograms. However, when we do not use technology in the lesson, we may be unable to integrate all these and present them to the students. Even if we do, it is in a non-permanent way. In the MCC 4 course, we had the opportunity to examine all of these in detail with the opportunities provided by integrating GeoGebra and other digital tools. Students were able to examine the concept from every angle. The fact that I adapted these suggestions in the curriculum to the dynamic learning environment and presented them to the students made a great contribution to their conceptual learning."

Cem's statements and actual teaching practice of MCC 4 aligned with the following indicator: "Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential." (Niess, Ronau, et al., 2009, p. 21). Therefore, all these could provide evidence to place him at *the advancing level* of TPACK for *the curriculum descriptor* of the Curriculum and Assessment Theme. Niess, Ronau, et al. (2009) pointed out that teachers at *the advancing level* for *curriculum descriptor* know that long-term change in modifying their curriculum to integrate technology into the teaching practice is necessary. Some teachers develop strategies to make the lesson captivating to students and develop mathematical thinking. They also alter and enhance a curriculum to seize opportunities presented through the effective use of technology in teaching and learning. The following characteristics were evident in Cem's practices in MCC 4. He used technology in his teaching and came up with other ways of teaching the area and height of parallelograms, such as converting one into the other. Cem employed GeoGebra activities to assist the students in investigating the height of a parallelogram. He urged the students to move the parallelograms and dynamically look at the heights to understand the concept more clearly. Cem used GeoGebra activities to teach the area of a parallelogram to engage students and stress the factors base and height. This practice depicted his competence in altering the curriculum to appropriately fit the use of technology. Cem employed application resources such as GeoGebra and WordWall Quiz Show to further explain and apply the area and height of parallelograms and engaged the students in group learning. When compared with the TPACK level of *curriculum descriptor* in MCC 3,

it was concluded that Cem progressed from *exploring level* (level 4) to *advancing level* (level 5) in terms of *curriculum descriptor* regarding curriculum and assessment theme in MCC 4. Cem demonstrated significant growth in integrating technology with his pedagogical methods and content knowledge, improving his TPACK.

The coach played a crucial role in identifying the critical areas of improvement in the curriculum modification and technology integration into the curriculum, considering the weaknesses and strengths of the teaching practice at the MCC 3. The coach began by discussing Cem's MCC 3 lesson, in which he effectively incorporated GeoGebra to teach the area and height of triangles. The coach focused on the best practices and noted what Cem should try to avoid in the future, specifically regarding curriculum integration. Cem's coach guided him in identifying the goals for MCC 4, particularly the requirement of linking area and height from triangles to parallelograms. The objective was to enhance students' learning by having them find out the area formula through different interactions. The coach presented Cem with the spreadsheet view of GeoGebra to help the students find the area of a parallelogram. This would be a creative application of technology since the students could manipulate stimuli and examine the relationship between variables. The coach and Cem created GeoGebra activities that the students would solve to discover parallelogram characteristics. They discussed how to manipulate the dynamic aspect of GeoGebra to show the height and area and how to move from the visual representation to the spreadsheet to look for a formula. Cem's coach's intervention was to assist him in progressing from using technology as a tool to support the visualization of learning to use it as a significant part of his learning process. Integrating the spreadsheet view of GeoGebra was another way of ensuring that technology was used to enhance learning and discovery. Cem's coach advised him to prepare activities that are technology-integrated and, at the same time, engage students in performing mathematical activities. This approach was appropriate given the progress of *the curriculum descriptor*, which shows that consistent improvement in the curriculum alteration is critical. Thus, Cem could give a more holistic view of assessing student's knowledge by integrating several technological tools. Cem used these assessments to complement his teaching and was able to correct any

misconceptions that were identified during the assessments. The pre-conference discussions, suggestions, examples, and lesson plans contributed to Cem's TPACK level in *the curriculum descriptor*. In other words, the coach could reflect his mathematical, pedagogical, and educative power with his beliefs about mathematics and mathematics teaching and learning to Cem.

Regarding *the assessment descriptor* within the curriculum and assessment theme in MCC 4, Cem has actively explored using various technology-based assessment items and questions since the MCC 3 session. Consequently, he formulated assessments where students demonstrated their comprehension of geometrical concepts using GeoGebra as an appropriate technological tool, surpassing traditional paper-and-pencil tasks like GeoGebra homework assignments. Cem evaluated students' learning through GeoGebra activities, provided feedback on their ideas and predictions regarding geometric concepts within these activities, conducted WordWall Quiz Show sessions for group work, and assessed students' geometric constructions and justifications within GeoGebra homework assignments.

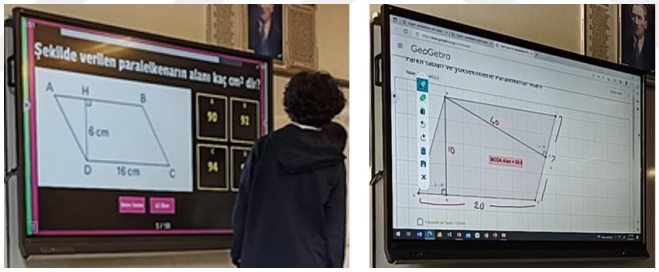


Figure 52. Different technology-based assessment items Cem used in his MCC 4 lesson

However, Cem needed to adjust assessment practices to thoroughly evaluate students' conceptual understanding of the subject matter in a manner necessitating the comprehensive and complete use of technology. He did not conduct assessment activities to capture students' comprehension of the mathematical principles embedded in specific technologies. This limitation hindered Cem from advancing by one level in *the assessment descriptor* within the TPACK development model compared to his level in MCC 3 which focused on the learning objective '*the area relation of a triangle and solves related problems*'. Cem elaborated on this issue during the pre-conference as follows:

> “It would be very effective to teach the subject in a way that requires the full use of technology. However, how I integrate technology into the assessment and evaluation process is limited by the current conditions of the school and classroom, technological infrastructure, and facilities. For example, I could teach much more effectively if every student had a tablet or a computer in front of them. Alternatively, I could teach completely through technology if my teaching lesson could be in a computer lab. I could use dynamic worksheets by sending various tasks to the students' screens or using both paper and pencil and dynamic worksheets in which they could make various manipulations in a dynamic learning environment and note the changes they observe. In this way, it would be possible for me to give instant feedback to students on correct and incorrect learning, systematic errors, and so on. That is why the home tasks we develop in GeoGebra are a great teaching opportunity as they involve the full use of technology and can be done easily from their computers.”

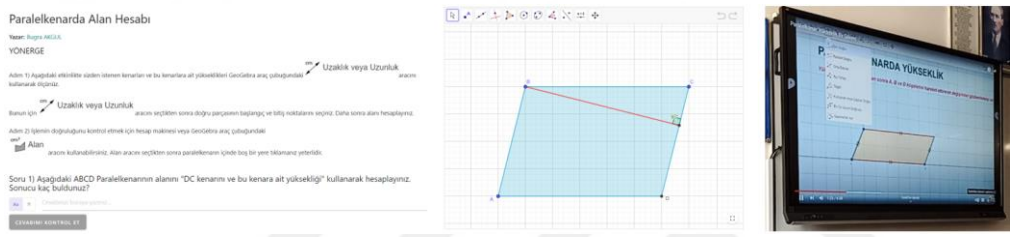


Figure 53. Screenshots regarding the GeoGebra homework assignment from the MCC 4 lesson

When Cem was asked to what extent and for what purpose he used technology to assess students' learning in the post-conference 4, he gave the following answer;

> “I integrated various technological tools such as GeoGebra, GeoGebra homework, WordWall Quiz Show, and Khan Academy into assessment practices and encouraged students to explore during the lesson. I assessed students' learning far beyond paper and pencil work. For example, even when the student uses a GeoGebra activity, he gives me many clues about his learning. Similarly, I assessed the student's learning in the lesson and provided real-time feedback. I did this by using many digital tools.”

Cem’s statements from the pre-conference and actual teaching practice of MCC 4 aligned with the indicator of: “Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).” (Niess, Ronau, et al., 2009, p. 21). Therefore, all these could provide evidence to place him at *the exploring level* of TPACK for *the assessment descriptor* of the Curriculum and Assessment Theme of the TPACK Development

Model. Niess, Ronau, et al. (2009) asserted that teachers at *the exploring level* for *Assessment descriptor* actively explore various forms of technology-based assessment items and questions (e. g., active, inactive, neutral, or passive). At this level, teachers develop tests and assignments where students must display their comprehension of the mathematical concepts using technology tools that are more advanced than just pen and paper. The following were the characteristics of Cem's practices in MCC 4: He also strived to incorporate different technological tools in assessing students' comprehension of mathematical concepts concerning parallelograms. Cem used GeoGebra activities and worksheets to gauge the student's comprehension of the height of a parallelogram. He also constantly gave feedback and asked conceptual questions to ensure students understood the topics. Cem employed GeoGebra activities to investigate the area relationship in parallelograms and gave the students a GeoGebra assignment to solve. He also used exit cards as a form of assessment at the end of the lesson to determine the extent of learning. Cem employed WordWall Quiz Show and Khan Academy activities to check students' knowledge of the area of the parallelogram and the height. He allowed students to share their knowledge with others and solve problems in groups, and the assessments were not limited to tests. Moreover, in *the assessment descriptor* in MCC 4, it was evident that there was a positive change in Cem's TPACK, which boosted TPK which entails the knowledge of how and in what manner teaching and learning changes when specific technologies are employed (Mishra & Koehler, 2006). Cem successfully used GeoGebra, GeoGebra home assignment, WordWall Quiz Show, and Khan Academy in his assessment activities and allowed students to explore, thus showcasing a better understanding of how technology can be of value in teaching and learning. This progression shows that he has developed the skills of integrating technological advancement into teaching to make it more efficient and informative.

In the pre-conference of the MCC 4, the coach and the mathematics coaching process contributed to Cem's improvement in *the assessment descriptor* of the Curriculum and Assessment Theme. This was evident as compared to his performance in MCC 3. In the pre-conference of MCC 4, the coach noted that it would be helpful to introduce the participants to various kinds of technology-based assessment items and questions. The coach also urged him to shift from the conventional evaluation

processes and incorporate more engaging and active techniques to evaluate the students' comprehension of mathematical concepts. This guidance made Cem realize the possibility of using technology to improve and enhance the assessment process to provide a broader, more interactive environment. The coach then introduced Cem to the spreadsheet view of GeoGebra and how it assists students in finding out mathematical contents. Thus, students could see and manipulate the formula for the area of a parallelogram in GeoGebra's spreadsheet view. This tool helped the students change different parameters and watch the results, which is why it helped develop conceptual understanding. Before the conference, the coach gave Cem practical GeoGebra exercises in the spreadsheet view and worked with him one-on-one. Some of them were illustrations and ideas that he used to incorporate this tool in his classes. The coach also provided feedback on Cem's lesson plans to improve their assessment by including more technological elements. Indeed, in harmony with the coach's input, Cem incorporated GeoGebra Spreadsheet View, WordWall Quiz Show, and Khan Academy activities, which a marked departure from conventional and multiple-choice assessment methods can characterize. This variety allowed a better assessment of the students' factual and conceptual knowledge and problem-solving skills. Cem was able to measure students' learning through interactive tools that gave instant feedback to students. This made it easier for him to correct wrong notions and cement right notions in the students' minds. It helped students to discover concepts practically because of the integration of technology. GeoGebra, which Cem used, enabled students to drag shapes and observe the results, allowing them to better understand parallelograms.

4.2.1.3. TPACK Level of Cem Regarding Curriculum and Assessment Theme After Mathematics Coaching

To determine Cem's final TPACK level regarding the Curriculum and Assessment Theme, his teaching practice after he participated in four coaching cycles of mathematics coaching which focused on the learning objective '*Problem solving related to the areas of compound shapes consisting of triangles, rectangles and parallelograms*,' were observed for three hours. To determine the final TPACK level, no pre-conference was held with Cem before the observed lesson, and no post-

conference was held after the lesson. The researcher did not provide support for mathematics coaching to Cem. Cem prepared the lessons independently by reflecting on the knowledge and experiences gained in the four coaching cycles he participated in.

Cem's actual teaching practice after mathematics coaching demonstrated *the advancing level* of TPACK for *the curriculum descriptor* while *the exploring level* of TPACK for *the assessment descriptor* of the Curriculum and Assessment Theme according to the TPACK Development Model.

Cem initiated the first session of the 3-hour class focused on problem-solving and area comprehension by engaging students in an inquiry into the methodologies for calculating the area of rectangles, squares, triangles, and parallelograms. This approach assessed the students' foundational understanding, vital for tackling subsequent area measurement challenges. Subsequently, he introduced three distinct Geoboard online interactive activities, prompting students to undertake basic area calculation exercises. Following this preparatory activity, Cem introduced an exercise involving calculating areas of irregular geometric shapes using GeoGebra. Students employed various methods, including counting unit squares within the shape or decomposing it into familiar components like triangles and rectangles, facilitating area calculation. Cem provided feedback on students' solutions and manipulated shape corners using GeoGebra's dynamic features to generate diverse irregular geometric shapes, encouraging broader student participation. Following this, students were tasked with verifying their calculated areas against those determined instantaneously using GeoGebra's area measurement tool. Cem facilitated similar activities, guiding students through the process and leveraging GeoGebra's dynamic features to introduce novel irregular geometric shapes, fostering diverse problem-solving strategies among students. As students grappled with problem-solving, they engaged in discussions regarding solution strategies. Cem solved a problem by himself and ended the lesson.

In the second lesson of the MCC 4 teaching practice, Cem began with a skill-based problem so that the students could suggest possible solutions. After a try by one

student, Cem used GeoGebra to display another version of the problem, and thus, another student could provide another approach. Eventually, Cem used GeoGebra to explain the details of solving the given problem due to the program's interactivity. Then, Cem and his students continued working with the transformation that folds a rectangle at the corner and uses GeoGebra to represent shapes graphically. Eight students solved geometric problems on GeoGebra to enhance their understanding of the subject. Cem then posed a problem to the students regarding how they could use the prior activity in the present context. Cem then presented another real-life problem to the students and asked them to present as many solutions as possible; after listening to everyone's solution and suggestions, Cem presented another problem. He then translated this problem into GeoGebra for the dynamic aspects of GeoGebra. He had students use the tools of GeoGebra to solve the problem and explain the solution in detail. It was the end of the lesson, and Cem gave area problems from different pages of the textbooks as homework. In the third lesson of the MCC 4 teaching practice, Cem brought practical problems that apply the textbook's area relations of triangles, squares, rectangles, and parallelograms into the class. It was expected that students would work on these problems and that they would exchange solutions by tutoring one another. Cem offered help to the students struggling to solve their problems. Cem then gave the students GeoGebra area problems to solve as homework for the day.

Concerning *the curriculum descriptor* of curriculum and assessment theme, the approach Cem adopted in the mathematics coaching regarding the curriculum and the way he transferred this approach into his teaching practice after the mathematics coaching, the way he transformed the curriculum into his classroom teaching, the integration of technology into the curriculum and the alteration and improvement of the activities within the curriculum were quite alike in his MCC 4 teaching practice. Cem improved the curriculum by integrating GeoGebra as one of the most useful resources for teachers and students. He also enhanced the students' problem-solving skills by applying GeoGebra in parallelograms and triangles to solve real-life problems. He used GeoGebra as a tool for problem-solving. Cem adapted the existing curriculum activities, and the real problems recommended by the Ministry of National Education and transformed them into more suitable for GeoGebra. In

addition, he came up with new teaching methods based on the current syllabus/lessons and incorporated GeoGebra to enhance the students' understanding. Another of Cem's significant changes concerned turning issues into active learning tasks; he encouraged the generation of various approaches and solutions and, consequently, the improvement of students' problem-solving abilities.



Figure 54. Interactive activities regarding the area of irregular shapes from post-MC lesson

It was observed that students solved problems traditionally using paper and pen and GeoGebra's tools like isometric paper, gridded paper, measuring area and length tools, and methods of constructing height on various sides. They also described the results and solutions they found to each other. Moreover, Cem and his students worked with areas of irregular polygons on GeoGebra worksheets; students could easily measure and move the vertices of geometric shapes to create irregular polygons and subdivide them into different geometric shapes to determine the total area. This allowed the students to work through many different situations and shapes within a fluid environment, which is more than what could have been achieved in a conventional static environment and thus enhances understanding.

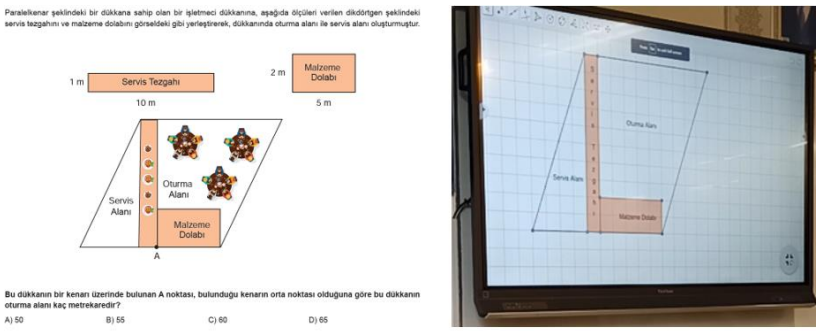


Figure 55. Problem 1 from the post-mathematics coaching lesson and its adaptation to GeoGebra

When students used GeoGebra to solve problems, besides checking the correctness of their answers, they also used the program's functions to help solve complex problems. It is important to note that, besides improving problem-solving skills, students were also introduced to different ways of solving problems. Moreover, besides adapting curriculum problems to suit GeoGebra, Cem provided students with opportunities to tackle problems related to the lesson content by researching and integrating activities aligned with the curriculum's objectives. For instance, he introduced a GeoGebra activity focusing on the geometric shapes formed by folding a rectangle from its corners in specific ways. This approach empowered students to apply the knowledge from the activity to solve relevant problems, reinforcing their understanding of the subject matter and fostering more profound engagement with the curriculum.

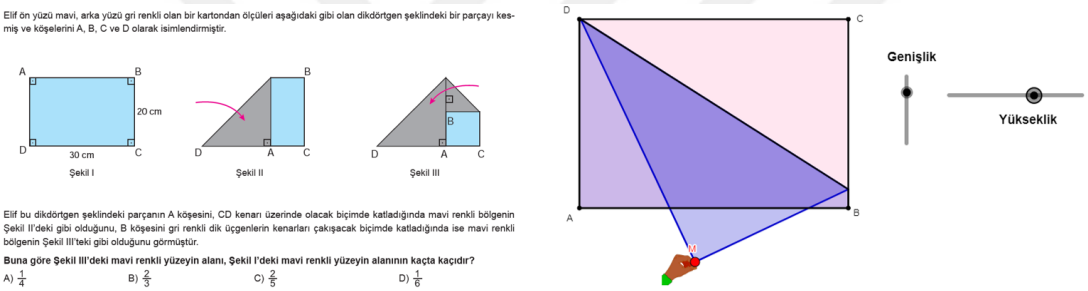


Figure 56. Rectangle folding activity in GeoGebra and a related problem from the curriculum

Cem's actual teaching practice after mathematics coaching aligned with the indicator of: "Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential." (Niess, Ronau, et al., 2009, p. 21). Therefore, all these could provide evidence to place him at *the advancing level* of TPACK for *the curriculum descriptor* of the Curriculum and Assessment Theme of the TPACK development model. Niess, Ronau, et al. (2009) pointed out that teachers at *the advancing level* of curriculum development appreciate the fact that continuous enhancement of the curriculum is necessary to enable them to effectively and efficiently integrate technology to teach and learn. They create new strategies for incorporating technology into the teaching and learning process of mathematics and frequently

update and enrich the curriculum regarding the application of technology as a powerful tool for teaching and learning. Cem's teaching practice after mathematics coaching was characterized by the following aspects of *the advancing level* of *the curriculum descriptor*. He was fully aware of the need to maintain effective technology integration into the curriculum. He came up with ways to embrace technologies that would foster the development of students' mathematical thinking. Cem employed GeoGebra to demonstrate to students how to solve the area of irregular geometrical shapes. He used shape corners to create different shapes, allowing the students to use various methods to solve the problem. This approach depicted the concept of sustained innovation and the development of mathematical thinking using technology. Cem introduced geometric shapes made by folding a rectangle with the help of GeoGebra and showed the students the transformations. Cem also integrated GeoGebra into his classes, improving his students' understanding of geometric concepts. Cem described specific situations relevant to area relations and got students to work in pairs to explain their solutions. This practice demonstrated his commitment to improving his curriculum by incorporating technology to support and encourage students' critical thinking skills.

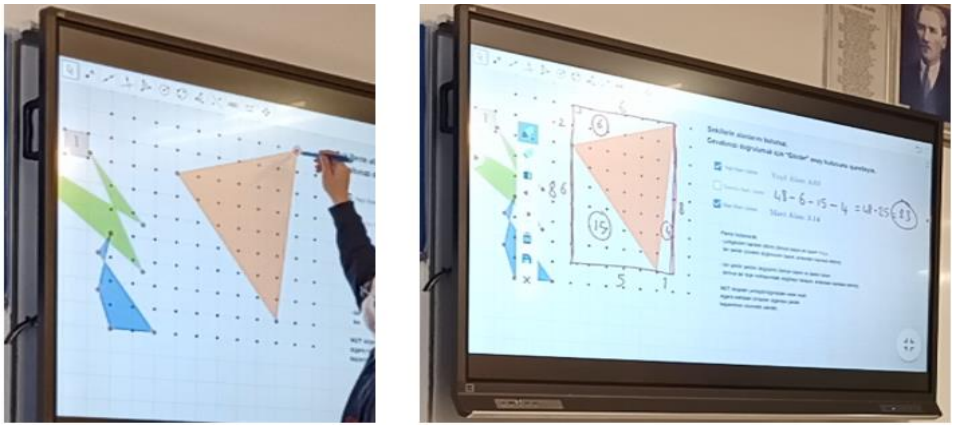


Figure 57. Technological assessment item from post-mathematics coaching lesson

In examining Cem's TPACK level in terms of *the assessment descriptor* within the curriculum and assessment theme following mathematics coaching, it becomes evident that his approach to assessing students' learning, as outlined in the TPACK Development model, closely mirrors that of MCC 4. After mathematics coaching, an analysis of his teaching practice and lesson plans reveals Cem's proactive exploration

of diverse technology-based assessment items and questions. Consequently, Cem crafted and implemented assessments beyond conventional paper-and-pencil exercises, leveraging GeoGebra as a suitable technological resource. Through these assessments, students demonstrated their comprehension of geometric concepts, elevating their problem-solving skills and technological proficiency. Cem's evaluation of students' learning extended to activities involving calculating irregular shape areas using GeoGebra and virtual Geoboard, accompanied by constructive feedback on students' utilization of technology and their conceptual understanding of geometry.

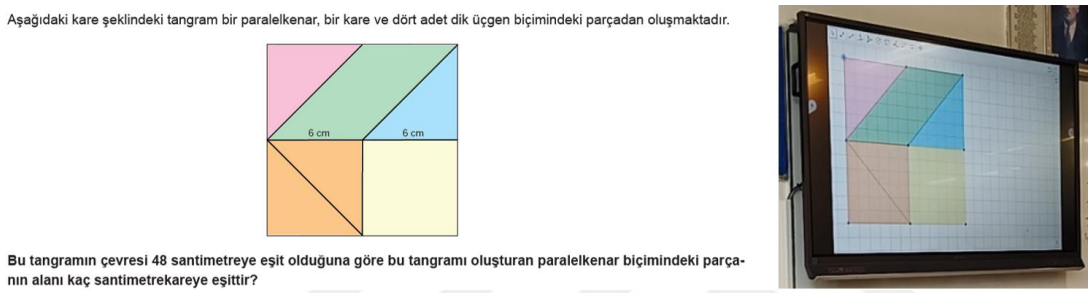


Figure 58. Problem 2 from the post-mathematics coaching lesson and its adaptation to GeoGebra

Furthermore, assessing student learning involved immersing them in real-world problem-solving contexts and critically evaluating their responses within GeoGebra homework assignments and the rationale behind their solutions. The inherently dynamic features of GeoGebra, coupled with its array of embedded tools, provided Cem with ample opportunities to assess student learning comprehensively within a dynamic learning environment, where problems could be approached from multiple angles, and solutions could vary considerably.

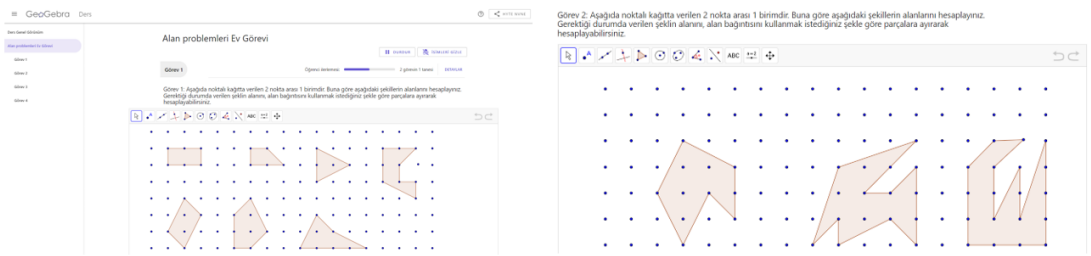


Figure 59. Sample assessment items from the GeoGebra home assignment on area problems

Cem's actual teaching practice after mathematics coaching aligned with the indicator of: "Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive)." (Niess, Ronau, et al., 2009, p. 21). Therefore, all these provided evidence to place him at *the exploring level* of TPACK for *the assessment descriptor* of the Curriculum and Assessment Theme. Niess, Ronau, et al. (2009) identified that teachers at *the exploring level* for *the assessment descriptor* examine the various types of technology-based assessment items and questions (e.g., technology active, inactive, neutral, or passive). They create tests in which students have to exhibit their comprehension of the mathematical concepts by applying the correct tool, which applies to produce written work. Cem's teaching practice after mathematics coaching was analyzed, and the teaching practice showed features of *the exploring level* for *the assessment descriptor*. He also researched different forms of technology-integrated assessments and created assessments that allowed students to use technology to exhibit their knowledge. Cem challenged students to compare the areas of irregular shapes they calculated to the areas obtained using GeoGebra's area measuring tool. This practice made the students use technology in assessment; this was not just a simple checklist of multiple-choice questions, as seen in most cases where the assessment is done on paper. Cem employed GeoGebra to model and justify problems so that the students could come up with solutions and check the validity of the solutions. In this manner, he integrated technology to create assessments built on and beyond students' knowledge through active and investigatory means. Cem gave GeoGebra area problems to be done for the homework so that the students could solve problems and show what they had learned independently. This approach was aligned with his research interest in technology-affiliated assessment techniques and his attempt to implement them in his classroom. However, Similar to MCC 4, Cem did not arrange his assessment practices to thoroughly evaluate students' conceptual understanding of the subject matter in a manner that required the comprehensive and complete use of technology. Essentially, he did not conduct assessment activities to capture students' understanding of the mathematical principles embedded in specific technologies. This limitation prevented Cem from advancing one level in *the assessment descriptor* within the TPACK development model compared to his level in MCC 4.

4.2.1.4. The Progress of Cem in TPACK Regarding Curriculum and Assessment Theme Through Mathematics Coaching in the Learning Area of 'Measurement of Area'

The figure 60 presents Cem's TPACK levels for *curriculum* and *assessment descriptor*s of Curriculum and Assessment Theme based on his teaching practices in technology-enriched lessons during and following mathematics coaching. According to the TPACK development model, Cem progressed in *the curriculum* and *assessment descriptor*s throughout the mathematics coaching cycles. The Measurement of the Area learning domain was analyzed within MCC 3, MCC 4, and post-mathematics coaching teaching practices. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

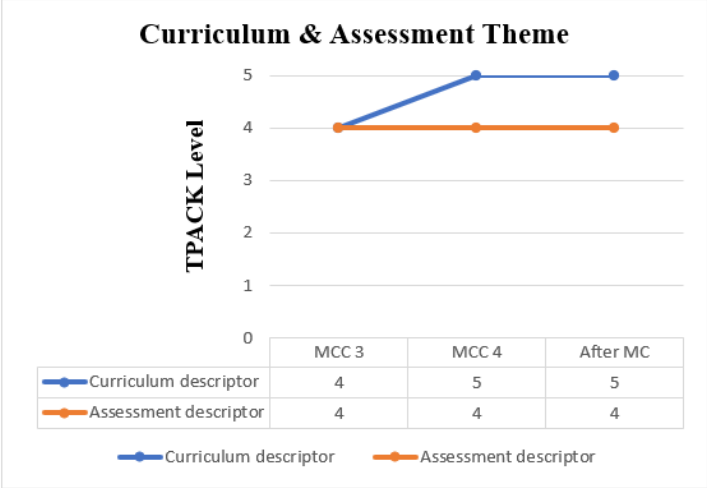


Figure 60. The progress of Cem of the Curriculum and Assessment Theme in the learning area of Measurement of Area

The curriculum descriptor pertains to understanding curriculum content and materials that incorporate technology into mathematics teaching and learning. Cem's level in *the curriculum descriptor* of the Curriculum and Assessment Theme at MCC 3 indicated *the exploring level* (level 4). In his instructional approach at MCC 3, Cem explored the integration of technology as a fundamental learning aid within his curriculum. He actively pursued innovative ideas and strategies to integrate

technology into a central role in students' conceptual understanding of mathematics. As a result, he restructured his previous mathematics lessons to integrate GeoGebra into exploring topics like the height in triangles and the area formula of triangles. Developing his unique methods, Cem harnessed technology to enhance the existing curriculum, adapting and creating activities that closely aligned with the current learning objectives on triangle properties such as height and base and area calculations of triangles.

He progressed in *the curriculum descriptor* throughout his involvement in the mathematics coaching cycles. Cem's level in *the curriculum descriptor* of the Curriculum and Assessment Theme at MCC 4 and after mathematics coaching indicated *the advancing level* (level 5). Cem recognized the ongoing need for innovation in adapting his curriculum to incorporate technology as a teaching and learning aid effectively. He crafted inventive strategies to leverage technology in cultivating students' understanding of mathematical concepts, particularly in exploring topics like the height in parallelograms and the area formula of parallelograms and solving area measurement problems. Utilizing technological tools such as GeoGebra, Cem facilitated dynamic investigations into parallelogram properties, including the relationship between height and area. By harnessing GeoGebra's dynamic features, students were empowered to manipulate parallelogram components, visualize geometric relationships, and solve real-world area measurement problems with greater clarity and insight. Moreover, Cem modified the curriculum to ensure that it aligns with all GeoGebra's features, thus enabling the students to learn interactively and thus improve their knowledge of parallelograms and their uses.

The assessment descriptor is concerned with the student's knowledge acquisition concerning technology. Cem's level in the TPACK Development model's *curriculum descriptor* at MCC 3, MCC 4, and after mathematics coaching, showed that he was at *the exploring level* (level 4). Cem also tried using all the available ICT tools and techniques for assessment and different questions. He developed assessments using technology in which students had to apply the concepts they had learned previously, thereby going beyond writing on paper. Some of these

assessments were GeoGebra homework, which contained open-ended questions, construction, multiple-choice questions, and explanations of conjectures and ideas in geometry. Cem used various techniques to assess the students' learning and support their critical thinking skills in MCC 3, MCC 4, and post-mathematics coaching. Regarding the assessment, the student's knowledge of the height and area of triangles and parallelograms and their problem-solving skills with the help of GeoGebra activities were evaluated. Nevertheless, Cem failed to incorporate reflective practices to modify assessment strategies to assess the student's understanding of the concepts and skills, especially those that entirely use technology. Therefore, he could not progress to the higher level of *the assessment descriptor* due to this constraint.

4.2.2. The Learning Theme

The learning theme contains two descriptors: Mathematics learning and the conception of students' thinking. The descriptor of mathematics learning examines whether the teachers use technology as a possible aid for teaching mathematics. The conception of students' thinking deals with how students' thinking skills are nurtured with technology. In the following sections, Cem's TPACK is examined regarding the learning theme of the TPACK Development Model by using *mathematics learning descriptor*s and *the conception of student thinking descriptor*s.

4.2.2.1. TPACK Level of Cem Regarding Learning Theme in the MCC 3

To assess Cem's TPACK level regarding the Learning Theme in the third mathematics coaching cycle, which focused on the learning objective '*Establishes the area relation of a triangle and solves related problems,*' his teaching practices were observed for three hours. The pre-conferences, post-conferences, and mathematics lessons done within the Mathematics Coaching Cycle 3 (MCC 3) provided the data for all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in the MCC 3 during mathematics coaching showed that he was at *the exploring level* of TPACK both for mathematics learning and *the conception of student thinking descriptor*s of the Learning Theme.

In MCC 2, Cem ended the learning area of ‘Angles’ and proceeded to teach the learning area of ‘Measurement of Area’ in MCC 3. Observation of Cem’s actual teaching in the classroom MCC 3 revealed that he could naturally apply the knowledge from the previous coaching cycles into his teaching of ‘Measurement of Area’, a learning domain that includes many topics. After the coaching sessions and pre- and post-conferences on technology integration in teaching topics within the learning area of angles, Cem effectively used the technology to support and enrich the students’ understanding of how to construct the height and calculate the triangle area. This proved his competence in transferring knowledge and coming up with new techniques in the teaching process for the students to grasp.

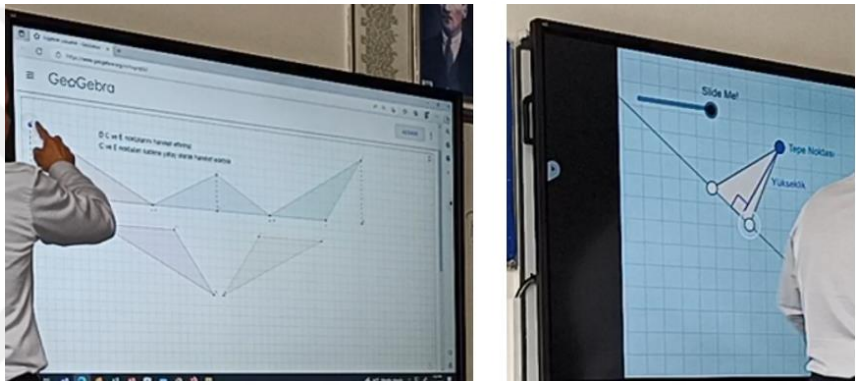


Figure 61. Cem while using the activities of height in a triangle on GeoGebra in MCC 3

Cem taught the steps to find the height of a triangle through GeoGebra. He also demonstrated to the students how to make a line from a vertex to the other side of the triangle, which is perpendicular to the base of the triangle, and said that this line represents the triangle's height. Cem helped the students discover the area relation of triangles with the help of the GeoGebra application. He provided various triangles for students to measure the base and height and then calculate the area using the formula. Students were encouraged to compare areas and find patterns. Cem also employed interactive activities where students manipulated triangles and their heights in GeoGebra to observe the effects on the area dynamically. By incorporating GeoGebra into the teaching of height and area of triangles, Cem engaged students in hands-on exploration, promoted a deeper understanding of geometric concepts, and provided opportunities for active learning and discovery.

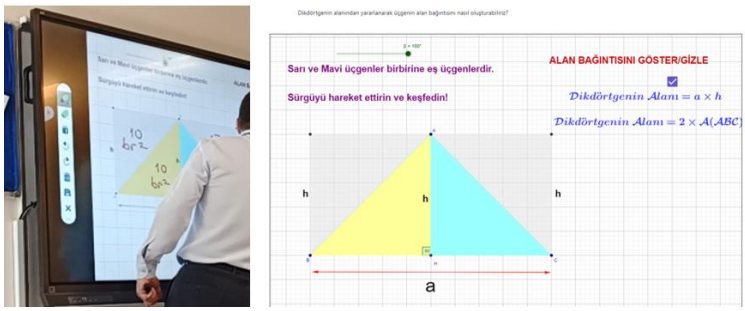


Figure 62. Cem associating the area of a triangle with the area of a rectangle in MCC 3

Cem employed various technologies to support the learning process across specific areas within the mathematics curriculum. Under his guidance, students engaged in an array of topics spanning from adjacent angles to constructing the height of a triangle and calculating triangle areas based on their relations. Through integrating technology, Cem facilitated an interactive and exploratory learning environment for his students, enabling them to delve into diverse mathematical concepts with greater depth and understanding. Students delved into various mathematics topics, constructing the height of a triangle, discovering the area relation of a triangle, and calculating the areas of various triangles using the area relation of a triangle).



Figure 63. A student exploring the area relation of a triangle on GeoGebra in MCC 3

Additionally, when asked during the MCC 3 pre-conference about the rationale behind using GeoGebra activities and other technological resources, Cem responded:

> "While instructing angles, I gained extensive insights into integrating technology into teaching. Through collaborative discussions, we explored various aspects together. I acquired knowledge regarding the suitability of different technologies for specific topics and the alignment of teaching methods with chosen technologies. Implementing these insights during the

> initial two coaching cycles allowed me to witness firsthand the efficacy of integrating technology into the classroom. I planned to utilize technology as an instructional aid, fostering students' exploration of concepts."

During the third pre-conference, the coach and Cem had planning sessions where they prepared for the lessons to be delivered. The coach demonstrated how to incorporate technology into the lessons and engage the students with GeoGebra and other tools to discover the area relations of triangles, for example. The coach focused on the student-centered approach and helped Cem develop activities focusing on students' needs. This entailed developing ways through which the students could discover mathematical concepts independently using technology in the process. The coach offered feedback and a general conversation with Cem regarding his teaching practices and technology integration. Cem could use these discussions to ponder the efficiency of his approaches and modify the strategies when needed to enhance the student's achievement. Cem's coach urged him to search for and test various technological tools that could be embedded into the teaching-learning process. The activities in this study also allowed Cem to become acclimated to and more competent with technology integration for mathematical instruction. As supported and suggested by the coach, Cem reviewed the previous topics and tested the students' knowledge through questions. The coach's questioning skills allowed Cem to check the students' knowledge and set a foundation for the technological activity. Thanks to the coach's performance of how GeoGebra can be used to reveal triangle heights, Cem supported student work. GeoGebra was used to assist the students in altering the position of the triangle heights, which made them better grasp the concept. The trainer stressed the concept of applying mathematical information in real-life situations. Cem ensured that he provided real-life examples of how the area of a triangle could be used in day-to-day life; this made the lessons more interesting and accessible for the students to comprehend. The coach's recommendations on employing GeoGebra to explain the connection between rectangle and triangle areas helped Cem design purposeful activities. The students manipulated the shape's height or base length and saw how the area was influenced, thus enhancing their learning with the help of technology. The coach's feedback on how to use technology in assessment strategies assisted Cem in developing good assessments. Cem employed GeoGebra and Khan Academy activities to check the students' understanding, gave

frequent feedback, and corrected misconceptions. In brief, it can be concluded that the pre-conference discussions, suggestions, examples, and lesson plans contributed to Cem's improvement in *the mathematics learning descriptor* of the Learning Theme. In other words, the coach could reflect his mathematical, pedagogical, and educative power with his beliefs about mathematics and mathematics teaching and learning to Cem.

When Cem was asked in post-conference 3 to what extent the use of technology in the lesson contributed to students' conceptual learning, he responded as follows;

> "Throughout the whole lesson, I observed that key learning about the height of a triangle and the area calculation of a triangle took place when the students used and analyzed GeoGebra activities, experimented with their mathematical ideas, and listened to their conclusions."

Cem's alignment with the indicator of *the exploring level* in the Learning Theme "Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum." (Niess, Ronau, et al., 2009, p. 21) further demonstrated his development in TPACK. Specifically, Cem's statements in pre-conference 3 and his teaching practice in MCC 3 showcased his engagement with technology-enhanced assessment strategies. This evidence placed him at *the exploring level* of TPACK for *the mathematics learning descriptor* of the Learning Theme. Niess, Ronau, et al. (2009) asserted that mathematics teachers at *the exploring level* apply technology to enhance particular topics in the mathematics curriculum. These teachers give the students a chance to learn many things concerning the topic at hand and, at times, even other topics that are not related to the current one. For example, during the implementation of MCC 3, Cem used technology to support learning specific mathematics topics and encouraged the students to solve related mathematical problems using the available technology tools. He employed GeoGebra to help the students explore the heights of various triangles and change the shapes to get a better feel for the concept. This activity allowed the students to move beyond identifying facts and use technology to investigate the topic. Moreover, Cem used GeoGebra activities to extend the student's understanding of the area concerning rectangles and triangles. Thus, by manipulating the height or base length with the help of the GeoGebra spreadsheet view, students' understanding of the topic was enhanced

through experimentation. Cem also used GeoGebra and Khan Academy activities to extend the assessment of the student's learning achievement as they solved different methods of finding the area of a triangle, thus increasing students' engagement through technology. Moreover, comparing Cem's TPACK level in MCC 2 and MCC 3 revealed his progression from *the adapting level* (level 3) to *the exploring level* (level 4) regarding *the mathematics learning descriptor*. The improvement in Cem's learning descriptor from *the adapting level* to *the exploring level* in *the mathematics learning descriptor* shows that he has developed his Technological Pedagogical Knowledge (TPK). This improvement is evident in his use of technology in teaching and learning processes to enhance the student's learning process, showing better and more efficient use of technology in teaching and learning.

As for *the conception of student thinking descriptor*, in the actual teaching practice of MCC 3, Cem's dedication to aiding students in grasping mathematical concepts was evident through his careful planning, execution, and reflective methods. He strategically integrated technology-enhanced activities that complemented the curriculum and promoted meaningful student involvement. Cem introduced technological tools and assessed their impact on students' mathematical learning outcomes and attitudes toward the subject. Through actively managing these activities, he aimed to guide and boost student engagement, encouraging self-directed learning. During the MCC 3 teaching sessions, it was observed that Cem's students actively participated in GeoGebra activities and displayed a high level of enjoyment in the learning process. Their enthusiasm and involvement were particularly notable, indicating that Cem's instructional approach effectively created a positive and engaging learning environment consistent with improving students' mathematical thinking skills and attitudes.

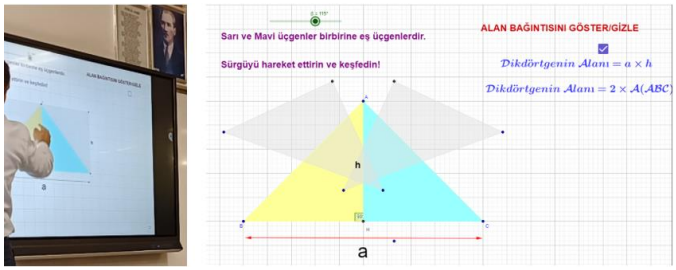


Figure 64. Cem while explaining the relationship between the area of a rectangle and a triangle in GeoGebra in the MCC 3 lesson

Cem's teaching style demonstrated a deep understanding of student's comprehension levels and grasp of mathematics, enabling him to customize and modify his teaching to address specific areas of difficulty. When faced with student questions, he skillfully probed to identify the precise direction confusing. For example, when a student was puzzled about why the result of base multiplied by height in the area relation of a triangle was divided by two, Cem first constructed a rectangle in GeoGebra and asked the student about the area relation of the rectangle. Upon receiving the answer "We multiply the base and height" from the student, Cem further questioned, "What happens when we draw a diagonal of this rectangle?" The student responded that two identical or congruent triangles are formed. Cem then prompted the student, "If half of the area of the rectangle is one of the congruent triangles, would not half of the area of the rectangle be equal to the area of the triangle?". Consequently, the student clarified the confusion and correctly applied the clarified concept in another example. This teaching approach underscored Cem's ability to diagnose learning difficulties, provide specific assistance, and encourage meaningful student reflection.



Figure 65. Cem while demonstrating how the area of a triangle changes with height in GeoGebra in the MCC 3 lesson

In pre-conference 3, when asked about the role of technology in students' learning for the planned lesson, Cem responded as follows:

> "Technology has a mediating role in the student's concept exploration. Therefore, I have planned to use GeoGebra activities, online quizzes, and Khan Academy studies as teaching and learning tools. Here, as a teacher, I will act as the exploration director. I plan to do this by asking questions, explaining, and providing feedback to students while I and my students use the technology. I believe this will deepen students' understanding of the concept. To summarize, the main role of GeoGebra and other technological

tools and activities will be as a tool that allows students to explore the concept and contributes to their conceptual learning."

Despite several positive aspects of Cem's teaching approach, there were notable areas where improvement could be made. One such area is the integration of technology, which appeared more as an add-on rather than an integral part of the mathematics curriculum. This limited integration may have hindered opportunities for deeper exploration and understanding of mathematical concepts through technology. Furthermore, there was a missed opportunity to engage students in higher-order thinking activities like project-based learning and complex problem-solving, which could have enhanced their problem-solving and critical thinking abilities. Despite the coach's suggestions during the pre-conference 3 sessions to engage students in problem-solving and higher-order thinking activities and integrate problem situations into the lesson, Cem preferred technology-supported activities to deepen students' understanding and support their conceptual learning. In the actual teaching practice of MCC 3, technology was not effectively utilized to promote advanced comprehension of mathematical concepts among students. Leveraging technology to its full potential could significantly enhance students' mathematical understanding and problem-solving capabilities. All these shortcomings prevented Cem from progressing at the TPACK level in *the conception of student thinking descriptor*.

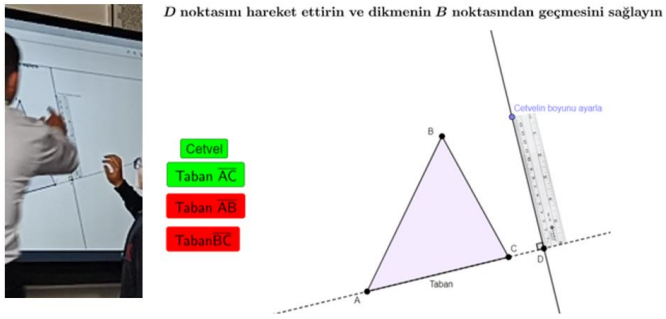


Figure 66. Cem while guiding a student during technology use regarding height in a triangle

When Cem was asked whether he used technology to support students' mathematical thinking in the lesson in post-conference 3, he responded as follows;

"While the students learned the area relation of a triangle through GeoGebra activities, they established the relationship between the area relation of a

> triangle and the area relation of a rectangle. This supported their mathematical thinking. This made it very easy for the students to learn the area of the triangle."

Cem's statements during the pre-conference of MCC 3 were consistent with his actual teaching practice at MCC 3 and his teaching practice aligned with the indicator of *the exploring level*: "Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding." (Niess, Ronau, et al., 2009, p. 21). Therefore, all these served as evidence to position him at *the exploring level* of TPACK for *the conception of student thinking descriptor* of the Learning Theme. Niess, Ronau, et al. (2009) explained that teachers at *the exploring level* for *the conception of student thinking descriptor* plan, implement, and reflect on teaching and learning to facilitate students' understanding. They deliver and assess technological interventions concerning the student's mathematics learning and their perceptions of the subject. These teachers coordinate technology-integrated activities to channel the learning process and encourage the students to be more self-reliant in mathematics. Specifically, the characteristics described above were evident in Cem's practice during MCC 3. He strategically organized and integrated technological tools into classroom activities and learning experiences, thus monitoring how these affected the students' achievement and expectations towards mathematics. Thus, by integrating GeoGebra activities to discover triangle heights, Cem ensured students correctly explained the concepts by asking questions and giving feedback. He assisted students in enhancing their knowledge of the heights of triangles by using technology. Also, Cem introduced GeoGebra activities to the students and guided them in finding triangles and rectangles and making them express their thoughts. He also employed GeoGebra and Khan Academy activities to check the students' comprehension, gave consistent feedback, and guided the students to engage and manage their learning in finding areas of triangles.

In pre-conference 3, the coach focused on working with Cem to develop lessons that would positively incorporate technology so that students would be led to the development of meaning-making skills. They also discussed the need to continually assess the student's level of understanding and manipulate the teaching procedures. The coach and Cem devised questions and activities that the students could ponder,

which would help them think deeply about the concepts being taught. For instance, they discussed how they would employ GeoGebra to show triangle heights and area relationships graphically; they made sure that these tools would be a means to improve the student's understanding. The coach also ensured that every lesson was followed by evaluating the processes that had been effective and the ones that needed to be adjusted. This involved thinking about the remarks made by the students, their participation, and their comprehension of the given topic. It was recommended that Cem employ technology activities and measure their impact by taking note of the students' reactions and attitudes toward the specific mathematics concepts being taught. This reflective practice assisted Cem in developing an awareness of the student's thinking process, thus enabling him to improve his teaching techniques. Thus, the coach offered tactics to ensure students stay engaged during technology-integrated activities and become self-directed students. This entailed outlining the expectations, giving the students a clear roadmap of how things would be done, and giving them opportunities to investigate concepts by themselves with the help of technology.

The coach also explained using GeoGebra and other technological tools to design activities that would be interesting for the students to solve and comprehend mathematics concepts. Cem was taught how to organize these activities to engage the students and help them concentrate on the mathematical aspects of the technology. The coach advised Cem to question and actively listen to the students during technology-integrated lessons. This approach enabled Cem to see the students' way of thinking and consequently allowed for correcting misconceptions and building deeper thinking. The coach proposed including the students' feedback in the further classes, thus making the learning process more student-oriented. Cem was informed on how to alter his instruction according to student feedback to make learning more receptive and efficient. Thus, Cem implemented these strategies and utilized GeoGebra in teaching triangle height, questioned students to determine their comprehension, and corrected them. He used the coach's strategies for handling technology-integrated activities and pondered the student's participation and comprehension. Cem guided GeoGebra activities involving the relation between a triangle and a rectangle area and asked the students to describe their ideas and apply

them to various situations. He executed the planned activities and assessed the appropriateness of the strategies based on the students' feedback. Cem employed GeoGebra, Khan Academy, and WordWall to check the students' level of understanding and, at the same time, improve it. These activities were coordinated to promote the active participation and independence of the students while assessing their effectiveness in changing student's attitudes.

4.2.2.2. TPACK Level of Cem Regarding Learning Theme in the MCC 4

To determine Cem's TPACK level regarding the Learning Theme at the fourth mathematics coaching cycle, which focused on the learning objective '*Establishes the area relation of a parallelogram and solves related problems,'* his classroom teaching practices were observed for three hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 4 (MCC 4) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in the MCC 4 during mathematics coaching indicated *the advancing level* of TPACK for *the mathematics learning descriptor* and *the exploring level* of TPACK for *the conception of student thinking descriptor* of the Learning Theme.

Cem's actual teaching practice in MCC 4 was more advanced regarding *the mathematics learning descriptor* of Learning Theme than in MCC 3. In MCC 4, Cem's teaching practices went beyond simply incorporating technology as a tool to teach specific mathematics topics. He skillfully utilized technologies like GeoGebra, WordWall online quiz show, and GeoGebra homework assignments to engage students in exploring geometrical concepts and meticulously plan, execute, and reflect on his teaching practices. Cem's approach was driven by deep concern, conviction, and motivation for fostering students' thinking and understanding of mathematics, which he believed could be significantly enhanced through the seamless integration of these diverse technologies. In MCC 3, Cem used technology as a tool to facilitate the learning of the subject. However, in MCC 4, Cem integrated

technology as a tool for students to explore concepts and improve their understanding and thinking, unlike in MCC 3, based on his personal belief and concern that students' learning should be conceptual. This was the main evidence that Cem progressed in *mathematics learning descriptor* of learning theme. Through Cem's guidance, students embarked on a learning experience where they delved into the topic while harnessing technology's power. Students seamlessly used various technological tools to investigate and develop a deep understanding of geometric concepts, such as height in a parallelogram and the area of a parallelogram. As a result, students not only gained a deeper understanding of these mathematical concepts but also developed a heightened appreciation for the role of technology in their learning experience.

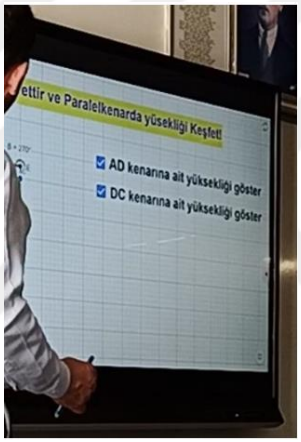


Figure 67. Cem while using the activity of height in a parallelogram in MCC 4

Cem taught how to determine the height of a parallelogram using GeoGebra as a dynamic learning and teaching tool. He showed students how to draw a perpendicular line from a vertex or any point on one of the parallelogram's sides to the opposite side, explaining that this line represents the height. Guiding them to investigate the area relation of parallelograms using GeoGebra, Cem presented several parallelograms for students to measure the base and height and then calculate the area using the formula. Students were encouraged to compare areas and identify repeating patterns. Cem also integrated interactive activities in which students manipulated parallelograms and their heights in GeoGebra to observe dynamic effects on the area. By integrating GeoGebra into teaching the area relationship of heights and parallelograms, Cem encouraged students' hands-on exploration,

facilitated a deeper understanding of geometric concepts, and provided opportunities for active learning and exploration.

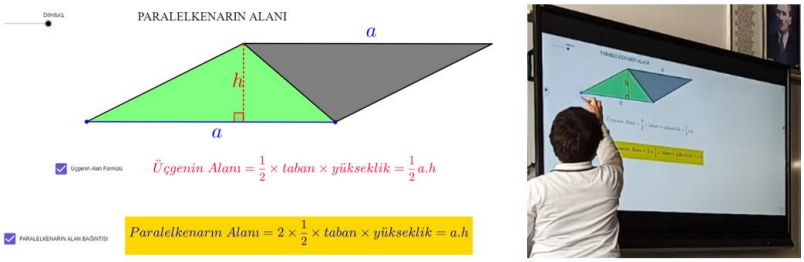


Figure 68. A student exploring the area relation of a parallelogram on GeoGebra in MCC 4

In Cem's MCC 4 teaching practice, he used technology to teach students a concept they had not learned before. During the warm-up activities, he listened to students' geometric predictions and ideas about the concept and encouraged them to share them with their classmates. Students explored the concept of height by using dynamic features such as dragging on the GeoGebra activity and then constructed heights on parallelograms both in the dynamic learning environment of GeoGebra and in a static environment on the works in the textbook. The students, who applied what they learned with GeoGebra homework in a dynamic environment, then discovered the area of relation of the parallelogram with GeoGebra activities. Cem encouraged students to share their geometric conclusions and generalizations while using the activities to explore the area relation of the parallelogram. Following the students' use of GeoGebra, he gave detailed explanations using GeoGebra while answering questions from the students. Cem, who taught a new concept (the area of a parallelogram) using the concepts that students had already learned (the area of a rectangle and triangle), followed an utterly student-centered approach to using technology. In this way, he contributed to students' conceptual learning and enabled them to establish relationships between geometric shapes.

When Cem was asked to what extent he thought that technology would contribute to the conceptual learning of this topic in pre-conference 4, he responded as follows;

> "We are in the fourth coaching cycle, and I have noticed changes in many things, from the students' answers to my questions to the inferences they

express while using the activity. Their mathematical communication skills and conceptual understanding have improved, and I can easily understand this. From the pre-coaching process until now, I know much better which activities they would like to do, what their learning needs are, how I should handle the subject, which teaching method, and which technologies I should integrate. Through personal experience, I have realized that when subject-specific technologies such as GeoGebra are combined with content selected by the subject and my teaching method, the quality of learning and teaching increases noticeably. In short, a concept I teach students does not remain in the air. When they do not understand something or have a question in mind, the whole lesson continues: Let us try and see if it is so."

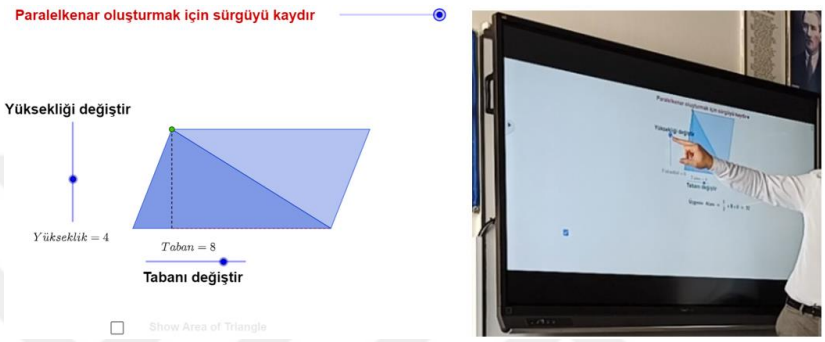


Figure 69. Cem associating the area of a parallelogram with the area of a triangle in MCC 4

Cem was asked what he thought about the use of technology in today's course at the post-conference 4 after the lesson, and he replied as follows:

> "I planned and implemented the lesson, and everything went as planned. For example, there was an activity in which I showed that the area was the same when different bases and heights were used. Integrating a few more activities like this into the lesson might be useful because students had some difficulties in this part. I believe that I can overcome this by using the power of technological tools such as GeoGebra since I care very much that my students learn well with technology."

Cem's statements in pre-conference and post-conference 4 and his teaching practice in MCC 4 aligned with the following indicator: "Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through the integration of the various technologies." (Niess, Ronau, et al., 2009, p. 22). Therefore, this evidence placed Cem at *the advancing level* of TPACK for *the mathematics learning descriptor* of the Learning Theme. Niess, Ronau, et al. (2009) expressed that teachers

at *the advancing level* of *mathematics learning descriptor* should manage the process of teaching and learning in mathematics with passion and concern for the students' thinking and understanding of the mathematics that is expected to be improved using technologies. The role of the teacher at this level is to create an atmosphere where students can discover mathematics learning topics, and the teacher incorporates various technologies in the teaching and learning process. The above features were evident in Cem's practice during MCC 4 as he integrated technology into his lessons and displayed concern about the students' mathematical understanding through technology. Thus, in the first lesson of MCC 4, Cem introduced the concept of height in parallelograms by asking questions and giving practical examples. He then employed GeoGebra to enable the students to manipulate the parallelogram and hence understand the heights of the particular. This dynamic exploration will assist the students in understanding the topic through the visualization of the concept. Moving to the second lesson of MCC 4, Cem used GeoGebra activities to explain the area of a parallelogram in relationship to the area of rectangles and triangles to the students. Thus, Cem helped the students learn area calculation by letting them change shapes and noting the results. In the final lesson of MCC 4, Cem used the Word Wall Quiz Show to increase students' motivation in finding solutions to area-related problems in groups. This also helped consolidate their knowledge and promoted peer and active learning through technology-integrated activities.

Moreover, comparing Cem's TPACK level in MCC 3 and MCC 4 revealed his progression from *the exploring level* (level 4) to *the advancing level* (level 5) regarding *Mathematics learning descriptor*. Cem's progression in *the mathematics learning descriptor* of Learning Theme revealed that the pre-conference discussions, suggestions, examples, and lesson plans contributed to Cem's TPACK level in *the mathematics learning descriptor* of Learning Theme. In other words, the coach could reflect his mathematical, pedagogical, and educative power with his beliefs about mathematics and mathematics teaching and learning to Cem. The progression of Cem from *the exploring level* to *the advancing level* in *the mathematics learning descriptor* between MCC 4 and MCC 3 relates to the TPACK dimension. This dimension focuses on using technology while using suitable methods to teach a subject matter (Mishra & Koehler, 2006). In MCC 3, Cem showed knowledge of

how to design, teach, and evaluate an activity supported by technology to improve student learning. However, in MCC 4, Cem moved to a higher order of technology integration where he found ways and means of integrating these tools to improve student's learning and understanding of mathematical concepts and, in the process, modify and enhance the curriculum to suit the use of technology better. This progression shows how Cem is developing competence in integrating technological knowledge with pedagogical and content knowledge to improve students' learning, which is the core of the TPACK framework.

The coach's role and the mathematics coaching process were significant for Cem's development in *Mathematics learning descriptor*. In pre-conference 4, the coach worked with Cem to plan lessons that would expand the use of technology to enhance the students' learning of area concepts and problems, especially the area of triangles and parallelograms. They identified certain GeoGebra activities and ways through which the activity can be used to enhance the student's learning of the concept being taught. The coach suggested ensuring the lessons include aspects where students can use technology to work with mathematical representations. The coach advised Cem to keep pondering the teaching practices and the effects of using technology in the classroom. The members assessed the previous lessons taught and discussed some of the things that may have been done in the wrong way. The coach assisted Cem in identifying concrete learning objectives for each class and in applying students' feedback and assessment information to modify his teaching approaches. This reflective practice was helpful as it assisted Cem to be mindful of the students' needs and the intended learning outcomes. The coach gave tips on how to create a classroom climate that enables the students to engage in the utilization of technology to solve mathematical problems. They shared ideas for asking questions without right or wrong answers and designing activities requiring the students to find the answers themselves. Cem was informed on how to conduct the class and assist the students in linking what they have observed with the mathematical concepts being learned. This approach was intended to improve the facets of student's rationality and analytical abilities. Following the coach's recommendations and guidance, Cem employed GeoGebra to help students interactively investigate the heights of parallelograms. He asked questions that would make the students reason

about the concept and its uses, allowing them to grasp the height of a parallelogram intuitively. GeoGebra activities led by Cem were used to demystify the connection between the areas of parallelograms, rectangles, and triangles. He prompted the students to alter shapes and note the results, stimulating their curiosity. Cem applied the WordWall Quiz Show, one of the collaborative learning strategies, to involve the students in the problem-solving process related to the area. This approach encouraged peer teaching and group work in solving problems, improving the students' comprehension.

As for *the conception of student thinking descriptor* of the Learning Theme, Cem's actual teaching practice in MCC 4 was very similar to MCC 3 in terms of *the conception of student thinking descriptor* of the Learning Theme. Cem's commitment to guiding students in grasping mathematical concepts was evident in his meticulous planning, execution, and reflective practices. He strategically incorporated technology-enhanced activities aligned with the curriculum and fostered active student participation. Cem introduced technological tools and evaluated their impact on students' mathematical learning outcomes and attitudes toward the subject. Through proactive management of these activities, he aimed to guide and enhance student engagement, fostering a culture of self-directed learning.

The coach's role and the mathematics coaching process greatly influenced Cem's development in *the conception of student thinking descriptor*. The coach advised Cem to distinguish the degree of technology integration and how to apply technology in class practically. They debated how these activities can be organized to relate to the learning outcomes and how the students can be directed through the discovery. The coach also stressed the need to properly define the rules and objectives associated with using technology so that the students would not be more distracted by the technology than the mathematical concepts. The coach advised Cem to focus more on students' participation and comprehension during technological activities. They discussed the strategies to apply questions and assessments to check for students' understanding and give feedback. Cem was informed on how to employ exit cards and other quick checks to assess the students' comprehension at the end of each lesson and thus modify the lesson delivery based on the students' feedback. The

coach gave pointers on creating a class culture that enables students to learn independently. They discussed how the level of independence of students could be enhanced, for instance, by giving the students a choice on how to approach a specific subject and having them state what they want to achieve from the lesson. Cem was introduced to ways and means of getting the students to reflect and express themselves and the learning process to their fellow students and take ownership of it. Similarly, Cem used the following GeoGebra activities to demonstrate to the students how to determine the height of a parallelogram. He also provided feedback and imposed questions that made the students reason at the conceptual level, hence enabling the students to be active and understand the mathematical concepts. Cem created the GeoGebra activities in which students altered the given figures and computed the area ratios of the parallelograms. He had a chance to utilize formative assessments such as exit tickets and provided feedback based on the students' answers. Cem used WordWall Quiz Show for the students to check their knowledge funnily and interactively as per the group division. He managed the technology-enhanced activity to ensure that the students' motivation and level of independence were high to foster group work and peer teaching.

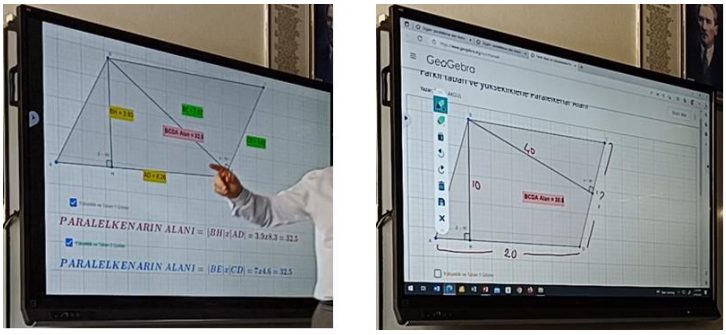


Figure 70. Cem while explaining the relationship between the area of a rectangle and a triangle in GeoGebra in the MCC 3 lesson

While implementing the MCC 4 lesson, it was observed that Cem's students participated in GeoGebra activities and seemed to have fun while learning. The students' interest and participation were exceptionally high, which can be interpreted as implementing Cem's pedagogical practices supporting a positive and active learning climate to develop students' mathematical thinking skills and dispositions. This particular teaching strategy used by Cem reflected a clear understanding of the

students' learning capabilities; hence, he could teach to help the students address their areas of weakness. In responding to student questions, he skillfully entered the students' worry zones to get to the root of the concern. For example, when a student was confused between the area of a parallelogram and that of a trapezium concerning dividing the product of the height and base by two or not dividing it, Cem asked a sequence of questions. He asked, "Which two shapes are congruent in a parallelogram?" the student replied 'triangle'. Capitalizing on this response, Cem asked further about finding the area of a triangle and learned that the product of base and height is half. He then asked, "If there are two congruent triangles in the parallelogram, is there a need to divide the base multiplied by height result by two?" to which he demonstrated using GeoGebra by dividing a parallelogram into two congruent triangles with a diagonal and proving the concept's applicability to any parallelogram. This was done to ensure that the students corrected the wrong notion and could use the clear concept in other problems. This teaching strategy proved Cem's ability to identify students' needs, address them, and encourage students' self-assessment.

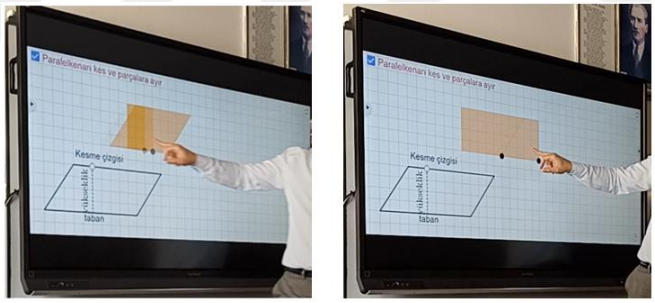


Figure 71. Cem, while elaborating on the relationship between parallelogram and rectangle areas in GeoGebra in the MCC 4 lesson

In addition, when asked what his role would be in the lesson he would present in the MCC 4 pre-conference, Cem answered as follows;

> "After many coaching cycles and technologically enriched lessons, students know what is expected of them when I open a technological tool or a technology-based activity. When I open a GeoGebra activity and ask various questions, the students take over the rest, and I provide them with feedback. Their generalizations, observations, and inferences already lead them to explore the concept. Therefore, in the course, I see myself as an exploration

director who engages students in various activities and provides them with learning environments where they can manage their learning."

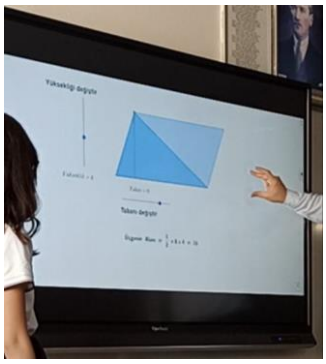


Figure 72. Cem, while guiding his student while exploring the area relation of a parallelogram in the MCC 4 lesson

However, Cem did not involve students in problem-solving activities except to present them with a parallelogram area calculation problem at the beginning of the first lesson. Even after the students had mastered height in parallelogram and area calculation, he did not have one of his students solve the problem or solve the problem himself. In other words, Cem did not engage students in higher-order thinking activities such as project-based activities, problem-solving, and decision-making activities to learn mathematics using technology as a learning tool. This case prevented Cem from being positioned at a more advanced level in *the conception of student thinking descriptor*.

At the post-conference of MCC 4, when asked if he would make any changes if he were to apply the lesson again, he answered as follows;

> "After finishing the lesson, I realized I did not include activities that develop higher-order thinking skills, such as problem-solving. I projected the area problem from a real-life context on the board at the beginning of the lesson. At the beginning of the lesson, I listened to the students' predictions about how to solve it. I planned to solve this problem at the end of the lesson, but time was not enough. If I were to do this lesson again, I would include some problem-solving activities like this problem at the end of the lesson."

Cem's statements during the pre-conference and post-conference of MCC 4 and his actual teaching practice in MCC 4 aligned with the indicator of: "Plans, implements,

and reflects on teaching and learning with concern for guiding students in understanding." (Niess, Ronau, et al., 2009, p. 21). Therefore, all these served as evidence to position him at *the exploring level* of TPACK for *the conception of student thinking descriptor* of the Learning Theme. Niess, Ronau, et al. (2009) noted that *the exploring level* of *the conception of the student thinking descriptor* is characterized by the teachers who plan, execute, and evaluate of teaching and learn to direct students' thinking. These teachers continue to carry out and assess technology activities regarding its effect on students' mathematics learning and attitudes towards the subject. These teachers also facilitate using technology in the classroom to guide students and encourage them to learn mathematics independently. The design of the tasks during MCC 4 revealed that Cem was in harmony with these features since he strategically integrated technology to support students' learning and their level of participation.

Cem began the first lesson of MCC 4 with the GeoGebra activity on the Height of a Parallelogram, where students could use the application's interactive tools to study the concept of height. He constantly gave feedback and asked conceptual questions to ensure the students were interested and grasped the mathematical concepts in the activity. In the second lesson, Cem engaged the students using GeoGebra activities to solve problems regarding the area of parallelograms. This way, he encouraged students to change shapes and look at the results, thus controlling and, at the same time, empowering student's learning. They assessed their learning using exit slips and gave feedback on what they thought they understood and their perception of the topic. In the last lesson of MCC 4, Cem applied the WordWall Quiz Show to check the students' knowledge and enabled peer teaching and collaborative problem-solving. This way, the students remained well-occupied and were able to work independently.

4.2.2.3. TPACK Level of Cem Regarding Learning Theme After Mathematics Coaching

To determine Cem's final TPACK level regarding the Learning Theme after participating in four mathematics coaching cycles, his teaching practices were

observed for three hours while teaching '*Problem solving related to the areas of compound shapes consisting of triangles, rectangles and parallelograms.*' To determine the final TPACK level, no pre-conference was held with Cem before the observed lesson, and no post-conference was held after the lesson. The researcher did not provide support for Cem in mathematics coaching. Cem prepared the lessons independently by reflecting on the knowledge and experiences gained in the four coaching cycles they participated in.

Cem's actual teaching practice after mathematics coaching indicated *the advancing level* of TPACK for both the mathematics learning and *conception of student thinking descriptor*s of the Learning Theme.

Cem's actual teaching practice after mathematics coaching was very similar to the MCC 4 in terms of *mathematics learning descriptor* of Learning Theme. Cem meticulously planned his lessons to incorporate GeoGebra, ensuring alignment with specific learning objectives related to problems of area calculations. He selected problems from real-life contexts, such as the amount of fabric required to cover the pillow face with pieces of fabric, the need for area calculation for the seating and service area of a restaurant, the need for paint according to the area of the door of a master who needs door painting, how much area is needed by someone designing a hut, etc. to contextualize mathematical concepts and engage students in practical applications. These real-world problems were transferred into GeoGebra, where Cem harnessed the software's dynamic features to provide interactive and visual representations. Throughout the implementation phase, Cem employed GeoGebra as a problem-solving tool, guiding students through solving area problems using the software's various features. Students were encouraged to explore and manipulate shapes, measure dimensions, and calculate areas using GeoGebra's measurement tools and formulas. By experiencing firsthand how mathematical concepts manifested in real-life scenarios and utilizing GeoGebra's dynamic capabilities, students developed a deeper understanding of area concepts and their practical significance. Furthermore, Cem emphasized reflective practices, encouraging students to contemplate their knowledge of area concepts and their use of GeoGebra.

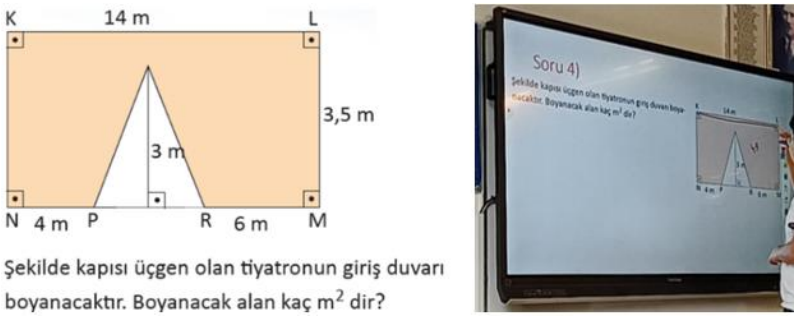


Figure 73. Sample problem from the lesson after mathematics coaching

Through class discussions and collaborative activities, students had the opportunity to share insights, address challenges, and deepen their comprehension of mathematical principles. By integrating GeoGebra into his teaching, Cem created a dynamic learning environment where students actively explored and applied mathematical concepts in real-world contexts. Through this approach, students not only developed proficiency in calculating areas but also cultivated problem-solving skills and a deeper appreciation for the role of technology in mathematics education. Cem's innovative use of GeoGebra served as a testament to the transformative impact of educational technologies in enhancing student learning experiences in mathematics.

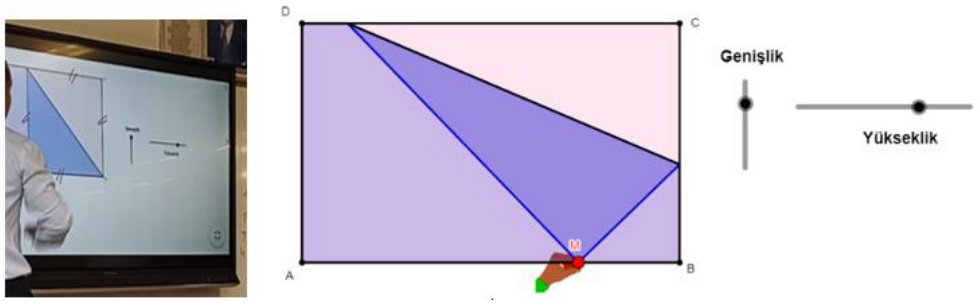


Figure 74. Rectangle folding activity of GeoGebra from the lesson after mathematics coaching

Cem attempted to enhance students' understanding of the fundamental concepts approach by integrating GeoGebra into his teaching practices to explore area problems involving rectangles, triangles, and parallelograms. Cem meticulously planned his lessons with deep concern and personal conviction for enhancing students' thinking and understanding of mathematics by integrating various

technologies. He ensured alignment with specific learning objectives related to area calculations, carefully selecting problems from real-life contexts to contextualize mathematical concepts and engage students in practical applications. With unwavering commitment, Cem transferred these real-world problems into GeoGebra, leveraging the software's dynamic features to provide interactive and visual representations. Throughout the implementation phase, Cem employed GeoGebra as a powerful problem-solving tool, guiding students with personalized attention and dedication. He facilitated students' exploration and manipulation of shapes, encouraging them to measure dimensions and calculate areas using GeoGebra's measurement tools and formulas. With a steadfast focus on student comprehension and growth, Cem facilitated hands-on learning experiences that fostered deep understanding and critical thinking skills. Furthermore, Cem fostered a reflective environment, nurturing students' ability to contemplate their understanding of area concepts and their utilization of GeoGebra.

Through meaningful class discussions and collaborative activities, students were encouraged to share insights, address challenges, and deepen their comprehension of mathematical principles under Cem's supportive guidance. By seamlessly integrating GeoGebra into his teaching with unwavering concern and personal conviction for student learning, Cem created a dynamic and enriching learning environment. Students actively explored and applied mathematical concepts in real-world contexts, developing proficiency in calculating areas and cultivating problem-solving skills with a profound understanding of the practical significance of mathematics. Cem's innovative and student-centered approach to technology integration underscored the transformative impact of educational technologies in enhancing student learning experiences in mathematics.

Cem's teaching practice following mathematics coaching was consistent with the following indicator: “Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through the integration of the various technologies.” (Niess, Ronau, et al., 2009, p. 22). Therefore, this evidence placed him at *the advancing level* of TPACK for *the mathematics learning descriptor* of the Learning

Theme. According to Niess, Ronau, et al. (2009), the teachers at *the advancing level* for *the mathematics learning descriptor* plan, implement, and reflect on teaching and learning with the conviction that different technologies should be used to improve students' thinking and understanding of mathematics. At this level, teachers assist students in discovering mathematical topics by using various forms of technology to help them understand the concepts being taught. According to these characteristics, Cem's teaching after the mathematics coaching was rich in technology use that supported students' mathematical learning and involvement. He began the sessions by determining students' prior knowledge of the area of various shapes through Geoboard online activities. He then showed the class GeoGebra how to find areas of irregular figures. This enabled the students to partition the shapes into more straightforward and familiar forms and check through GeoGebra's area app. These activities enhanced the students' participation in learning and enhanced their comprehension of the area using technology. Cem used GeoGebra to describe different techniques for solving problems related to skill-based questions and working with geometric shapes' transformations. Thus, by putting the students through cases where they come up with various solutions, Cem fostered critical thinking and total understanding. He also made many efforts to make the students relate the mathematics being taught to them with real-life situations. Cem brought a lot of group work into the class and used the techniques of peer tutoring and problem-solving using practical scenarios. He presented GeoGebra problems in the area to solve. Therefore, he kept on incorporating the use of technology in teaching and learning as well as the frequent experiences of the students on the area concept in mathematics.

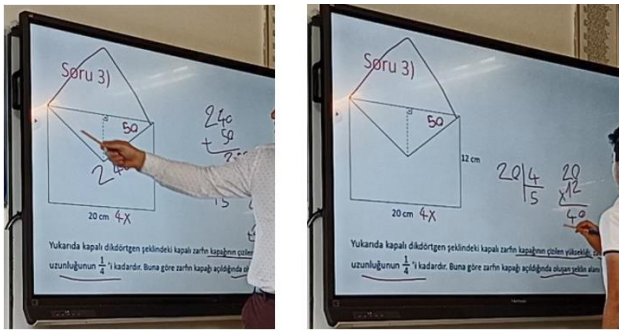


Figure 75. Cem, while elaborating on a student's solution to the area-related problem

In terms of *the conception of student thinking descriptor* of the Learning Theme, Cem's approach to teaching after the mathematics coaching indeed reflected a more advanced knowledge of student thinking regarding the Learning Theme than the MCC 4. Cem's instructional approach to teaching area problems concerning rectangles, triangles, and parallelograms with GeoGebra indicated a remarkable shift in his perception of student thinking as opposed to his teaching practice at MCC 4; he was also able to incorporate GeoGebra in teaching students at the higher order of thinking of mathematical concepts. Cem fully utilized the dynamism of GeoGebra to ensure that the students got the best learning experience. Including GeoGebra, virtual Geoboard, and a WordWall quiz as part of the class activity, Cem's lessons enabled the students to analyze mathematics concepts and better understand fundamental geometric relations carefully. For instance, Cem did not use this teaching strategy during the practical teaching at MCC 4; nevertheless, after receiving the mathematics coaching, he changed the course of his teaching practice dramatically. He had shown improvement in many teaching areas, including using GeoGebra to enhance students' hands-on learning of mathematical concepts. Another area that was well illustrated by Cem's use of GeoGebra was how he was able to present the students with the shapes of geometric figures and characteristics. Using the dynamic features of GeoGebra, it was easier for students to appreciate the relationship and effects of dimension and configuration changes. For instance, Cem provided the students with a hands-on experience of changing the base and the height of a parallelogram to show the changes in the shape area.

Also, Cem included problem-solving activities to help students solve context-based problems, where they had to use GeoGebra to find the solution. This way, Cem made the students use mathematical ideas in real-life situations, enhancing their comprehension. For example, students solved actual problems related to finding the area and made rational decisions based on the mathematical concepts. Cem benefited from GeoGebra's collaboration tools to improve student learning and knowledge sharing. Students in a group had to complete geometric tasks and think about how to solve them. This helped the students thoroughly comprehend the material by interacting with their peers and exchanging ideas. Thus, Cem's successful incorporation of GeoGebra allowed students to achieve the highest levels of

understanding of the subject matter. Cem used GeoGebra's dynamic features and problems from real-life situations to ensure that students are actively exploring, making connections, and constructing their knowledge, developing a deeper and broader understanding of mathematics. Cem has shown growth in instructional practice within the classroom, which betrays the benefits of continuous professional development and reflective teaching strategies for the students.

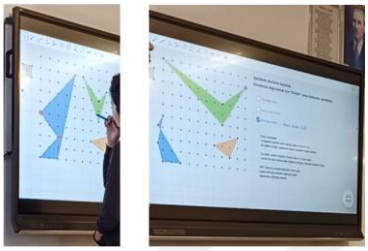


Figure 76. Cem extending the problem via dynamic features of GeoGebra

Cem integrated technology into mathematics teaching to foster high-order thinking skills in the students with the aid of GeoGebra. He used GeoGebra's interactive features to give one-on-one guidance to students depending on their learning capabilities, thus enabling them to have a deeper understanding of mathematical concepts. Additionally, Cem leveraged GeoGebra's collaborative features to facilitate peer learning and discussion, creating a supportive learning environment where students could share insights and deepen their understanding through collaborative problem-solving tasks. Moreover, Cem employed GeoGebra as a dynamic learning tool to cultivate his students' advanced understanding of mathematical concepts. Overall, Cem's innovative use of technology-empowered students to engage in active exploration, make meaningful connections, and construct their knowledge, ultimately enhancing their grasp of mathematical concepts to advanced levels.

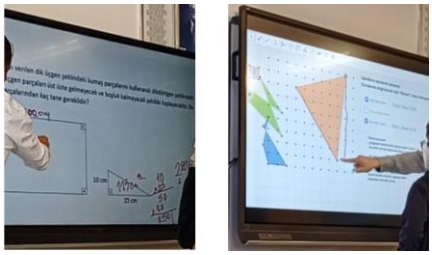


Figure 77. Cem guides his students during problem-solving during the lesson after mathematics coaching

For instance, Cem noticed during the lesson that some students struggled with understanding the concept of congruent shapes within parallelograms (two congruent triangles), which affected their ability to calculate the area accurately (students' confusion about whether to divide it in half or not). Instead of directly providing answers or repeating explanations, Cem guided the students through a reflective discussion. He encouraged them to visualize the parallelogram and identify which shapes within it were congruent. When students hesitated or made mistakes, Cem prompted them with questions to promote critical thinking about geometric properties. Through this interactive process, Cem helped students correct their misunderstandings and deepened their comprehension of congruence and its role in calculating areas. This example illustrates Cem's approach to fostering student thinking and problem-solving skills within area calculations, showcasing his dedication to guiding students toward a deeper understanding of mathematical concepts.

Cem's teaching practice after mathematics coaching aligned with the indicator of *the advancing level* in *the conception of student thinking descriptor* within the Learning Theme as follows: "Technology-integration is integral (rather than in addition) to development of the mathematics students are learning." (Niess, Ronau, et al., 2009, p. 22) and the example of this descriptor: "Engages students in high-level thinking activities (such as project-based and problem-solving and decision making activities) for learning mathematics using the technology as a learning tool." (Niess, Ronau, et al., 2009, p. 22). Niess, Ronau, et al. (2009) asserted that teachers at *the exploring level* for the conception of student thinking plan, implement, and assess the teaching and learning in the classroom, focusing on the student's understanding. It defines and assesses technology applications in connection with students' mathematics learning and attitudes toward mathematics. These teachers coordinate the technology-integrated tasks to facilitate students' learning and development of self-regulation skills in the context of mathematics. Thus, it is possible to state that Cem's practice demonstrates the following characteristics significantly. He ensured that all the activities supported by technology were effectively coordinated while observing the students' level of understanding and participation. Cem used GeoGebra to construct engaging and visual activities, which helped the students learn how to calculate the

area of irregular geometrical shapes. Thus, Cem was able to support students' technology use and promote its effective application for learning. His utilization of GeoGebra to provide other approaches to solving problems and depicting geometric transformations reflected his focus on the student's comprehension. Cem also ensured that students were engaged in the learning process by helping them solve certain cases and share their solutions with the class, making them more self-directed. Using peer tutoring and collaborative problem-solving, Cem integrated technology-enhanced activities to engage the students and encourage them to direct their learning. His concern for the student's comprehension and attitude toward mathematics was evident in his application for the WordWall Quiz Show in group work and continuous feedback for learning.

Furthermore, all these observations regarding Cem's teaching practice indicated that he progressed to *the advancing level* in *the conception of student thinking descriptor* within the Learning Theme. Furthermore, Cem's improvement in the conception of the student thinking descriptor is related to the Pedagogical Knowledge (PK) component of the TPACK framework. This dimension deals with identifying students' learning processes, the most suitable teaching approaches, and organizing classroom procedures to support learning (Mishra & Koehler, 2006). As a result of progress in this descriptor, Cem displayed enhanced professional development in students' thinking and participation, specifically insofar as technology is concerned in the classroom.

4.2.2.4. The Progress of Cem in TPACK Regarding Learning Theme Through Mathematics Coaching in Learning Area of 'Measurement of Area'

The figure 78 presents Cem's TPACK levels for *mathematics learning* and *conception of student thinking descriptors* of Learning Theme based on his teaching practices in technology-enriched lessons during and following mathematics coaching. Cem progressed in mathematics learning and *conception of student thinking descriptor*s according to the TPACK development model throughout the mathematics coaching cycles. The Measurement of the Area learning domain was analyzed within MCC 3, MCC 4, and post-mathematics coaching teaching practices.

The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

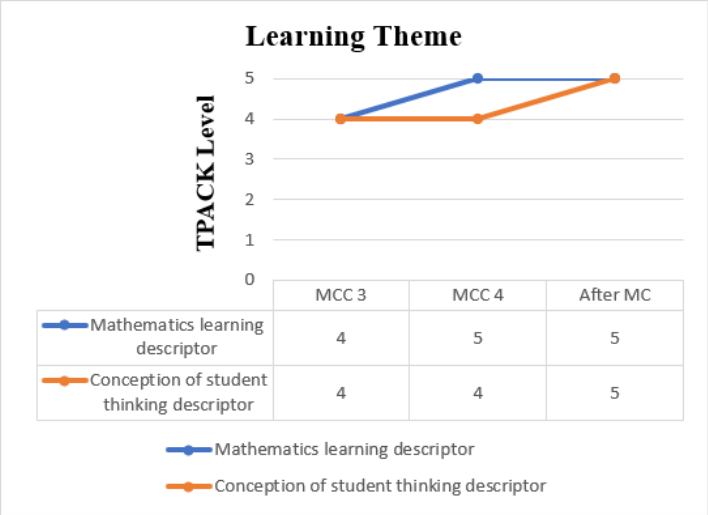


Figure 78. The progress of Cem of the Learning Theme in the learning area of Measurement of Area

The mathematics learning descriptor pertains to the concentration on the subject matter, which involves students' learning of mathematical topics. This descriptor also focuses on the teacher's viewpoint regarding teaching mathematics using technology and their methods for incorporating technology into students' learning. Essentially, it concerns the teacher's stance and actions regarding the fusion of technology into mathematics education.

Cem's level in the TPACK development model's *mathematics learning descriptor* at MCC 3 indicated *the exploring level* (level 4). Cem employed GeoGebra as a learning and teaching tool to enhance the mathematics curriculum's understanding of particular geometry topics. These involved students engaging in various activities using GeoGebra to delve into concepts like heights of different types of triangles, bases, and area calculations, utilizing the software's dynamic capabilities. Cem guided students to uncover the relationship between the area of a triangle and that of a rectangle or square, leveraging their prior knowledge of these shapes to deepen their comprehension through deductive and inductive reasoning.

According to his teaching practices, Cem progressed in *mathematics learning descriptor* at MCC 4 and after mathematics coaching. Cem's level in the TPACK development model's *mathematics learning descriptor* at MCC 4 and after mathematics coaching indicated *the advancing level* (level 5). Cem meticulously designed, executed, and pondered upon teaching and learning activities, prioritizing students' comprehension and grasp of concepts concerning the height and area of parallelograms. He incorporated GeoGebra activities and tools to enrich student understanding, ensuring they grasped the dynamic properties of GeoGebra to establish connections between the area of a parallelogram and that of a rectangle. Through this integration, students explored the calculation of the area of parallelograms and their heights and gained insight into how these concepts relate to the area of rectangles. Furthermore, Cem employed a consistent approach when teaching area calculation problems that required utilizing the areas of triangles, parallelograms, rectangles, and squares. By consistently applying the same methodology, students were able to develop a deeper understanding of how these geometric shapes interrelate and how their respective areas can be calculated. Cem's methodical approach encouraged students to make connections between different geometric concepts, strengthening their overall understanding of geometry. Alongside GeoGebra, Cem utilized technologies like Khan Academy and WordWall quiz shows to assess student learning and gain deeper insights into mathematical concepts.

As for *the conception of student thinking descriptor*, this descriptor is related to the demonstration of conceptions of how students learn and develop their thinking skills. Cem's level in the TPACK development model's conception of student thinking at MCC 3 and MCC 4 indicated *the exploring level* (level 4). In teaching the height of a triangle and parallelogram and the area relations of a triangle and parallelogram, Cem planned, carried out, and evaluated teaching and learning with a concern for guiding students' conceptual understanding. Cem carried out and assessed GeoGebra activities regarding student learning of concepts related to triangles and parallelograms and student attitudes toward learning mathematics with technology. He managed GeoGebra activities toward directing student engagement and self-direction in learning concepts related to triangles and parallelograms.

Cem's level in the TPACK development model's conception of student thinking after mathematics coaching indicated *the advancing level* (level 5). Cem considered the integration of GeoGebra into instruction as fundamental rather than supplementary to students' conceptual development. He involved students in challenging cognitive tasks like solving real-world problems and making decisions in the context of learning area calculation problems, employing GeoGebra as a multifaceted tool for learning, teaching, and problem-solving. Using technology, he aimed to cultivate a more profound comprehension of mathematical concepts at an advanced level.

4.2.3. Teaching Theme

The teaching theme centers on teachers' opinions of technology, their proficiency in implementing instructional approaches that utilize technology, and their ability to create mathematical environments that use technology to facilitate individual and group-based mathematical learning and creativity. The teaching theme encompasses three descriptors: *mathematics learning descriptor*, *instructional descriptor*, and *environment descriptor*. The TPACK of Cem is analyzed in the following sections based on the TPACK Development Model's teaching theme.

4.2.3.1. TPACK Level of Cem Regarding Teaching Theme in the MCC 3

Cem’s teaching practices were observed for three hours to determine his TPACK level regarding the Teaching Theme at the third mathematics coaching cycle which focused on the learning objective ‘*Establishes the area relation of a triangle and solves related problems.*’ The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 3 (MCC 3) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem’s actual teaching practice in the MCC 3 indicated *the exploring level* of TPACK for *the mathematics learning descriptor* and *environment descriptor* while *the advancing level* for *the instructional descriptor* of the Teaching Theme.

In the MCC 3 actual teaching practice, Cem's approach to addressing the subject matter was notably more sophisticated than in the MCC 2 actual teaching practice,

particularly concerning *the mathematics learning descriptor* within the Teaching theme. In the MCC 2 actual teaching practice, Cem employed technology to enhance and solidify students' understanding of mathematical concepts they had already been introduced to. Likewise, students utilized technology to strengthen their grasp of previously taught concepts. Unlike MCC 2 which focused on the learning objective *'Exploring the properties of complementary, supplementary, adjacent, opposite angles, and solving related problems'*, in MCC 3 actual teaching practice, Cem took a different approach by involving students in more advanced cognitive tasks, such as problem-solving and decision-making activities, leveraging technology to learn mathematics.

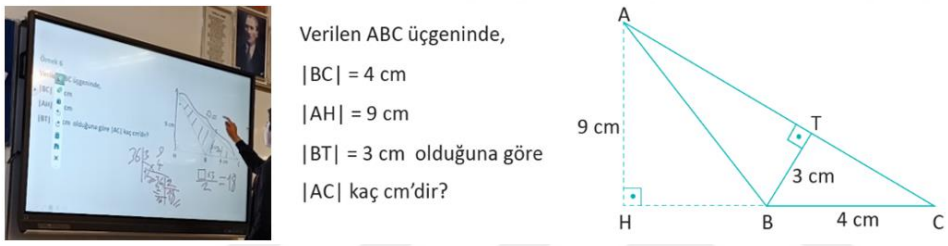


Figure 79. Cem's problem for calculating the area of a triangle using different bases and heights in MCC 3

For instance, at one point in the lesson, Cem presented a problem to the students and asked them to explore how the area of a triangle can be calculated using different base lengths and heights of the triangle. He then selected a student to solve the problem on the board, listened carefully to their guesses, reasoning, and mathematical deductions, and encouraged them to share their thinking with the class. The student struggled to decide which base and height to solve the problem.

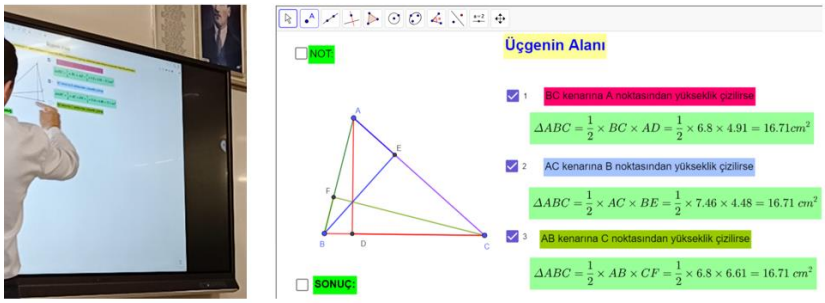


Figure 80. The GeoGebra activity regarding the calculation of the area of a triangle using different bases and heights in MCC 3

Subsequently, Cem used a GeoGebra activity to help students discover mathematical ideas to solve this problem. He allowed another student to work on the GeoGebra activity to discover that the area of a triangle can be calculated using different base lengths and the height of the base. Cem continued by engaging the whole class in a large group discussion and worked on developing students' higher-order thinking skills through decision-making and problem-solving activities.

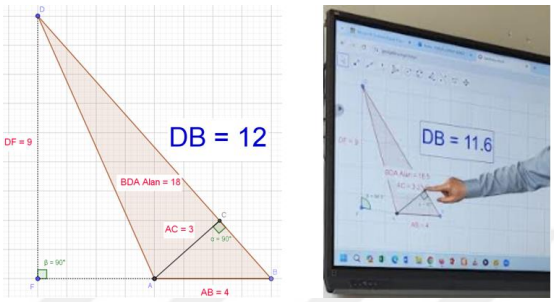


Figure 81. Area problem in triangle solved in GeoGebra in MCC 3 lesson

Cem then solved the problem by opening the problem in GeoGebra and using GeoGebra as a learning tool. While solving the problem, he examined with the students how to obtain solutions for different triangles by using various dynamic features of GeoGebra (area, side length, angle measurement, dragging, moving, manipulating, etc.).

When Cem was asked at the MCC 3 pre-conference how he thought technology would contribute to students' learning of this topic, he replied as follows;

> "With the GeoGebra activities I chose in this lesson, I aimed for students to explore the concept of height in a triangle and the area relation of a triangle. I planned to do this by engaging students in problem-solving situations where they would think and test their mathematical ideas and activities that would contribute to their conceptual learning. I hope they will reach various generalizations by testing different situations with GeoGebra's opportunities."

During the post-conference session after each lesson, Cem and the coach shared the lesson's positive and negative aspects, emphasizing students' interest and comprehension. In the post-conference 3, when Cem was asked about the impact of the use of technology in the lesson, his response was as follows:

> “I think the use of GeoGebra in the lesson affected the students' learning by acting as a learning tool that develops students' higher-order thinking skills. For example, students were involved in GeoGebra activities to calculate the area of a triangle using different bases and different side lengths. They then reflected on their learning in different problem situations and decided which side and height to use for area calculation.”

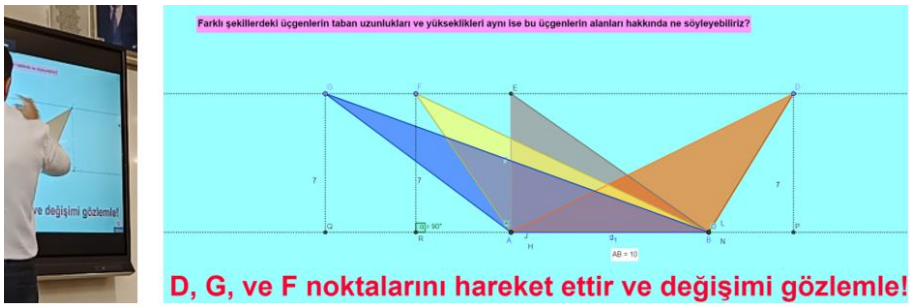


Figure 82. Cem while utilizing dynamic features of GeoGebra in the MCC 3 lesson

In pre-conference 3, the coach was instrumental in Cem’s development in *the mathematics learning descriptor* of the Teaching Theme. The coach introduced Cem to activities involving high-level thinking, like projects, problems, and decisions, and stressed the role of such activities in mathematics education. The coach showed the teacher how technology can be used to impart information and help students discover mathematics on their own. The coach showed how GeoGebra activities and other technological tools that can be used in class to help students discover mathematical concepts on their own can be used. The coach met with Cem to produce a plan that would include technology in class in a relevant way. The main idea of the discussion lay in the possibility of applying GeoGebra activities to teach specific concepts, like the height and area of a triangle, with stress on students’ discovery and engagement. The coach urged Cem to analyze his teaching and how the use of technology affects the students’ achievement. Cem’s coach supported him in defining specific goals for his lessons concerning students’ learning and motivation and the use of technology. He stressed the students' ability to develop their understanding of mathematical ideas. As the coach recommended, Cem started working more on stimulating students’ critical thinking and problem-solving skills rather than memorizing the information. This change in his thinking made him introduce new mathematical ideas to his students. He started to appreciate that technology, especially GeoGebra was an effective tool to capture students’ interest as it provided them with real-time

demonstrations of concepts and allowed them students to change the variables as they wished. Thus, following the coach's recommendation, Cem continued to create a set of more engaging and student-oriented activities using technology for investigations. Thus, this reflective practice helped improve Cem's teaching strategies, promote learning among the students, and increase their awareness of the value of higher-order thinking skills. His managerial approach organized the lessons to ensure that the students would spend most of their time on the computer and, therefore, gain a better understanding of the mathematical concepts.

Cem's instructional choices and decisions in MCC 3 involved high-level thinking activities and technology, corresponding with *the exploring level* for *the mathematics learning descriptor* of the Teaching Theme. This alignment was evident in Cem's practice and statements, reflecting the indicator: "Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool." (Niess, Ronau, et al., 2009, p. 23). Niess, Ronau, et al. (2009) posited that teachers at *the exploring level* incorporate students with higher-order thinking skills (such as project-based and problem-solving and decision-making activities) in teaching mathematics using technology as a medium of instruction. These teachers use technology-integrated activities, lesson plans, and successes in teaching and learning in their classrooms and disseminate them to other teachers. In the same way, Cem's teaching practice contained these characteristics. Cem started using real-life examples to introduce the students to triangle heights. He employed GeoGebra activities to make students work with the concept of heights of triangles and their relationship with different aspects of triangles, followed by investigation and application of the findings to the worksheets. This approach encouraged critical and creative thinking and solving problems through technology. Cem utilized GeoGebra to get students to discover the connection between the area of rectangles and the area of triangles, manipulate the base length or height, and observe the changes that occur—this kind of use of concepts with technology allowed for the development of understanding and critical analysis. Cem posed a problem that required the determination of the area of the triangle with different bases and heights. He encouraged students to present solutions, moderate group activities, and improve

their problem-solving with technology. Cem's progression can be described as relating to the Technological Content Knowledge (TCK) dimension of the TPACK framework (Mishra & Koehler, 2006). It also indicated Cem's improving proficiency in utilizing technology to support teaching particular mathematical topics.

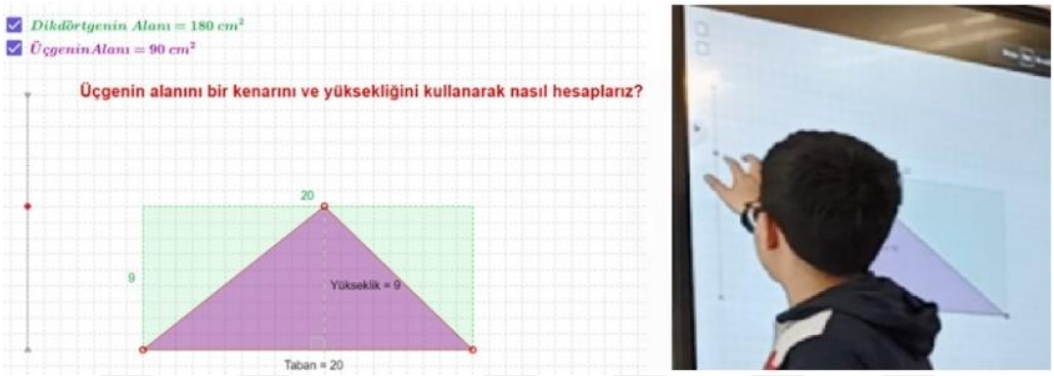


Figure 83. A student, while discovering the relation of triangle area in the MCC 3 lesson

Regarding *the instructional descriptor* related to the teaching theme in MCC 3, Cem took a more active part in shaping the students' learning process. He did not only follow what the coach told him to do. Instead, he researched and made directives on what he was to do. This change helped him modify the teaching strategies he used in his mathematics class to match the characteristics of the class and the students he was facing. Another critical component of Cem's strategy in MCC 3 was his focus on building active learning spaces enabled by technology. Instead of focusing only on conventional methods, he used digital tools such as GeoGebra, Khan Academy, WordWall, and GeoGebra assignments during the teaching process. This way, Cem enabled the students to engage in mathematical activities and problem-solving and put their hypotheses to the test. Taking a relatively passive role instead of being the one who leads the exploration, Cem allowed his students to explore on their own. He created an environment where students felt free to explore mathematical concepts with the help of GeoGebra functions. This method ensured that the students had curious minds as they solved various problems, which included finding the areas of triangles and heights of particular objects. In MCC 3, Cem generally used research-based decisions for instruction, technology integration, and student-centered teaching.

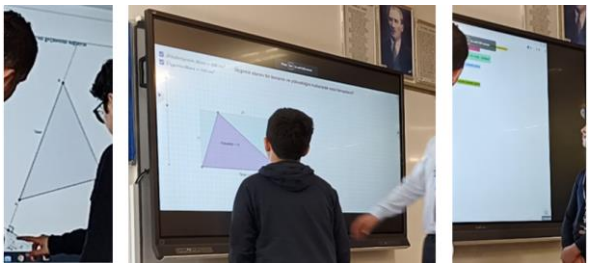


Figure 84. Cem while guiding students during technology use in MCC 3

Through guiding instead of directing, he helped the students to gain a better insight into the subject and, at the same time, to sharpen their analytical faculties in a student-centered environment. In addition, Cem effectively helped the students navigate the different tools and gave the impression of valuing the mathematical understanding. He also used deductive and inductive approaches and integrated technology to enhance students' mathematical thinking. Cem's teaching strategies in the MCC 3 lesson were discovery learning, Q-A methods, large group discussions, real-life contexts, and peer teaching. All of these methods and strategies were developed using the constructivist approach. For instance, Cem helped students discover the triangle area using the deductive approach and the area of a rectangle with base and height as the fundamental idea, where GeoGebra was used to show activities. This approach reveals the logical thinking methods typical for mathematics teaching. Likewise, Cem employed a teaching technique that involved technology to make the students derive the correlation between triangle heights of different classifications by focusing on the angles that form the triangles, which is another example of the deductive strategy. Cem used GeoGebra and ensured that students made frequent generalizations by participating in the activities.

Cem reflected with the coach about each lesson to determine what strategies were successful and which ones were not to ensure that the use of technology was more effective the next time. As for the extent to which Cem believed that the teaching method and the instructional strategies he employed in the Lesson were effective in promoting the students' learning, he responded in post-conference 3 as follows:

> "In the lesson, I tried to support students' learning with many teaching methods such as discovery learning, peer teaching, collaborative problem

> solving, etc. In the pre-conference, we planned these teaching methods to work harmoniously with the technology and the concepts we will teach. For example, discovering a triangle's area based on a rectangle's area was very effective with the GeoGebra activity. The deductive and inductive strategies I used in the lesson greatly improved the quality of learning and provided conceptual learning."

Cem's observed improvements and instructional choices in MCC 3 showed that he progressed to *the advancing level* of the *instructional descriptor*. His practice aligned with the indicator: "Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics." (Niess, Ronau, et al., 2009, p. 23). Niess, Ronau, et al. (2009) noted that teachers at *the advancing level* of *the instructional descriptor* integrate several teaching methods (both deductive and inductive methods) coupled with technologies to encourage students' thinking on mathematical concepts. Such teachers assist the students to transition easily from one tool to the other while showing evident passion and interest in the subject area of mathematics. These characteristics were also observable in Cem's classroom practice. Cem employed both deductive and inductive approaches. In the beginning, he presented questions to teach the students, and then he let the students learn about triangle heights with the help of GeoGebra. He managed to present students' projects and give his comments and suggestions; he tried to enhance the strategies students could comprehend well. Cem incorporated direct instruction and student exploration in GeoGebra activities to help the students determine the connection between the area and the triangles' height. His approach made the students easily transition from one technological tool to another while ensuring they understood the concepts. Cem supplemented the conventional activities with technology-based ones, including Khan Academy and WordWall quizzes. He promoted peer teaching and group problem-solving and showed a very liberal and modern teaching style. Moreover, compared to the TPACK level of Cem in MCC 2, it was determined that he progressed from *the exploring level* (level 4) to *the advancing level* (level 5) regarding *the instructional descriptor* in MCC 3. This progression relates to the TPK component of Mishra and Koehler's (2006) TPACK framework. It shows how Cem has progressed in integrating various technological tools and resources to enhance the student's active participation in understanding mathematical concepts.

Pre-conference 3 was aimed at the concrete assistance that the coach and the coaching process offered to Cem to progress from *the exploring level* to *the advancing level* in *the instructional descriptor* of the Teaching Theme in the TPACK development model. The assistance was based on helping Cem understand how different instructional methods can be integrated with technology to increase the student's participation and understanding of mathematics. The coach, jointly with Cem, elaborated on specific instructional goals that had to be set for the lesson, focusing on the area of relation of a triangle and the solution of related activities. The reasons for the need to use deductive and inductive teaching methods to help students with a variety of learning styles and to increase their comprehension level were explained. The coach supported Cem in designing and organizing the classes and ensured that there was variety in using technological tools and instructional strategies. They underlined the importance of a logical flow so that one activity and technology would smoothly transition into another. The coach suggested that Cem should help students take control of their learning by providing situations requiring thinking and problem-solving. The topics covered included ways students can be encouraged to think and work independently. The coach emphasized the importance of self-assessment of the pedagogical practices to find areas for improvement and celebrate success. They urged Cem to contemplate the outcomes of various instructional processes and technological tools that he uses for teaching. For instance, the coach helped Cem develop lessons that started with real-world scenarios and questions to engage the students, followed by activities that involved using GeoGebra and other gadgets. They described how to introduce GeoGebra as a tool to investigate triangle heights and areas so that students can interact with the concepts.

As for *the environment descriptor*, there were notable advancements in Cem's instructional approach in MCC 3 compared to his practices in MCC 2. The learning environment, encompassing the roles of both students and teachers, organizational methods, handling of subject matter and concepts, and the purpose of integrating technology, demonstrated a more advanced and constructivist nature. In MCC 2, Cem predominantly employed technology in a deductive and teacher-directed manner to maintain control over the learning activities. However, in MCC 3, he

diversified his approach in MCC 3 by employing a mix of deductive and inductive strategies with technology to prompt his students' engagement in understanding triangle area and height concepts. To fulfill this objective, Cem integrated various technologies, such as GeoGebra, Khan Academy, WordWall, and GeoGebra homework assignments, into the lesson. Cem cultivated a classroom environment encouraging students to express their mathematical ideas openly. Leveraging peer teaching, he had one student explain a topic, concept, problem, or situation to another student, promoting the sharing of answers and fostering collaborative learning. Cem allowed students to share their solutions to a problem with the class.

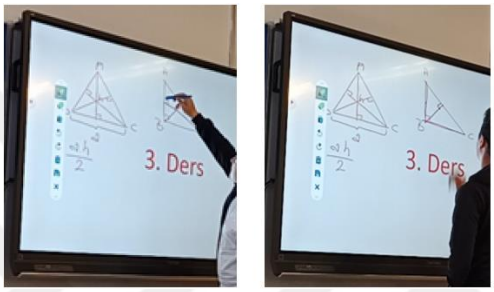


Figure 85. Students sharing their mathematical ideas about the topic in MCC 3

In contrast to the MCC 2 lesson, Cem's utilization of technology in the MCC 3 lesson shifted towards a more student-centered and discovery-oriented approach. Cem employed various instructional methods, incorporating deductive and inductive techniques alongside technology, to stimulate his students' critical thinking regarding triangle area and height concepts. Cem integrated various technologies into the lesson to achieve this goal, including GeoGebra, Khan Academy, Wordwall, and GeoGebra homework assignments. Furthermore, students were actively involved in utilizing technology, such as the dynamic features of GeoGebra, to explore the mathematical topic of triangle area and height through actions like dragging, manipulating, sliding, and observing changes. Throughout the teaching sessions, Cem encouraged student exploration, aiming to deepen understanding of geometric concepts and provide opportunities for active learning and discovery. Cem structured the classroom environment to focus on individual student work, peer-teaching, fostering idea-sharing, and facilitating exploratory questions during large group discussions or through personalized questioning. During the three-hour lesson, Cem

allowed various students to engage with concepts using smart board technology. They tackled exercises from the textbook, solved problems, and responded to individual and group questions. Students worked independently rather than in groups during the MCC 3 teaching practice.

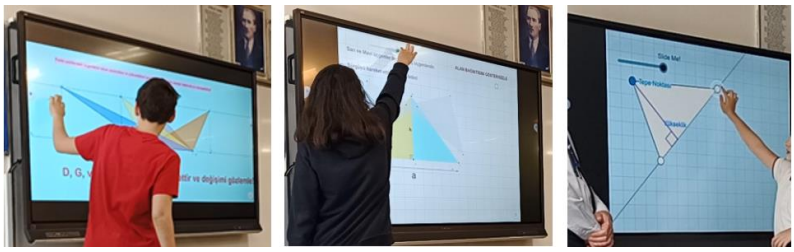


Figure 86. Students use technology to explore concepts taught in MCC 3

When Cem was asked in the pre-conference of MCC 3 about his role in the lesson and how he would utilize deductive or inductive strategies in the course, he replied as follows:

> "I will act as a guide, aiming to facilitate students' understanding of the area relationship of triangles by engaging them in situations using both deductive and inductive approaches. Essentially, we will incorporate strategies that traverse from the specific to the general and vice versa, which hold significant importance in geometry. As the teacher, my role will be that of a facilitator who presents to the students the scenarios and activities we have devised in line with implementing these strategies. For instance, we will utilize GeoGebra to create multiple triangles with a given base and corresponding height, measuring their areas using the 'area calculation' tool. I anticipate that students will uncover the method of calculating triangle area based on the results from various triangles. Likewise, I expect them to recognize the relationship between the triangle area and the area of a rectangle through a deductive example. In essence, I will solely guide them throughout this entire process."

All these showed that discussions, suggestions, examples, and shared teaching practices and lesson plans significantly contributed to Cem's TPACK level in *the environment descriptor*. In other words, the coach could reflect his mathematical, pedagogical, and educative power with his beliefs about mathematics and mathematics teaching and learning to Cem. Moreover, these were considered evidence that Cem moved to a higher TPACK level for *the environment descriptor* and showed development in TPACK.

In post-conference 3, when Cem was asked what the roles of the students and himself as the teacher in the lesson were and how the technology affected these roles, he responded as follows:

> "My role was to guide and instruct the students to explore. My students were students who followed the instructions and, through this, explored mathematical relationships, ideas, and concepts. I would say that the technology, namely the GeoGebra activities in the lesson, served as a bridge between my role and the role of my students. Technology was a learning tool used by the students to explore the concepts."

All this provided evidence that Cem's practice in MCC 3 aligned with *the exploring level* of TPACK for *the environment descriptor* within the teaching theme since his varied use of instructional strategies and technologies to engage students in mathematical thinking was consistent with the indicator: "Discusses a range of ways of teaching with technologies for linking students into thinking about what mathematics is (Niess, Ronau, et al., 2009, p. 23). At *the exploring level*, teachers examine several teaching methods, such as the deductive and the inductive, to engage students in mathematical thinking with the help of technological tools (Niess, Ronau, et al., 2009). Thus, teachers at this level incorporate technology into teaching different subjects and topics. These characteristics of the teacher's pedagogical approach were evident in the MCC 3 classroom with Cem as the teacher. Cem used GeoGebra to help students explore the heights of triangles; it is for this purpose that both deductive and inductive approaches were encouraged. He made it a point to incorporate different technologies to enhance the students' interactions with mathematical constructs. Cem used GeoGebra to support students in their investigation of area relations; he provided direct teaching and, at the same time, let students discover on their own. Thus, using many technological tools enabled him to employ various teaching techniques. Cem used Khan Academy, GeoGebra, and WordWall to make the students conduct several activities. These were peer teaching, group problem-solving, and technology-based activities, all indicative of an exploring teaching strategy.

Cem's progression is related to enhancing the TPACK aspect of the TPACK framework formulated by Mishra and Koehler (2006). It relates to improving Cem's

skills in designing an effective learning context that incorporates technology, pedagogy, and content harmoniously and productively. During the pre-conference 3 session, the coach was instrumental in supporting Cem's progression from adapting to *the exploring level* of the *environment descriptor* of the TPACK development model used to depict the students' development in technology, pedagogy, content, and learning. The coach focused on explaining to Cem different methods of teaching, which can be divided into deductive and inductive teaching. Thus, the coach enriched Cem's understanding of various methodologies and their connection with technology by presenting different techniques and their application. Cem used both explicit teaching and learning by doing approaches with the students. For instance, he first provided quiz questions to guide the students and then allowed the students to explore triangle heights using GeoGebra. This blended approach incorporates all the instructional strategies highlighted during the pre-conference. The coach suggested that Cem use various technological tools to increase students' interest and understanding. By identifying the benefits of various technologies and their possible uses, the coach helped Cem see the importance of having a more diverse technology toolbox. Cem used GeoGebra, Khan Academy, and WordWall quizzes in his classes. Since the different technologies were employed, the students could interact with the content differently, thus enhancing their understanding of the area relation of triangles. He stressed the need to change from a classroom where the teacher is the focal point to a class where the students are the focal point. This entailed orienting Cem on implementing activities requiring the students to be very involved in learning. Cem allowed students to share their work and findings with the board, discuss with their peers, and solve some area-related problems. As such, the observed change of focus on the student, as per the coach's recommendation, enhances the learning process and makes it more stimulating. The coach modeled reflective practice, making Cem think about the effectiveness of his strategies and the technologies involved. Cem's reflection helped him define the weaknesses and the methods for enhancing student interaction with technology. Cem's soliciting students' problem-solving approaches and leading group discussions indicate he is a reflective teacher. He always assessed the effectiveness of the strategies that he used in the classroom and made changes that would benefit the students. The coach encouraged Cem to try various techniques and tools during the lesson and stressed

flexibility and timely adjustment to the student's needs and feedback. This reassured Cem and gave him more confidence to try new ideas and technologies. Cem could alternate his teaching methods and use a broad range of technological tools in teaching. This was evident in his willingness to accept new methods and technologies influenced by the coach's encouragement.

4.2.3.2. TPACK Level of Cem Regarding Teaching Theme in the MCC 4

To determine Cem's TPACK level of the Teaching Theme at the fourth mathematics coaching cycle, which focused on the learning objective '*Establishes the area relation of a parallelogram and solves related problems,'* his classroom teaching practices were observed for three hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 4 (MCC 4) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in MCC 4 indicated *the advancing level* of TPACK for the mathematics learning, environment, and *instructional descriptor*s of the Teaching Theme.

In the MCC 4 actual teaching practice, Cem's approach to addressing the subject matter was notably more advanced than in the MCC 3 actual teaching practice, particularly concerning *the mathematics learning descriptor* within the Teaching theme. In the MCC 3 lesson, Cem engaged students in high-level thinking activities for learning and teaching height in triangles and area relation of triangles (such as exploring the concept, finding relationships among different concepts and shapes, and making and testing geometrical conjectures, decision making activities) using the GeoGebra as a learning and teaching tool. In MCC 4 teaching practice, from the outset, Cem recognized the potential of GeoGebra's dynamic features as transformative tools for enhancing students' understanding of mathematical concepts. By seamlessly integrating GeoGebra into the introductory phase of the lesson, Cem harnessed its dynamic capabilities to engage students and facilitate the accurate translation of mathematical concepts into understandable forms. In an activity where

students manipulated parallelogram shapes through dragging, they made decisions as incorrect shapes failed to fit within the parallelogram, offering a dynamic learning environment where students managed their learning and received instant feedback. Similarly, in another task where students incorrectly drew the height in GeoGebra, they used the angle measuring tool to realize that the angle formed by the drawn line segment and the base was not 90 degrees. Consequently, they adjusted their geometric construction to adhere to the height definition. During the assessment phase, students worked in pairs to answer various questions on the WordWall quiz show. Those who provided incorrect solutions revised their work, or Cem facilitated discussions to present correct solutions, fostering dynamic learning interactions.

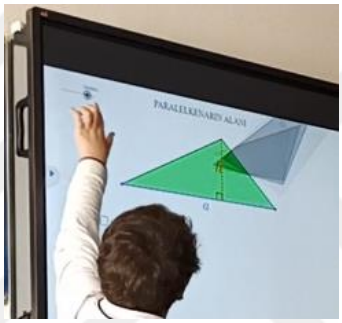


Figure 87. A student using GeoGebra's dynamic features for learning the area relation of parallelogram

Utilizing GeoGebra's interactive features, Cem invited students to actively participate in the learning process by manipulating shapes through dragging, changing sliders, and resizing figures. Through these dynamic actions, students could directly observe the impact of altering parameters on parallelogram properties, such as height and area. By allowing students to interact with geometrical objects in real time, GeoGebra provided a tangible and intuitive representation of abstract concepts, fostering deeper comprehension and engagement. As the lesson progressed, Cem continued to leverage GeoGebra's dynamic features as a central component of instructional delivery. Guided by inquiry-based principles, Cem encouraged students to explore mathematical relationships through hands-on interactions with GeoGebra. By enabling students to move corners and adjust dimensions, GeoGebra empowered them to independently investigate the geometric properties of parallelograms and their associated heights. Students gained insight into the underlying mathematical

processes through dynamic manipulations of geometric figures, leading to a more robust understanding of the relationship between height and area. In the lessons, GeoGebra was a versatile teaching and learning tool easily incorporated into any activity. Regardless of whether he was engaging the whole class in a discussion, small groups in an activity, or individual students in an investigation, Cem ensured that GeoGebra's dynamic nature included everybody.

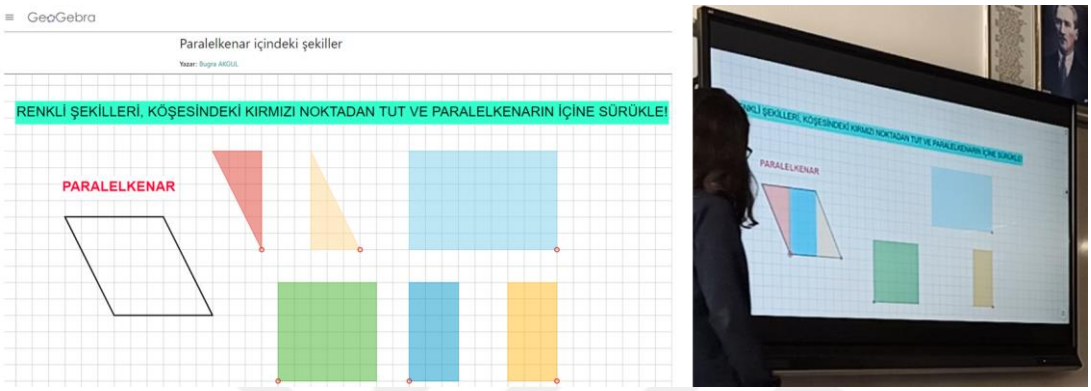


Figure 88. A student using the shapes inside parallelogram GeoGebra activity

In this regard, Cem used visual aids and interactive activities, simulations, and feedback, thus involving every student and motivating them to learn at their own pace. In addition, Cem implemented GeoGebra's dynamic aspects in the post-lesson evaluation and thus exhibited an awareness of the proper way to integrate technology into the classroom. Cem used formative assessments with the help of GeoGebra's tools, which allowed the students to engage with mathematics in real-time. Thus, observing the students' interactions with GeoGebra, Cem better understood their perception of the subject and the difficulties they faced, which could be addressed. Therefore, Cem demonstrated the uptake and integration of technologies as tools for learning and teaching mathematics by using GeoGebra's dynamic aspects to translate mathematical ideas into forms that students could understand. With GeoGebra's dragging, changing the sliders, resizing the figures, and moving the points, the students could physically engage with the mathematical objects and concepts and thus learn more effectively. By allowing students to engage the dynamic features of GeoGebra, Cem allowed the students to be more than just recipients of mathematical knowledge but, instead, students who can actively participate in the learning process.

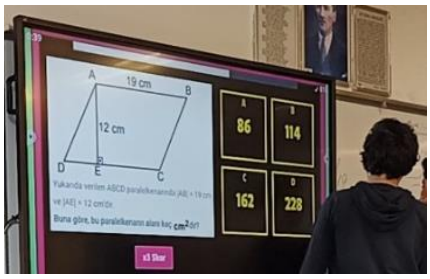


Figure 89. WordWall activity involving peer teaching and group work from the MCC 4 lesson

In post-conference 4, Cem was asked if and how he integrated technology into his teaching to develop students' mathematical thinking and conceptual understanding, and he said the following.

> "Yes, I believe that I did. I understood that I actively transformed the concepts into more understandable ones with GeoGebra's dynamic features and other tools. I understood that I enhanced students' conceptual understanding from their responses in the lesson, from the end-of-course assessment results such as exit cards and WordWall, and their responses to the GeoGebra home task. I tried to use technology at every stage of the lesson and in all aspects of mathematics, from the introduction to the evaluation of student learning at the end."

All these provided evidence that Cem's actual teaching practice in MCC 4 aligned with the following indicator of *the advancing level* in *mathematics learning descriptor* of Teaching Theme: "Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students." (Niess, Ronau, et al., 2009, p. 23). Based on the study of Niess, Ronau, et al. (2009), the teachers at *the advancing level* for *the mathematics learning descriptor* are more proactive and continuous in integrating technology into the learning and teaching of mathematics. It helps to comprehend the mathematical concepts and methods, considering they are presented and comprehensibly. Hence, a high level of technology use was identified in Cem's teaching practice in MCC 4, and he frequently and consistently employed applications to facilitate students to learn mathematical concepts. For instance, Cem always employed GeoGebra to support students' activities and help them understand and investigate the height and area of

parallelograms. Cem gave the students the freedom to change the measurement of the parallelogram's side using GeoGebra, which helped the students understand and apply the idea of height to any parallelogram shape. This practice is in accord with the feature of integrating the use of technology to enhance comprehension of mathematical ideas. Cem also used concrete objects to relate the students' learning to real life; for instance, he used the Tower of Pisa, tents, and sailboats to illustrate how mathematics was applicable in the real world. Thus, using examples related to parallelograms and their characteristics, Cem helped students understand the usefulness of the mathematical concepts they were learning.

Compared with the TPACK level in MCC 3, it was concluded that Cem progressed from *the exploring level* (level 4) to *the advancing level* (level 5) regarding *the mathematics learning descriptor* in MCC 4. Cem's observed improvements in MCC 4, stemming from pre-conference discussions, suggestions, examples, and lesson plans the coach provided, demonstrate an advancement in his TPACK level within *the mathematics learning descriptor* of the Teaching Theme. In other words, the coach could reflect his mathematical, pedagogical, and educative power with his beliefs about mathematics and mathematics teaching and learning to Cem. Also, the progression in *the mathematics learning descriptor* of the Teaching Theme from exploring to *advancing level* in Cem's case relates to the development of TPACK, as asserted by Mishra and Koehler (2006). This dimension refers to how Cem has gained the capacity to incorporate technology in teaching and learning mathematics, improving student achievement.

During the pre-conference 4 session, the coach facilitated Cem's progression from *exploring* to *advancing level*, according to *the mathematics learning descriptor* of Teaching Theme. In this regard, the coach employed several measures to facilitate Cem's learning and concentrated on enhancing the quality of the instructional process and understanding when and how to utilize technology to improve mathematics education. Cem's coach recommended that he use the reflective teaching approach. Hence, he had to critically analyze his past classes, students' perceptions, and their performance in MCC 3 to establish the impact of technology on learning. Thus, this reflection made Cem aware that technology could help

elaborate on more complicated mathematical concepts. The coach introduced Cem to the advanced techniques of using GeoGebra and other tools for the area and height of the parallelogram and how it makes learning theoretical concepts easier. Furthermore, the coach noted that it should be explained how the mathematical ideas work in practice and pointed at the Leaning Tower of Pisa and sailboats as examples of parallelogram properties. The coach endorsed the development of activities derived from the students' requirements and assisted them in identifying reasoning and innovating solutions. Several GeoGebra tasks were proposed to be solved individually and in groups regarding characteristics of a parallelogram, thus indicating the transition from the teacher's guiding role to the students. Thus, this regular feedback on the lesson plans that Cem had prepared proved beneficial to him as it assisted him in enhancing his thinking. The coach then reviewed the strategy and made some observations regarding the language used, the rate at which the questions were asked, and the kinds of questions being asked to elicit critical thinking. Cem strengthened this aspect by having this provision to enhance technology integration in his teaching.

As for *the instructional descriptor* of MCC 4, Cem's teaching approach they mirrored that of MCC 3, employing a diverse array of strategies such as group work, peer teaching, discovery learning, and both deductive and inductive methods, complemented by technology to engage students in mathematical thinking. For instance, Cem utilized GeoGebra activities to guide students in exploring concepts like area relation and the height of parallelograms through hands-on discovery. Furthermore, Cem embraced a constructivist teaching approach and employed inductive strategies, leveraging students' existing knowledge to introduce new concepts, as demonstrated when students discovered the area relation of parallelograms based on triangles. Additionally, he facilitated collaborative learning by pairing students to solve problems, exemplified in activities like the WordWall Quiz show, where one student could teach the other, enhancing their comprehension. Moreover, Cem fostered a culture of sharing and discussion, encouraging students to articulate their geometric insights and solutions during WordWall activities, GeoGebra exercises, and large group discussions. Overall, Cem adeptly navigated

various technological tools, ensuring students remained engaged and developed a deep understanding of mathematical concepts.

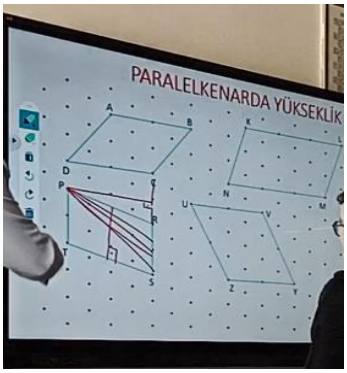


Figure 90. Cem clarifies a common misconception about the height of a parallelogram

When Cem was asked in post-conference four whether he used technology while utilizing deductive or inductive strategies in the lesson, he answered as follows;

> "Of course I did. I used it to teach the area relation of the parallelogram while using the area of the triangle. I moved two congruent triangles side by side in the GeoGebra activity. More precisely, by dynamically sliding the slider, the twin of a triangle was positioned next to it. The sum of 2 congruent triangles turned into a parallelogram. Based on this, the students discovered that two of the area relations of the triangle are equal to the area relation of one parallelogram. They observed that the product of base and height in a parallelogram is not divided by two. Students then verified this in the GeoGebra spreadsheet view. As a result, they realized that this area relation, valid for a parallelogram, is valid for all shapes with parallelogram properties."

In line with the coach's suggestions and support during pre-conference, Cem adeptly harnessed the power of GeoGebra to teach geometrical concepts with a blend of deductive and inductive strategies, particularly focusing on the height in parallelogram and its area relation. Cem engaged his students with interactive demonstrations using GeoGebra, immersing them in visually exploring parallelograms and their properties. Through GeoGebra's dynamic features, students observed firsthand the geometric transformations that occurred as the height of a parallelogram changed, laying the groundwork for deductive reasoning. Through guided inquiry and group discussions, Cem encouraged students to make conjectures

and formulate hypotheses about the relationship between height and area, fostering a learning culture by discovery. As students delved deeper into their investigations, Cem facilitated meaningful discourse, guiding them toward discovering the area formula through inductive reasoning. Drawing upon their observations and insights from GeoGebra simulations, students began to discern patterns and connections between height and area, gradually unraveling the mathematical principles underlying parallelogram geometry. GeoGebra served as a dynamic catalyst for learning, providing students with real-time feedback and opportunities for hands-on exploration. Whether through deductive presentations of established formulas or inductive inquiries into geometric relationships. His enthusiasm and dedication to fostering mathematical understanding created an environment where students were empowered to explore and learn confidently. Cem orchestrated collaborative learning experiences, where students collaboratively explored challenging problems and refined their problem-solving skills through group work and peer teaching.

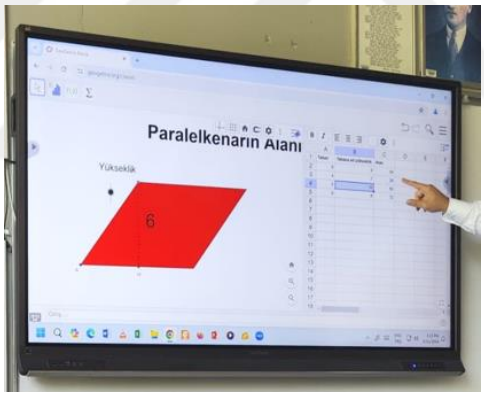


Figure 91. Cem while using the spreadsheet for the area relation of a parallelogram

All these provided evidence to place Cem at *the advancing level* regarding *instructional descriptor* of Teaching Theme since Cem's actual teaching practice at MCC 4 aligned with the following indicator of this level: "Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics." (Niess, Ronau, et al., 2009, p. 23). Niess, Ronau, et al. (2009) stated that the teachers at *the advancing level* for *instructional descriptor* integrate several teaching approaches (both deductive and inductive) with technologies to involve students in the critical

thinking of mathematical ideas. Cem's instructional decision-making in MCC 4 indicated a comprehensive understanding of the full array of strategies appropriate for use in the classroom, along with the proper integration of technology. Cem taught students by telling them directly and engaging them in activities that would make them discover the concepts of height and area in parallelograms. Cem started with direct questions to check prior knowledge, then used GeoGebra activities in which students found out properties on their own. This combination of strategies aligns with the concept of adopting several teaching strategies. Cem allowed students to present the findings and have peer-to-peer discussions. Letting students solve the problems with GeoGebra on their own, share their solutions with the board, and explain the results to their classmates fits the concept of a student-centered approach in which the teacher acts as a guide rather than a lecturer, which corresponds to the higher level of *the instructional descriptor*.

During the pre-conference discussions, the coach took the time to provide detailed, individualized feedback on Cem's previous lessons, both the positives and the room for improvement. The coach explained numerous teaching methods based on deductive and inductive reasoning to Cem. For instance, the coach introduced direct instruction and student-led discovery by showing the students how it is done. The coach then demonstrated how to start the lesson with direct questions to check the students' prior knowledge and then moved on to more interactive activities where students were asked to use GeoGebra to understand the properties of parallelograms. The coach further highlighted the importance of technology in the learning process, not just as a tool to display figures but also as a tool critical to the learning process. The coach recommended that Cem make the learning process more student-oriented; that is, students should actively participate in the learning process. The mentor stressed the importance of self-observation and made Cem think about the strategies used in the class and the effectiveness of the technologies. Cem's coach advised him to try various methods and tools to teach students and stressed that it is crucial to be ready for change and to be sensitive to students' reactions. Therefore, based on these factors, which include the coach's support, diversified teaching methods, focus on technology, student-centered approach, demonstration of reflective practices, and flexibility and adaptation, Cem improved his performance in *the instructional*

descriptor of the Teaching Theme. Cem's coaching strategies moved the students from *the exploring level* to *the advancing level*, thus helping students to think mathematically using diverse tools and techniques.

Regarding *the environment descriptor*, there were notable advancements in Cem's teaching approach from MCC 3 to MCC 4. The learning environment, which encompasses the roles of both students and teachers, organizational methods, handling of subject matter and concepts, and the purpose of integrating technology, demonstrated a more advanced and constructivist nature in MCC 4. In MCC 4, he refined his approach to technology-enhanced activities to maintain student engagement and foster self-directed learning of mathematics. The students actively participated in the lesson, directed their learning, and enjoyed the learning process. Compared to MCC 3, there were no passive listeners or note-takers. Cem mentioned this in the pre-conference interview of MCC 4 as follows;

> "In the post-conference interview of MCC 3, we discussed the parts that need to be improved about the learning environment and the role of students. In the lesson where I will explain the area and height of the parallelogram, I will organize and improve the learning environment accordingly."

When Cem was asked in post-conference 4 about the impact of technology on the role of students in the lesson, he replied as follows:

> "I encouraged students to learn both individually and in groups. There was much participation in the class. I tried to provide a learning environment in which students could manage and direct their learning through GeoGebra activities, and my use of technology acted as a tool that bridged my goals and actions. In other words, GeoGebra and other digital tools were tools for students to realize their role as discovering students."

All these could provide evidence that Cem's teaching practice in MCC 4 aligned with the following indicator of *the advancing level* regarding *environment descriptor* of Teaching Theme: "Manages technology-enhanced activities in a way that maintains student engagement and self-direction in learning mathematics." (Niess, Ronau, et al., 2009, p. 23). Niess, Ronau, et al. (2009) defined teachers at *the advancing level* for *the environment descriptor* as integrating technology into learning activities

about mathematics while ensuring the students' active and autonomous learning. At this level, teachers create and alter the groups where learning occurs for each student and the group. Cem also showed how he could use technology to enhance engagement and independent learning during a class held on MCC 4. As for the class activities, Cem used GeoGebra and WordWall quizzes to ensure active participation from the students. Using GeoGebra to explore the mathematical features and the spirit of rivalry in WordWall quizzes enhanced the students' motivation. The students' motivation levels were high and keen to stick to the set learning goals. Cem grouped the students and made sure that students teach each other and solve problems in groups. Cem encouraged the students to be self-directed and valued individual and group learning when he arranged the students into pairs and groups to solve the problems. This aligned with creating and modifying the learning groups to increase interest and comprehension.

A comparison with Cem's TPACK level in MCC 3 revealed his progression from *the exploring level* (level 4) to *the advancing level* (level 5) regarding *environment descriptor* in MCC 4. Cem's transition from *the exploring level* to *the advancing level* within *the environment descriptor* of the Teaching Theme can be associated with improving the Technological Pedagogical Knowledge (TPK) aspect of the TPACK framework. This dimension focuses on understanding the changes that can occur in teaching and learning processes and their results based on using particular technologies in specific ways (Mishra & Koehler, 2006). Cem's development in MCC 4 teaching practice, along with the help of the pre-conference discussions, suggestions, examples, and lesson plans that the coach offered, showed an improvement in the TPACK level within *the environment descriptor* of the Teaching Theme. The coach connected Cem's mathematical background, teaching perspective, and technological knowledge with the coach's views on mathematics teaching.

The coach was effective in supporting Cem's development of technology-enhanced classroom management skills in pre-conference 4, specifically concerning *the environment descriptor* of the Teaching Theme. The coach gave general instructions, presented the best practices, and gave specific feedback concerning combining the deductive and inductive teaching styles. This involved showing that there is a middle

ground between instructing the students and encouraging them to discover concepts on their own; in this case, through GeoGebra to introduce the height and area of parallelograms. The coach fostered a student-centered approach: this informed Cem to engage the students to actively participate in their learning by using GeoGebra on their own, presenting solutions, and participating in group discussions. This approach helped the students own up to the learning activities. To enhance the students' motivation and level of independence, the coach incorporated modern technologies like WordWall that enabled the students to engage in a quiz competition. The coach showed how learning groups can be formed and transformed and how the students should work in pairs and groups to solve problems during GeoGebra lessons. This encouraged the students to be independent and active students. This way, the coach encouraged reflection, which helped Cem point out weaknesses and modify actions. The coach focused on technology use in the classroom and how Cem could better manage the GeoGebra activities and the students' discovery process. Cem's coach supported him in designing GeoGebra activities to engage with the lesson objectives of finding the height and area of parallelograms. Cem also created and modified the learning groups in MCC 4 and appreciated the concept of learning as a group and as an individual. As for the teaching practice, the students fully managed the learning process through activities using technological tools such as GeoGebra, GeoBoard, and WordWall. The students could change misconceptions and systematic learning errors for the better either on their own or with their classmates' help by teaching them. In this regard, Cem acted as an exploratory director in MCC 4, focusing on the direction of the learning environment for the entire MCC in compliance with the lesson flow. He fluently moved from one activity, from one concept to another, and from one teaching tool to another without losing students' interest and making sure they progressed. Cem encouraged and supported students' involvement and independent studying of the topics related to parallelograms, height in parallelograms, and area of a parallelogram.

4.2.3.3. TPACK Level of Cem Regarding Teaching Theme After Mathematics Coaching

To determine Cem's final TPACK level of the Teaching Theme after he participated in four mathematics coaching cycles, his teaching practices, which focused on the

learning objective '*Problem solving related to the areas of compound shapes consisting of triangles, rectangles and parallelograms,'* were observed for three hours while teaching the 'problem solving related to the areas of irregular shapes consisting of triangles, rectangles and parallelograms'. To determine the final TPACK level, no pre-conference was held with Cem before the observed lesson, and no post-conference was held after the lesson. The researcher did not provide support for mathematics coaching to Cem. Thus, Cem prepared the lessons independently by reflecting on the knowledge and experiences gained in the four coaching cycles they participated in.

Cem's actual teaching practice after mathematics coaching indicated *the advancing level* of TPACK for the mathematics learning, environment, and *instructional descriptor*s of the Teaching Theme.

Regarding *the mathematics learning descriptor* after mathematics coaching, it was observed that Cem's teaching practice after mathematics coaching was very similar to that in MCC 4 regarding *the mathematics learning descriptor*. It was an innovative approach for Cem that students calculated the area of irregular shapes prepared in GeoGebra and tried to solve problems from the context of real life by transferring them to the dynamic learning environment of GeoGebra. Cem actively used the dynamic features of GeoGebra to create different shapes and offered students the opportunity to work on more examples. This way, the problems studied were solved using different solutions thanks to GeoGebra tools. GeoGebra was used as a learning, teaching, and problem-solving tool to translate students' problems and concepts into understandable forms accurately. Leveraging GeoGebra's versatility, Cem translated real-life challenges— calculating the area illuminated by a flashlight and the restaurant's seating area and hut design—into interactive GeoGebra simulations. By immersing students in these practical contexts, Cem facilitated meaningful connections between abstract mathematical concepts and their real-world relevance, igniting curiosity and motivation among his students. Cem's presentation of skill-based and real-life contextualized problems in the curriculum to his students by adapting them to technology enabled them to develop a different perspective on problem-solving with the advantages of GeoGebra's dynamic learning environment.

For example, in the tangram problem, the students solved the problem in the static environment, transferred the problem to the square worksheet in GeoGebra, and solved it using GeoGebra's various measurement tools and were able to check the accuracy of their results. The students also had the opportunity to make geometric conjectures and test them in GeoGebra.



Figure 92. A student while solving a problem through GeoGebra

Cem used the immense potential of GeoGebra as a catalyst for enhancing students' problem-solving skills. Embracing GeoGebra not solely as a supplementary tool but as a central component of his teaching, Cem utilized its capabilities to engage and empower his students in problem-solving. Cem leveraged its functionalities to create interactive visualizations that facilitated problem-solving in novel and effective ways. One of the foremost advantages offered by GeoGebra was its ability to transform abstract mathematical concepts into tangible, interactive models. In teaching area problems, Cem utilized GeoGebra to provide students with dynamic representations of geometric shapes, allowing them to manipulate dimensions and observe real-time changes in area calculations. Through this hands-on exploration, students gained more profound insights into the computation of areas, fostering a more intuitive understanding of mathematical concepts.

All these could provide evidence to place Cem at *the advancing level* of TPACK for *mathematics learning descriptor* of the Teaching Theme since Cem's actual teaching practice following mathematics coaching aligned with the following indicator: "Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students." (Niess, Ronau, et al., 2009, p. 23). Niess, Ronau, et al. (2009) defined *the mathematics learning descriptor* for teachers at *the*

advancing level as the active and consistent use of technology for enhancing mathematics learning and teaching meanings that the students can easily understand. The teacher at this level is expected to provide new strategies on how students can learn mathematics using technology. Thus, Cem's characteristics were consistent with those of teachers at this level. Cem frequently used GeoGebra and Geoboard virtual manipulatives to enhance students' geometry learning. He employed these technologies to give the students practical and hardly perceptible images of the area of various shapes. Cem also used GeoGebra to move shapes around and solve for areas, making the students easily comprehend the lessons. For instance, he helped students compute the area of irregular geometric figures by dissecting them into simpler figures, thus making the abstract concrete. Cem was able to help his students and present various techniques to apply to solve a certain kind of problem using technology. His capacity to provide several ways of finding the area of a shape, including using GeoGebra's dynamic capabilities and Geoboard activities, illustrated his awareness of technology integration in teaching.

Concerning *the instructional descriptor* of MCC 4, Cem's teaching practice after mathematics coaching was very similar to that in MCC 4 in terms of *the instructional descriptor*. Cem initiated the lesson with a dynamic group discussion on area calculation problems involving parallelograms, rectangles, and triangles. He skillfully prompted students with thought-provoking questions to spark critical thinking and encouraged them to contribute their insights and prior knowledge about these geometric shapes. Through inquiries about similarities, differences, and area calculation methods, Cem fostered a collaborative learning environment, setting the stage for active participation in subsequent activities.

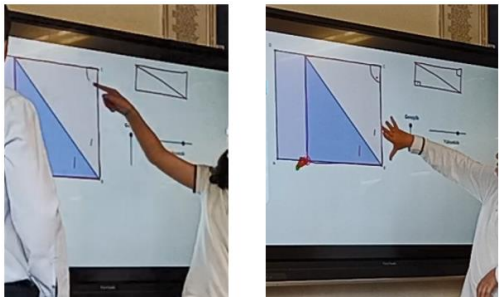


Figure 93. Students while sharing their geometrical ideas using GeoGebra

Throughout the lesson, Cem implemented an innovative teaching method where various students individually tackled problems on the board. This strategy promoted active engagement and facilitated peer learning and cooperation. Cem guided each student through the problem-solving process, using insightful questions to enhance comprehension and urging them to articulate their thought processes to their peers. By showcasing diverse solution approaches and perspectives, Cem cultivated a dynamic learning atmosphere where students benefitted from each other's methods and strategies. Cem utilized GeoGebra's dynamic capabilities to enrich the problem-solving process. Students could manipulate dimensions and observe immediate changes in area calculations by creating interactive visualizations of geometric shapes and properties. This hands-on engagement with dynamic models deepened their comprehension of dimension-area relationships, reinforcing mathematical concepts. Cem further enhanced the learning experience by integrating hands-on activities and real-life scenarios. Cem fostered a dynamic learning atmosphere by employing diverse instructional techniques, such as student-led problem-solving, probing inquiries, and real-life applications. By incorporating collaborative activities, interactive discussions, and hands-on exploration with GeoGebra, students actively engaged in tackling area calculation problems involving geometric shapes. As a result, they honed their problem-solving abilities, enhanced their grasp of mathematical concepts, and nurtured a genuine enthusiasm for learning.

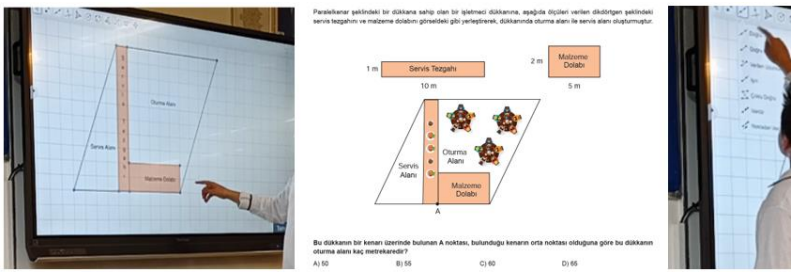


Figure 94. A student while solving another problem through GeoGebra

Cem's commitment to GeoGebra and technology in teaching marked a significant shift in educational practice, emphasizing problem-solving skills. Through GeoGebra's versatile visualization, exploration, and collaboration applications, Cem went beyond traditional teaching, fostering the development of robust problem-

solving abilities in his students. Cem utilized GeoGebra effectively in teaching area calculation problems through both inductive and deductive strategies. With GeoGebra, students engaged in hands-on exploration of geometric shapes, observing real-time changes in dimensions and area calculations. This facilitated inductive reasoning as students observed and identified patterns and generalized rules for calculating areas. Additionally, GeoGebra supported deductive reasoning by providing a platform for logical proofs and explanations based on established mathematical principles. Through guided discussions and logical arguments backed by GeoGebra visualizations, Cem helped students deepen their understanding of area calculation problems and develop deductive reasoning skills.

Therefore, all these could provide evidence to place Cem at *the advancing level* of TPACK for *the instructional descriptor* of the Teaching Theme since Cem's actual teaching practice following mathematics coaching aligned with the following indicator: "Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics." (Niess, Ronau, et al., 2009, p. 23). Niess, Ronau, et al. (2009) pointed out that the teachers at *the advancing level* for *the instructional descriptor* apply several instructional methods (including deductive and inductive methods), combining them with technologies to involve students' thinking about mathematics. These teachers allow the students to easily transition from one tool to another and portray a passion for mathematics. Cem's characteristics can be associated with teachers at this level. Cem employed the deductive and inductive approaches to teaching. He began with the whole group instruction, followed by the student's independent work with the help of GeoGebra. After mathematics coaching, it was observed that Cem employed diverse teaching methods alongside technology to stimulate students' engagement with mathematical concepts during his lessons. To illustrate, he gave a skill-based problem, let the students make predictions on the solution, and through GeoGebra, they tried other possible approaches. Cem's smooth movement from one tool to another, such as GeoGebra, Geoboard, and WordWall quizzes, exhibited his ability to involve students in various tools. This approach aided the students in enhancing their comprehension of mathematical concepts by presenting them with various approaches and techniques for solving problems.

Cem’s enthusiasm for mathematics was seen in his thorough explanations and the fun group activities he had the class do. He urged the students to reflect on their approaches and to share them with the class, creating a climate of learning that valued depth and analysis.

Regarding *the environment descriptor* of the Teaching Theme, Cem's teaching practice following mathematics coaching was similar to that in MCC 4. GeoGebra enabled Cem to use different collaborative tools that helped to involve the students and create interaction between them. Using GeoGebra, Cem engaged the students in group work and assigned them group activities and problem-solving where students could share ideas, help each other, and solve problems together. Cem also ensured that students worked in groups and shared knowledge, fostering a collaborative learning culture. Cem formed and reformed learning groups during the lesson by mathematics coaching since he understood the importance of the students engaging in individual lessons and group activities.

The students were encouraged to own their learning process throughout his teaching process, especially using technology applications such as GeoGebra and Geoboard. Here, they could correct the misconceptions and procedural errors either through self-adjustment or with the help of their friends through peer teaching. Learning became an active process, and students were assigned various tasks that involved exploration and problem-solving with Cem’s help. As a facilitator, Cem could be a student’s guide and control the learning process's dynamics based on the lesson's course. He could move from one activity, concept, or tool to the next smoothly and coherently, promoting progressive advancement in solving problems. Cem’s orientation towards students and their learning, including encouraging them to be proactive students, was most noticeable in his explanation of such topics as area calculation problems.

Considering the instructional strategies Cem employed, he created a classroom setting where the students were not mere receivers of information but rather players in the learning process. Cem enabled students to use technology, work in groups, and learn independently to develop into problem-solvers and critical thinkers. Thus,

Cem's approach revealed the possibilities of student-centered teaching; that is, students are not only passive recipients of the knowledge but also its creators. The students themselves recognized students' errors and misconceptions in problem-solving, and they tried to correct them on their own or with help from their classmates. This made the students feel they owned their results, thus making them responsible for their learning. Thus, Cem used an effective teaching strategy that engaged the students and allowed them to participate actively in the learning process. As a result of encouraging students to explore, work in groups, solve problems, and think critically, Cem offered a stimulating and productive learning experience in his classroom. All the students participated in the class by contributing to the discussion, asking questions, explaining, and justifying their ideas. These interactions improved their knowledge of mathematical concepts and also boosted their speaking and critical thinking abilities.

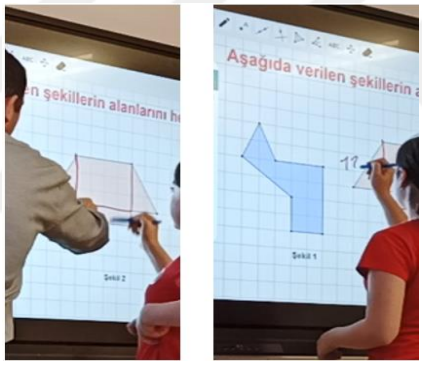


Figure 95. Cem while guiding a student during area calculation of irregular shapes

Therefore, all these could provide evidence to indicate that Cem was at *the advancing level* of TPACK for *the environment descriptor* of the Teaching Theme as his teaching practice following mathematics coaching was consistent with the indicator of: "Manages technology-enhanced activities in ways that maintain student engagement and self-direction in learning mathematics." (Niess, Ronau, et al., 2009, p. 23).

Niess, Ronau, et al. (2009) pointed out that the teachers at *the advancing level* for *the environment descriptor* collaboratively facilitate the technology-supported activities that ensure the students' autonomy and interest in learning mathematics. All these

teachers embraced individual and group learning to teach the students. Cem's teaching practice, observed after the mathematics coaching, was consistent with the characteristics of teachers at *the advancing level* of *the environment descriptor* of Teaching Theme. Likewise, Cem engaged the students actively by using activities and discussions in the class. He employed the dynamic aspects of GeoGebra to grab students' attention and make them participate in the learning process. Cem wanted the students to be proactive in their learning and gave them GeoGebra and other tools to work on problems independently or in pairs. This approach enhanced the student's initiative since they had to find concepts independently and check their answers.

Cem used learning groups to encourage students to teach each other and solve problems in groups. For instance, while carrying out the WordWall quiz activities, the students were divided into groups of two, where they were required to brainstorm and come up with solutions, hence learning individually and as a group.

4.2.3.4 The Progress of Cem in TPACK Regarding Teaching Theme Through Mathematics Coaching in Learning Area of 'Measurement of Area'

The figure 96 presents Cem's TPACK levels for *mathematics learning, instructional,* and *environment descriptors* of Teaching Theme based on his teaching practices in technology-enriched lessons during MCC 3, MCC 4, and after mathematics coaching (After MC). Cem reached the highest level in mathematics learning, *instructional, environment, and professional development descriptors* according to the TPACK development model throughout the mathematics coaching cycles.

The level of professional development descriptor of the teaching theme was determined from the TPACK interview conducted before and after mathematics coaching. The Measurement of the Area learning domain was analyzed within MCC 3, MCC 4, and post-mathematics coaching teaching practices. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

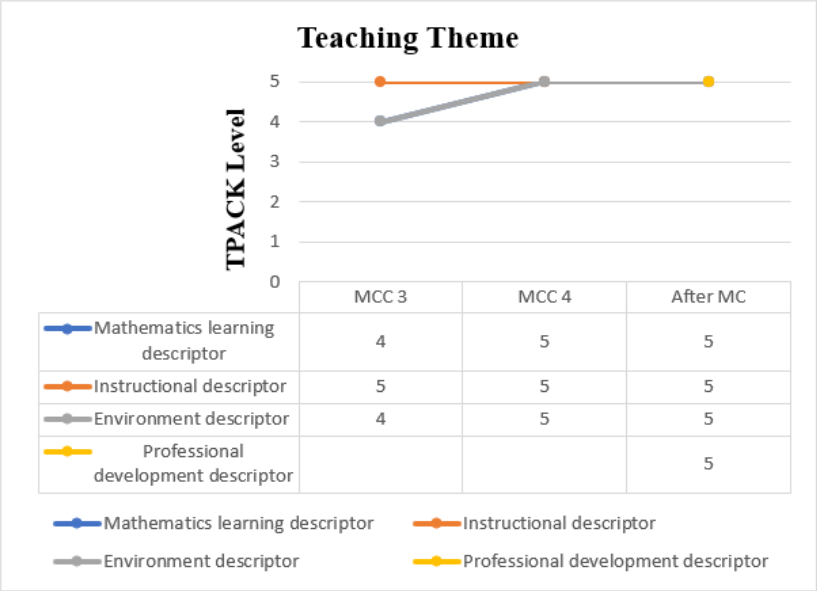


Figure 96. The progress of Cem of the Teaching Theme in the learning area of Measurement of Area

Mathematics learning descriptor focuses on how technology facilitates mathematics learning, such as reinforcing concepts, enhancing understanding, or engaging students in high-level thinking activities. Through his involvement in the mathematics coaching cycles, Cem demonstrated advancement in *Mathematics learning descriptor*.

Cem's level in the TPACK development model's *Mathematics learning descriptor* at MCC 3 indicated *the exploring level* (level 4). Cem engaged students in high-level thinking activities such as decision-making, making inferences and mathematical connections for learning height in a triangle, and area calculation relation in a triangle using technology as a learning tool. Cem taught the students in a way that contributed to developing their high-level thinking skills by asking them questions. At the same time, they worked on GeoGebra activities, answering the questions asked by the students and testing their mathematical conjectures and ideas. Additionally, Cem exchanged classroom-proven, technology-driven teaching strategies, concepts, and achievements with his colleagues.

Cem's level in the TPACK development model's *mathematics learning descriptor* at MCC 4 and after mathematics coaching indicated *the advancing level* (level 5), the

highest level of the Teaching Theme regarding *mathematics learning descriptor*. Cem actively and consistently embraced technology as a valuable tool for teaching and learning various mathematical concepts, such as calculating the area of parallelograms, determining their heights, and solving problems involving the areas of triangles, parallelograms, squares, and rectangles. He adeptly utilized GeoGebra's dynamic features and multiple representations to effectively translate complex mathematical ideas into formats that were accessible and understandable for students. As a result of his innovative approach, students began to perceive Cem as a go-to resource for novel methods of using technology to enhance their mathematical understanding and learning experiences.

Instructional descriptor assesses the teacher's instructional strategies when integrating technology into mathematics teaching, ranging from direct instruction to more student-centered approaches. Through his involvement in the mathematics coaching cycles, Cem demonstrated advancement in *instructional descriptor*. Cem's level in the TPACK development model's *instructional descriptor* at MCC 3, MCC 4, and after mathematics coaching indicated *the advancing level* (level 5). Cem utilized a diverse range of instructional approaches (peer teaching, learning by discovery, group work, large group discussion), incorporating both deductive and inductive strategies alongside technologies such as GeoGebra, GeoGebra homework assignments, and Khan Academy to actively involve students in exploring concepts such as triangle height and area calculation, as well as parallelogram height and area calculation relationships, along with various area calculation problems. He skillfully guided students in seamlessly transitioning between different tools while demonstrating a keen emphasis on and enthusiasm for a thorough understanding of geometry topics.

Environment descriptor examines how the classroom environment is managed when technology is incorporated, including the level of control exerted by the teacher and the degree of student engagement and self-direction. Cem's level in the TPACK development model's *Environment descriptor* at MCC 3 indicated *the exploring level* (level 4). Cem delved into various instructional strategies, encompassing deductive and inductive methods, synergistically integrating technology to captivate students' interest and cognitive engagement in exploring triangle height and area calculation

concepts. Central to his pedagogical approach was integrating GeoGebra as a main technological tool. Through GeoGebra's interactive platform, Cem fostered an environment where students could actively manipulate geometric figures, visualize mathematical relationships, and experiment with different scenarios, deepening their understanding of triangle height and area calculation concepts. With GeoGebra's versatile and dynamic features, including dynamic representations and interactive simulations, Cem allowed students to explore and construct their understanding of geometric principles actively, enhancing their conceptual comprehension and problem-solving skills.

Cem's level in the TPACK development model's *environment descriptor* at MCC 4 and after mathematics coaching indicated *the advancing level* (level 5). Cem skillfully utilized technology, specifically GeoGebra, to enhance student engagement and foster self-directed learning in area calculation, area calculation relation of parallelogram, and parallelogram height. Through GeoGebra's dynamic feautres, students actively explored mathematical concepts, manipulating models and engaging in interactive simulations. Additionally, Cem organized learning groups where individual and collective learning were valued, leveraging GeoGebra's collaborative features to facilitate group discussions and problem-solving. In summary, GeoGebra played a crucial role in Cem's approach, empowering students to take ownership of their learning while promoting collaboration and exploration in mathematics.

4.2.4. Access Theme

The access theme comprises three descriptors: Usage, barrier, and availability. *The usage descriptor* assesses whether the teacher permits students to utilize GeoGebra or any other technology. The barriers descriptor highlights how the teacher addresses difficulties linked to technology integration. Moreover, *the availability descriptor* examines how technology provides access to higher levels of mathematics and more significant opportunities for student investigation (Niess, Ronau, et al., 2009). The following sections analyze Cem's TPACK regarding the access theme according to the TPACK Development Model.

4.2.4.1. TPACK Level of Cem Regarding Access Theme in the MCC 3

To determine Cem's TPACK level of the Access Theme at the third mathematics coaching cycle, which focused on the learning objective '*Establishes the area relation of a triangle and solves related problems,*' his teaching practices were observed for three hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 3 (MCC 3) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice in MCC 3 indicated *the advancing level* for usage and *barrier descriptors,* while *the exploring level* for *availability descriptor* of TPACK of the Access Theme.

In MCC 2, Cem allowed students to utilize technology to delve into specific mathematical topics, such as exploring different types of angles (adjacent, opposite angles, and so on). During the actual teaching practice of MCC 2, Cem ensured that technology was readily available and accessible to students, actively encouraging its use for mathematical exploration throughout the class. Furthermore, Cem's instructional approach, as indicated by the Access theme in MCC 3, extended to permitting students to incorporate technology into every aspect of the mathematics class (from the introduction to the lesson to the assessment and evaluation of the learning). This was evident in Cem's integration of technology into various learning domains and in his and his students' use of GeoGebra as a representation of concepts, a dynamic teaching and learning tool for conceptual learning, and a tool for assessing student learning. In the observed teaching practices of MCC 3, it was evident that Cem viewed technology as an opportunity to challenge preconceived notions regarding the scope of mathematical concepts students could grasp. Using GeoGebra innovatively (discovering the area of a triangle from the area of a rectangle) was an opportunity for the students to master geometrical concepts.

When Cem was asked at the pre-conference of MCC 3 what he intended to achieve by having students use technology in the classroom, he responded as follows:

"In our previous discussions on math coaching, we have extensively explored the dual role of technology as both a learning aid and a teaching tool, accompanied by practical examples. Through my research, I have delved into various instances, lesson plans, and technology-driven activities that illustrate the seamless integration of technology across all mathematical domains. Students must uncover concepts and their interrelationships through technology carefully selected to complement the subject matter and instructional approach. This approach can lead to diverse learning scenarios. For instance, when tackling the concept of polygons, students may express curiosity about the geometric progression towards a circle as the number of sides increases. What if it took on such a form? "Let us explore this," the teacher encourages the class. In essence, leveraging technology allows students to experiment with their ideas, validate mathematical hypotheses, and engage in challenging learning experiences."

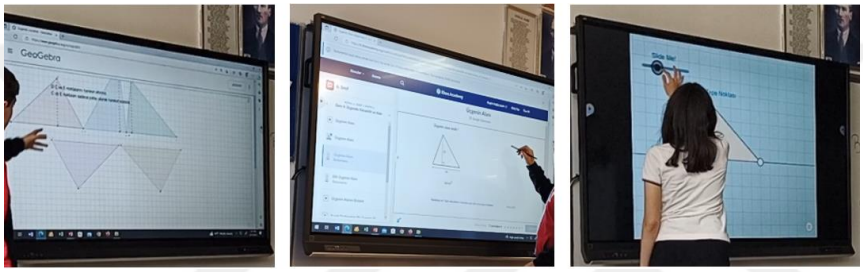


Figure 97. Students while engaging in technology in the MCC 3 lesson

Cem's statements in pre-conference 3 were consistent with his MCC 3 lesson teaching performance. Cem and his students spontaneously constructed triangles to experience challenging situations and to test different students' mathematical estimations and ideas. They examined heights and areas in these triangles. In this way, Cem allowed students to master the fundamental concepts of the lesson through the dynamic features of GeoGebra. When Cem was asked about the students' use of technology in the lesson in post-conference 3, he replied as follows:

"The students used the technology actively from the beginning to the end of the lesson. They reached generalizations regarding the area relation of triangles with GeoGebra activities, tested their geometric ideas, and had the chance to correct their misconceptions. In addition, they used technology to gain fluency in the subject and concepts and to reinforce what they had learned. In short, I considered technology an opportunity for all these and used it as a tool for students' conceptual learning."

All these could provide evidence to place Cem at *the advancing level* of *the usage descriptor* as Cem's actual teaching practice in MCC 3 and his statements at the pre-

conference and post-conference 4 aligned with the following indicator: “Permit students to use technology in every aspect of mathematics class.” (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) asserted that teachers at *the advancing level* of *the usage descriptor* allow students to use technology in every aspect of the mathematics classroom. Teachers at this level see technology as a tool that can be used to question existing paradigms regarding the extent of mathematical comprehension. The following are the features of the contextual features of teaching, which Cem’s teaching practice at MCC 3 embodied. Cem allowed students to use technology in every aspect of the mathematics lesson. For instance, he incorporated GeoGebra activities to help students discover the heights of triangles and find the location of height in various triangle configurations. Cem’s approach enabled the students to engage the geometric concepts dynamically, which helped them achieve a deeper understanding of the subject through practical activities. Cem used the GeoGebra software to explore the concept of triangles’ heights; students altered the shapes and watched the changes in height. This approach engaged the student’s attention to the concept through practical and visual methods. Cem allowed different students to write their answers on the board and use technology to explain them and enhance learning. He gave students GeoGebra assignments and provided videos for students to follow up on the work at home. The integration of technology in every activity in the mathematics class showed that the level of teaching had risen where technology was used to challenge the student’s knowledge of mathematics.

When compared with the TPACK level in the MCC 2 *the usage descriptor*, it was concluded that Cem progressed from *the exploring level* (level 4) to *the advancing level* (level 5) in MCC 3. The development in *the usage descriptor* might be related to the improvement in Cem’s Technological Pedagogical Knowledge (Mishra & Koehler, 2006), which was characterized as the understanding of how pedagogical practices can be changed when particular technologies are used in specific ways (Mishra & Koehler, 2006). Thus, it might be concluded that Cem continued to build his knowledge and skills in the appropriate use of technology support for teaching and learning mathematics so that students can apply technology in all areas of mathematical learning.

At the pre-conference of MCC 3, the coach was highly influential in enhancing Cem's performance, particularly in *the usage descriptor* of the Access Theme. The coach informed Cem that he should integrate technology into the mathematics lessons he will teach. In this particular situation, the coach presented the possibilities of how technological tools could be incorporated into the learning process so that it would not interfere with classroom functioning and would improve it. The coach gave good practical examples of how technology can be used in mathematics teaching. This comprised an illustration of how tools like GeoGebra, Khan Academy, and WordWall could assist in delivering several mathematical concepts, including the area relation of a triangle. Cem's coach advised the latter to allow the students to use technology in every facet of their mathematics learning, from discovery to assessment. This way, students were allowed to use technology for virtually every aspect of the learning process, thus improving their foundation and application of mathematical knowledge. In addition to that, the coach also stressed the aspect of giving feedback as often as possible when using technology in teaching. Cem was advised to use technology to track the students' progress and give them real-time feedback, thus enhancing the learning process. Accordingly, Cem's level of TPACK was positively impacted by the pre-conference discussions, recommendations, illustrations, and instructional materials. Essentially, the coach effectively translated their mathematical, pedagogical, and educational expertise and their convictions about mathematics and its instruction to Cem. This holistic approach cultivated an atmosphere where technology access and utilization were provided and actively endorsed for student exploration in mathematics.

Concerning *the barrier descriptor* of the Access Theme of the TPACK development model, Cem's approach to addressing and minimizing obstacles to technology integration in the actual teaching practice of MCC 3 was more advanced than how he did in MCC 2 teaching practice. While Cem solely recognized challenges for teaching mathematics with technologies and explored strategies and ideas to minimize the impact of those challenges in MCC 2, Cem put the knowledge, ideas, and strategies from his research and coach's suggestions into practice in his MCC 3 teaching practice. Cem recognized the challenges of teaching with technology and demonstrated his readiness to address them through extensive planning and

preparation to maximize the use of available resources and tools. Furthermore, by leveraging technology in these ways, Cem attempted to expand the accessibility and depth of mathematical concepts, such as the area calculation of a triangle, providing students with engaging and interactive learning experiences that promote deeper understanding and retention. He effectively employed methods for technology management issues, which he created based on the coach's suggestions and his research.

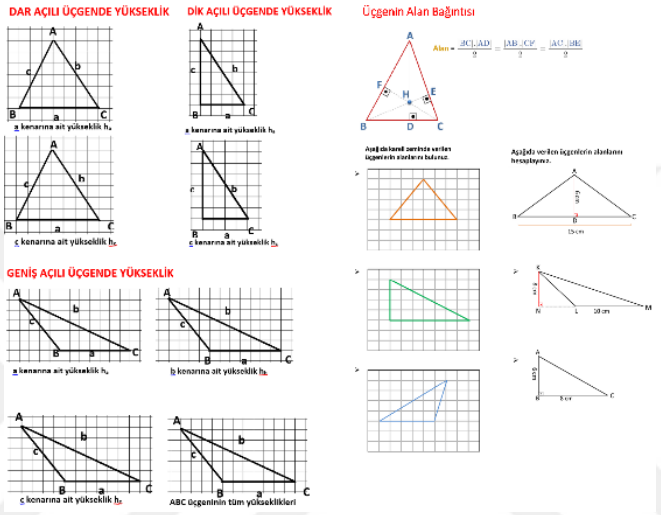


Figure 98. A section from Cem's printed material worksheet for the topic in MCC 3

Cem ensured that the technologies he selected were understandable and user-friendly for students. When students encountered difficulties, such as momentary internet disruptions or issues with scrolling through GeoGebra worksheets, he assisted them calmly and professionally. In unforeseen situations arising from the dynamic nature of GeoGebra, he adeptly refreshed or restarted activities to keep the lesson on track. Throughout MCC 3, Cem encountered no challenges or obstacles in teaching with technology. As students were accustomed to frequent technology use during coaching sessions, they navigated technological tools effortlessly. Moreover, before each lesson, Cem meticulously inspected the classroom's computer and smart board. He initiated GeoGebra activities by accessing the file from a portable USB drive or utilizing a website link within the lesson presentation. He proactively anticipated potential technical glitches (e.g., internet connections, power outages, hardware or software malfunctions). Cem also maintained a worksheet to distribute to students in

the event of a power outage, ensuring continuity of instruction. Before lessons, he reminded students to bring essential tools like protractors, compasses, and rulers required for the subject matter.

When questioned during pre-conference 3 about potential challenges he could encounter while using technology in the lesson and the precautions he had implemented to address them beforehand, Cem replied as follows;

> "Throughout our collaborative coaching cycles and my teaching practice, I have recognized the potential challenges my students and I may encounter when incorporating technology into instruction. Nevertheless, I believe I can effectively address these challenges through meticulous planning and thorough preparation, optimizing the utilization of available resources and tools. Typically, a foreseeable challenge in my use of technology could involve software malfunctions or internet connectivity issues. While I have not encountered such problems thus far, if they arise, the school's technical support staff promptly attend to resolve the issue within minutes. Additionally, I strategize and rehearse the content and sequence of activities planned for the lesson, outlining the topics to be covered and the questions to be posed well before stepping into the classroom. For instance, I simulate GeoGebra activities and other technological materials, familiarizing myself with their functionality as if conducting the lesson in person. Concerning student engagement, certain difficulties may arise when students navigate technology. In such instances, I provide clear and understandable instructions to guide them through the tasks. If students encounter challenges, I rephrase the expected actions in GeoGebra activities (such as dragging corners, sliding sliders, adjusting angles, etc.) to facilitate understanding and progress."

When Cem was asked about the precautions he took in post-conference 3 to address possible obstacles that might hinder the use of technology in the lesson, he replied as follows:

> "In the pre-conference, we discussed this issue with you in detail. After detailed preparation and planning in the pre-conference, I was prepared to eliminate all possible obstacles during the lesson. I gave the students clear and precise instructions and ensured they did not deviate from the learning objectives using GeoGebra or other digital tools. In this way, the students used the technology only to expand the concepts they had learned and to think about the next stage. I also guided them continuously while they were using the activities. Apart from this, I planned what could be done to solve technical problems. I backed up the activities to my portable memory so that they would work without the internet, as there might be situations such as

> internet outages. I also printed worksheets and made the necessary checks in the classroom before the lesson."

Cem's teaching practice aligned with the following indicator regarding *the advancing level* of *the barrier descriptor* of the Access Theme: "Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) expressed that teachers at *the advancing level* of *the barrier descriptor* understand the challenges that come with integrating technology into teaching and learning. Therefore, the teachers use extended planning and preparation, properly utilizing the available resources, tools, and other strategies to overcome such challenges. They utilize devices to make mathematics concepts more understandable for students. Cem displayed the characteristics of a teacher at the next level of practice by pointing out the issues encountered in integrating technology into teaching. He made substantial preparations to utilize technological tools because he clearly described and demonstrated how to do the "GeoGebra-Height in a Triangle" homework assignment, accompanied by a video the coach posted on YouTube. The second aspect that can be attributed to Cem's planning skills was his systematic incorporation of GeoGebra into the lessons on the area relationship of a triangle.

Compared with the TPACK level in the MCC 2 *the barrier descriptor*, it was concluded that Cem progressed from *the exploring level* (level 4) to *the advancing level* (level 5) in MCC 3. It could be suggested that the improvement of Cem's Technological Knowledge (TK) and Technological Pedagogical Knowledge (TPK) (Mishra & Koehler, 2006) in *the barrier descriptor* could be observed. Technological Knowledge (TK) is the knowledge about the technology, including understanding the tools used in teaching. At the same time, Technological Pedagogical Knowledge (TPK) is the knowledge concerning the use of technology in teaching for effective learning (Mishra & Koehler, 2006). This might suggest that Cem enhanced his ability to identify and address the issues related to the use of technology in teaching. Thus, he learned how to strategically prepare to address these tools to enhance the student's understanding of mathematical concepts. Cem's

progression in *the barrier descriptor* might refer to his enhanced TK and TPK (Mishra & Koehler, 2006). TPK refers to understanding how teaching and learning can be transformed when particular technologies are employed in specific ways (Mishra & Koehler, 2006). This signifies that Cem has enhanced his capacity to effectively integrate technology into his pedagogical strategies, enabling students to utilize technology comprehensively across all facets of the mathematics class.

During the pre-conference of MCC 3, the coach helped Cem identify and solve the problems of using technology in the teaching process. The coach helped him solve these challenges by establishing a more detailed planning and preparation process. The coach, therefore, worked with Cem to determine the possible problems that could arise from integrating technology, such as technical problems, students' experience with the tools to be used, and the extent to which the technology supports the learning objectives stated in the curriculum. The coach stressed that the challenges can be met only with more planning and preparation than in the previous cases. This entailed developing lesson plans incorporating technology, ensuring all the needed resources and tools were available, and developing plans in case of technical hitches. The coach recommended the efficient use of the available resources and tools. This entailed utilizing tools like GeoGebra on dynamic geometry and Khan Academy for problem-solving and quizzes.

As for *the availability descriptor*, Cem's technology integration in MCC 3 facilitated a more advanced exploration of mathematics, catering to a broader range of students than his approach in MCC 2. Cem's utilization of technology allowed students to delve into and apply fundamental concepts. He evaluated students' comprehension by incorporating various representations and connections of these concepts.

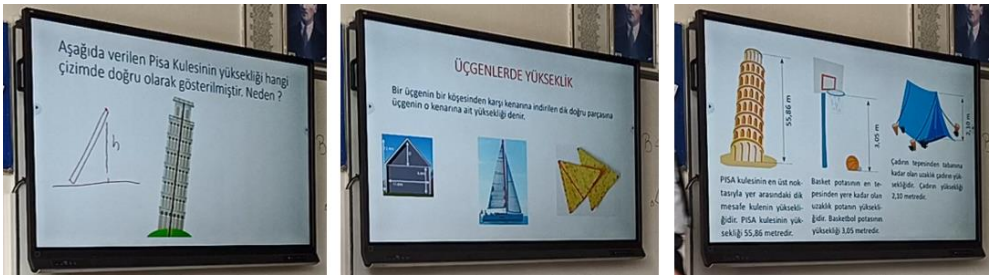


Figure 99. The real-life examples from the MCC 3 lesson as multiple representations

Additionally, he utilized visual aids, real-life examples, and verbal explanations to present concepts with a variety of representations, thereby enabling students to relate them to everyday scenarios. Moreover, unlike in MCC 2, Cem used the spreadsheet feature in GeoGebra to guide students in discovering how the base and height of a triangle are used to calculate the triangle area and the area relation of a triangle in MCC 3. The spreadsheet, which used sliders to simultaneously calculate the areas of triangles with different side lengths and heights, contributed to students' conceptual learning and allowed them to interpret the area relation of a triangle. In other words, Cem and his students spontaneously formed different situations with different triangles, calculated the area of a triangle, and interpreted the results using tables. In GeoGebra, Cem and his students used sliders to manipulate triangles, varying their areas, side lengths, and heights. This approach encouraged exploration of the topic at a more advanced level and from diverse perspectives. Leveraging GeoGebra's dynamic capabilities, Cem and his students could analyze numerous triangles quickly, enhancing their ability to generalize and deepen their conceptual understanding.

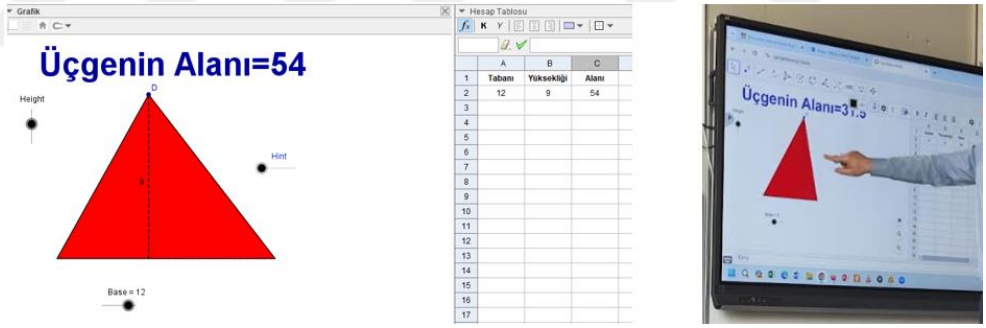


Figure 100. An activity using the spreadsheet feature of GeoGebra from the MCC 3 lesson

When Cem was asked at the MCC 3 pre-conference what the function of technology would be in the use of multiple representations and how he would utilize technology in transitions between representations, he replied as follows;

> "The function of technology in multiple representations will allow me to represent the concept in many ways and switch between them. For example, I will start the lesson using daily life situations and related visuals. After that, explaining the concept verbally will support students' understanding. Then,

> they will be able to see both the graphic view of a triangle created in GeoGebra and its values, such as area, perimeter, side length, and height, in the algebra view. We will also have a study using spreadsheets. Here, they will also see a table representation. In this sense, technology will serve as a tool that will allow me to easily switch between daily life context, graphical, tabular, visual, and algebraic representations."

Moreover, Cem replied as follows when he was asked in post-conference 3 how he utilized technology in the lesson to make multiple representations and transitions between representations.

> "I used it in a variety of ways. Dynamic examination of basic concepts through technology allowed these concepts to be represented with multiple representations. We started with images and videos of examples of daily life. From here, we examined the area of the triangle in the graphic view of GeoGebra, that is, on the blank worksheet. We also examined the heights of different triangles based on their angles. Thanks to the spreadsheet view, I benefited from teaching the area formula. Here, students noticed the relationship between base, height, and area. They established the formula themselves. They also viewed area-related values in GeoGebra as symbolic via algebra view in the left side of GeoGebra interface."

Cem's teaching practice demonstrated the use of technology to explore key mathematical topics with multiple representations, in alignment with the following indicator of *the exploring level* of *the availability descriptor* of the Access Theme: "Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) defined teachers in *the exploring level* of *the availability descriptor* as using technology to inquire and experiment on the essential mathematical concepts and their relationship and how these can be represented in various forms. For example, situations that give rise to the system of equations are given, the system is solved, and the solution is presented graphically in tables, symbols, and data. The features of *the exploring level* can be observed in Cem's teaching practice in MCC 3. He used technology to support the identification of concepts, their use, and evaluation of significant topics, as well as to provide different views of the concepts and their relationships. In the second lesson, where the teacher's goal was to teach the area relationship of a triangle, Cem used GeoGebra activities to help the students learn the relationship between the area of a rectangle and a triangle. In this way, students could see how height and base length

dynamically affect the triangles with the same base. Cem asked students to try various bases and heights of a triangle to estimate the area and then to explain the connection between the area and the other two sides of the triangle. This was done after seeking the area relation of a triangle through a spreadsheet view of GeoGebra. Compared with the TPACK level in the MCC 2 *availability descriptor*, Cem's teaching practice in MCC 3 provided evidence of his progression from *the adapting level* (level 3) to *the exploring level* (level 4) in *the availability descriptor* of the Access Theme. Cem's improvement in *the availability descriptor* might be indicated by his Technological Pedagogical Knowledge (TPK) development. TPK means comprehending how different technologies can improve or strengthen the teaching-learning processes (Mishra & Koehler, 2006). This means Cem has mastered using technology to identify, implement, and evaluate key mathematical concepts and ideas in different formats, thus enhancing students' understanding.

During pre-conference 3, the coach offered the following feedback to Cem to help him improve the availability of the descriptor. This directed him to research, apply, and reflect on the main concepts through the application of technology while using models of triangle heights and area of a triangle. The coach showed how he uses technology to discover and display different concepts using multiple representations. For instance, GeoGebra was applied to show how triangles' heights and areas worked dynamically and made it possible for students to visualize the relationship between various geometric characteristics. Thus, the coach supported Cem in applying technology to solve real-world issues. This entailed the use of GeoGebra to design shapes on the screen and manipulate them, hence providing students with a practical way of solving problems. Cem's coach prompted him to incorporate technology into the assessments. For instance, using GeoGebra and Khan Academy, Cem could easily test the student's understanding of key concepts by using dynamic assessments and receiving instant results and information about the students' learning.

4.2.4.2. TPACK Level of Cem Regarding Access Theme in the MCC 4

To determine Cem's TPACK level regarding the Access Theme at the fourth mathematics coaching cycle, which focused on the learning objective '*Establishes*

the area relation of a parallelogram and solves related problems,' his classroom teaching practices were observed for three hours. The pre-conferences, post-conferences, and mathematics lessons conducted within the Mathematics Coaching Cycle 4 (MCC 4) constituted the sources of all the quotations and dialogues in the following paragraphs.

Cem's actual teaching practice during MCC 4 indicated *the advancing level* for the *usage*, *barrier*, and *availability descriptor*s of TPACK of the Access Theme.

As for *the usage descriptor*, Cem allowed students to use technology at all lesson stages. These stages were an introduction, exploration of the concept, deepening the understanding, eliminating misconceptions, applying what has been learned, and assessing students' learning. Cem's students' purpose of using technology in MCC 4, his perspective on using technology in the lesson, and how often and how they used technology was similar to the MCC 3 teaching practice in many aspects. In MC4, students' purposeful utilization of technology led by Cem catalyzed several key objectives: discovery, making conjectures and testing them, mastery of geometrical concepts, and establishing relationships among shapes. Students actively engaged with GeoGebra's dynamic interface, employing its tools to unlock geometric insights. Through hands-on exploration, they manipulated shapes, adjusted parameters, and observed real-time transformations, all aimed at fostering a culture of discovery. By leveraging GeoGebra's interactive features, students embarked on a process of self-discovery, unraveling the intricacies of height in parallelograms, its relationship to the area, and how the area relation of a parallelogram is formed. This process deepened their understanding and instilled a sense of ownership and agency in their learning process.

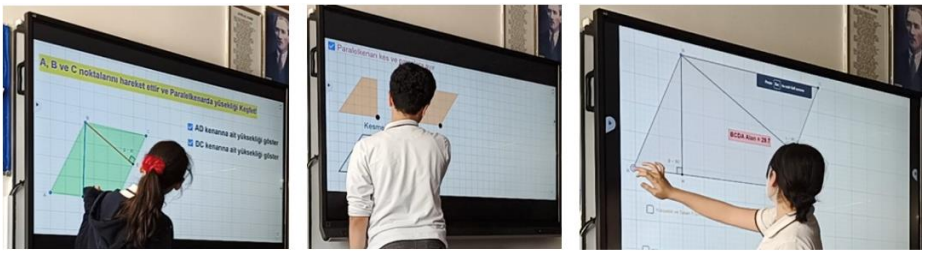


Figure 101. Students while using technology for discovery purposes in the MCC 4 lesson

Cem's strategic integration of GeoGebra enabled students to master geometrical concepts through experiential learning. By immersing themselves in interactive activities, students internalized abstract concepts by observing their tangible manifestations on screen. For instance, as they manipulated and formed shapes and observed corresponding changes in dimensions and area, students enhanced their spatial reasoning skills and developed an intuitive understanding of geometric principles. Through this process, technology was a powerful tool for scaffolding learning, facilitating mastery at each student's pace. Cem encouraged students to use GeoGebra to establish relationships among shapes, fostering critical thinking and inquiry. By exploring geometric properties across different figures, students discerned patterns and formulated conjectures within GeoGebra. This collaborative process nurtured a deeper appreciation for the interconnectedness of mathematical concepts and cultivated a culture of inquiry and exploration within the classroom. The students' use of technology provided Cem with ideas, feedback, and insights into their learning. Cem saw it as an opportunity for his students to reflect on their learning using the technology.

When Cem was asked at the pre-conference of MCC 4 what he intended to achieve by having students use technology in the classroom, he responded as follows:

> "I will ensure that students use technology at every stage of the lesson because I think the most important part of the learning experience is to concretize what is expressed verbally, to present it to the students in forms that will convince them to learn by doing and experiencing what is taught by using technology, to test it, to ask questions after testing it and to make inferences. For this reason, I will allow students to use technology from the introduction to the lesson to the online quiz stage with WordWall at the end of the lesson. Students enjoy this, and their learning experience is enriched."

Moreover, when Cem was asked about the student's use of technology in the lesson in post-conference 4, he replied as follows:

> "In the lesson, we did GeoGebra activities related to the height and area of a parallelogram, and we calculated the area using different bases and the heights of these bases. Students were actively engaged with GeoGebra throughout the whole lesson. They found answers to their why questions and correct misconceptions. They dynamically realized why the product of base

> and height is not divided by 2, as in the triangle. Instead of memorizing the area relation of the parallelogram, they manipulated the shapes and dynamically examined concepts such as area and height. For example, they felt that counting unit squares using squared paper cannot be continued and that a relation is needed for area calculation. I already taught this using a spreadsheet later in the lesson. The students found the area relation themselves. In short, I viewed GeoGebra and other digital tools as an opportunity for students to explore concepts and used them as a tool for students' conceptual learning."

All these provided evidence that Cem's actual teaching practice at MCC 4 and statements at pre-conference 4 aligned with *the usage descriptor* of *the advancing level* of the Access Theme: "Permit students to use technology in every aspect of mathematics class." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) stated that the teachers at *the advancing level* of *Usage descriptor* facilitate the use of technology in all aspects of mathematics teaching and learning. These teachers see technology as extending and creating new challenges for students' learning in mathematics. All the lessons conducted at this level are technologically enhanced, allowing the students to immerse themselves in the mathematical concepts. These characteristics were observed in Cem's teaching practice in MCC 4. *The usage descriptor* of TPACK in Cem's teaching practice in MCC 4 showed an improvement in the level of teaching. Cem always used technology in all aspects of his mathematics lessons. For instance, he employed GeoGebra activities to assist the students in discovering the idea of height in a parallelogram. Cem allowed the students to change the shapes' sizes and alter the bases' height. GeoGebra was used to set exercises in the form of constructions to be made by the students, and this made the students think critically and deeply about the topic and challenged their understanding of the topic. Also, Cem adopted other technological tools in the classroom to enhance the teaching and learning process, including GeoGebra, WordWall Quiz Show, and other online tools.

In pre-conference 4, the coach highlighted the need to ensure that Cem incorporates technology in the mathematics teaching process. The coach also discussed examples and techniques for using technology naturally in the various classroom tasks. The coach urged Cem to view technology as extending the possibilities of what kinds of mathematics students can tackle. Details that the coach shared about different lessons

that can be enriched with technology gave Cem a better idea of how students can be made to comprehend more. Thus, in MCC 4, Cem's teaching practice revealed a significant enhancement in the application of technology. He used GeoGebra and other technologies and applied them in the class to facilitate the students' visual discovery and learning of the concepts. Cem's approach allowed the students to apply technology to solve problems interactively and investigate, showing his progress to *the advancing level* in using descriptors.

As for *the barrier descriptor*, Cem's actual teaching practice at MCC 4 was similar to MCC 3 regarding *the barrier descriptor* of access theme. Cem acknowledged the difficulties of teaching with technology and showed his preparedness to tackle them by meticulously planning and preparing to make the most of the resources and tools at his disposal. Additionally, by utilizing technology in these ways, Cem aimed to broaden the accessibility and comprehension of mathematical concepts, such as calculating the area of a parallelogram, offering students interactive learning opportunities that fostered deeper and conceptual understanding. He adeptly employed strategies for managing technology-related issues, drawing from his coach's suggestions and research. Cem selected technologies that were easily understandable and user-friendly for his students. Whenever students faced challenges like temporary internet disruptions or navigating GeoGebra worksheets, he assisted them calmly and professionally. When unforeseen issues arose due to GeoGebra's dynamic nature, Cem adeptly refreshed or restarted activities to maintain the lesson's momentum. Throughout his experience with MCC 4, Cem encountered no obstacles or hindrances to teaching with technology. Thanks to the students' familiarity with technology from their previous math coaching, they felt at ease using tools like GeoGebra. Additionally, Cem diligently checked the classroom computer and smart board before each lesson, ensuring they correctly functioned. He accessed files from a portable USB stick or integrated website links into the lesson presentation to initiate GeoGebra activities. Anticipating potential technical disruptions like internet disconnection or power outages, Cem took proactive measures to prevent them. In the event of a power failure, he ensured continuity of learning by providing worksheets, square paper, and dot paper to distribute to students to examine the concepts of height and area in parallelogram.

When questioned during pre-conference 4 about possible challenges he and his students could encounter while using technology in the lesson and the precautions he had implemented to address them beforehand, Cem replied as follows;

> "In the fourth coaching cycle, I can now foresee what disruptions may occur when implementing a technology-supported lesson and what obstacles I or the students may encounter. As we have already discussed, almost all possible obstacles turn into obstacles that can be avoided with meticulous preparation and comprehensive planning. After each coaching cycle, I think about the things that hindered or prevented the use of technology in that lesson. We always do this in post-conference meetings with you. Then, for my next lesson, considering the obstacles in the previous lesson, if there were such a situation, of course, I would take precautions against it. As a teacher, I minimize the possible obstacles to using technology in the lesson by preparing and rehearsing before the lesson. I also prevent the things that may be obstacles to my student's use of technology with detailed instructions and explanations because I take care that my students do not use technology beyond the limit of achievement. In this way, I use the resources and tools for the current lesson most effectively."

Furthermore, when Cem was asked about the precautions he took in post-conference 4 to address possible obstacles that might prevent the use of technology in the lesson, he replied as follows:

> "Until the fourth coaching cycle, I integrated technology many times. As the coaching cycles progressed, my knowledge about possible barriers and how to resolve these barriers increased gradually. After a detailed preparation and planning process before the lesson, I was prepared to resolve all possible barriers to my technology integration during the lesson. For example, I know how to intervene if the student has difficulty using a GeoGebra activity. Alternatively, I uploaded my activities to the portable memory to run GeoGebra offline during an internet outage. In case of a power outage or other disruption, the school's technical team solves the problem quickly. I also printed a worksheet for this."

All these could provide evidence that Cem's actual teaching practice at MCC 4 and statements at pre-conference 4 aligned with the indicator of *the barrier descriptor* of *the advancing level* of the Access Theme: "Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) stated that teachers at *the advancing level* of *the*

barrier descriptor understand the difficulties of integrating technology into teaching and learning; hence, they plan and prepare to meet these difficulties. They make the most of resources and tools to guarantee that technology helps broaden the scope of mathematical knowledge introduced to students. The teachers at this level actively remove barriers and work towards improving the learning process. Cem's teaching practice in MCC 4 showed the enhancement of the TPACK for *the barrier descriptor*. Cem effectively found solutions to the issues that arose in teaching with technology due to proper preparation. He previewed the possible difficulties by giving clear instructions and constant guidance while using GeoGebra. For instance, Cem demonstrated to students activities that entailed finding the area of a parallelogram with the area of a rectangle, shifting triangles within the parallelogram, and counting unit squares. His proactive measures in planning and overseeing these activities made sure that technology was appropriately applied toward enhancing the students' comprehension of more mathematical ideas. Cem being knowledgeable about and capable of resolving the obstacles that could occur during technology integration enabled the students to engage in and understand more advanced mathematical concepts effortlessly.

In pre-conference 4, the coach introduced Cem to potential barriers that may encounter in applying technology and ways to address them. This encompassed strategies on how to use technology tools, how to solve common issues, and how to prepare for the worst. The coach stressed the necessity of further planning and preparation to maximize the available resources and tools. This involved the preparation of individual lessons, identifying the students' requirements, and having backup strategies for any possible issues with technology. Thus, in MCC 4, Cem showed that he understood potential obstacles and their solutions through proper preparation. Cem gave clear directions and encouraged the class using GeoGebra, giving feedback occasionally. His lessons were well planned, and he integrated various forms of technology to help students grasp mathematical concepts, showing that he progressed to *the advancing level* in *the barrier descriptor*.

Regarding *the availability descriptor,* due to the incorporation of technology in MCC 4, Cem could cover more topics in mathematics and address a larger number of

students than in MCC 3. Cem used technology so students could explore and implement basic principles. He assessed the students' understanding using different relationships and linkages of these concepts. Cem utilized multiple representations in the teaching practice of MCC 4 as follows: 1) Geometric Representation (using diagrams to explain concepts visually), 2) Algebraic Representation (expressing concepts in terms of variables and equations: b for base and h for height and formulating the area in b x h), 3) Numerical Representation (Providing numerical examples to illustrate concepts such as students being able to calculate the area as 8 x 5 = 40 square units for a parallelogram with base eight units and height five units), 4) Graphical Representation (GeoGebra) (Utilizing technologies such as GeoGebra to illustrate concepts dynamically). This approach encouraged exploration of the topic at a more advanced level and from diverse perspectives.

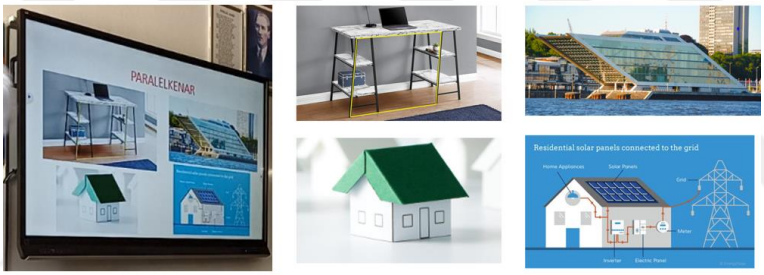


Figure 102. The real-life examples from the MCC 4 lesson as multiple representation

Also, he employed diagrams, examples from daily life, and word descriptions to explain the given concepts in different forms to make it easier for the students to understand. This approach made the students think and research the topic more deeply and from various viewpoints. Thus, using GeoGebra's dynamic features, Cem and his students could explore many examples of height in parallelograms and the area of a parallelogram in a short time, thereby improving their ability to reason and develop their abstract concepts. Cem, therefore, introduced his students to more advanced concepts and cases or mathematical relations in their usual learning and let them investigate these. Students altered the positions of the vertices of the parallelogram on GeoGebra and noticed the changes in the height. GeoGebra offered a way to visually see how the area of a parallelogram is constant as long as the base and height are constant regardless of the shape of the parallelogram. Through

GeoGebra, students could see and interpret the graphs and equations of the constructed parallelogram, including the angles, lengths, area, height, and the like, as depicted in the following figure.

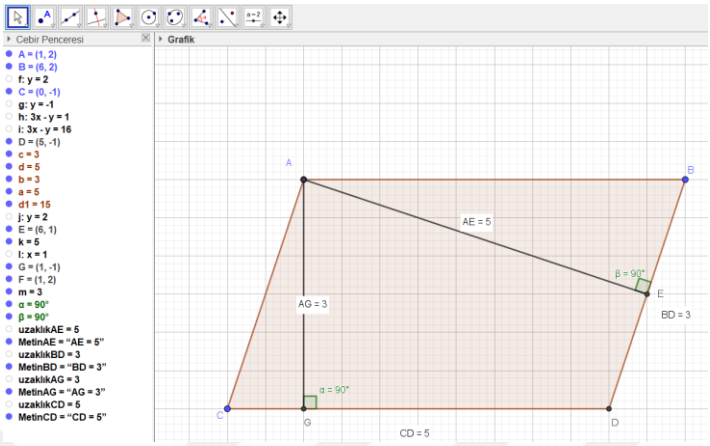


Figure 103. Graphical (right side) and algebraic (left side) representations of the parallelogram from the MCC 4 lesson

At post-conference 4, Cem was further probed on the role of technology in the use of multiple representations and how he integrated technology in the transition between the representations, and he replied as follows;

> "Everything we discussed and arranged during the pre-conference before the lesson unfolded according to plan. I designated a role for technology to facilitate multiple representations, conveying the concept through various means and facilitating transitions between them. This approach aimed to bolster students' grasp of the concept by establishing interconnected relationships. To illustrate, I commenced the lesson by presenting real-life scenarios alongside relevant visuals. Subsequently, verbal explanations further solidified students' comprehension. Following this, they observed parallelogram values using GeoGebra, including area, perimeter, side length, and height, presented both graphically and algebraically. By utilizing technology in this manner, I aimed to seamlessly transition between real-life contexts, graphical representations, tabular data, visual aids, and algebraic expressions. In doing so, I aimed to democratize access to technology, allowing more students to engage with the concept in its more intricate forms."

Cem responded as follows when asked in post-conference 4 how he used technology in the lesson to make multiple representations and transitions between representations;

> "The use of GeoGebra facilitated multiple representations of the concepts I was teaching. It allowed me to move smoothly between different representations of the parallelogram. I mean that the dynamic exploration of basic concepts through technology-enabled concepts can be represented differently. I started the lesson with pictures, videos of real-life examples, and verbal explanations. From there, we explored the height and area of the parallelogram in the GeoGebra graph view, essentially using a blank worksheet. Using the spreadsheet view, I could effectively teach the area relationship of the parallelogram. Students observed the relationship between base, height, and area and derived the formula. They also saw the area-related values symbolically in GeoGebra's algebra view on the left side of the window."

Cem's actual teaching practice at MCC 4 and statements at the post-conference of MCC 4 were aligned with *the availability descriptor* of *the advancing level* of the Access Theme: "Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience." (Niess, Ronau, et al., 2009, p. 24). Therefore, all these provided evidence that Cem progressed from *the exploring level* (level 4) to *the advancing level* (level 5) in MCC 4 compared to MCC 3 regarding *the availability descriptor*. Niess, Ronau, et al. (2009) pointed out that teachers at *the advancing level* of *the availability descriptor* allow the students investigate more complicated mathematical content or mathematical relationships within the conventional framework. These teachers apply various technological tools to expand what students can learn and the manner of their learning. Teachers at *the advancing level* expand on the use of resources and the internet in searching for interesting mathematical problems, and the part technology plays in solving the problems. Cem's teaching practice in MCC 4 showed the following characteristics in this case. Cem allowed more advanced mathematical concepts and mathematical relationships to be incorporated into the student's learning process. In this way, Cem extended the possibilities of what students could learn by technology and different activities. For instance, students applied GeoGebra to investigate the area of parallelograms and triangles, changing the shape and adjusting parameters with the help of sliders for the connection between base and height. Cem also helped the students search for interesting mathematical problems on the internet and thus introduced them to the use of technology in problem-solving. This way, students could manipulate mathematical ideas and gain a deeper understanding through technology.

In pre-conference 4, the coach introduced Cem to more tools and resources for applying technological pedagogy in an inquisition of sophisticated mathematical concepts. This involved trying to find problems and assigning them to the students to help them deal with different forms of mathematical concepts. The coach suggested that Cem allow the students to use technology to find mathematical concepts in the daily curriculum. This entailed helping the students to engage in solving different problems that required the use of various technologies to represent and solve mathematical activities. Accordingly, the coach supported the idea, and Cem's practice in MCC 4 revealed that he provided students with opportunities to engage in more elaborate mathematical activities using tools. For instance, the students solved problems in parallelograms and triangles using GeoGebra and adopted different ways of presentation. Cem allowed the students to use various technological tools to investigate mathematical concepts individually and in pairs, corresponding to *the advancing level* in *the availability descriptor*. Cem utilized technology, particularly GeoGebra, to provide access to higher levels of mathematics and foster exploration among a diverse student population. Through GeoGebra's interactive features, Cem created dynamic learning environments where students could engage with complex mathematical concepts regardless of their prior knowledge or background. By integrating technology into his teaching approach, Cem provided students with tools to visualize abstract mathematical ideas, making them more accessible and understandable. Moreover, Cem regulated online resources and simulations that catered to varying learning styles and abilities, ensuring that all students could participate in investigating advanced mathematical topics. By embracing technology, Cem transcended traditional barriers to learning, enabling more students, including those with diverse needs and backgrounds, to explore and engage with mathematics at higher levels. This approach expanded students' mathematical horizons and empowered them to take ownership of their learning processes.

4.2.4.3. TPACK Level of Cem Regarding Access Theme After Mathematics Coaching

To determine Cem's final TPACK level regarding the Access Theme after he participated in four mathematics coaching cycles, which focused on the learning

objective '*Problem solving related to the areas of compound shapes consisting of triangles, rectangles and parallelograms,'* his teaching practices in the classroom were observed for three hours while teaching the 'problem solving related to the areas of compound shapes consisting of triangles, rectangles and parallelograms'. To determine the final TPACK level, no pre-conference was held with Cem before the observed lesson, and no post-conference was held after the lesson. The researcher did not provide support for mathematics coaching to Cem. Cem prepared the lessons independently by reflecting on the knowledge and experiences gained in the four coaching cycles they participated in.

Cem's actual teaching practice after mathematics coaching indicated *the advancing level* in usage, barrier, and availability of descriptors of TPACK of the Access Theme.

As for *the usage descriptor*, Cem allowed students to use technology at all lesson stages. The purpose of using technology during Cem's teaching practice after mathematics coaching, the student's perspective on the use of technology in the lesson, and how often and how they used technology was similar to MCC 4 teaching practice in many aspects. In the first of the three lessons in which Cem taught area measurement problems, he first allowed the students to use various technologies to calculate the area of irregular shapes. At the beginning of the lesson, Cem prepared the students to calculate the area of more complex shapes (parallelogram, triangle, rectangle, square, etc.) with simple warm-up activities (shapes formed by combining rectangles and triangles). Students calculated the areas of various geometric shapes that Cem created on the online Geoboard. Afterwards, they did similar exercises on GeoGebra. In lesson 2, students solved context-based problems in a static environment, and when the problem was transferred to GeoGebra's dynamic learning environment. In addition, students also used activities that would contribute to the solution of context-based problems (e.g., rectangle folding activity). In the last lesson, students solved the textbook problems and transferred them to the smart board. Students used GeoGebra as both a learning and teaching tool and a problem-solving tool. The technology allowed students to analyze many different situations, problems, and geometric shapes, test their predictions, and verify their solutions via

the dynamic features of GeoGebra in a very short period. In addition, students also used activities that would contribute to the solution of context-based problems (rectangle folding activity).

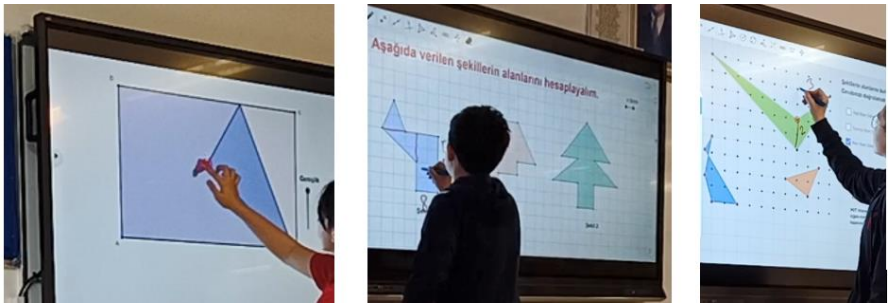


Figure 104. Students while using technology in lessons after mathematics coaching

All these could provide evidence that Cem's actual teaching practice after mathematics coaching aligned with the indicator of *the advancing level* of *usage descriptor* regarding Access Theme: "Permit students to use technology in every aspect of mathematics class." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) indicated that the teachers at *the advancing level* of *the usage descriptor* let the students use technology in every aspect of mathematics teaching. As with the previous case, these teachers see technology as a tool that can help students overcome conventional wisdom regarding the level of understanding students can achieve in mathematics. Cem's teaching practice after the mathematics coaching reflected the higher level of *the usage descriptor*. He permitted students to use technology and GeoGebra in almost every part of the mathematics lessons. This way, Cem used technology to provoke and deepen the students' thinking in mathematics. For example, Cem introduced the lesson by getting students to work on online activities on Geoboard to determine areas of the shapes. He then carried out activities that involved calculating areas of irregular geometrical shapes using the GeoGebra application. This activity challenged the students to find different ways of finding the area of the given shapes; they used to count unit squares and break down complex shapes into simpler ones like triangles and rectangles. Cem encouraged more students to provide feedback and control the edges of shapes with the help of GeoGebra's dynamic attributes. This was the case with his approach, which showed that technology was essential to teaching and learning.

As for *the barrier descriptor*, Cem's actual teaching practice after mathematics coaching was similar to MCC 4 regarding *the barrier descriptor* of access theme. Cem acknowledged the difficulties of teaching with technology and showed his preparedness to tackle them by meticulously planning and preparing to make the most of the resources and tools at his disposal. Additionally, by utilizing technology in these ways, Cem aimed to broaden the accessibility and comprehension of mathematical concepts, such as area calculation problems based on real-life context, offering students interactive learning opportunities that fostered deeper conceptual understanding and new approaches for problem-solving. He adeptly employed strategies for managing technology-related issues, drawing from his coach's suggestions and research. Moreover, Cem selected technologies that were easily understandable and user-friendly for his students as a preventive measure. Whenever students faced challenges like temporary internet disruptions or difficulties navigating GeoGebra worksheets or working on certain GeoGebra activities, he guided them calmly and professionally.

When unforeseen issues arose due to GeoGebra's dynamic nature, Cem adeptly refreshed or restarted activities to maintain the lesson's momentum. Throughout his teaching practice after mathematics coaching, Cem encountered no obstacles or hindrances to teaching with technology. Due to the students' background exposure to technology via mathematics coaching, the students were comfortable using tools like GeoGebra, Geoboard, and WordWall. Besides, Cem always inspected the computer and the classroom's smart board before each class to confirm they were working correctly. When starting the GeoGebra activities, he either opened files from a portable USB flash drive or input web links into the lesson plan. Regarding the technical challenges, Cem prepared himself well to avoid technical challenges such as internet connection failure or power outages. He was prepared for a power outage and ensured that learning was not interrupted by giving the students worksheets, square paper, and dot paper to help them analyze the concepts of height and area in a parallelogram.

Therefore, it was concluded that Cem's actual teaching practice after mathematics coaching aligned with the indicator of *the advancing level* of *barrier descriptor* regarding Access Theme: "Recognizes challenges in teaching with technology and

resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools." (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) stated that the teachers at *the advancing level* in *the barrier descriptor* comprehend the challenges of teaching with technology. To overcome them, they prepare and plan more than the other levels; they include all the resources and tools. These teachers use technology to expand the kinds of mathematics that can be presented to the students. Cem improved *the barrier descriptor* by addressing issues concerning teaching with technology. He realized this through effective planning and organization to guarantee that the right resources and equipment were well utilized. Cem ensured that the students used digital tools to improve their chances of understanding the mathematical concepts taught. This is the case in the second MCC 4 lesson, where Cem presented a skills-based problem and introduced GeoGebra to model different solutions. Thus, with the help of the dynamics tools available in GeoGebra, he was able to show the students the transformations and help them understand geometry. This allowed the students to apply the newly learned concepts and exercise problem-solving by employing various strategies. Cem's preparation of the lessons and using GeoGebra to present different forms of the problem was an example of how he addressed the possible challenges and efficiently employed the tools to enhance the students' learning.

Regarding *the availability descriptor*, Cem's actual teaching practice after mathematics coaching was very similar to MCC 4 in the context of the Access Theme. Cem provided problem-solving to many students using GeoGebra and other technological tools in the classroom. He developed activities in GeoGebra where students can solve different area problems and change the shapes, dimensions, and formulas using technology. This helped students of varying learning abilities to manipulate the material in a way they understood and at their own pace. Cem also made several online resources and problem sets that met different levels of students and their preferences and gave them complex but solvable area measurement problems. Thus, Cem enriched the problems from real-life situations and provided opportunities for all students to participate in problem-solving activities. Thus, Cem integrated technology into the teaching process, allowing more students to engage in problem-solving and enhance their mathematical learning.

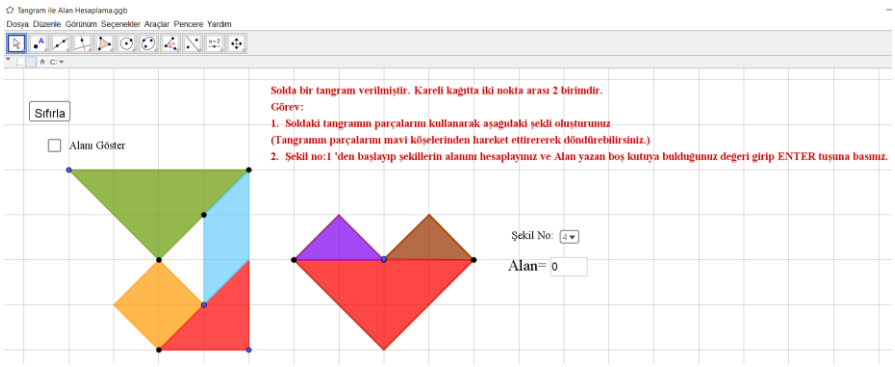


Figure 105. Area calculation using tangram activity of GeoGebra

In his teaching, Cem used problem-solving in mathematics and GeoGebra to solve area calculation problems and explained them to the students. Thus, the learning process on the area measurement through various problem-solving activities in the GeoGebra environment allowed students to try different solution methods and manipulate shapes, dimensions, and formulas in real-time. Cem also provided group activities, which were solved using the technology since it created an interactive platform for the students to work in groups as they learn. Through GeoGebra's dynamic platform, Cem emphasized the relevance of mathematics to students' daily lives by engaging them in scenarios based on real-life contexts. GeoGebra also played a crucial role in helping students make mathematical connections through multiple representations, enabling them to explore concepts from various perspectives and develop a deeper understanding of mathematical principles in different contexts. Overall, Cem's innovative use of technology transformed problem-solving into a dynamic and immersive experience, empowering students to confidently and creatively apply their mathematical knowledge to solve real-world problems.

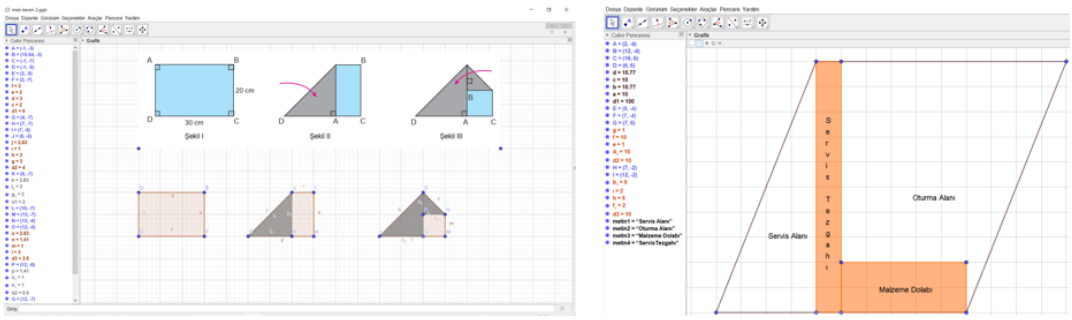


Figure 106. Graphics and algebra view of problems provided by GeoGebra

All these could be considered as evidence to place Cem at *the advancing level* since his teaching practice after the mathematics coaching was consistent with the following indicator of *the availability descriptor* at *the advancing level* within the Access theme: "Students are taught and allowed to explore more complex mathematical topics or mathematical connections as part of their normal learning experience" (Niess, Ronau, et al., 2009, p. 24). Niess, Ronau, et al. (2009) asserted that the teachers at *the advancing level* of *the availability descriptor* enable the students to engage in more detailed mathematical topics or even explore mathematical connections within the typical learning environment. For instance, these teachers use the internet to search for interesting mathematical problems, and the part technology can take in solving these problems. Another aspect of Cem's teaching practice regarding *the availability descriptor* that he made it possible for the students to engage in more advanced mathematics topics and relations between the topics within normal learning activities. Cem used GeoGebra to help students identify the area relations of triangles, squares, rectangles, and parallelograms and asked them to solve problems and explain them to their peers. In the third lesson of the MCC 4 lesson plan, Cem added authentic problems that called for the application of area relations of various geometric shapes. He challenged those students to solve those problems and helped the struggling students. Thus, by assigning the students GeoGebra area problems as homework, Cem still kept on challenging the students with other complex mathematical problems. Also, GeoGebra and the Internet to search for interesting problems motivated the students to discover how technology can solve various problems and enhance their achievement.

4.2.4.4. The Progress of Cem in TPACK Regarding Access Theme Through Mathematics Coaching in Learning Area of "Measurement of Area"

The figure 107 depicts Cem's TPACK levels for *usage*, *barrier*, and *availability descriptor*s of Access Theme concerning his instruction in technology-enhanced lessons in MCC 3, MCC 4, and after the mathematics coaching (After MC). Cem reached the highest level in the *usage*, *barrier*, and *availability descriptor*s of the TPACK development model across the mathematics coaching cycles. The learning domain of Measurement of Area was examined in the context of MCC 3, MCC 4,

and the practices of post-mathematics coaching teaching. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

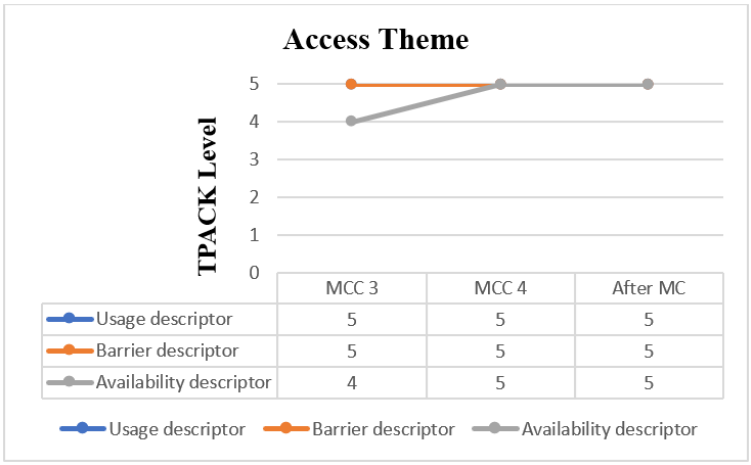


Figure 107. The progress of Cem of the Access Theme in the learning area of Measurement of Area

In the TPACK development model, *the usage descriptor* is based on the extent to which students are permitted to use technology as a tool in their learning within the classroom context (Niess, Ronau, et al., 2009). Cem's level in *the usage descriptor* at MCC 3, MCC 4, and after mathematics coaching indicated *the advancing level* (level 5). During the teaching practices of MCC 3 and MCC 4 and following mathematics coaching, Cem engaged his students in learning geometry concerning the height and area of triangles and parallelograms and area calculations in mathematics classes. GeoGebra was utilized in every aspect of the lessons, from dynamic construction to evaluating students' comprehension and introducing new ideas. It was the student's choice to work with GeoGebra, supported by Cem, to understand these geometry concepts, thus allowing them to learn them actively. Cem's approach shifted the conventional paradigms of how mathematics can be learned by using technology to increase the students' possibilities in mathematics.

The barrier descriptor explores teachers' difficulties when incorporating technology into teaching and how these difficulties are overcome (Niess, Ronau, et al., 2009).

Referring to the TPACK development model, it was identified that Cem's level in *the barrier descriptor* at MCC 3, MCC 4, and after mathematics coaching, was at Level 5. Thus, the GeoGebra application in triangles and parallelograms demonstrated a promising way for the students to develop their geometry mastery. It also ensured easier control while integrating the technology and increased the students' participation in the lesson. Using GeoGebra activities incorporated into the class, the students appear more interested in the topic. With close supervision, Cem thus explained and solved the difficulties associated with teaching geometry using GeoGebra to the students. Although there are various difficulties in incorporating technology into teaching, Cem already planned to address these problems. Thus, he ensured the adequate and efficient use of available resources and tools by thoroughly preparing and planning. GeoGebra, therefore, turned out to be a useful tool in enhancing students' conceptual knowledge of geometry, especially in finding areas of triangles and parallelograms and solving other related problems. Cem ensured that GeoGebra was readily available to the students, allowing them to work through these concepts independently and at their own pace.

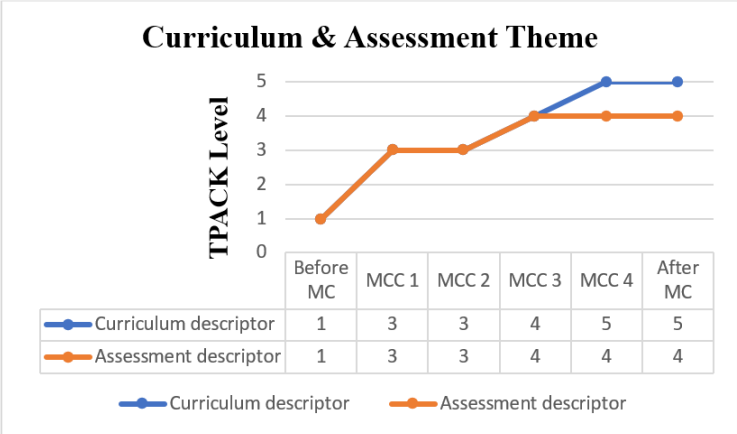
The availability descriptor shows how, through technology, the extent of mathematics and how students can engage in mathematics is greatly expanded. Through digital tools, virtual manipulatives, and specialized software, students with diverse backgrounds are allowed to learn advanced mathematics and tools that were once unavailable to them (Niess et al., 2009). According to the TPACK development model, in *the availability descriptor* at MCC 3, Cem's level was at *the exploring level* (level 4). GeoGebra was especially useful in determining, implementing, and analyzing significant concepts by providing different views of the concepts and how they are related, especially regarding the area and height of triangles. As for the area of triangles, the students used spreadsheets to recognize the correlation between the triangle's area, height, and base. In addition, the students changed the checkboxes and saw the algebraic expression of the triangle area formula and the figure's graph.

The results showed that Cem was at *the advancing level* (level 5) in the TPACK development model's *availability descriptor* at MCC 4 and after the mathematics coaching. The students are encouraged and stimulated to explore the more complex

geometrical concepts and relationships as a fundamental aspect of their learning. Cem used GeoGebra as a mathematical tool for teaching, learning, and solving problems, mainly in parallelograms and area problems. All the tools provided by GeoGebra, many examples, the ability to work with the concepts at a high level, and the dynamic features helped students understand the possibilities of using technology to solve problems.

4.2.5. The Overall Progress of Cem in All Themes and Descriptors of the TPACK Development Model

The figures 108 to 111 below depict Cem's development in TPACK from before to after mathematics coaching, organized according to the themes and corresponding descriptors of TPACK development model. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).



	Before MC	MCC 1	MCC 2	MCC 3	MCC 4	After MC
Curriculum descriptor	1	3	3	4	5	5
Assessment descriptor	1	3	3	4	4	4

Figure 108. The progress of Cem of the Curriculum and Assessment Theme

Regarding the overall progress of Cem in Curriculum and Assessment Theme, as illustrated in the figure 108, Cem was at *the recognizing level* (level 1) in *the curriculum descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the adapting level* (level 3) in MCC 1 and continued at this level in MCC 2. Cem progressed to *the exploring level* (level 4) in

MCC 3 and then to *the advancing level* (level 5) in MCC 4. After the mathematics coaching, he completed the study at the highest level, *the advancing* (level 5) in *the curriculum descriptor*. Moreover, Cem was at *the recognizing level* (level 1) in *the assessment descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the adapting level* (level 3) in MCC 1 and continued at this level in MCC 2. Cem progressed to *the exploring level* (level 4) in MCC 3 and continued at *this level* in MCC 4. After the mathematics coaching, he completed the study at the exploring level (level 4) in *the assessment descriptor*.

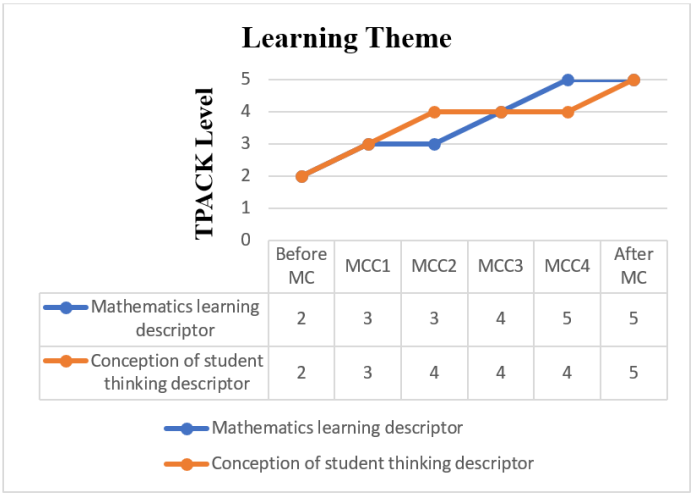


Figure 109. The progress of Cem of the Learning Theme

Concerning the overall progress of Cem in the Learning Theme, as seen in the figure 109, Cem was at *the accepting level* (level 2) in *the mathematics learning descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the adapting level* (level 3) in MCC 1 and continued at this level in MCC 2. Cem progressed to *the exploring level* (level 4) in MCC 3 and then to *the advancing level* (level 5) in MCC 4. After the mathematics coaching, he completed the study at the highest level, *the advancing* (level 5) in *the mathematics learning descriptor*. Furthermore, Cem was at *the accepting level* (level 2) in *the conception of student thinking descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the adapting level* (level 3) in MCC 1. Cem progressed to *the exploring level* (level 4) in MCC 2 and continued at this level in MCC 3 and MCC 4. After the mathematics coaching, he completed the study

at the highest level, *the advancing* (level 5) in *the conception of student thinking descriptor*.

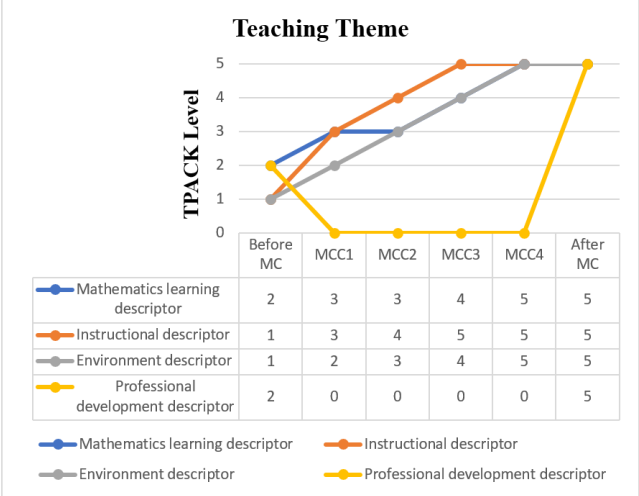


Figure 110. The Progress of Cem of the Teaching Theme

As for the overall progress of Cem in the Teaching Theme, as seen in the figure 110, Cem was at *the accepting level* (level 2) in *the mathematics learning descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the adapting level* (level 3) in MCC 1 and continued at this level in MCC 2. Cem progressed to *the exploring level* (level 4) in MCC 3 and then to *the advancing level* (level 5) in MCC 4. After the mathematics coaching, he completed the study at the highest level, *the advancing* (level 5) in *the mathematics learning descriptor*. Also, Cem was at *the recognizing level* (level 1) in *the instructional descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the adapting level* (level 3) in MCC 1 and then to *the exploring level* (level 4) in MCC 2. Cem progressed to *the advancing level* (level 5) in MCC 3 and continued at this level in MCC 4. After the mathematics coaching, he completed the study at the highest level, *the advancing* (level 5) in *the instructional descriptor*. In addition, Cem was at *the recognizing level* (level 1) in *the environment descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the accepting level* (level 2) in MCC 1 and then to *the adapting level* (level 3) in MCC 2. Cem progressed to *the exploring level* (level 4) in MCC 3 and then to *the advancing level* (level 5) in MCC 4. After the mathematics

coaching, he completed the study at the highest level, *the advancing* (level 5) in *the environment descriptor*. Lastly, Cem was at *the accepting level* (level 2) in *the professional development descriptor* before mathematics coaching. Through the mathematics coaching cycles, he progressed in this descriptor and completed the study at the highest level, *the advancing* (level 5) in *the professional development descriptor.*

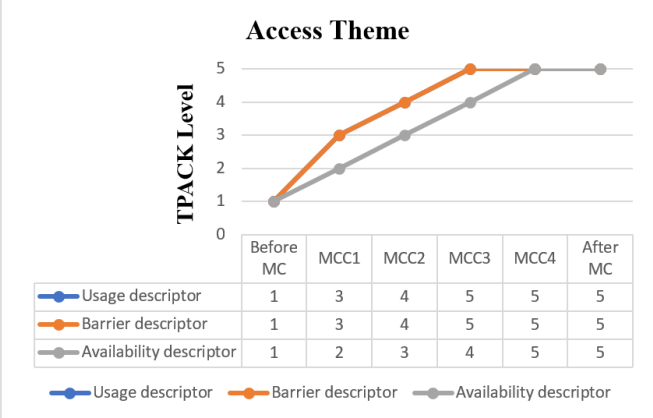


Figure 111. The progress of Cem of the Access Theme

With respect to the overall progress of Cem in the Access Theme, as seen in the figure 111, Cem was at *the recognizing level* (level 1) in *the usage descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the adapting level* (level 3) in MCC 1 and to *the exploring level* (level 4) in MCC 2. Cem progressed to *the advancing level* (level 5) in MCC 3 and continued at this level in MCC 4. After the mathematics coaching, he completed the study at the highest level, *the advancing* (level 5) in *the usage descriptor*. Similarly, Cem was at *the recognizing level* (level 1) in *the barrier descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the adapting level* (level 3) in MCC 1. Cem progressed to *the exploring level* (level 4) in MCC 2. Cem progressed to *the advancing level* (level 5) in MCC 3 and continued at this level in MCC 4. After the mathematics coaching, he completed the study at the highest level, *the advancing* (level 5) in *the barrier descriptor*. Also, Cem was at *the recognizing level* (level 1) in *the availability descriptor* before mathematics coaching. When he started to participate in coaching cycles, he progressed to *the accepting level* (level 2) in MCC 1 and then to *the adapting level*

(level 3) in MCC 2. Cem progressed to *the exploring level* (level 4) in MCC 3 and then to *the advancing level* (level 5) in MCC 4. After the mathematics coaching, he completed the study at the highest level, *the advancing* (level 5).

4.3. Teacher's Perceptions Regarding Mathematics Coaching

This section relates to findings that respond to the study's second research question and revolve around teacher's perceptions of mathematics coaching concerning TPACK. The following quotations are from the mathematics coaching interview conducted after the teacher participated in cycles of mathematics coaching. This section aimed to understand the participant teacher's perception of mathematics coaching as it is directed at improving his TPACK in the 'Angles and Measurement of Area' unit.

The analysis revealed four prominent themes, including (1) Contributions of mathematics coaching to professional development, (2) Differentiating aspects of mathematics coaching compared to traditional in-service training, (3) The difficulties of mathematics coaching, and (4) Recommendations for mathematics coaching. First, teachers pointed to several positive impacts of mathematics coaching: improving their own TPACK, developing new and technological teaching methods, raising achievement and motivation in mathematics, and promoting cooperation between the teachers and students. Secondly, some differences between conventional in-service training and mathematics coaching were pointed out: differences in learning results, the permanent and sustainable impact, and the approach combining theory and practice. Thirdly, the assigned teachers mentioned some drawbacks of mathematics coaching, such as the time it takes to prepare the lessons. Lastly, the participant teacher provided useful recommendations for the future, underlining that it should be used in more schools for a longer time, with different units of mathematics, different technologies used in math classrooms, involving colleagues' group dynamics, and using AI tools and Gamification. In this way, the findings offered a more complex picture of teacher's attitudes toward mathematics coaching and the potential benefits and drawbacks of the intervention. In this paper, the four themes are discussed, and subthemes that are related to the themes are identified and summarized in Table 8.

Table 8. Themes and Codes in the Mathematics Coaching Interview

Themes	Subthemes
Contributions of mathematics coaching to PD	Improving his TPACK Improving instructional practice Increasing student motivation and achievement Fostering research on professional development Increasing co-operation
Differentiating aspects of mathematics coaching compared to traditional in-service trainings	Observable differences in learning outcomes of students Permanent and sustainable effect on transforming teacher Combining theory and practice
The difficulties of mathematics coaching	Time spent on lesson preparation
Recommendations for mathematics coaching	Spreading in schools Spread over a longer period Application in different mathematics units Incorporating different subject-specific technologies Attending with colleagues Incorporation of AI tools and Gamification

4.3.1. Contributions of Mathematics Coaching to Professional Development

Cem described his mathematics coaching experience as very positive and identified it as having a substantial positive effect on his professional development and the achievement of expectations. When reflecting on how mathematics coaching influenced his TPACK, Cem shared the following insights:

> "Previously, I sporadically incorporated technology into my lessons, but through the coaching process, I recognized that my approach is haphazard and unplanned. The coaching sessions enhanced my content knowledge, delving into comprehensive discussions to deepen my understanding of teaching the subject and broadened my repertoire of teaching methods applicable across various subjects. Throughout the coaching, we extensively utilized diverse technological tools, integrating them seamlessly into subject instruction. This experience expanded my technological proficiency and cultivated discernment in selecting the most suitable technology for specific teaching methodologies. Personally, witnessing how conceptual learning and exploration preemptively address misconceptions and learning challenges in the classroom was enlightening. I acquired a wealth of new insights spanning technology, subject matter expertise, and pedagogical strategies. Most

> notably, I developed the ability to integrate these disparate elements into cohesive lesson plans seamlessly. Consequently, I believe my TPACK has significantly evolved."

Cem's statements in the mathematics coaching interview supported his view that mathematics coaching *improved his TPACK*. Cem also explained another vital advantage of the mathematics coaching process and said it significantly influenced *his instructional practice*. He shared how the changes that he made in his teaching style greatly enhanced the learning process. When discussing the strengths of the coaching process, Cem articulated:

> "As you know, TPACK cannot exist in isolation from instructional practice. Through the extensive discussions, research, and planning conducted both pre- and post-conference, I had the opportunity to delve into theoretical concepts. Subsequently, I applied these insights directly in my teaching practice, engaging in a cycle of application, observation, and revision. This iterative process allowed me to identify and address improvement areas within my teaching practice. Moreover, mathematics coaching instilled a sense of discipline and organization regarding lesson planning and instructional methodology. It established structured routines in my lesson preparation process, akin to a checklist. I meticulously plan and rehearse each stage of lesson introduction, deepening understanding and assessment. This includes formulating targeted questions for students, devising strategies to address potential misconceptions, and anticipating student responses. All these facets have markedly improved through the development of my teaching practice. Thus, the coaching process significantly contributed to improving my instructional practice."

Cem's statements showed that the coach assisted and directed the teacher to enhance his TPACK. Cem also highlighted several changes in his teaching practices after the mathematics coaching process, notably increasing *the student's motivation, engagement, and academic achievement*. When reflecting on the strengths of the coaching process, Cem shared the following:

> "The classroom environment we worked in was lively yet comprised students with notably low math achievement levels. Before the coaching process, I often had to coax participation from students, who predominantly followed lessons as passive observers. However, following integrating technology into the curriculum with increased vigor post-coaching, I witnessed a remarkable shift in student attitudes. The introduction of technology sparked a sense of novelty among students, leading to a noticeable surge in their motivation and

> interest levels. In a classroom where participation was previously a challenge, there was a remarkable uptick in students' eagerness to engage with the material, utilize technology, tackle problems, and contribute to discussions. This newfound enthusiasm translated into tangible academic progress. Following the coaching process, the students underwent a school-wide written examination focusing on angles and area measurement. Remarkably, the class, which had trailed the 6th-grade school average by 9 points before coaching, surpassed the school average by 1 point in the subsequent examination. This significant improvement served as a testament to the effectiveness of the coaching process. Witnessing my efforts' tangible outcomes and positive impact was immensely gratifying and was a powerful motivator for me in my journey through mathematics coaching."

Cem's statements indicated that mathematics coaching positively affected Cem's instructional practice and the students' attitudes towards mathematics, interest, engagement, and achievement. Cem stressed the significance of mathematics coaching in *fostering research on teachers' professional development*. In his interview regarding this aspect of mathematics coaching, he stated:

> "Throughout my teaching career, my focus was refining my teaching methods and supporting my students' learning. While occasional discussions with the mathematics department at my school touched upon instructional and curricular matters, I lacked the intrinsic motivation to delve deeper into questions such as how to improve my practice, the most effective teaching approaches for various subjects, and the integration of technology. However, my involvement in the coaching process ignited a newfound curiosity. I now actively research many topics, including how other teachers teach specific subjects, integrating technology with various teaching methodologies, and the seamless alignment of content, pedagogy, technology, and student needs. I strive to design student-centered, conceptually-driven lessons of the highest caliber while addressing common misconceptions and exploring solutions. Before each lesson, I engage in extensive research to inform my instructional decisions. Additionally, I actively seek professional development opportunities such as technology-centered training, seminars, and workshops to enhance my teaching practice. The coaching process has underscored the vast knowledge yet to be acquired, motivating me to seek growth and improvement continuously."

Cem stressed that mathematics coaching helped the participants to *increase cooperation*. In his interview on this issue, he remarked as follows:

> "Throughout the mathematics coaching process, you and I, as colleagues, engaged in extensive collaborative efforts. We collaborated on lesson

> planning, evaluated lessons, and exchanged suggestions and ideas during pre- and post-conference sessions and interviews. This collaborative experience significantly enhanced my willingness and ability to work effectively within a team. Collaborative work is undeniably one of the most crucial skills for teachers, especially when collaborating with peers who are teaching at the same level. It involves constructive discussions, joint activities, and the exchange of academic content. Mathematics coaching has notably bolstered my capacity for collaborative instructional planning, emphasizing the value of working with others to enhance teaching practices."

4.3.2. Differentiating Aspects of Mathematics Coaching Compared to Traditional In-Service Trainings

During the mathematics coaching interview, the coach asked Cem about the difference between mathematics coaching and traditional in-service training. Cem said that Mathematics coaching is more effective than other forms of in-service training because it produces more *observable differences in learning outcomes*. When asked about the strengths of mathematics coaching, Cem expressed:

> "I witnessed the tangible effectiveness of everything discussed and planned during the coaching sessions. From student interest, motivation, and success to their participation in lessons, variations in mathematical thinking, questions asked, and responses given, I observed changes across all these areas during the coaching process. These observable learning outcomes surpass the results typically seen after regular in-service training. Through coaching, I had the opportunity to observe and adjust my teaching based on these learning outcomes, leading to more impactful and sustainable professional development."

Cem also added that mathematics coaching could bring about *a permanent and sustainable effect on transforming teachers* by imparting knowledge and skills that can be applied in the teachers' practice throughout their careers. He elaborated on this point during the mathematics coaching interview, saying:

> "The teacher's conviction and willingness to implement are crucial factors. To achieve this, teachers must first experiment with new approaches and witness the results firsthand. In this regard, math coaching is a job-embedded program offering ample opportunities for such experimentation. Through post-conference sessions and post-lesson evaluations, I discovered that many assumptions regarding TPACK were not as accurate as I had believed. Your insights during these sessions shed light on aspects I had overlooked as a

> teacher, prompting significant realizations. Applying the lessons learned across various subjects throughout my teaching career ensures that the impact of math coaching remains continuous and enduring. For instance, considerations regarding the effective use of technology in lessons and the alignment of content with chosen technologies have become ingrained in my instructional planning routine. This integration of learning into routine practice contributes to the lasting and sustainable nature of the knowledge gained through mathematics coaching."

Cem underlined that compared to traditional in-service trainings, one of the main characteristics of mathematics coaching is *the combination of theoretical knowledge and practice.* Cem stated the following in the mathematics coaching interview on this issue:

> "As a teacher, this aspect deeply impressed me, convinced me, and resonated with me because it remained closely connected to the classroom environment, allowing me to witness the effectiveness of planned strategies through practical application directly. I could assess whether everything discussed and planned before the coaching sessions proved useful and effective. Unlike ordinary in-service training, which often focuses primarily on theoretical information, math coaching allows theoretical knowledge to be translated into practice. I could gauge the quality of teaching by evaluating the resulting learning outcomes."

4.3.3. The Difficulties of Mathematics Coaching

Cem agreed that mathematics coaching has no major drawbacks and that the approach has a few shortcomings; however, he noted some difficulties. He stated that the most significant problem associated with coaching is the *time spent on lesson preparation*. In the interview, he remarked as follows:

> "I gained extensive knowledge about technology, pedagogy, and content throughout the coaching process. However, this learning occurred through numerous meetings, discussions, and meticulous preparations. Planning lessons with meticulous detail, from introduction to evaluation, is time-consuming. Considering the workload in schools, this demands considerable time and undivided attention. Therefore, the primary challenge of participating in math coaching is the time and concentration required for thorough lesson planning. The significant amount of time spent on lesson preparation is indeed noteworthy."

4.3.4. Recommendations for Mathematics Coaching

Cem suggested increasing awareness and *widespread adoption of mathematics coaching in schools* to enhance teachers' professional development. He mentioned this issue as follows:

> "Mathematics coaching represents a valuable opportunity for practical teacher development. Before my involvement in this study, I was unaware of math coaching. Reflecting on my experience, I wish I had the opportunity to participate in such a program earlier in my career. Therefore, all teachers must conduct coaching programs periodically to witness firsthand tangible results. I strongly recommend math coaching to other math teacher colleagues in my school and my friends in professional communities."

From Cem's statements, it was understood that he supported adopting mathematics coaching in most schools. Cem's other recommendation concerning mathematics coaching was the study's time frame. Cem also highlighted the fact that the process of mathematics coaching should be *spread over a longer period*. He elaborated on this issue as follows;

> "I believe participating in the coaching process for a longer duration would be more beneficial in reinforcing and expanding upon what I have learned and accomplished in specific subjects. I think engaging in coaching for a semester or even a year would afford me greater opportunities to learn and acquire additional skills."

Cem stated that he agreed that mathematics coaching should be delivered over a more extended period. Moreover, Cem stressed that integrating mathematics coaching into different mathematics units can positively impact the teachers' knowledge and practices in diverse learning domains. He articulated this issue in the mathematics coaching interview as follows:

> "Extending the coaching process to encompass diverse units such as algebra, data analysis, and numbers and operations would greatly enhance technology integration across various learning domains. As teachers, applying the theoretical and practical knowledge gained from the coaching process to different units would be immensely beneficial."

Cem's statements indicated that mathematics coaching could be useful for the teacher in the other mathematics units to enhance the teacher's professional

development. Cem indicated that *incorporating various mathematics-specific technologies* into the content of mathematics coaching would improve the coaching outcome. He expressed:

> "Throughout the coaching sessions, I gained insight into mathematics-specific technologies as part of our training in TPACK. While GeoGebra stood out as particularly user-friendly, there is potential to include other technologies specific to mathematics lessons in the coaching curriculum. This expansion would enrich the learning experience and deepen student and teacher knowledge. The focus could be introducing different technological tools to broaden the teacher's skills."

Cem's remarks indicated that it would be more useful and diverse for the teachers in mathematics coaching to include other subject-specific technologies such as Geometry Expressions, Desmos, The Geometer's Sketchpad, Cabri, and Cinderella in addition to GeoGebra. Cem revealed that collaboration, teaching, and the coaching process might improve significantly by *engaging in mathematics coaching with colleagues*. Cem highlighted the idea that it is possible that engaging in a collaborative process might increase the level of participation in a group context and the sharing of ideas and experiences, as well as promote self and others' reflection. Cem described this matter in the mathematics coaching interview as follows:

> "Participating in mathematics coaching alongside all mathematics teachers within the school can facilitate the exchange of experiences through interactive engagement and enhance instructional planning. I believe that forming distinct teacher groups, each focused-on planning different learning areas and units at various times with the coach, can yield excellent outcomes in developing TPACK. Suppose each teacher shares their experiences with different technologies across diverse units, such as sharing insights about particularly effective technological tools or strategies to mitigate misconceptions."

Cem also stressed the need for group-based mathematics coaching since teachers can discuss with each other. Therefore, he stated that such an environment of professional development could enhance the team spirit. Lastly, Cem mentioned that one of the most influential and innovative methods that could be used in teaching mathematics while promising much more for the teacher is *incorporating artificial intelligence tools and gamification (game-based learning)* in teaching and learning

mathematics. Cem described his opinion on this matter in the mathematics coaching interview in the following manner.

> "Recognizing that today's students are digital natives who enjoy gaming, I believe learning how to deliver content through gamified approaches during coaching sessions would be highly beneficial. Additionally, artificial intelligence (AI) tools that tailor teaching to students' individual learning styles and skill levels are available. Integrating such tools into mathematics coaching could equip teachers with valuable skills and resources. For instance, these AI tools could also assess student learning. I am confident these tools can positively impact student engagement and academic achievement."

Cem's statements revealed how integrating AI tools and gamification in the mathematics coaching process could be useful in enhancing the teaching practice of mathematics teachers and students' achievement.

CHAPTER 5

CONCLUSION, DISCUSSION, and IMPLICATIONS

This chapter provides a detailed discussion of Cem's TPACK development, which was analyzed through each of the themes of the TPACK development model: Curriculum and Assessment, Learning, Teaching, and Access. At the end of these discussions, the chapter presents the conclusions drawn from the study and Cem's perceptions of mathematics coaching regarding TPACK development. Finally, implications for the TPACK development model and mathematics coaching are provided before presenting the study's limitations and suggesting future research directions.

5.1. Discussion on Cem's TPACK Development

This study aimed to investigate the changes in Cem's TPACK in the context of geometry, particularly in the learning areas of 'Angles and Measurement of Area', through his participation in the professional development (PD) program of mathematics coaching. The Mathematics Teacher TPACK Standards and Development Model of Niess, Ronau, et al. (2009) was adopted in the current study to determine Cem's TPACK level before and after his participation in mathematics coaching and to monitor changes in Cem's TPACK during coaching cycles. Another aim of this study was to investigate Cem's perceptions of mathematics coaching regarding developing TPACK.

The study's findings suggested an answer to the first research question: Cem's TPACK development and instructional practices in the learning area of 'Angles and Measurement of Area' were positively and significantly influenced by his participation in mathematics coaching. Regarding the teacher's knowledge, Cem demonstrated notable progression from PCK to TPACK in learning areas such as

‘Angles and Measurement of Area.’ This progression entailed transitioning from a content-centric approach to a more balanced one incorporating content-centric and techno-centric approaches. Before mathematics coaching, Cem had predominantly low levels of TPACK (*recognizing* and *accepting*) for all four themes and their associated descriptors of the TPACK Development Model. Through his participation in mathematics coaching, Cem progressed to the higher levels in all themes and related descriptors in the TPACK Development Model. At the end of the mathematics coaching, Cem reached the highest level in the TPACK Development Model in all four themes and their descriptors, excluding *the assessment descriptor* of the Curriculum and Assessment Theme.

The study’s results also suggested an answer to the second research question: Cem perceived the effects of mathematics coaching on TPACK development as highly positive. Cem’s viewpoints clearly explained the effects of mathematics coaching: enhancing TPACK, improving teaching practices, enhancing students’ motivation and achievement in mathematics, encouraging research on PD, and fostering collaboration. When comparing mathematics coaching to regular in-service teacher training, Cem emphasized noticeable differences in student learning outcomes, the permanent and sustainable effects of transforming teaching practices, and the effective integration of theory and practice. However, Cem also highlighted some challenges, particularly the significant time required for lesson preparation. To improve the implementation of mathematics coaching, Cem suggested its dissemination in schools, extending the duration of coaching, applying it across various mathematics units, integrating different subject-specific technologies, participating with colleagues, incorporating AI tools, and utilizing gamification strategies.

As a summary of Cem’s TPACK levels before mathematics coaching, among the eleven descriptors of themes in the TPACK Development Model by Niess, Ronau et al. (2009), Cem's teaching practice was at *the recognizing level* (level 1, the lowest level in the TPACK Development Model) for seven descriptors: Curriculum and *assessment descriptors* of the Curriculum and Assessment Theme, instructional and *environment descriptors* of the Teaching Theme, and usage, barrier, and *availability*

descriptors of the Access Theme. Additionally, he was at *the accepting level* (level 2) for four descriptors: Mathematics learning and *professional development descriptors* of the Teaching Theme and the conception of student thinking and *mathematics learning descriptors* of the Learning Theme. By the end of the mathematics coaching cycles, Cem attained the highest level of TPACK in all themes and related descriptors except for the *assessment descriptor* in the Curriculum and Assessment Theme. In the subsequent sections, the results of this study are presented, considering themes of the TPACK development model.

5.1.1. Discussion on Cem's TPACK Development Regarding Curriculum and Assessment Theme

Before participating in mathematics coaching, Cem was at the lowest level (*recognizing*) of TPACK for *the curriculum* and *assessment descriptors* of the Curriculum and Assessment Theme. However, Cem progressed to the highest level of the *curriculum descriptor* and *the exploring level* of *the assessment descriptor* after participating in the mathematics coaching cycles (MCCs). Table 2 shows the TPACK levels of Cem in six technology-integrated lessons taught before, within, and after the mathematics coaching. In Table 9, the abbreviation MCC stands for Mathematics Coaching Cycle.

Table 9. The progress of Cem in Curriculum and Assessment Theme and related descriptors

Theme	Descriptor	Before MC	MCC 1	MCC 2	MCC 3	MCC 4	After MC
Curriculum & Assessment Theme	***Curriculum descriptor***	1	3	3	4	5	5
	Assessment descriptor	1	3	3	4	4	4

Note. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

Based on the TPACK development model, Cem's TPACK level regarding the Curriculum and Assessment Theme and associated descriptors was assessed before

he participated in mathematics. This assessment was based on his teaching of the learning objective: 'Understanding that two rays with the same starting point form an angle and representing them with symbols.' Therefore, his TPACK level was evaluated in the learning domain of the 'Angles.'

Before the study, Cem agreed that using mathematical concepts through technological tools can assist in comprehending the curriculum content as stated in the descriptions of *the recognizing level* of *the curriculum descriptor* of Curriculum and Assessment Theme. Even though Cem admitted that technology could potentially enrich the curriculum, he did not use it in teaching angles and other mathematical concepts and relations as he did not know how to use it effectively and appropriately. For instance, he employed the smart board to display various images of ideas at the beginning of the lesson while explaining the concept of angles with multiple representations. However, he doubted the strategies' ability to support the students in identifying the basic concept as stated in the descriptions of *the recognizing level*. Cem did not use interactive tools such as GeoGebra to explain angles. Instead of employing subject-specific technologies that could have deepened students' understanding, he showed a video of a real-life example of the angles. The lack of enthusiasm to change and the use of technology in developing the curriculum revealed that Cem had a low level of TPACK, which aligns with research that links teachers' confidence and competency in using technology to inadequate training, as Ertmer et al. (2012) and Ottenbreit-Leftwich et al. (2010) found in their studies. Furthermore, Cem had no exposure to GeoGebra training during his undergraduate study, and this might have limited Cem's vision and knowledge on how to integrate this tool into the curriculum, which is consistent with previous research that indicates that teachers' experience and training often limit their ability to effectively use educational technology, as Albion & Ertmer (2002) and Mayes and De Freitas (2013) indicated in their studies.

Concerning *the assessment descriptor* of the curriculum and assessment theme, Cem had a negative attitude towards using GeoGebra to assess students' learning before mathematics coaching. His belief that technology was unnecessary for evaluating students' understanding of geometry aligns with the studies by Ertmer et al. (2012)

and Niess (2005, 2005a), who pointed to the fact that teachers with low TPACK often resist using technology in assessment because they lack necessary knowledge and skills of technology in assessment. Cem's use of traditional assessment techniques and items, such as multiple-choice questions irrelevant to angles, aligns with conventional practices. This phenomenon is shared with teachers who have not undergone PD on the effective and proper use of technology for assessment practices, as Ertmer (2005) and Lawless and Pellegrino (2007) revealed. Despite significant advancements in his instructional strategies, his ability to fully integrate technology into assessments lagged, likely due to the increasing complexity of tasks and the limited technological infrastructure in his classroom, as Kim et al. (2013) and Kopcha (2012) found in their studies.

Upon participating in the first mathematics coaching cycle, Cem's TPACK improved significantly and rapidly in the Curriculum and Assessment Theme. Initially, Cem did not use the technology to the full extent he could, owing to the concerns that technology might disrupt the learning process, as identified by Warschauer and Matuchniak (2010). This aligns with the learning curve in PD, where teachers tend to learn quickly initially, as Desimone (2009) and Guskey (2002) revealed in their studies. In the case of Cem, it is possible to explain the early progression of the first two coaching cycles that focused on imposing the foundational ideas and practices of coaching, which underscored integrating technology as an effective tool into his existing pedagogical patterns and approaches. Following this initial rapid progression, Cem could come into a consolidation phase in which he built and reinforced his knowledge and skill in the area, as mentioned in the studies by Desimone and Garet (2015) and Opfer and Pedder (2011). Cem did not make much observable development during this period because he concentrated on internalizing and enhancing the skills and knowledge he had gained. This plateau could also be due to the consolidation phase, in which Cem gained more understanding before advancing to the next level, as mentioned in the study by Clarke and Hollingsworth (2002). This was not the end of Cem's learning curve, even if it might seem like a plateau, because it is essential for deeper understanding and building the capacity to use these new approaches naturally. In Cem's case, the initial learning showed that the new tools had been introduced effectively to him through coaching, and the

stabilization phase most likely showed that he naturally adopted the new practices in his teaching. It is crucial to provide continuous coaching during this phase to help Cem maintain his progress and be ready for the next level, as identified by Clarke and Hollingsworth (2002), Guskey (2002), and Knight (2007). However, as he was involved in the coaching process, Cem realized that applying tools such as GeoGebra in teaching is helpful, mainly to involve students and help them conceptually understand the subject. Cem's development through the TPACK levels in mathematics coaching clearly illustrated how targeted and structured PD could improve teachers' technology integration into teaching and assessment. During his participation in mathematics coaching cycles, Cem progressed from *the recognizing level* to *the adapting level* by MCC 1 and maintained his TPACK level by MCC 2. At this level, he began acknowledging the advantages and benefits of using tools such as GeoGebra in teaching and learning mathematics. Cem's development during mathematics coaching cycles aligns with the previous research that revealed that coaching enhances teachers' self-confidence and proficiency in integrating technology into teaching, as found by Gibbons and Cobb (2016, 2017) and Liao et al. (2021).

Cem started using GeoGebra and MathsPad to construct angles and solve problems related to angles with the help of the dynamic features of these tools. This change in practice is consistent with studies that show that coaching results in using more efficient and effective teaching strategies that foster conceptual learning among students, as stated in the studies by Solarski (2019) and Ward (2020). During MCC 3, Cem progressed to *the exploring level* for the curriculum and *assessment descriptors*. He used GeoGebra to illustrate the area relationship of a triangle and had the students do some activities based on the concept. This starkly contrasted his earlier reluctance and concerns regarding the effectiveness of using technology in his teaching to proactively look for how it could be appropriately integrated. This aligns with studies that show how coaching may assist teachers in progressing from using technology simply as a tool to using digital resources in teaching and learning more comprehensively, as found in Russell et al. (2020) and Solarski (2019) studies. By MCC 4, Cem applied GeoGebra in teaching the area relation of the parallelogram and related problem solving and progressed to *the advancing level* for the *curriculum*

descriptor. Cem might be said to have progressed to the highest level of TPACK since technology is incorporated into the curriculum in a unique way that improves the content to be taught and the student's understanding, as Aygün (2016) found in her study.

Nevertheless, Cem could not progress to the highest level of *the assessment descriptor*, stemming from limitations such as insufficient classroom equipment and access to technology, as Kopcha (2012) and Warschauer et al. (2004) found. He could not fully implement technological-based assessments because of the limited technological facilities in his classroom. This aligns with the research done by Chien et al. (2014), Clarke-Midura and Dede (2010)**,** and Elmahdi et al. (2018) that underlined the need for necessary technological infrastructure, resources, and opportunities to employ effective technology-based assessments. As a result, the fact that the students did not have access to internet-connected devices during the lesson and Cem could not display the student's work and give them feedback prevented him from applying innovative assessment methods that demand full use of technology as found in the studies by Ertmer and Ottenbreit-Leftwich (2010), Hew and Brush (2007), and Inan and Lowther (2010). Therefore, Cem could not progress to the highest level of the *assessment descriptor*. Cem's view that the innovative technology-based assessment could be done when every student has access to devices aligns with the studies by Hew and Brush (2007) and Ertmer et al. (2012), who stated that teachers and students need appropriate tools and equipment. Therefore, this study also revealed that the failure to progress to the highest level of *the assessment descriptor* within the Curriculum and Assessment Theme might have been due to similar constraints, such as unequal access to technological devices among students, as indicated in the studies by Hew and Brush (2007) and Ertmer et al. (2012). On the other hand, Aygün's (2016) study, which also employed mathematics coaching to enhance in-service teachers' TPACK levels using the TPACK development model of Niess, Ronau, et al. (2009) focused on the learning domains of Geometry with a similar period of study and teaching hours, revealed that one of the participants achieved the highest level of *the assessment descriptor*. Aygün's (2016) provision of a computer for every 3 or 4 students for teaching and assessment was critical in participant teacher's progression to *the advancing level* in

the assessment descriptor, given that adequate technological infrastructure is crucial for supporting progression to high levels of teachers' TPACK in assessments. This comparison shows how crucial it is for teachers and students to have adequate technological resources and affordances to implement technology in teaching and assessment practices and achieve high TPACK levels. In conclusion, Cem's experiences in mathematics coaching shed light on the main issues of challenges and opportunities discussed in the literature regarding the use of technology in mathematics teaching and assessing students' learning. Cem's early reluctance and resistance to integrating technology into the curriculum, followed by gradual progress, highlighted the importance of targeted, structured, and effective PD in improving TPACK and fostering meaningful use of technology with proper pedagogy in mathematics teaching and learning. Cem's transformation was consistent with research in the literature, which emphasizes the pivotal role of coaching as PD in helping teachers address obstacles to technology integration and ultimately enhancing the learning outcomes of students, as indicated by Liao et al. (2021) and Saclarides and Harbour (2023).

Regarding the modifications in curriculum, mathematics coaching fostered Cem to change and enhance the curriculum through GeoGebra, making the curriculum more dynamic and interactive for mathematical explorations by adapting problems and activities presented in the curriculum. This modification is consistent with the study by Gibbons and Cobb (2017), who argued that coaching assists teachers in integrating digital tools into their teachings, enhancing and enriching the curriculum and making the students' learning more engaging. Mathematics coaching positively and significantly affected Cem's TPACK and technology-based assessment knowledge and skills. Through mathematics coaching, Cem progressed from a teacher using technology as a visual aid to teaching to a teacher who employed technology effectively with proper pedagogy in teaching and assessment practices by using GeoGebra in his classes. This progression highlighted the essential function of coaching in supporting the progress of Cem's TPACK and improving teaching effectiveness, ultimately leading to higher student achievement. Cem's advancement in the use of technology for enriching the curriculum and employing innovative technology-based assessment practices is in line with the studies by Aygün (2016),

Gibbons and Cobb (2017), Liao et al. (2021), Solarski (2019), and Ward (2020) who highlighted the benefits of coaching on teachers' instructional strategies, skills, and on their capacity to integrate technology within their teaching and assessment processes

To sum up, before the mathematics coaching, Cem adopted a traditional and content-centric approach to curriculum and assessment, emphasizing memorization and standardized assessment. This is by the study of Liao et al. (2021), who noted that teachers typically begin participation in PD with more content-based and then embrace technology use and other forms of more interactive assessment through PD. During his participation in coaching cycles, Cem changed his approach to learning and teaching and shifted his focus to integrating technology into curriculum modifications and interactive assessment practices. Cem's shift is in line with Gibbons and Cobb's (2017) findings, who noted that coaching assists teachers in changing the curriculum to incorporate digital tools and make teaching and learning more interactive, engaging, and effective in line with students' comprehension. Therefore, mathematics coaching was crucial in advancing Cem's TPACK, particularly in integrating GeoGebra into his curriculum, assessment, and instructional practices. However, his progress in the Curriculum and Assessment Theme suggested that ongoing support and targeted interventions are necessary to overcome the plateau effect and continue advancing TPACK levels. This reflected broader findings in the literature that emphasize the need for sustained PD to fully realize the potential of technology in education, as indicated by Hew and Brush (2007) and Mishra and Koehler (2006).

5.1.2. Discussion on Cem's TPACK Development Regarding Learning Theme

Before mathematics coaching, Cem's TPACK levels were at the *accepting level* in the Learning Theme for *mathematics learning* and *the conception of student thinking descriptors*. However, with the help of the mathematics coaching cycles (MCCs), Cem achieved the highest level of the Learning Theme in all its descriptors. Table 10 presents the TPACK levels of Cem during the six technology-integrated lessons taught before, during, and after the MCC.

Table 10. The progress of Cem in Learning Theme and related descriptors

Themes	Descriptor	Before MC	MCC 1	MCC 2	MCC 3	MCC 4	After MC
Learning Theme	***Mathematics learning descriptor***	2	3	3	4	5	5
	Conception of student thinking descriptor	2	3	4	4	4	5

Note. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

Before mathematics coaching, Cem's teaching practice regarding the learning theme was consistent with the *accepting level* of TPACK for *mathematics learning* and *the conception of student thinking descriptors*. Cem initially considered technology a barrier and was concerned that students would be distracted from learning basic mathematical concepts, as supported by the studies of Spangenberg and De Freitas (2019) and Zamir and Ali (2023). This concern led him not to integrate technology in teaching angles but to use traditional methods, such as the 'houses without angles' group activity without digital tools. The reluctance of Cem and his resistance to using technology in his teaching practices are indications that he requires training to effectively use technology in teaching, as indicated by Adipat et al. (2023). Cem's use of direct instruction and traditional approaches led to students not engaging in technology, as found by La Fleur and Dlamini (2022), who pointed out that teachers might not be able to determine suitable contexts for technology integration when they are not confident about their skills and proficiency regarding technology integration. This limited approach of Cem revealed his low TPACK level, which hindered the possibility of utilizing the potential benefits of technology for improving students' engagement and learning of mathematics, as Angeli and Valanides (2009) suggested. Similarly, Cem's teaching approach before mathematics coaching regarding *the conception of student thinking descriptor* also indicated *the accepting level* of TPACK.

As a result, he avoided using subject-specific technology almost entirely for concern that its use would limit students' ability to think mathematically and provided

detailed verbal instructions and real-life models of angles in place of technology, as Chai et al. (2013) and Harris and Hofer (2011) found. His lack of enthusiasm in harnessing technology to identify and address students' misconceptions and their cognitive processes is due to low TPACK, which might have been occasioned by ineffective and inadequate PD, as supported by the studies of Lawless and Pellegrino (2007) and Polly et al. (2010). Cem preferred not to use technology and instead relied on conventional and teacher-centered strategies. This aligns with the studies of Niess (2005) and Mishra and Koehler (2006), who revealed that teachers with low TK often underutilize or employ technologies effectively. His approach without technology might also indicate that he lacked confidence in using technology to support higher-order thinking and problem-solving skills, a problem evident in teachers with low TPACK, as Agyei et al. (2022) and Saubern et al. (2020) found in their studies. Cem's earlier hesitance in the use of technology underlined a problem that is typical of many teachers who have not been well-prepared or supported, and which returns the importance of the need for ongoing PD to foster the required skills and assurance in the use of technology, as identified by Ertmer et al. (2012) and Harris et al. (2009).

During mathematics coaching, Cem improved significantly in the Learning Theme. Initially, he considered technology as a factor that hinders learning, which aligns with the study of Hamutoglu (2021), who pointed out that teachers consider technology as an interfering factor rather than a tool for learning. This perception restricted him from using digital tools to the extent of typing presentations and projecting them. However, during the coaching cycles, Cem transformed and understood that technologies could be used to improve students' engagement and learning. With the following coaching cycles, Cem's development in specific areas, such as the conception of student thinking, seemed to stagnate. This was observed in the third and fourth coaching cycles and indicated a learning plateau where the learning rate slows down after an initial steep learning curve. Such plateaus are common, particularly when tasks become more complex, requiring higher-order thinking skills and deliberative processes, which are time-consuming, as found by Borko (2004) and Opfer and Pedder (2011). In the first and second coaching cycles, Cem incorporated GeoGebra and MathsPad to teach students how to construct

congruent angles and shift from simple to more complex teaching techniques. This aligns with the studies of Liao et al. (2021) and Gibbons and Cobb (2017), who revealed that coaching enhances teachers' self-efficacy in integrating technology into teaching. During the coaching process, Cem continued to expand on these tools in the lessons on supplementary, complementary, adjacent, and opposite angles and encouraged students to find the solutions on their own, consistent with Gibbons and Cobb (2017), who pointed out that coaching assists teachers in adopting more creative teaching approaches. In the third cycle, Cem progressed to *the exploring level* and applied technology to teach advanced concepts, such as the area relations of a triangle. This shift in technology integration aligns with the study conducted by Russell et al. (2020), which indicated that coaching helps teachers progress from using technology in its simplest form to a comprehensive level of technology integration. By MCC 4, Cem progressed to *the advancing level*, where he could plan, execute, and evaluate his use of GeoGebra in the teaching and learning process and curriculum. This progression aligns with Aygün (2016), who noted that the highest levels of TPACK are reached when technology is used to enhance teaching and learning and increase the depth of student understanding. On the other hand, unlike some studies that describe how teachers' progression stagnates after making initial gains, as Ertmer et al. (2012) indicated, Cem's continuous learning suggested that there is lasting improvement with sustained coaching. Hence, the improvements of Cem helped to elucidate how mathematics coaching might positively affect a teacher's approach to the use of tools and their TPACK, as identified by prior research, and how it also offers insights into the potential for further enhancement of TPACK through ongoing coaching.

During the coaching process, Cem was supported by the coach in deepening his knowledge about using GeoGebra to enhance students' understanding of angles and how to measure area. This support, which likely enhanced his TPACK, aligns with the findings by Buser (2018), who noted that coaching assists teachers in enhancing their content knowledge and improving students' ability to understand mathematical concepts. The coach's recommendations to move problems to GeoGebra's environment to address students' difficulties and misconceptions are consistent with Larsen (2012) and Bengo (2013), who emphasized the role of coaching in

implementing new technologies and improving teaching strategies. Nevertheless, there is agreement with Downing (2020) in the fact that Cem could not reach the highest level in *the conception of student thinking descriptor* in MCC 4 since he did not encourage his students to develop higher-order thinking skills and use GeoGebra to do so. Downing (2020) stated that PD is vital for teachers to fully harness the potential of technological tools in developing students' thinking skills, suggesting that Cem may have needed more time or more focused mentorship. This is because Cem did not reach the highest level in *the conception of student thinking descriptor* by MCC 4, which could be attributed to his perceived ineffectiveness and lack of confidence when handling complex, high-level cognitive activities that involve GeoGebra, which often discourages teachers from fully integrating such tools, as found by Ertmer and Ottenbreit-Leftwich (2013).

In conclusion, Cem's learning process and student engagement ideas were mainly based on the content, with little emphasis on technology for providing interaction or critical thinking skills before joining Mathematics Coaching. This aligns with Harris et al.'s (2009) study, which revealed that teachers start the teaching process with the traditional, didactic approach, followed by teacher training that promotes student-centered teaching. During the coaching program, Cem started using GeoGebra in his classroom, making lessons more interesting and participatory for students. This transformation aligns with the view of Angeli and Valanides (2009), who pointed to the use of technology in enhancing critical thinking skills and collaborative learning in teaching Mathematics. Nonetheless, Cem faced some challenges, especially in mathematics learning and the conception of student thinking, which showed minimal progress. This finding does not entirely align with the study by Agyei and Voogt (2012), who noted a smooth transition among the teachers in the collaborative design-based PD programs. It may be that the individualization of the coaching model in the present study introduced difficulties that led to this plateau, in the sense that such change embodies the conceptual demand of moving student learning from content focus to process focus. After coaching, Cem had a much-improved application of technology to enable student learning; however, the stagnation in *mathematics learning* and *conception of student thinking descriptors* continued. This plateau implies that although Cem was gradually learning to use technology in the

classroom and improving the student's learning, a complete change of paradigm and more guidance than what was offered in the coaching process was needed to enhance the students' engagement. As indicated by Aygün (2016), more frequent and continuous coaching and reflective practices are essential as they help the teachers overcome such plateaus, which may be the reason for the differences in the progress of Cem compared to other research. Concerning student engagement and instructional strategies, the use of GeoGebra by Cem to solve high-level thinking problems such as problem-solving and decision-making is in congruence with the study done by Harris et al. (2009) and Angeli & Valanides (2009), who stressed on the aspect of technology as an effective tool to enhance critical thinking skills of students. Also, as a guide, Cem uses instructional strategies with GeoGebra, consistent with Downing (2020), who found that coaching assists teachers in embracing the student-centered approach. Cem was also able to address the multiple learning modalities by having the students work on GeoGebra-supported activities and using both inductive and deductive methods in line with the study of Niess et al. (2009), which suggested that technology helps support the teaching of diverse students and promoting student learning.

5.1.3. Discussion on Cem's TPACK Development Regarding Teaching Theme

Before mathematics coaching, Cem had relatively low levels of TPACK (accepting and recognizing) in *the mathematics learning*, *instructional*, *environment*, and *professional development descriptors* of the Teaching Theme. However, during his participation in the mathematics coaching cycles (MCCs), Cem progressed to the highest level in all descriptors of the Teaching Theme. Table 11 presents Cem's TPACK levels in six technology-integrated activities taught before, during, and after the MCC.

Table 11. The progress of Cem in Teaching Theme and related descriptors

Theme	Descriptor	Before MC	MCC 1	MCC 2	MCC 3	MCC 4	After MC
Teaching Theme	***Mathematics learning descriptor***	2	3	3	4	5	5

Table 11. (continued)

Instructional descriptor	1	3	4	5	5	5
Environment descriptor	1	2	3	4	5	5
Professional development descriptor	2	-	-	-	-	5

Note. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

Before mathematics coaching, Cem demonstrated *the accepting level* in *the mathematics learning descriptor* and *the recognizing level* in *the instructional* and *environment descriptors* of the Teaching Theme. His approach to teaching angles was highly teacher-centered, emphasizing direct instruction without incorporating technology-enhanced activities. This is consistent with Tondeur et al. (2017), who found that teachers with low technological knowledge and skills often rely on conventional techniques. Cem thought technology was unimportant and only used it to display real-life situations to the students. This aligns with Kopcha's (2012) observation that teachers implement technology as complementary to teaching. His limited prior experience with GeoGebra and his lack of ideas on how to make the students understand the mathematical concepts through the use of GeoGebra reflect the findings by Inan and Lowther (2010) that teachers tend to underutilize the available technologies in their teaching due to perceived irrelevance or if they see no use of the technology. Cem's reluctance to implement student-centered approaches to teaching and his inclination to retain control over students' behaviors during classroom lessons by employing a teacher-centered approach aligns with Ertmer and Ottenbreit-Leftwich (2013), who pointed to the fact that concerns about class management make teachers avoid technology integration. This aligns with Hattie's (2009) findings, where some teachers cited that direct instruction is the most effective way of guaranteeing that students cover the given curriculum and manage the classroom. These factors may discourage the use of more interactive and student-centered technologies. The similarity of these findings with previous research supports that Cem's low TPACK and inactivity in PD might have led him to use

conventional methods. His instructional environment, characterized by a lack of group work and interactive learning, aligns with Niess et al. (2010), who discovered that instructors with minimal knowledge of student thinking and low self-assurance in using technology are less likely to integrate student-centered teaching approaches. Cem's teaching practice before mathematics coaching was congruous with the content-centric approach identified in the literature. The content-centric approach focuses on mastering the discipline's content, where the main aim is to ensure that the student understands and can apply the various concepts and information within the subject area. Gess-Newsome (1999) and Shulman (1986) have documented similar practices whereby lessons are developed based on curricular and content standards and where content is emphasized. Before mathematics coaching, Cem used conventional teaching methods, including lecturing, reading from textbooks, and assignment writing, with the teacher as the source of knowledge. This feature is in line with Shulman's (1986) teacher-centered approach. With the teacher as the focal point in delivering knowledge, this method can be characterized as a content-centric teaching approach, as Harris and Hofer (2009) expressed. The lack of the use of technology in Cem's teaching strategies is consistent with the research done by Niess (2005) and Harris and Hofer (2009), in which the researchers discovered that teachers who have a content-centric teaching approach tend to use conventional materials such as textbooks and whiteboards and not technology.

In MCC 1 and MCC 2, Cem improved his TPACK level to *the adapting level* in *the mathematics learning descriptor*. This shift marked the beginning of his exploration and practice in the effective use of technology in teaching and learning mathematics in line with the TPACK development model. Cem started to use GeoGebra and MathsPad to teach students to define and identify congruent angles. As a result, he transitioned from solely recognizing the potential of applying technology in the classroom to actively using technology to support the learning process. This change is consistent with Liao et al. (2021), who pointed out that coaching assists teachers in integrating technology into their teaching practice. By MCC 2, Cem progressed to applying technology for supplementary, complementary, adjacent, and opposite angles, where GeoGebra was used to help students discover and solve problems. This aligns with the study by Gibbons and Cobb (2017), who noted that teachers are

assisted in using technology to teach students more interactively through coaching. Cem's development to *the exploring level* by MCC 3, where he employed GeoGebra to explain the area relationship of a triangle and had the students solve problems, was consistent with Solarski (2019), who has pointed out that coaching assists teachers in incorporating technology as a significant component of teaching and learning process. In MCC 4, Cem progressed to the highest level of *the mathematics learning descriptor*. At this level, he seamlessly integrated technology in planning, executing, and evaluating his teaching for student thinking and learning. This comprehensive and effective integration of technology with proper pedagogy that Niess, Ronau, et al. (2009) described is consistent with the findings of Bengo (2013) and Ward (2020), who observed that effective coaching assists teachers in attaining higher levels of technology use in teaching and learning, that is, technology-enhanced learning.

Regarding Cem's development in *the instructional descriptor*, he began at *the recognizing level* before coaching, which means that he was aware of the benefits of technology but did not put it into practice. This initial reluctance aligns with Niess, Ronau, et al. (2009), who stated that the teachers did not use technology to build mathematical concepts at *the recognizing level*. According to MCC 1, Cem progressed to *the adapting level*, where he began incorporating technology and engaged students in low-level cognitive activities after the coaching sessions. This development aligns with Lawless and Pellegrino's (2007) assertion that the initial phases of PD entail teachers' mimicking technology integration into teaching lessons. By MCC 2, Cem progressed to *the exploring level*, where he actively integrated GeoGebra into his teaching and invited students to explore angles together, taking on a guiding role rather than directing. In this light, the shift to student-centered learning with the use of technology aligns with the study by Gibbons and Cobb (2017), which showed that coaching assists teachers in transforming from directing learning to facilitating, that is, from controlling to supporting the learning of the students on mathematical concepts. Cem's progression to *the advancing level* by MCC 4, in which he used several instructional strategies with technology to enable students to think critically about mathematics, is consistent with Solarski's (2019) and Ward's (2020) studies that suggested coaching assists

teachers in growing their teaching effectiveness and sophistication by implementing practical technology-based strategies.

As for *the environment descriptor* of the Learning Theme, Cem was at *the recognizing level* at the beginning of the study. He acknowledged the potential benefits and facilities of technology usage but did not use it in the learning environment, as stated in the TPACK development model. Niess, Ronau, et al. (2009) stated that teachers at this level use technology to consolidate the concepts taught without technology. His classroom environment was, however, characterized by a traditional, teacher-centered delivery format where students were passive listeners and note-takers through lessons, as observed by Becker and Ravitz (1999), where teachers with traditional orientation kept class control and restricted student use of technology. In MCC 1, Cem progressed to the *accepting level*, where he formed a positive attitude towards integrating GeoGebra into his teaching and began to use GeoGebra in his class in a controlled manner, in a teacher-directed approach. This controlled use of technology is consistent with Tondeur et al. (2017), who pointed out that teachers limit technology use at the beginning of their integration process. Thus, by MCC 2, Cem was at the *adapting level*; he changed his teaching environment by using technology more interactively and allowing the students to work with GeoGebra to learn mathematics. This aligns with Ward (2020), who noted that coaching helps teachers develop more interactive lessons with the technology used in teaching. Cem progressed to *the exploring level* of the TPACK model by MCC 3, in which he integrated GeoGebra to create a compelling and stimulating learning environment. This integration of deductive and inductive methods to have the students reason and solve problems through technology conforms with the research conducted by Gibbons and Cobb (2017) and Russell et al. (2020), who argued that good coaching helps in the creation of classrooms where students can reason about and discover mathematics with the use of technology. Therefore, according to MCC 4, Cem shifted to *the advancing level* with technology as an integral part of the classroom and realized that applying technology could enhance students' interest and their capability to manage their learning. This aligns with Liao et al. (2021), who said that coaching helped teachers develop technological pedagogical actions that kept students active and self-directed.

In conclusion, by participating in mathematics coaching cycles, Cem progressed in the *mathematics learning, instructional,* and *environment descriptors* of the TPACK development model. Initially, Cem considered technology to be a factor that hindered learning and, thus, relied only on traditional teaching methods. However, after participating in mathematics coaching, he transformed his teaching approach and incorporated technology into his teaching more effectively. Mathematics coaching played a significant role in transforming Cem's pedagogical practices, especially the use of GeoGebra as a tool for teaching and learning. This transformation aligns with the studies by Liao et al. (2021), Solarski (2019), Gibbons and Cobb (2017), and Ward (2020) that highlighted the pivotal role of coaching in helping teachers transition from basic to more comprehensive levels of technology integration. After Cem's involvement in mathematics coaching, GeoGebra became a central tool in his teaching, encouraging students to think critically and solve complex problems, demonstrating how technology enhances mathematics teaching and learning. In addition, it is essential to note that Cem wanted to improve his professional qualifications, which also influenced the successful implementation of GeoGebra in the teaching process. His participation in several PD courses to explore new ideas for teaching mathematics with multiple technologies is consistent with findings by Ertmer and Ottenbreit-Leftwich (2010) and Larsen (2012), who noted that PD is crucial for teachers to employ new tools and strategies. This benefited Cem as he continued learning, implemented new technologies in his class, and enhanced his teaching practice.

Before mathematics coaching, Cem's teaching strategies were teacher-centered, relying on direct instruction with minimal technology integration. This is a common starting point for many teachers, as highlighted by Harris et al. (2009) and Angeli and Valanides (2009), who observed that traditional instructional strategies often dominate before PD interventions. During the coaching process, Cem began to explore a broader range of instructional strategies, incorporating both deductive and inductive methods supported by technology. This development is consistent with the findings of Açıkgül and Aslaner (2020), who revealed that GeoGebra-supported PD enhances teachers' instructional strategies by promoting more active student engagement. Niess, Ronau, et al. (2009) also pointed out that teachers at *the*

advancing level of the TPACK framework incorporate a wide range of instructional methods to enhance the students' mathematical thinking through technology. However, Cem experienced a plateau in *the mathematics learning descriptor* within the Teaching Theme, indicating that while he made initial progress, fully adopting these new instructional strategies was challenging. This plateau contrasts with the more positive outcomes reported by Campbell and Griffin (2017), who observed smoother transitions in teachers' instructional practices through consistent and comprehensive coaching. After the coaching cycles, Cem significantly improved his instructional strategies, incorporating technology with a more balanced and effective approach. Despite these improvements, the plateau in *the mathematics learning descriptor* persisted, suggesting that while Cem gradually progressed, fully embracing and sustaining new instructional practices required more ongoing support than was available during the coaching process. Aygün (2016) consistently highlighted the importance of continuous and reflective coaching to help teachers overcome these challenges, a factor that might have mitigated the plateau had it been more emphasized in the present study.

5.1.4. Discussion on Cem's TPACK Development Access Theme

Before mathematics coaching, Cem had the lowest level (recognizing) of TPACK in *the Access Theme's usage, barrier, and availability descriptors*. However, Cem reached the highest level of all descriptors of the Access Theme by way of his participation in the mathematics coaching cycles (MCCs). Table 12 presents the TPACK levels of Cem in six technology-enriched lessons implemented before, during, and after the MCC.

Table 12. The progress of Cem in Access Theme and related descriptors

Theme	Descriptor	Before MC	MCC 1	MCC 2	MCC 3	MCC 4	After MC
Access Theme	***Usage descriptor***	1	3	4	5	5	5
	Barrier descriptor	1	3	4	5	5	5
	Availability descriptor	1	2	3	4	5	5

Note. The levels of the TPACK Development Model, ranging from the lowest (level 1) to the highest (level 5), are as follows: Recognizing (level 1), Accepting (level 2), Adapting (level 3), Exploring (level 4), and Advancing (level 5).

Before mathematics coaching, Cem's teaching practice demonstrated the *recognizing level* in the Access Theme's *usage*, *barrier*, and *availability descriptors*, indicating the lowest level. Cem encountered barriers to integrating technology into his teaching practices, primarily due to limited access, insufficient knowledge, and familiarity with subject-specific technologies. This is a common issue highlighted by Inan and Lowther (2010), who found that initial barriers often stem from a lack of confidence, proficiency, and available resources. Cem did not allow students to use technology even after he had explained the concept of angles through direct instruction and a teacher-centered approach. Cem's beliefs align with Koehler and Mishra's (2009) conclusion, which found that teachers with low TPACK are likely to prefer traditional teaching methods because they lack confidence in using educational technologies. Although technology was perceived as a tool that could help provide less challenging content to many students, Cem resisted changing his teaching practices. This resistance aligns with Hébert et al. (2021), who found that teachers' concerns about technology accessibility in teaching result in using conventional teaching methods. Despite realizing its benefits, such reluctance to embrace technology in the classroom aligns with Kihoza et al. (2016), who noted that reasons such as a lack of resources and training remain the primary reasons teachers are reluctant to incorporate technology into their teaching. Therefore, Cem preferred content-centric teaching, mainly using direct instructional strategies and teacher-centered activities without technology. This is consistent with the research of Shulman (1986) and Gess-Newsome (1999), who noted that teachers with insufficient knowledge of technology tend to focus on content coverage rather than incorporating new instructional tools. This aligns with the studies of Koehler and Mishra (2009) and Kihoza et al. (2016), where the authors found that teachers' use of traditional methods results from a lack of resources and inadequate training to use new technological tools. Therefore, Cem did not use any strategies to overcome these barriers, which indicates low TPACK. This study aligns with the study by Kafyulilo and Fisser (2019), who pointed out that teachers with low TPACK levels may struggle with barriers to technology integration due to a lack of experience and comfort in using technology in the classroom. In addition, Cem's failure to support the students to engage in higher mathematical thinking using technology, although he identified the importance of integrating technology into teaching, is consistent with

Kushner Benson et al. (2015), who stated that a good teacher should have the necessary pedagogical knowledge that enables them to integrate technology into the teaching and learning process effectively. The inadequacy of technological resources in Cem's classroom, whereby students could not have personal computers or tablets, might have contributed to Cem's reluctance to integrate technology into teaching. This aligns with the study done by Ertmer et al. (2012), who pointed out that many teachers are hesitant to incorporate technology into their teaching because of issues related to equity and accessibility, especially where they might not ensure that every student has access to the technology.

Cem's low levels in the Access Theme also align with findings on insufficient PD's role. According to Lyublinskaya and Kaplon-Schilis (2022), many teachers are not well equipped with the necessary skills and knowledge on how to incorporate technological tools with the pedagogical and content knowledge they possess, which leads them to employ more conventional approaches to technology use. For instance, Niess, Ronau, et al. (2009) noted that in-service teachers who lack effective PD tend to remain at the initial levels of TPACK, where they acknowledge technology's affordances but do not fully implement them in their teaching. Koehler and Mishra (2009) asserted that classroom management is another problem associated with Cem's attitude toward integrating technology in his class. Koehler and Mishra (2009) found that teachers are often concerned with classroom management when students use technology, which can lead to a preference for conventional, teacher-centered methods. This concern is consistent with findings by Fernández et al. (2020), who noted that teachers with low TPACK, as Cem, often avoid bringing technology into the classroom due to concerns about potential classroom management issues.

Cem did not attend any PD program to improve his TPACK or learn how to incorporate GeoGebra into his mathematics lessons, which might have contributed to his low TPACK levels, as Lyublinskaya and Tournaki (2013) underscored. Lyublinskaya and Tournaki (2013) stated that teachers' TPACK was generally lower without PD, which affected their capacity to design technology-enhanced lessons. This aligns with the study by Jaipal and Figg (2010), who stressed that PD encompassing practical, classroom-based applications is vital for teachers to

incorporate technology. Consistently, Cem did not attend any training during his undergraduate studies in Türkiye, where he did not take any courses on the effective use of dynamic mathematics or geometry software such as GeoGebra in the classroom. The above studies found that teacher training programs focus on specific technical skills but often fail to demonstrate the practical value of technology in practice. This means that the TPACK of teachers is low or underdeveloped. This is consistent with the findings of Harris and Hofer (2011) and Jaipal-Jamani and Figg (2015), who argued that without PD that includes practical classroom application, teachers struggle to build the higher levels of TPACK needed to integrate technology into teaching. Finally, the conventional approaches to assessment that Cem most likely used align with Hargreaves et al. (2004). They also discovered that many mathematics teachers think that traditional assessment methods are the most effective ways of determining the extent of students' understanding and, therefore, restrict the use of technology in both teaching and assessment. This also explains why Cem prefers using paper-and-pencil methods over technology in his teaching and assessment practices.

With his participation in coaching cycles, Cem progressed to *the adapting level* in the *usage descriptor* by MCC 1. He started permitting students to use GeoGebra in specifically designed lessons, such as constructing congruent angles to given angles. This was a shift to a more purposeful use of technology, as described by Liao et al. (2021), where coaching helps teachers use technology strategically in their teaching with specific instructional purposes. By MCC 2, Cem progressed to *the exploring level*, and students were allowed to use GeoGebra to explore certain mathematical concepts dynamically, such as supplementary, complementary, adjacent, and opposite angles. This is evident in encouraging students to discover properties of angles using GeoGebra. This aligns with Gibbons and Cobb's (2017) study, which revealed that coaching assists teachers in embracing more interactive instructional approaches where students deeply explore mathematical concepts through technology. By MCC 3, Cem progressed to *the advancing level*, where he permitted students to use GeoGebra in every aspect of the mathematics lesson, including introduction, exploration, verification, problem-solving, testing conjectures, reinforcing the concepts, and assessment. This total immersion in the technology-

enriched learning environment, where the lesson focused on exploring and discovering mathematical concepts, aligns with Bengo (2013), who found that coaching improves teachers' capacity to incorporate technology into teaching as an integral part of every lesson. Hence, it is vital for teaching and learning. By MCC 4, Cem remained in the moving phase, expanding the possibilities of the students and what they could examine and learn through technology. This aligned with the study by Saclarides and Harbour (2023), who pointed out that coaching enables teachers to utilize all affordances of technology to improve student's learning outcomes in various contexts.

At the beginning of the study, Cem was at *the recognizing level* in *the barrier descriptor* of Access Theme. He opposed alterations in content taught even though he recognized that technology could enhance students' access to the content. His reluctance is in line with the findings revealed by Koehler and Mishra (2009), who argued that teachers with low TPACK often resist using technology in teaching and learning because they feel that it might interfere with the learning of the concepts. By MCC 1, Cem progressed to *the adapting level*, where he began to consider technology to improve the teaching and learning of mathematics by offering the students new ways of learning the concepts. At first, he thought the concepts learned with GeoGebra were ineffective when taught without technology. However, he realized the effectiveness of using GeoGebra in improving the student's understanding of concepts. This aligns with the study of Ertmer and Ottenbreit-Leftwich (2010), who noted that PD enables teachers to start recognizing and embracing that technology plays a crucial role in enhancing students' learning. By MCC 2, Cem progressed to *the exploring level*, including GeoGebra for assessing students' learning and seeking strategies to manage technology integration effectively in teaching. This is in line with the study by Polly et al. (2010), who noted that to address barriers to technology integration, teachers should seek new strategies and solutions to minimize challenges, as Cem did. Through MCC 3, Cem progressed to *the advancing level*, whereby he tackled problems by comprehensive planning and preparing for them in detail. He also ensured that he maximized using GeoGebra and other digital tools such as MathsPad, WordWall, and Khan Academy. This level of purposeful and planned integration of digital tools where technology is

used to extend the range of concepts that can be taught and learned aligns with the study done by Bengo (2013), in which coaching was found to help teachers overcome barriers through strategic planning and resource management. By MCC 4, Cem continued at *the advancing level*, actively engaging in the management of technological challenges and the optimization of resources, in line with Larsen (2012), who stated that coaching assists teachers in enhancing their approaches to the use of technology in the classroom.

Considering Cem's progression in Access Theme through coaching cycles, there was no sign of a plateau in this theme through his participation in mathematics coaching. That Cem gradually learned how to overcome barriers to technology integration, utilize technology to make concepts more accessible to students, and allow students to use technology in every aspect of mathematics classes might be the underlying reason, as found by Clarke and Hollingsworth (2002) and Opfer and Pedder (2011). On the other hand, the other themes showed plateaus throughout this study. The learning curve of the Access Theme in which Cem's TPACK level increased in each coaching cycle and did not remain at the same level might suggest that the complexity of tasks in different themes of the TPACK development model might vary. Moreover, the knowledge and skills gained that the Access theme requires might be more straightforward to acquire than the changes in teacher practices required for the other themes. Albion and Ertmer (2002) noted that after initial obstacles are faced, teachers might still enhance their technology integration. This could explain why Cem did not experience a plateau in this theme. The development observed in Cem's TPACK levels, especially in *the usage, barrier,* and *availability descriptors*, aligns with the related literature that underscores the effects of targeted PD on teachers' knowledge for allowing students to use technology, overcoming barriers to technology integration, and using technology to make more mathematics available for students. Consistently, Aygün (2016) found that mathematics coaching could contribute significantly to enhancing teachers' TPACK, helping them overcome barriers to technology use and making the best use of available resources. This is consistent with Cem's development in these domains as he progressed to the highest level of the TPACK model. Similarly, Açıkgül and Aslaner (2020) underlined the need for structured and effective PD, such as GeoGebra-supported

microteaching, to improve teachers' TPACK. Their study indicated that when teachers are provided with personalized support and opportunities to practice using technologies, they can enhance their TPACK using the available technologies and resources. This finding aligns with the present study's findings, which revealed that Cem could overcome barriers, effectively utilize all the technological tools available to him, and achieve the highest level of *the barrier descriptor* through the personalized support that mathematics coaching provided as structured and effective PD. Furthermore, Polly (2011) elaborated on the importance of PD in enhancing teachers' TPACK, mainly as they work on the complexities of implementing technology in teaching. Polly (2011) revealed that to improve the TPACK of teachers as Cem, it is necessary to provide them with long-term, practical PD, such as mathematics coaching, especially in areas related to technology use and access.

5.2. The Summary of Discussion

Mathematics coaching improved Cem's teaching practice and enabled him to use subject-specific technologies effectively. Through mathematics coaching, Cem progressed from possessing PCK to developing strong TPACK, as Aygün (2016) found. This transition was supported and facilitated by his involvement in mathematics coaching cycles, which led him to the highest levels of TPACK in all themes and related descriptors, except *the assessment descriptor* in the Curriculum and Assessment Theme. As Aygün (2016) indicated, change in teachers' TPACK regarding *the assessment descriptor* and practices was the slowest transformation compared to other descriptors. The results of this study also aligned with the findings of studies of Borko (2004) and Desimone (2009), who asserted that targeted, personalized, and sustainable PD could result in improved teaching practices and effective technology integration. The results of the current study are consistent with the previous studies on TPACK development through PD and coaching, yet there are some inconsistencies. The consistencies, such as the increased focus on interactive and technology-enriched approaches to teaching, emphasized mathematics coaching's potential for transforming teachers and their teaching practices, as Desimone (2009) found. Nevertheless, the plateaus in specific themes, such as Curriculum and Assessment, Learning, and Teaching Themes, emphasized that the

development of TPACK is a complex process. Hence, the plateaus in specific themes underscored that progression in different themes of the TPACK development model might require differentiated functions of the coach's support and modifications in TPACK training for the teacher, as Costa and Garmston (2004) revealed. These plateaus also indicated that although Cem achieved significantly across themes and descriptors, specific cognitive and pedagogical barriers could not be eliminated in the period allocated and with the available resources. As noted by Aygün (2016), available resources with reflective practices are instrumental not only for an initial improvement but also for further enhancement of teachers' development in all the areas of TPACK. The absence of plateauing in the Access Theme to other themes highlights the need to customize support in PD programs. Whereas issues regarding access theme could be solved more quickly and easily, embedding technologies in the teaching and learning process requires more systematic and sustained teacher professional learning.

In summary, the present study's findings provided insights into the process of TPACK development in mathematics coaching. The consistency with prior research reinforces the importance of ongoing, targeted, and strategic PD, while the inconsistencies, particularly the plateaus, underscore the need to consider contextual variables, including available resources and the approach to PD used. Therefore, these factors might be considered in designing future PD efforts to assist teachers in achieving higher levels of TPACK and thus enhance their teaching practices.

5.3. Discussion on Cem's Perceptions of Mathematics Coaching

The findings of this study concerning Cem's perceptions of mathematics coaching on his TPACK development in the 'Angles and Measuring Area' learning areas yielded four main themes: (1) Contributions of mathematics coaching to professional development, (2) Differentiating aspects of mathematics coaching compared to traditional in-service trainings (3) The difficulties of mathematics coaching, and (4) Recommendations for mathematics coaching.

Cem's perceptions of the role of mathematics coaching in PD align with existing literature. For instance, his view that mathematics coaching enhanced his TPACK is

supported by Larsen (2012), who established that collaborative design of coaching activities enhances teachers' TPACK. In the same way, the impact of coaching on instructional practice is consistent with the study of Buser (2018), who found that teachers who were provided with coaching use more effective teaching methods, hence improving instructional practices. Another researcher, Bengo (2013), has pointed out that coaching helps create positive changes in teachers' perceptions of their pedagogical strategies and subsequently enhances student achievement.

The previous research supporting Cem's perception that mathematics coaching enhances student motivation and achievement includes Campbell and Malkus (2010, 2011), who noted that coaching for mathematics leads to enhanced student achievement. This aligns with the study of Kraft et al. (2018), who also found that coaching enhances student achievement in mathematics. This is by Obara and Sloan (2009), who claimed that coaching practices help provide insights into teachers' knowledge and PD of teachers. In addition, Yopp et al. (2014) stressed that coaching supports the development of a collaborative culture among teachers, thus increasing teamwork and creating a solid professional learning community, which was also seen in Cem's views.

Cem's perceptions align with several studies that compare mathematics coaching with regular in-service training. For instance, Kraft et al. (2018) illustrated that schools that benefited from mathematics coaching observed more explicit and tangible outputs and effects on student achievement than schools utilizing solely traditional in-service training, consistent with Cem's perceptions. Campbell and Malkus (2010, 2011) provided further evidence by showing that students taught by teachers who participated in coaching achieved higher than their peers, showing the direct effect of coaching on student learning outcomes. Moreover, Cem perceived coaching as a transformative PD program that brings about a permanent and sustainable change in teachers' instructional practices, which aligns with the existing literature. Saclarides and Harbour (2023) stated that coaching has implications for long-term changes in teachers' instructional practices. Costa and Garmston (2004) noted that cognitive coaching enhances reflective practice and self-directed learning, which is crucial in sustaining PD.

Cem perceived coaching as a PD program that incorporates theory and practice; as Price-Cole (2018) suggested, theory needs to be put into practice in a classroom setting to enhance the effectiveness of PD. Consistent with Cem's perception of coaching, Buser (2018) also pointed out that coaching assists in connecting theory to actual practice in the classroom and, thus, improves teachers' instructional practices. As for the difficulties of mathematics coaching, especially the time spent on lesson preparation, Cem's perception aligns with previous studies. Warnock et al. (2022) and Downing (2020) also noted that planning lessons in mathematics coaching is relatively time-consuming and challenging, particularly when developing TPACK and dealing with students' misconceptions. Mudzimiri et al. (2014) also stressed that the cyclic process of lesson planning and improving the lesson plan prepared is rather time-consuming to produce the desired impact.

As for the recommendations for mathematics coaching, the existing literature supports Cem's suggestions of expanding mathematics coaching across schools, increasing the duration of coaching, applying it to other mathematics units, integrating AI tools and gamification, and engaging in coaching sessions with other teachers as Aygün (2016) found in her study. First, research has supported integrating AI tools and gamification to enhance student engagement and academic performance. Holmes et al. (2019) highlighted that AI-driven gamification can create engaging learning environments and improve academic outcomes by tailoring challenges to students' individual needs.

Regarding long-term coaching, Darling-Hammond et al. (2017) also emphasized that systematic, sustained professional development is crucial for achieving lasting changes in instructional practices. Finally, Cem's recommendation of collaborative coaching among colleagues aligns with the findings of Vescio et al. (2008), who noted that shared coaching experiences foster a community of practice, allowing teachers to improve instructional methods collectively. In conclusion, Cem's perceptions of the roles, the distinctions, the difficulties, and the recommendations for mathematics coaching are consistent with the related research studies in the literature, which, in turn, supported that mathematics coaching was an instrumental model for PD.

5.4. Implications of the Study

The findings of this study on a middle school mathematics teacher's TPACK development have numerous implications for educational practice, professional development, and theoretical frameworks. Through the analysis of a teacher's development within the mathematics coaching professional development program, this study illustrated the transformative potential of coaching to affect the teacher's TPACK positively. The implications drawn from the findings can help advance the understanding of TPACK, mathematics coaching, and teacher education and inform how educators, professional development providers, and theoreticians can support improving teaching practices through strategic and theoretical advancements.

5.4.1. Implications for TPACK Development and Theory Refinement

The findings of this study provided important information regarding the non-linear and context-based nature of TPACK development with an emphasis on technology integration in geometry teaching. The study's findings revealed variability in the teacher's TPACK for integrating technology, pedagogy, and content knowledge in different themes and descriptors. This variability indicates that current TPACK frameworks should be revised to capture the complex and more nuanced development of teachers' TPACK.

The current TPACK Development Model by Niess, Ronau, et al. (2009) should be expanded and have better-defined descriptors, particularly for those between technology and content knowledge, such as technology-based assessments. The study's findings also point out that there are still some gaps in the TPACK development and, thus, highlight the need to refine the assessment tools that can be more applicable in authentic classroom contexts. To enhance the assessment of TPACK for pre-service and early-career teachers, schools and other educational institutions should try to incorporate other assessment tools like peer observations, reflective practice, and simulated teaching environments.

Theory developers who intend to enhance the TPACK framework should consider these implications to ensure the model can easily adapt to different contexts. The

findings of this study also suggest that TPACK is not a linear process, and it is different depending on the context of the instruction. Thus, the current models should be expanded to include a more complex framework that recognizes TPACK development as a dynamic and contextual process. Furthermore, the descriptors associated with technology-based assessments in the TPACK development model used in this study should be further elaborated to help the researchers. Other contextual variables, including subject area, teaching experience, and availability of resources, can also be integrated into the TPACK model to increase its adaptability and applicability.

5.4.2. Implications for Mathematics Coaching

As evidenced by this study, mathematics coaching can be a powerful tool for changing teachers' teaching practices and moving from the content-based approach to a teaching approach incorporating technology, pedagogy, and content knowledge. This study also highlighted that the crucial components of coaching, such as individualized pre- and post-conference sessions, ongoing feedback, and meeting the teacher's needs, should be considered essential in improving the teacher's TPACK. The findings of this study underscore the importance of sustaining coaching support, given that brief intervention periods may not allow teachers sufficient opportunity to internalize new knowledge. As concluded in this study, coaching programs should focus on long-term support, especially in areas of concern such as technology-based assessments. Moreover, the customization of the process can further improve the approach and better fit the specific needs of individual teachers.

Coaching can also be enhanced by providing teachers with targeted help based on their experience and technological knowledge. Ensuring that the teachers participate in coaching that is relevant to their needs should, in turn, result in better changes in their teaching practices. This highlights the need for versatile coaching models since every teacher is different and cannot be coached in the same manner as the next teacher. The wider school environment also shapes mathematics coaching, so its effectiveness depends on it. Future studies and practice should consider the impact of school policies, administration support, and available resources on the success of the

coaching programs. Thus, it is possible to improve coaches' effectiveness and impact on developing teachers' TPACK by freeing them from non-instructional responsibilities and allowing them more time to focus on planning, observation, and feedback.

Furthermore, it is essential to consider the collaborative models in which instructional coaches work alongside teacher leaders or peers. These can help spread the effects of coaching through a school, thus increasing the effectiveness of coaching on teaching practices. An additional aspect that should not be overlooked is the emotional and psychological support in mathematics coaching. Based on this study, it is possible to state that coaching can enhance teachers' self-confidence, motivation, and propensity to implement new teaching methods. This implies that coaching should not only focus on technical and pedagogical support but also on support for positive emotions and creating a climate for learning.

5.4.3. Implications for Teacher Education and Professional Development Programs

Coaching, reflective practice, and collaborative learning environments should be incorporated into teacher education and professional development programs to enhance the implementation of TPACK in classroom practice. Coaching programs should be part of both pre-service and in-service teacher education; the former to ensure that teachers are exposed to the concept of coaching as a tool to enhance teaching practice, the latter as a way of enhancing teacher's practice through ongoing support and guidance in the incorporation of content and technology into planning and teaching.

Teacher educators should encourage teachers to be reflective, helping them devise ways to modify the teaching approaches used in light of classroom experiences. The findings of this study suggest that professional development programs should focus on developing a collaborative learning environment where teachers can share and learn from each other as they solve problems, share ideas, and develop strategies for incorporating technology in teaching.

Designing professional development programs to cater to the needs of teachers can help cater to the different TPACK levels present in today's classrooms. Continued support should empower the teachers in confidence and skills that enable them to integrate technology into their teaching effectively. In addition, coaching programs should depict appropriate technology integration so that teachers can assess themselves and monitor their improvement. Professional development providers should focus on sustainable, self-reflective, and cooperative strategies more likely to result in substantial changes in teaching practices.

5.5. Limitations and Recommendations of the Study

5.5.1. Limitations of the Study

This study has the following limitations that might affect the validity and transferability of the results. First, the research setting is relatively narrow to the extent that the study was done with one mathematics teacher in a private middle school, which affects the transferability of the findings to other teachers. Furthermore, the study focused on particular topics in the learning area of geometry: 'Angles and Measurement of Area,' and the findings might not be generalized to TPACK across the whole of geometry or other mathematical learning areas. A limitation of the study is that the focal point of GeoGebra is the primary technological tool used; thus, the findings may not be generalizable to other forms of dynamic mathematics software.

In addition, it is essential to mention the limitations of mathematics coaching for future studies. Several factors could affect its effectiveness within the broader perspective of mathematics coaching. In this regard, a central issue is the diversity in the quality of coaching contingent on the expertise and relationship between coaches and teachers (McGatha, 2008). Moreover, scaling coaching programs might be challenging due to the time and resources required, especially in resource-limited areas (Gibbons & Cobb, 2017). Contextual differences between schools, such as culture and administrative support, may also affect the standardization and success of coaching practices (Kraft et al., 2018).

5.5.2. Recommendations for Further Studies

Given these limitations of this study, the following research propositions are offered for future research. This study aimed to describe the process of developing a teacher's TPACK in geometry through mathematics coaching sessions and illustrate the value of a specially designed mathematics coaching strategy for PD. This mathematics coaching strategy enhanced the teacher's knowledge of incorporating GeoGebra into geometry lessons using his pedagogical, content, and technological knowledge. To confirm and extend the current study's findings, the recommended future research should use this type of coaching strategy for a more extended period, for instance, a whole semester.

It is also recommended that future studies extend the focus of mathematics coaching to encompass other areas of mathematics, for example, algebra, data analysis, and numerical operations, as well as to investigate other dynamic geometry software, including Geometry Expressions, The Geometer's Sketchpad, Cabri, and Cinderella. Expanding the lens might provide a more comprehensive understanding of the effects of mathematics coaching on teachers' TPACK. It is also suggested that mathematics coaching should be included in the teachers' PD in Türkiye to help overcome the gap between theoretical and practical knowledge often missed during university studies. This approach could help in-service and pre-service teachers by providing a more integrated and realistic learning process. Lastly, it is suggested that the mathematics coaching strategy used in this study should be tested in other educational contexts to examine its transferability and possibility of being included in the teacher education system of Türkiye.

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APPENDICES

A. TPACK SURVEY

TEKNOLOJİK PEDAGOJİK İÇERİK BİLGİSİ ÖLÇEĞİ

Sevgili Öğretmen;
Elinizdeki veri toplama aracı, öğretmenlerin teknolojik pedagojik içerik bilgilerini belirlemek amacıyla kullanılacaktır. Bu amaçla aşağıda yer alan soruların her birini okuyunuz ve o maddeye ne kadar katıldığınızı gösteren ifadelerinden hangisi size uygunsa oraya X işareti koyunuz. Verdiğiniz cevaplar sadece araştırmacılar tarafından görülecek ve kesinlikle başka bir durumla ilişkilendirilmeyecektir. Değerli katkılarınız için teşekkür eder, saygılar sunarız.

Cinsiyetiniz : ( ) Kadın ( ) Erkek **Yaşınız: Hizmet Süreniz : Branşınız:**

Görev yaptığınız okulda ihtiyaç duyduğunuz teknolojiye erişebiliyor musunuz? ( ) Evet ( ) Hayır

Mezun Olduğunuz Fakülte Türü : ( ) Eğitim ( ) TEF ( ) Fen Edebiyat ( ) Diğer

Teknoloji kullanma Seviyeniz : ( ) Yetersiz ( ) Yeterli

Teknoloji kullanımıyla ilgili hizmet içi eğitim aldınız mı? : ( ) Evet ( ) Hayır

	Maddeler	Tamamen katılmıyorum	Katılmıyorum	Kararsızım	Katılıyorum	Tamamen katılıyorum
Teknoloji Bilgisi	Teknoloji ile ilgili problemlerimi nasıl çözebileceğimi bilirim.	( )	( )	( )	( )	( )
	Teknolojiyi kolaylıkla öğrenebilirim.	( )	( )	( )	( )	( )
	Önemli yeni teknolojilere uyum sağlayabilirim.	( )	( )	( )	( )	( )
	Teknoloji ile oldukça sık ilgilenirim.	( )	( )	( )	( )	( )
	Birçok farklı teknoloji hakkında bilgi sahibiyim.	( )	( )	( )	( )	( )
	İhtiyaç duyduğum teknolojiyi kullanma becerilerine sahibim.	( )	( )	( )	( )	( )
	Farklı teknolojilerle yeteri kadar çalışma fırsatlarına sahip oldum.	( )	( )	( )	( )	( )
İçerik Bilgisi	Matematik hakkında yeterli bilgiye sahibim.	( )	( )	( )	( )	( )
	Matematiksel düşünebilirim.	( )	( )	( )	( )	( )
	Matematiği anlamamı geliştirecek çeşitli strateji ve yollara sahibim.	( )	( )	( )	( )	( )
	Sosyal bilgiler hakkında yeterli bilgiye sahibim.	( )	( )	( )	( )	( )
	Tarihsel düşünebilirim.	( )	( )	( )	( )	( )
	Sosyal bilgileri anlamamı geliştirecek çeşitli strateji ve yollara sahibim.	( )	( )	( )	( )	( )
	Fen bilimleri hakkında yeterli bilgiye sahibim.	( )	( )	( )	( )	( )
	Bilimsel düşünebilirim.	( )	( )	( )	( )	( )
	Fen bilimlerini anlamamı geliştirecek çeşitli strateji ve yollara sahibim.	( )	( )	( )	( )	( )
	Okuryazarlık hakkında yeterli bilgiye sahibim.	( )	( )	( )	( )	( )
	Edebi düşünebilirim.	( )	( )	( )	( )	( )
	Okuryazarlığı anlamamı geliştirecek çeşitli strateji yollara sahibim.	( )	( )	( )	( )	( )
Pedagojik Bilgi	Sınıfta öğrenci performansının nasıl değerlendirileceğini bilirim.	( )	( )	( )	( )	( )
	Öğretim etkinliklerini mevcut durumda öğrencilerin neyi anlayıp anlamadıklarına bağlı olarak değiştirebilirim.	( )	( )	( )	( )	( )

	Öğretim stilimi farklı öğrenenlere uygun şekilde değiştirebilirim.	( )	( )	( )	( )	( )
	Öğrencilerin öğrenmelerini birçok yolla değerlendirebilirim.	( )	( )	( )	( )	( )
	Sınıf ortamında, birçok farklı öğretim yaklaşımlarını (İşbirlikli öğrenme, doğrudan öğrenme, Sorgulayıcı öğrenme, problem/proje temelli öğrenme vb.) kullanabilirim.	( )	( )	( )	( )	( )
	Sıkça karşılaşılan öğrenci anlamaları/yanlış anlamaları ve kavram yanılgılarına aşinayım.	( )	( )	( )	( )	( )
	Sınıf yönetiminin nasıl organize edileceğini ve sürdürüleceğini bilirim.	( )	( )	( )	( )	( )

	Maddeler	**Tamamen katılmıyorum**	**Katılmıyorum**	**Kararsızım**	**Katılıyorum**	**Tamamen katılıyorum**
Pedagojik İçerik Bilgisi	Öğrencilerin matematik öğrenmelerine ve matematiksel düşünmelerine rehberlik etmek için etkili öğretim yaklaşımlarını nasıl seçeceğimi bilirim.	( )	( )	( )	( )	( )
	Öğrencilerin okuryazarlığı öğrenme ve düşünmelerine rehberlik etmek için etkili öğretim yaklaşımlarını nasıl seçeceğimi bilirim.	( )	( )	( )	( )	( )
	Öğrencilerin fen bilimlerini öğrenme ve düşünmelerine rehberlik etmek için etkili öğretim yaklaşımlarını nasıl seçeceğimi bilirim.	( )	( )	( )	( )	( )
	Öğrencilerin sosyal bilgileri öğrenme ve düşünmelerine rehberlik etmek için etkili öğretim yaklaşımlarını nasıl seçeceğimi bilirim.	( )	( )	( )	( )	( )
Teknolojik İçerik Bilgisi	Matematik çalışmak ve matematiği anlamak için kullanabileceğim teknolojiler hakkında bilgi sahibiyim.	( )	( )	( )	( )	( )
	Okuryazarlık çalışmak ve okuryazarlığı anlamak için kullanabileceğim teknolojiler hakkında bilgi sahibiyim.	( )	( )	( )	( )	( )
	Fen bilimlerini çalışmak ve anlamak için kullanabileceğim teknolojiler hakkında bilgi sahibiyim.	( )	( )	( )	( )	( )

	Sosyal bilgileri çalışma ve anlamak için kullanabileceğim teknolojiler hakkında bilgi sahibiyim.	( )	( )	( )	( )	( )
Teknolojik Pedagojik Bilgi	Bir ders için öğretim yaklaşımlarının etkisini artıracak teknolojileri seçebilirim.	( )	( )	( )	( )	( )
	Bir ders için öğrencilerin öğrenmelerini artıracak teknolojileri seçebilirim.	( )	( )	( )	( )	( )
	Aldığım öğretmenlik eğitimi, teknoloji kullanımının öğretim yaklaşımlarını nasıl etkileyeceği hakkında derinlemesine düşünmeme neden olmuştur.	( )	( )	( )	( )	( )
	Sınıfımda teknolojiyi nasıl kullanacağım hakkında eleştirel biçimde düşünüyorum.	( )	( )	( )	( )	( )
	Farklı öğretim etkinlikleri ile ilgili öğrenmekte olduğum teknolojilerin kullanımını uyarlayabilirim.	( )	( )	( )	( )	( )
Teknolojik Pedagojik İçerik Bilgisi	Matematik ile ilgili teknolojiler ve öğretim yaklaşımlarını uygun bir şekilde birleştirerek ders anlatabilirim.	( )	( )	( )	( )	( )
	Okuryazarlık ile ilgili teknolojiler ve öğretim yaklaşımlarını uygun bir şekilde birleştirerek ders anlatabilirim.	( )	( )	( )	( )	( )
	Fen bilimleri ile ilgili teknolojiler ve öğretim yaklaşımlarını uygun bir şekilde birleştirerek ders anlatabilirim.	( )	( )	( )	( )	( )
	Sosyal bilgiler ile ilgili teknolojiler ve öğretim yaklaşımlarını uygun bir şekilde birleştirerek ders anlatabilirim.	( )	( )	( )	( )	( )
	Sınıfımda kullanabileceğim teknolojileri, öğrencilerin ne öğreneceği, nasıl öğreteceğim ve öğreteceğimi geliştirecek nitelikte seçebilirim.	( )	( )	( )	( )	( )
	Sınıfımda çalışmalarım hakkında öğrendiğim; içerik, teknoloji ve öğretim yaklaşımlarının bir arada olduğu stratejileri kullanabilirim.	( )	( )	( )	( )	( )
	Okulumda; içerik, teknoloji ve öğretim yaklaşımlarının kullanımını koordine etmeleri için arkadaşlarıma yardımcı olacak liderlik edebilirim.	( )	( )	( )	( )	( )
	Bir dersin içeriğini zenginleştirebilecek teknolojileri seçebilirim.	( )	( )	( )	( )	( )

B. INTERVIEW PROTOCOL OF TPACK (IN TURKISH)

1. "Açılar ve Alan Ölçme" konusunun öğretimine teknolojiyi nasıl entegre etmeyi planladın? Açıklar mısın?

CURRICULUM & ASSESSMENT THEME -C: **Curriculum descriptor**

a. "Açılar ve Alan Ölçme" konusunu teknoloji destekli öğretirken ulusal müfredata ve ders kitabına ne düzeyde bağlı kalırsın? Değişiklik yapar mısın? Neden?
b. "Açılar ve Alan Ölçme" konusunun öğretiminde hangi teknolojik araçları kullanırsın? Neden bu araçları seçersin? Açıklar mısın?

2. Öğrencilerin "Açılar ve Alan Ölçme" konusuna ilişkin öğrenmelerini değerlendirirken kullandığın ölçme ve değerlendirme teknikleri nelerdir?

CURRICULUM & ASSESSMENT THEME -A: Assessment descriptor

a. Öğrencilerin öğrenmelerini değerlendirirken teknolojiden yararlanır mısın? Nasıl yararlandığını açıklayabilir misin?
b. Ölçme ve değerlendirme sürecinde teknolojiye nasıl bir rol biçersin?

3. "Açılar ve Alan Ölçme" konusunun öğrenciler tarafından öğrenilmesinde teknolojiyi hangi amaçla kullanırsın?

LEARNING-M: Mathematics learning descriptor

a. Teknoloji öğretim sürecinde bir öğrenme aracı mı yoksa bir öğretme aracı mıdır? Neden?
b. "Açılar ve Alan Ölçme" konusunun öğretiminde teknoloji ne şekilde ve ne zaman devreye girecek? Detaylı bir örnek verebilir misin?
c. Teknolojinin öğrencilerin öğrenmelerinde nasıl bir rolü var?
d. Öğrencilere bir kavramı keşfettirirken teknolojiyi kullanır mısın? Teknoloji bu süreçte nasıl bir rol üstlenir? Neden? Ayrıntılı olarak açıklayabilir misin?

4. "Açılar ve Alan Ölçme" konusunda bir öğrenme aracı olarak teknolojiyi nasıl kullanırsın?

LEARNING THEME-M: Mathematics learning descriptor

a. "Açılar ve Alan Ölçme" konusuna ilişkin anahtar kavramlar nelerdir?
b. "Açılar ve Alan Ölçme" konusuna özel hangi teknolojiler öğrencilerin öğrenmelerine katkıda bulunabilir? Neden?
c. "Açılar ve Alan Ölçme" konusunda teknoloji kullanımı öğrencilerin konuyu anlamalarını nasıl etkiler?

d. Öğrencilerin konuyu kavramsal öğrenmesi için teknolojiyi nasıl kullanırsın? Öğrencilerin anlamalarını derinleştirmek için teknoloji kullanımına bir örnek verir misin?

5. "Açılar ve Alan Ölçme" konusuna ilişkin öğrencilerin olası zorlukları ve kavram yanılgıları nelerdir?

LEARNING THEME -C: Conception of student thinking descriptor

a. Öğrencilerin kavram yanılgılarını gidermek veya anlamakta zorlandığı kavramları anlamaları için teknolojiden nasıl yararlanırsın? Açılar ve Alan Ölçme konusuna özgün (eş açı çizme, açı çeşitleri, üçgenin alanı, paralelkenarın alanı) detaylı birkaç örnekle açıklayabilir misin?

6. "Açılar ve Alan Ölçme" konusunu teknoloji destekli öğretmek için kullanacağın teknolojiyi seçerken nelere dikkat edersin?

TEACHING THEME-M: **Mathematics learning descriptor**

a. Bir öğretme aracı olarak teknolojiden nasıl yararlanırsın? Detaylı olarak açıklar mısın?
b. Teknolojinin öğrencilerin bu konuyu öğrenmesine ne şekilde katkıda bulunacağını düşünüyorsun?
c. Öğrencilerine daha önce öğrenmedikleri bir kavramı öğretirken teknolojiden yararlanır mısın? Neden? Bir örnekle açıklar mısın?

7. "Açılar ve Alan Ölçme" konusunun öğretiminde etkili olduğunu düşündüğün öğretim yöntem ve teknikleri nelerdir?

TEACHING THEME -I: Instructional descriptor

a. Hangi öğretim yöntem ve tekniği, teknoloji entegrasyonu ile etkili öğrenme sağlar? Bu süreçte teknolojinin öğretim yöntemindeki işlevi nedir?
b. Konunun öğretiminde tümdengelim veya tümevarım stratejilerinden faydalanır mısın? Nedenini örnekle açıklar mısın?
c. Öğretmen olarak dersteki rolün nedir? Dersinde öğretim süreci nasıl ilerler? Teknolojiyi nasıl kullandığını detaylı olarak açıklar mısın?
d. Teknolojiyle, seçtiğin öğretim yöntemi ders akışı boyunca nasıl bir uyum içinde olur? Kullanacağın teknoloji ile öğretmen-öğrenci rolü nasıl örtüşüyor? Bir örnekle anlatır mısın?
e. Dersin hangi aşamasında ne tür sorular sorarsın? Örnek sorularla açıklar mısın?

8. "Açılar ve Alan Ölçme" konusunun öğretiminde nasıl bir öğrenme ortamı tasarlarsın?

TEACHING THEME -E: Environment descriptor

a. Teknoloji bu derste nasıl bir role sahip? Bu konunun öğretiminde teknoloji hangi amaca hizmet edecek?
b. Teknolojiyi derste kullanırken nelere dikkat edersin? Neden?
c. Öğrencinin dersteki rolü nedir? Aktif öğrenci katılımı sağlayacak bir öğrenme ortamı hazırlar mısın? Bunu nasıl sağlarsın?
d. Öğrenciler ders içi etkinliklerini bireysel mi grup olarak mı yürütecekler? Neden?

9. "Açılar ve Alan Ölçme" konusunu öğretirken öğrenciler teknolojiyi kullanırlar mı? Nasıl kullandıklarını ayrıntılı olarak açıklar mısın?

ACCESS THEME -U: Usage descriptor

a. "Açılar ve Alan Ölçme" konusunu öğretirken öğrencilerin ne zaman teknoloji kullanır? Neden? (Temel kavramlar iyice öğrenildikten sonra/öğretim boyunca sınırlı ölçüde/teknoloji kullanımı için özellikle hazırlanmış konu ve ünitelerde/belirli matematik konularında/her matematik dersinde)
b. Öğrencilerin derste teknoloji kullanırken nelere dikkat edersin? Neden?
c. Öğrencilerin derste teknoloji kullanmalarının amacı nedir? Açıklar mısın?
d. "Açılar ve Alan Ölçme" konusunu teknoloji kullanarak öğretmek ile teknoloji kullanmadan öğretmek arasındaki farklılıklar nedir?
e. Teknoloji kullanımının avantajları ve sınırlılıkları nelerdir?

10. "Açılar ve Alan Ölçme" konusunda teknoloji entegrasyonunda hangi zorluklar ile karşılaşılabilir?

ACCESS THEME -B: Barrier descriptor

a. Öğrencilerin teknoloji kullanımında karşılaşabilecekleri zorluklar neler olabilir?
b. Öğrencilerin teknoloji kullanımında karşılaşabileceği zorluklarla baş etmeye yönelik ders öncesinde nasıl önlemler alırsın?
c. Öğrencilerin teknoloji kullanımında bir zorluk ya da teknik aksaklık anında ders esnasında nasıl müdahale edersin?
d. Öğrenciler teknoloji kullanırken sınıf yönetiminde nasıl bir strateji kullanırsın? Bu konuda endişelerin var mı?

11. "Açılar ve Alan Ölçme" konusunda faydalanabileceğin çoklu temsiller nelerdir?

ACCESS THEME - A: Availability descriptor

a. Derste çoklu temsiller kullanmak için teknolojiden faydalanır mısın? Nasıl?
b. Çoklu temsillerin kullanımında teknolojinin nasıl bir işlevi var?
c. Temsiller arası geçişte teknolojiden nasıl faydalanırsın?
d. Öğrencilerin daha karmaşık matematiksel durumlara yönelik genellemelere varmaları için teknolojiyi kullanır mısınız?

12. Mesleki gelişim çalışmaları (Hizmet içi eğitim, seminer, atölye vb.) hakkında ne düşünüyorsun?

TEACHING THEME - PD: Professional development descriptor

a. Mesleki gelişimine yönelik teknoloji odaklı bir etkinliğe (seminer, atölye, eğitim vb.) katılmak ister misin? Neden?
b. Katılmak istediğin teknoloji odaklı bir eğitimin içeriğinde neler olmasını ya da bu eğitimin nelere yönelik olmasını istersin?

C. INTERVIEW PROTOCOL OF TPACK (IN ENGLISH)

1. How do you plan to integrate technology into the teaching of "Angles and Measurement of Area"? Can you explain?

CURRICULUM & ASSESSMENT THEME -C: **Curriculum descriptor**

a. To what extent do you adhere to the national curriculum and textbook when teaching "Angles and Measurement of Area" with technology? Do you make changes? Why?
b. Which technological tools would you use in teaching "Angles and Measurement of Area"? Why do you choose these tools? Can you explain?

2. What are the assessment and evaluation techniques you use to evaluate students' learning about "Angles and Measurement of Area"?

CURRICULUM & ASSESSMENT THEME -A: Assessment descriptor

a. Do you use technology to assess students' learning? Can you explain how?
b. What role do you assign to technology in the assessment and evaluation process?

3. For what purpose would you use technology in students' learning of the topic "Angles and Measurement of Area"?

LEARNING-M: Mathematics learning descriptor

a. Is technology a learning tool or a teaching tool in the teaching process? Why?
b. How and when will technology come into play in teaching "Angles and Measurement of Area"? Can you give a detailed example?
c. What role does technology play in students' learning?
d. Do you use technology to help students explore a concept? What role does technology play in this process? Why? Can you explain in detail?

4. How do you use technology as a learning tool in "Angles and Measurement of Area"?

LEARNING THEME-M: Mathematics learning descriptor

a. What are the key concepts related to the topic "Angles and Measurement of Area"?
b. Which technologies specific to "Angles and Measurement of Area" can contribute to students' learning? Why?
c. How does the use of technology in "Angles and Measurement of Area" affect students' understanding of the topic?
d. How do you use technology for students' conceptual learning of the topic? Can you give an example of using technology to deepen students' understanding?

5. What are the possible difficulties and misconceptions of students regarding the topic "Angles and Measurement of Area"?

LEARNING THEME -C: Conception of student thinking descriptor

a. How do you use technology to eliminate students' misconceptions or help them understand concepts they have difficulty understanding? Can you explain with a few detailed examples specific to the topic of Angles and Measurement of Area (drawing congruent angles, angle types, area of a triangle, area of a parallelogram)?

6. What do you consider when choosing the technology you will use to teach "Angles and Measurement of Area" with technology support?

TEACHING THEME-M: Mathematics learning descriptor

a. How do you use technology as a teaching tool? Can you explain in detail?
b. How do you think technology will contribute to students' learning of this topic?
c. Would you use technology to teach your students a concept they have not learned before? Why? Can you explain with an example?

7. What are the teaching methods and techniques that you think are effective in teaching "Angles and Measurement of Area"?

TEACHING THEME -I: Instructional descriptor

a. Which teaching method and technique provides effective learning with technology integration? In this process, what is the function of technology in the teaching method?
b. Do you use deductive or inductive strategies in teaching the subject? Can you explain why with an example?
c. What is your role as a teacher in the lesson? How does the teaching process progress in your lesson? Can you explain in detail how you use technology?
d. How is the technology and the teaching method you choose in harmony during the lesson flow? How do the technology you will use and the teacher-student role overlap? Can you explain with an example?
e. What kind of questions do you ask at which stage of the lesson? Can you explain with sample questions?

8. What kind of learning environment do you design for teaching "Angles and Measurement of Area"?

TEACHING THEME -E: Environment descriptor

a. What role does technology play in this lesson? What purpose will technology serve in teaching this subject?
b. What do you pay attention to when using technology in the lesson? Why?
c. What is the role of the student in the lesson? Would you prepare a learning environment that will ensure active student participation? How would you ensure this?
d. Will students carry out in-class activities individually or in groups? Why?

9. Do students use technology when teaching "Angles and Measurement of Area"? Can you explain in detail how they use it?

ACCESS THEME -U: Usage descriptor

a. When do students use technology when teaching the topic "Angles and Measurement of Area"? Why (after the basic concepts have been thoroughly learned/to a limited extent throughout instruction/in subjects and units specifically prepared for the use of technology/in specific mathematics subjects/in every mathematics lesson)?
b. What do you pay attention to when students use technology in class? Why?
c. What is the purpose of students using technology in class? Can you explain?
d. What are the differences between teaching "Angles and Measurement of Area" using technology and teaching it without technology?
e. What are the advantages and limitations of using technology?

10. What difficulties can be encountered in integrating technology in "Angles and Measurement of Area"?

ACCESS THEME -B: Barrier descriptor

a. What are the difficulties that students may encounter in using technology?
b. How do you take precautions before the lesson to deal with the difficulties students may encounter in using technology?
c. How do you intervene during the lesson in case of a difficulty or technical glitch in students' use of technology?
d. What strategies do you use in classroom management when students use technology? Do you have any concerns about this?

11. What are the multiple representations you can use in the topic "Angles and Measurement of Area"?

ACCESS THEME - A: Availability descriptor

a. Do you use technology to use multiple representations in class? How?
b. What is the function of technology in the use of multiple representations?
c. How do you use technology to move between representations?
d. Do you use technology to help students make generalizations for more complex mathematical situations?

12. What do you think about professional development activities (in-service training, seminars, workshops, etc.)?

TEACHING THEME - PD: Professional development descriptor

a. Would you like to participate in a technology-oriented activity (seminar, workshop, training, etc.) for your professional development? Why?
b. What would you like to see in the content of a technology-oriented training you would like to attend or what would you like this training to be about?

D. OBSERVATION PROTOCOL OF TPACK

I. Temel Bilgiler

Öğretmen Adı-Soyadı:
Gözlem tarihi ve süresi:
Öğrenme alanı ve konu başlığı:
Öğretim programı kazanım (lar)ı:
Sınıf Mevcudu:
Öğrenme Ortamı: (Araştırmacının yeri, öğrencilerin yeri, teknolojik araçlar)

II. TPAB değerlendirmesi
(*Observed:* 1 point *Not Observed:* 0 point)

Levels	**CURRICULUM & ASSESSMENT-C: Curriculum descriptor**	Write 0 or 1 to each behavior
Level 1 Recognizing	Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.	
Level 2 Accepting	Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning	
Level 3 Adapting	Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.	
Level 4 Exploring	Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning	
Level 5 Advancing	Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.	
Field Notes (behaviors that are not mentioned above etc.):		

CURRICULUM & ASSESSMENT -A: Assessment descriptor	Write 0 or 1 to each behavior
Resists idea of technology use in assessment indicating that technology interferes with determining students‟ understanding of mathematics.	
Acknowledges that it might be appropriate to allow technology use	

as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).	
Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).	
Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).	
Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.	
Field Notes (behaviors that are not mentioned above etc.):	

LEARNING-M: Mathematics learning descriptor	Write 0 or 1 to each behavior
Views mathematics as being learned in specific ways and that technology often gets in the way of learning.	
Has concerns about students‟ attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.	
Begins to explore, experiment and practice integrating technologies as mathematics learning tools.	
Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.	
Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.	
Field Notes (behaviors that are not mentioned above etc.):	

LEARNING -C: Conception of student thinking descriptor	Write 0 or 1 to each behavior
More apt to accept the technology as a teaching tool rather than a learning tool.	
Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.	
Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.	
Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.	
Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.	
Field Notes (behaviors that are not mentioned above etc.):	

TEACHING -M: **Mathematics learning descriptor**	Write 0 or 1 to each behavior
Concerned that the need to teach about the technology will take away time from teaching mathematics	
Uses technology activities at the end of units, for "days off," or for activities peripheral to classroom instruction.	
Uses technology to enhance or reinforce mathematics ideas that students have learned previously.	
Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.	
Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.	

Field Notes (behaviors that are not mentioned above etc.):

TEACHING -I: Instructional descriptor	Write 0 or 1 to each behavior
Does not use technology to develop mathematical concepts.	
Merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies.	
Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.	
Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.	
Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.	
Field Notes (behaviors that are not mentioned above etc.):	

TEACHING -E: Environment descriptor	Write 0 or 1 to each behavior
Uses technology to reinforce concepts taught without technology.	
Tightly manages and orchestrates instruction using technology.	
Instructional strategies with technologies are primarily deductive, teacher-directed in order to maintain control of the how the activity progresses.	
Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.	
Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics.	
Field Notes (behaviors that are not mentioned above etc.):	

TEACHING - PD: Professional development descriptor	Write 0 or 1 to each behavior
Considers attending local professional development to learn more about technologies	
Recognizes the need to participate in technology related PD.	
Continues to learn and explore ideas for teaching and learning mathematics using only one type of technology (such as spreadsheets).	
Seeks out and works with others who are engaged in incorporating technology in mathematics.	
Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.	
Field Notes (behaviors that are not mentioned above etc.):	

ACCESS -U: Usage descriptor	Write 0 or 1 to each behavior
Permits students to use technology „only" after mastering certain concepts	
Students use technology in limited ways during regular instructional periods.	
Permits students to use technology in specifically designed units.	
Permits students to use technology for exploring specific mathematical topics.	
Permit students to use technology in every aspect of mathematics class.	
Field Notes (behaviors that are not mentioned above etc.):	

ACCESS -B: Barrier descriptor	Write 0 or 1 to each behavior
Resists consideration of changes in content taught although it becomes accessible to more students through technology.	
Worries about access and management issues with respect to incorporating technology in the classroom.	
Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.	
Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges	
Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.	
Field Notes (behaviors that are not mentioned above etc.):	

ACCESS - A: Availability descriptor	Write 0 or 1 to each behavior
Notices that authentic problems are more likely to involve „unfriendly numbers" and may be more easily solved if students had calculators.	
Calculators permit greater number of examples to be explored by students.	
Concepts are taught differently since technology provides access to connections formerly out of reach.	
Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections.	
Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience	
Field Notes (behaviors that are not mentioned above etc.):	

E. INTERVIEW PROTOCOL OF PRE-CONFERENCE (IN TURKISH)

1. Bölüm: Ders planının incelenmesi

1. Öğreteceğin konuya ait kazanımlardan bahseder misin?
2. Konuya ilişkin anahtar kavramlardan bahseder misin?
3. Ders içeriğini hangi kaynaklardan yararlanarak oluşturdun?
4. Ders içeriğini oluştururken ortaokul matematik müfredatından ne düzeyde yararlandın?
5. Ders içeriğini oluştururken ortaokul matematik müfredatında herhangi bir değişiklik ya da uyarlama yapma ihtiyacı duydun mu? Açıklar mısın?
6. Öğrencilerin bu konuya ilişkin olası zorlukları ve kavram yanılgıları nelerdir?
7. Öğrencilerin kavram yanılgılarını gidermek veya anlamakta zorlandığı kavramları anlamaları için teknolojiden yararlanmayı planladın mı? Evet ise ders planından bir örnekle açıklar mısın?
8. Konunun öğretimi için nasıl bir öğrenme ortamı tasarladın? Açıklar mısın?
9. Bu konunun öğretimine özel hangi teknolojiler öğrencilerin öğrenmelerine katkıda bulunabilir? Neden?
10. Derste hangi teknolojik araçları kullanacaksın? Neden bu araçları seçtin?
11. Derste kullanacağın teknolojiyi seçerken hangi kriterleri göz önünde bulundurdun? Neden?
12. Derste teknolojiyi ne zaman ve nasıl kullanmayı planladın? Dersin belli bir kısmına ait (Dersin başında, ders sonu Ölçme & Değerlendirme, dersin tümünde vb.) kullanımı/kullanımlarını örneklendirir misin?
13. Teknolojinin bu dersteki yeri nedir? Konunun öğretiminde teknolojiyi ne amaçla kullanmayı planladın?
14. Konuyu öğretirken teknolojiyi bir öğrenme aracı mı yoksa bir öğretme aracı olarak mı kullanacaksın? Neden?
15. Teknolojiyi bir öğrenme aracı olarak nasıl kullanacaksın? Bir örnekle açıklar mısın?
16. Teknolojiyi bir öğretme aracı olarak nasıl kullanacaksın? Bir örnekle açıklar mısın?
17. Planladığın bu derste öğretmen olarak rolün nedir?
18. Teknolojiyi derste kullanırken nelere dikkat edeceksin? Neden?
19. Derste öğrenciler teknolojiyi kullanacak mı? Evet ise ne zaman ve nasıl kullanacaklarını açıklar mısın?
20. Öğrencilerin derste teknoloji kullanmalarıyla neyi amaçladın?
21. Öğrencilerin derste teknoloji kullanırken nelere dikkat edeceksin? Neden?
22. Öğrencilerinin bu derste teknoloji kullanımında karşılaşabileceği zorluklar neler olabilir?
23. Öğrencinin teknoloji kullanımında karşılaşabileceği zorluklarla baş etmeye yönelik ders öncesinde önlemler aldın mı? Evet ise ne tür önlemler aldığını açıklar mısın?
24. Öğrencinin teknoloji kullanımında bir zorluk ya da teknik aksaklık yaşaması durumunda ders esnasında nasıl müdahale edeceksin?

25. Bu konunun öğretiminde çoklu temsillerden faydalanacak mısın? Evet ise nasıl faydalanmayı planladın?
26. Derste çoklu temsiller kullanmak için teknolojiden faydalanacak mısın? Evet ise nasıl faydalanmayı planladın?
27. Çoklu temsillerin kullanımında teknolojiden yararlanacaksan teknolojinin nasıl bir işlevi olacak? Temsil arası geçişlerde teknolojiden yararlanacak mısın? Evet ise nasıl faydalanmayı planladın?
28. Öğrencilerin öğrenmelerini değerlendirirken teknolojiden yararlanmayı planladın mı? Evet ise nasıl yararlanmayı planladığını açıklayabilir misin? Ölçme ve değerlendirme sürecinde kullanacağın teknolojinin rolü ne olacak?
29. Teknolojiye, öğrencilerin öğrenmelerinde nasıl bir rol biçtin?
30. Öğrencilere bir kavramı keşfettirmek için teknolojiyi kullanmayı planladın mı? Teknoloji bu süreçte nasıl bir rol üstlenecek? Neden?
31. Sence bu konunun öğretiminde teknoloji kullanımının avantajları ve sınırlılıkları nelerdir?
32. Derste kullanmayı planladığın teknoloji öğrencilerin anlamalarını nasıl etkileyecek?
33. Teknolojiyi konunun öğrenciler tarafından kavramsal öğrenilmesine katkıda bulunması bakımından kullanmayı planladın mı? Evet ise nasıl?
34. Teknolojinin bu konunun kavramsal olarak öğrenilmesine ne ölçüde katkı sağlayacağını düşünüyorsun?
35. Derste kullanacağın teknoloji öğrenci öğrenmelerini nasıl derinleştirecek? Örnek verir misin?
36. Öğrencilerin bu derste ilk kez öğrenecekleri bir kavram var mı? Bu kavramı öğretirken teknolojiyi kullanacak mısın? Evet ise nasıl kullanacaksın?
37. Derste hangi öğretim yöntem ve tekniğini kullanacaksın? Ders planında teknolojiyi bu öğretim yöntem ve tekniğine nasıl entegre ettin?
38. Derste tümdengelim veya tümevarım stratejilerinden faydalanacak mısın? Evet ise bu stratejilerinden nasıl yararlanmayı planladın? Teknolojinin bu stratejileri kullanırken nasıl bir işlevi olacak?
39. Teknolojiyle, seçtiğin öğretim yöntemi ders akışı boyunca nasıl bir uyum içinde olacak? Kullanacağın teknoloji ile öğretmen-öğrenci rolü nasıl örtüşecek?
40. Dersin hangi aşamasında hangi soruları soracaksın? Neden?
41. Öğrenciler ders içi etkinliklerini bireysel mi grup olarak mı yürütecekler? Nedenini açıklar mısın?
42. Öğrencilerin dersteki rolü ne olacak? Öğrencilerin derse katılımını sağlamayı nasıl planladın? Öğrencilerin öğrenme süreci senin yönetiminde mi yoksa kendi yönetimlerinde mi olacak?
43. Bu konuyu derste teknoloji ile öğretirken öğretmen olarak yaşayabileceğin olası zorluklar neler?
44. Öğrenciler teknoloji kullanırken sınıf yönetiminde nasıl bir strateji kullanacaksın? Bu konuda endişelerin var mı?
45. Ders planı hazırlama sürecinde mesleki olarak desteğe ihtiyaç duydun mu? Bunun nasıl üstesinden geldin? İyi olduğun ve gelişime dönük tarafların sence neler?

Bölüm 2: Ders planının iyileştirilmesi

- Ders planındaki etkili öğretim uygulamaları vurgulanır. Öğretimin geliştirilmesi için önerilerde bulunulur.

  1. Dersin başında konunun önemine dikkat çekmek ve öğrencilerin hazır bulunuşluğunu belirlemek için nasıl bir soru sorulabilir? Örneğin …….. gibi bir soru hakkında ne düşünürsün?

  2. …… kısımda bunu kullanmak iyi bir fikir. Peki …… gibi bir etkinliği öğretime dahil etmek hakkında ne düşünürsün? Sence öğrencilerin öğrenmelerine katkıda bulunur mu?

- Öğrencilerin kavramsal öğrenmelerine yeterli katkıyı sağlamayacağı düşünülen etkinlikler tartışılır.
  Derste kullanmayı planladığın bu teknoloji destekli etkinliğin öğrencilerin konuya ilişkin anlamalarını ne ölçüde derinleştirebileceğini düşündün? Bu etkinliğe alternatif olarak ne önerebilirsin? Peki …… gibi bir etkinliği öğretime dahil etmek hakkında ne düşünürsün?
- Etkili teknoloji entegrasyonuna örnek uygulamalar paylaşılır ve tartışılır.

  3. ……. gibi etkinlikler öğrencilerin kavramsal öğrenmelerine katkıda bulunur mu? Bu konuda ne düşünüyorsun?

- Teknolojik pedagojik alan bilgisindeki eksiklere yönelik önerilerde bulunulur.

  4. Kavram yanılgılarını ortadan kaldırmak ve olası kavram yanılgılarını önlemek için teknolojiyi …….. gibi kullanmanın ne ölçüde etkili olacağını düşünüyorsun?

  5. …….. gibi tümevarım ve tümdengelim stratejileri ile bütünleştirilebilecek etkinlikler derste kullanılabilir mi? Bu konudaki fikrin nedir?

  6. Çoklu Temsilleri ……. şeklinde teknolojiden faydalanarak kullanabilir misin?
  - Ön-konferans'ta yukarıda belirtilenler dışında, Niess, Ronau vd.'nin (2009) TPAB gelişim modelinde belirtildiği şekliyle modelin her bir temasına ait düzey tanımlayıcılarının en ileri düzeyindeki davranışlar öğretmen eğitiminin ve ön-konferansların odağını oluşturmaktadır;

  ❖ Öğrencilerin konuyu kavramsal öğrenmelerini değerlendiren teknoloji tabanlı uygulamalar eklenir.
  ❖ Öğrencilerin matematiksel düşünmelerini ve anlamalarını artıracak çeşitli teknoloji entegrasyonları üzerine düşünülür, ders planına eklenir.
  ❖ Teknoloji entegrasyonunu derse, dersi zenginleştiren bir öğe olarak eklemek yerine, teknoloji öğrencilerinin öğrenmelerini geliştirmek ve anlamalarını derinleştirmek için ders ile bütünlük içinde olan dersin ayrılmaz bir parçası haline getirilir.

- Teknoloji, öğrencilerin matematiksel kavram ve süreçleri anlayacağı forma dönüştürmek için etkin bir öğretme ve öğrenme aracı olarak konumlandırılır.
- Teknoloji içeren çeşitli öğretim yöntemlerine tümdengelim ve tümevarım stratejileri de entegre edilerek öğrencilerin konuya ilişkin derinlemesine düşünmeleri sağlanır.
- Ders, öğrencilerin teknoloji ile zenginleştirilmiş etkinlikler üzerinde kendi başlarına çalışabilecekleri ve kendi kendilerine öğrenmelerine imkân tanıyan öğrenme ortamı olarak düzenlenir.
- Etkili teknoloji entegrasyonuna yönelik mesleki gelişim programları, etkinlikleri ve seminerler önerilir.
- Öğrencilerin teknolojiyi matematiğin her alanında kullanmalarına izin vereceği durumlar eklenir.
- Derste teknoloji entegrasyonu sırasında ortaya çıkabilecek zorluklara yönelik kapsamlı çözüm önerileri ve önlemler planlanır ve derse eklenir.
- Öğrencilerin daha karmaşık matematiksel durumlar, matematiksel bağlantılar ve genellemeleri keşfedebilmesi için öğrencilere teknoloji kullanma fırsatı veren çalışmalar eklenir.

F. INTERVIEW PROTOCOL OF PRE-CONFERENCE (IN ENGLISH)

Section 1: Review of the lesson plan

1. Can you tell us about the learning outcomes of the subject you will teach?
2. Can you tell us about the key concepts related to the content of the lesson?
3. From which sources did you create the lesson content?
4. To what extent did you benefit from the middle school mathematics curriculum while creating the lesson content?
5. Did you need to make any changes or adaptations in the middle school mathematics curriculum while creating the lesson content? Can you explain?
6. What are the students' possible difficulties and misconceptions about this topic?
7. Did you plan to make use of technology to eliminate students' misconceptions or help them understand the concepts they have difficulty understanding? If yes, can you explain with an example from your lesson plan?
8. What kind of learning environment did you design for teaching the subject? Can you explain?
9. Which technologies specific to teaching this topic can contribute to students' learning? Why?
10. Which technological tools will you use in the lesson? Why did you choose these tools?
11. What criteria did you consider when choosing the technology you will use in the lesson? Why?
12. When and how did you plan to use the technology in the lesson? Can you exemplify its use(s) for a certain part of the lesson (at the beginning of the lesson, at the end of the lesson, during the whole lesson, etc.)?
13. What is the role of technology in this lesson? For what purpose did you plan to use technology in teaching the subject?
14. Will you use technology as a learning tool or a teaching tool while teaching the subject? Why?
15. How will you use technology as a learning tool? Can you explain with an example?
16. How will you use technology as a teaching tool? Can you explain with an example?
17. What is your role as a teacher in this lesson you have planned?
18. What will you pay attention to when using technology in the lesson? Why?
19. Will students use technology in the lesson? If yes, can you explain when and how they will use it?
20. What do you aim for students to use technology in the lesson?
21. What will you pay attention to when students use technology in the lesson? Why?
22. What are the difficulties your students may encounter in using technology in this lesson?

23. Did you take precautions before the lesson to cope with the difficulties that students may encounter in using technology? If yes, can you explain what kind of measures you took?
24. How will you intervene during the lesson if the student has a difficulty or technical glitch in the use of technology?
25. Will you utilize multiple representations in teaching this topic? If yes, how did you plan to utilize them?
26. Will you utilize technology to use multiple representations in the lesson? If yes, how did you plan to utilize it?
27. If you will utilize technology in the use of multiple representations, what will be the function of technology? Will you use technology for transitions between representations? If yes, how did you plan to use it?
28. Did you plan to use technology to assess students' learning? If yes, can you explain how? What will be the role of the technology you will use in the assessment and evaluation process?
29. What role have you assigned to technology in students' learning?
30. Have you planned to use technology to help students explore a concept? What role will technology play in this process? Why?
31. What do you think are the advantages and limitations of using technology in teaching this topic?
32. How will the technology you plan to use in the lesson affect students' understanding?
33. Did you plan to use technology in terms of contributing to conceptual learning of the topic by students? If yes, how?
34. To what extent do you think technology will contribute to the conceptual learning of this topic?
35. How will the technology you will use in the lesson deepen student learning? Can you give an example?
36. Is there a concept that students will learn for the first time in this lesson? Will you use technology to teach this concept? If yes, how will you use it?
37. Which teaching method and technique will you use in the lesson? How did you integrate technology into this teaching method and technique in the lesson plan?
38. Will you use deductive or inductive strategies in the lesson? If yes, how did you plan to utilize these strategies? What will be the function of technology when using these strategies?
39. How will the technology and the teaching method you choose be in harmony throughout the lesson? How will the teacher-student role overlap with the technology you will use?
40. Which questions will you ask at which stage of the lesson? Why?
41. Will students carry out their in-class activities individually or as a group? Can you explain why?
42. What will be the role of the students in the lesson? How did you plan to ensure students' participation in the lesson? Will the students' learning process be under your direction or their own?
43. What are the possible difficulties you may experience as a teacher when teaching this topic with technology in class?
44. What strategies will you use in classroom management when students use technology? Do you have any concerns about this?

45. Did you need professional support during the lesson plan preparation process? How did you overcome this? What do you think are your strengths and aspects for improvement?

Section 2: Enhancing the lesson plan

- Effective teaching practices in the lesson plan are emphasized. Suggestions are made for improving teaching.

  1. What kind of question can be asked at the beginning of the lesson to draw attention to the importance of the subject and to determine the readiness of the students? For example, what do you think about a question like .....?
  2. It is a good idea to use this in the ..... section. What do you think about incorporating an activity like ..... into the teaching? Do you think it would contribute to students' learning?

- Discuss the activities that are not thought to contribute sufficiently to students' conceptual learning.

  To what extent do you think this technology-supported activity that you plan to use in the lesson can deepen students' understanding of the subject? What can you suggest as an alternative to this activity? What do you think about incorporating an activity like ..... into your teaching?

- Examples of effective technology integration are shared and discussed.

  3. Do activities like ..... contribute to students' conceptual learning? What do you think about this?

- Suggestions are made for the deficiencies in technological pedagogical content knowledge.

  4. To what extent do you think it would be effective to use technology like ..... to eliminate misconceptions and prevent possible misconceptions?
  5. Can activities that can be integrated with inductive and deductive strategies such as ..... be used in the lesson? What is your opinion about this?

6. Can you use Multiple Representations such as ..... by utilizing technology?

- In the pre-conference, apart from the ones mentioned above, the behaviors at the most advanced level of the level descriptors of each theme of the model as stated in Niess, Ronau et al.'s (2009) TPACK development model constitute the focus of teacher training and pre-conferences;

  - ❖ Technology-based applications that assess students' conceptual learning of the subject matter are added.
  - ❖ Various technology integrations to enhance students' mathematical thinking and understanding are considered and added to the lesson plan.
  - ❖ Instead of adding technology integration as an enrichment to the lesson, technology is made an integral part of the lesson that is integrated into the lesson to enhance students' learning and deepen their understanding.
  - ❖ Technology is positioned as an effective teaching and learning tool to transform mathematical concepts and processes into a form that students can understand.

- Deductive and inductive strategies are integrated into various teaching methods that include technology, enabling students to think deeply about the subject matter.
- The course is organized as a learning environment where students can work on technology-enriched activities on their own and learn on their own.
- Professional development programs, activities and seminars on effective technology integration are recommended.
- Situations are added to allow students to use technology in all areas of mathematics.
- Comprehensive solutions and measures for difficulties that may arise during the integration of technology in the lesson are planned and added to the lesson.
- Activities that give students the opportunity to use technology to explore more complex mathematical situations, mathematical connections and generalizations are added.

G. POST- CONFERENCE PROTOCOL (IN TURKISH)

1. Bugünkü dersi baştan sona düşündüğünde akış olarak nasıl özetlersin?
2. Ders genel olarak nasıl geçti? Bugünkü dersi nasıl değerlendirirsin?
3. Bugünkü dersteki teknoloji kullanımı hakkında ne düşünüyorsun? Neler iyi gitti? Neler geliştirilebilir? Nasıl geliştirilebilir, önerilerin neler?
4. Ders planını uygularken bir zorluk yaşadın mı? Evet ise hangi kısımlarda zorlandın?
5. Dersi planladığın gibi mi işledin? Ders planında belirtmediğin ancak uygulamada yaşadığın bir şey oldu mu? Neden?
6. Ders sırasında beklenmedik bir olay oldu mu? Evet ise bu durumla nasıl baş ettin? Baş etmende teknoloji ne ölçüde destek oldu?
7. Derse teknoloji entegre etmene engel olan bir şey var mıydı? Evet ise ne olduğunu açıklar mısın?
8. Derste teknoloji kullanımının öğretim sürecini engellediği bir durum oldu mu? Örneklendirir misin?
9. Bugünkü dersteki teknoloji kullanımının öğrencilerin kavramsal öğrenmelerine ne ölçüde katkı sağladığını düşünüyorsun? Öğrencilerden kavramı keşfettiklerine dair bir geri bildirim (yorum, cevap vb.) aldın mı? Dersin bir kısmından örnek vererek açıklayabilir misin?
   (*Dersin video kaydından bir kısım izletilerek*) …… burada öğrencilerin daha iyi anlamaları nasıl sağlanabilirdi? Daha farklı ne yapılabilirdi?
10. Derste teknolojiyi öğrencilerin matematiksel düşünmelerini desteklemek için kullandığını düşünüyor musun? Evet ise nasıl kullandığına dersin bir bölümünden örnek verebilir misin?
11. Derste öğrencilerin teknolojiyi kullandılar mı? Evet ise nasıl ve ne amaçla kullandıklarını dersin bir bölümünden örnek vererek açıklar mısın? Öğrenciler teknolojiyi kullanırken bir zorluk yaşadı mı? Öğrenciler teknolojiyi kullanırken beklenmedik bir şey oldu mu?
12. Konuya ilişkin olası zorluklar ve kavram yanılgılarını ortadan kaldırmak için teknolojiyi kullandın mı? Evet ise nasıl kullandın? Teknoloji sence hangi yönüyle kavram yanılgıları ve zorlukları gidermede yeterli ve hangi yönüyle yetersiz kaldı?
    (*Dersin video kaydından başka bir kısım izletilerek*) ……. sence burada teknolojiyi kavramsal öğrenmeyi destekleyecek bir şekilde kullandın mı? Bu amaçla teknolojiden başka nasıl yararlanabilirdin? Ya da hangi soruları sorman veya açıklamaları yapman öğrencilerin anlamalarını derinleştirmeye yardımcı olurdu?
13. Derste bu konuyla ilgili önceden bahsettiğin anahtar kavramları öğrencilere keşfettirmek için teknolojiden ne ölçüde yararlandığını düşünüyorsun?
14. Derste teknoloji kullanımını düşündüğünde teknolojiyi hangi amaçlarla kullandığını söylersin?
    (*Dersin video kaydından bir kısım izletilerek*) ……...teknolojiyi burada kullanma amacın neydi? Buradaki kullanımının öğrencilerin kavramsal öğrenmelerine ne ölçüde katkı sağladığını düşünüyorsun?

15. Derste sorduğun soruların öğrencilerin konuyu anlamalarına ne ölçüde katkı sağladığını düşünüyorsun? Dersi tekrar yapsan bu sorulardan başka hangi soruları sorardın? Neden?
    (Dersin video kaydından bir kısım izletilerek) öğrencilere burada sorduğun soru yerine …… şeklinde bir soru sorsan öğrenciler sence nasıl yanıt verirlerdi? Bunun hakkında ne düşünüyorsun? *(önerilerde bulunulur)*

16. Bu derste kullandığın öğretim yöntemi öğrencilerin öğrenmelerinde ne ölçüde etkili olduğunu düşünüyorsun? Neden?
17. Sence başka hangi stratejiler öğrencilerin kavramsal öğrenmelerine katkıda bulunabilirdi?
18. Derste tümdengelim veya tümevarım stratejilerini kullandın mı? Bu stratejilerden faydalanırken teknolojiyi kullandın mı?
    (Dersin video kaydından bir kısım izletilerek) bu kısımda öğrencinin sorusunu …….. şeklinde yanıtladın. Başka hangi yöntem veya teknikle anlatabilirdin? …….. şeklinde bir açıklamayı düşündüğünde ne dersin? *(önerilerde bulunulur)*
19. Dersteki öğrenme ortamını tanımlar mısın? Öğrenme ortamı başka bir şekilde düzenlenseydi daha etkili hale gelebilir miydi?
20. Öğretmenin ve öğrencinin dersteki rolü neydi? Teknolojinin bu roller üzerine nasıl bir etkisi vardı?
21. Sence teknoloji kullanımın dersi ne yönde etkiledi? Teknoloji kullanmanın ne ölçüde bir fark yarattığını düşünüyorsun? Teknoloji kullanmasaydın ders nasıl olurdu? Ne eksik kalırdı? Ne değişmezdi?
22. Teknolojiyi derse entegre ederken herhangi bir zorluk yaşadın mı? Sınıf yönetiminde zorluk çektin mi? Evet ise bu zorluklarla nasıl baş ettin?
    Eğer bir zorluk yaşadıysa *(Dersin video kaydından bir kısım izletilerek)*……. bu bölümde zorluk yaşadığını düşünüyor musun? Evet ise bunun üstesinden gelebildiğini düşünüyor musun? …… şeklinde bir çözüm getirmek daha etkili olabilir miydi? *(önerilerde bulunulur)*
23. Derste teknolojiden çoklu temsiller açısından yararlandın mı? Evet ise nasıl yararlandın?
    [Dersin video kaydından bir kısım izletilerek] …….. sence bu bölümde çoklu temsiller için teknolojiden yararlanman öğrencilerin anlamalarına nasıl katkıda bulundu?
24. Derste teknolojiden temsiller arası geçiş açısından yararlandın mı?
    [Dersten bir video bölümü izletilerek] bu kısımda teknolojiden …….. şeklinde faydalansan öğrencilerin kavramsal öğrenmelerine daha çok katkıda bulunabilir miydi? Bu konuda ne düşünüyorsun? *(önerilerde bulunulur)*
25. Öğrencilerin öğrenmelerini değerlendirmek amacıyla teknolojiyi ne ölçüde kullandığını düşünüyorsun? Neden?
    [Dersten bir video bölümü izletilerek] ……burada neyi ölçmeyi amaçladın ve nasıl yaptın? Daha etkili bir şekilde yapabilir miydin? Ölçme ve değerlendirme sürecinde teknolojiyi……... şeklinde kullanman daha etkili olabilir miydi? Bu durum hakkındaki düşüncelerin nelerdir? *(önerilerde bulunulur)*

26. Uyguladığın bu dersi bir başka sınıfta uygulamadan önce neleri değiştirir neleri aynı bırakırdın? Neden? Derste düzenlemeler (ekleme veya çıkarma) yapar mıydın? Açıklar mısın?
27. Bu dersi mesleki gelişimin açısından nasıl değerlendirirsin? Dersi uygulamak mesleki gelişimine nasıl katkı sağladı? Teknoloji odaklı mesleki gelişim çalışmalarına katılımda bulunmak ister misin? Neden?

H. POST- CONFERENCE PROTOCOL (IN ENGLISH)

1. How would you summarize the flow of today's lesson when you think about it from beginning to end?
2. How was the lesson overall? How would you evaluate today's lesson?
3. What do you think about the use of technology in today's lesson? What went well? What could be improved? How can it be improved, what are your suggestions?
4. Did you have any difficulties while implementing the lesson plan? If yes, in which parts did you have difficulties?
5. Did you teach the lesson as you planned? Was there anything you did not specify in the lesson plan but you experienced in practice? Why?
6. Did anything unexpected happen during the lesson? If yes, how did you cope with this situation? To what extent did technology support you in coping?
7. Was there anything preventing you from integrating technology into the lesson?
8. Was there a situation where the use of technology in the lesson interfered with the teaching process? Can you give an example?
9. To what extent do you think the use of technology in today's lesson contributed to students' conceptual learning? Did you receive any feedback (comments, answers, etc.) from the students that they discovered the concept? Can you explain by giving an example from a part of the lesson?
[By showing a part of the video recording of the lesson] ..... How could students' understanding have been better here? What could have been done differently?
10. Do you think that you use technology in the lesson to support students' mathematical thinking? If yes, can you give an example of how you use it from a part of the lesson?
11. Did the students use technology in the lesson? If yes, can you explain how and for what purpose by giving an example from a part of the lesson? Did the students experience any difficulties while using the technology? Did anything unexpected happen while the students were using the technology?
12. Did you use technology to eliminate possible difficulties and misconceptions about the topic? If yes, how did you use it? In which aspect do you think the technology was sufficient in eliminating misconceptions and difficulties and in which aspect was it insufficient?
[By showing another part of the video recording of the lesson]..... Do you think you used technology in a way to support conceptual learning here? How else could you use technology for this purpose? Or what questions or explanations would have helped deepen students' understanding?
13. To what extent do you think you used technology in the lesson to help students explore the key concepts you mentioned earlier about this topic?
14. When you think about the use of technology in the lesson, for what purposes would you say you used technology?
[By showing a part of the video recording of the lesson].....What was the purpose of using technology here? To what extent do you think its use here contributed to students' conceptual learning?

15. To what extent do you think the questions you asked in the lesson contributed to the students' understanding of the topic? Which questions other than these questions would you ask if you were to repeat the lesson? Why?
[By showing a part of the video recording of the lesson] How do you think students would respond if you asked them a question like ..... instead of the question you asked here? What do you think about that? *(suggestions are made)*
16. To what extent do you think the teaching method you used in this lesson was effective in students' learning? Why?
17. Which other strategies do you think could have contributed to students' conceptual learning?
18. Did you use deductive or inductive strategies in the lesson? Did you use technology while using these strategies?
[By showing a part of the video recording of the lesson] You answered the student's question in the form of ..... in this part. With which other method or technique could you have explained it? What do you say when you think of an explanation in the form of .....? (*suggestions are made*)
19. Can you describe the learning environment in the lesson? Could the learning environment have been more effective if it had been organized in a different way?
20. What was the role of the teacher and the student in the lesson? What impact did technology have on these roles?
21. How do you think your use of technology affected the lesson? To what extent do you think the use of technology made a difference? What would the lesson have been like if you had not used technology? What would have been missing? What would not have changed?
22. Did you have any difficulties integrating technology into the lesson? Did you have difficulties in classroom management? If yes, how did you overcome these difficulties?
(If there were any difficulties) *[By showing a part of the video recording of the lesson]*..... do you think you had difficulties in this part? If yes, do you think you were able to overcome it? ..... would it have been more effective to bring a solution? (suggestions are made)
23. Did you make use of technology in terms of multiple representations in the lesson? If yes, how did you utilize it?
[By showing a part of the video recording of the lesson] ..... How do you think your use of technology for multiple representations in this section contributed to students' understanding?
24. Did you make use of technology in the lesson in terms of transition between representations?
[by showing a video section from the lesson] Would it have contributed more to students' conceptual learning if you had utilized technology in this section as .....? What do you think about this? (*suggestions are made*)
25. To what extent do you think you have used technology to assess students' learning and why?
[By showing a part of the video recording of the lesson] .....what did you aim to assess here and how did you do it? Could you have done it more effectively? Could it be more effective to use technology in the measurement and evaluation process in the form of .....? What are your thoughts about this situation? (*suggestions are made*)

26. What would you change and what would you leave the same before implementing this lesson in another class? Why? Would you make adjustments (adding or removing) to the lesson? Can you explain?
27. How would you evaluate this lesson in terms of your professional development? How did practicing the lesson contribute to your professional development? Would you like to participate in technology-oriented professional development activities? Why?

I. MATHEMATICS COACHING INTERVIEW (IN TURKISH)

1. Matematik koçluğu deneyimini genel olarak nasıl değerlendirirsin?
2. Matematik koçluğunun teknolojik, pedagojik ve alan bilgini kullanmanı nasıl etkilediğini düşünüyorsun?
3. Matematik dersine teknolojiyi entegre etme açısından Matematik koçluğunu etkili buldun mu?
4. Matematik koçluğunun hangi açıdan geliştirilmeye ihtiyacı var? Neden?
5. Matematik öğretimine teknoloji entegrasyonunun gelişimi bakımından koçluk sürecinde ne yapmak ya da bu sürece neyi dahil etmek matematik koçluğunu daha etkili hale getirirdi? Açıklar mısın?
6. Matematik koçluğunun güçlü yönlerinin neler olduğunu düşünüyorsun? Açıklar mısın?
7. Matematik öğretimine teknoloji entegrasyonunun gelişimi bakımından matematik koçluğunun en çok hangi yönünü beğendin?
8. Matematik koçluğunu beklentilerini karşılaması açısından nasıl değerlendirirsin?
9. Matematik koçluğu süreci mesleki gelişimini nasıl etkiledi?
10. Bir meslektaşına matematik koçluğuna katılmasını tavsiye eder misin? Neden?
11. Bunlar dışında Matematik koçluğu deneyimine dair eklemek istediğin bir şey var mı?

J. MATHEMATICS COACHING INTERVIEW (IN ENGLISH)

1. How would you evaluate your mathematics coaching experience in general?
2. How do you think mathematics coaching has affected your use of technological, pedagogical and content knowledge?
3. Did you find the Mathematics coaching effective in terms of integrating technology into the mathematics lesson?
4. In what ways does mathematics coaching need to be improved? Why?
5. In terms of the development of technology integration in mathematics teaching, what would have made the coaching process more effective or what could have been included in the coaching process? Can you explain?
6. What do you think are the strengths of mathematics coaching, can you explain?
7. Which aspect of mathematics coaching did you like the most in terms of the development of technology integration in mathematics teaching?
8. How would you evaluate mathematics coaching in terms of meeting your expectations?
9. How did the mathematics coaching process affect your professional development?
10. Would you recommend a colleague to participate in mathematics coaching? Why?
11. Apart from these, is there anything else you would like to add about your mathematics coaching experience?

K. MATHEMATICS TEACHER TPACK DEVELOPMENT MODEL

Level 1: Recognizing Level (knowledge)
Level 2: Accepting Level (persuasion)
Level 3: Adapting Level (decision)
Level 4: Exploring Level (implementation)
Level 5: Advancing Level (confirmation)

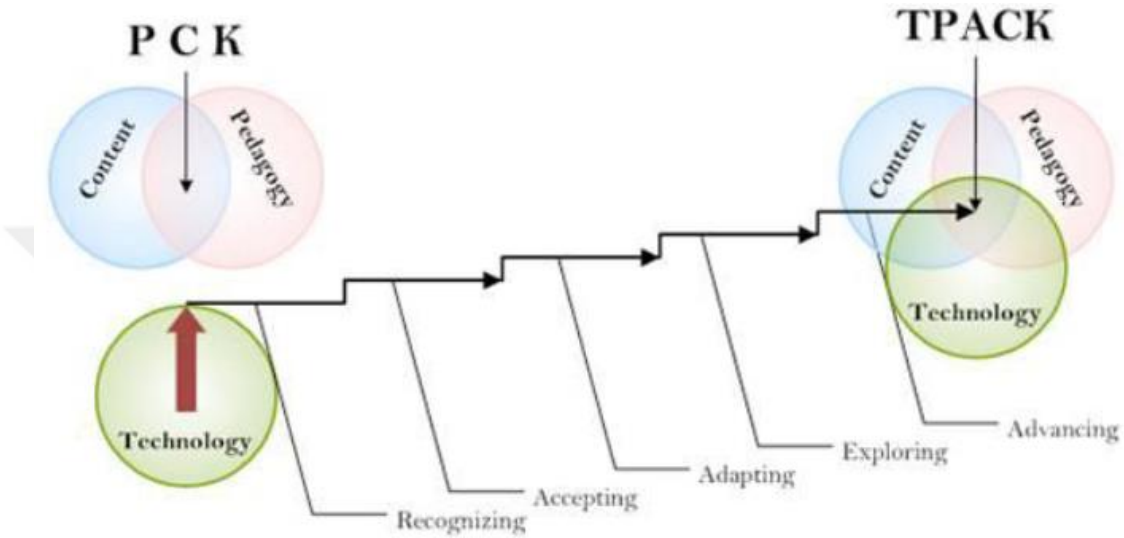


Figure 2. TPACK Development Model for Mathematics Teachers
(Niess, Ronau et al., 2009, p. 10)

Themes, Levels, Descriptors, Examples of TPACK Development Model:

CURRICULUM & ASSESSMENT THEME

C: Curriculum descriptor Ex: Mathematics Example

Recognizing
C: Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.
Ex: Creates graphs of multiple linear functions using graphing calculators to provide a visual representation for varying slopes. Considers these visuals as making sense of the idea of slope but is unsure of how this might help students learn the basic concept.

Accepting
C: Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning.
Ex: Attends and participates in mathematics dynamic geometry system workshop to identify curricular ideas for incorporating the technologies as learning tools. Mimics the incorporation of a dynamic geometry system idea from the workshop to display

measuring the sum of the angles of a triangle that upon multiple changes of the triangle suggests that the sum of the angles of any triangle is 180 degrees.

Adapting
C: Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.
Ex: Targets key topics students investigate with technology. Develops lessons to demonstrate mathematics concepts with technology and activities for students to use technology to verify or reinforce those concepts. After students have learned to create graphs of specific linear functions, students are challenged to use the spreadsheet to verify the graphical representation of the ordered pairs.

Exploring
C: Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning.
Ex: Adapts own previous mathematics lesson to include technology.
Ex: Develops own ideas about using technology to enhance current curriculum; thus, begins altering preexisting activities or creating new activities for current curriculum.

Advancing
C: Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.
Ex: Develops innovative ways to use technology to develop mathematical thinking in students such as using virtual algebra tiles to extend ideas of handheld manipulatives to focus on variables in algebraic expressions.
Ex: Modifies and advances curriculum to take advantage of technology as a tool for teaching and learning such as using CAS to explore more complex algebraic expressions.

CURRICULUM & ASSESSMENT THEME

A: Assessment descriptor Ex: Mathematics Example

Recognizing
A: Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.
Ex: Does not allow calculator use when assessing students' understanding of solving linear equations.

Accepting
A: Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).
Ex: Attends and participates in a mathematics assessment professional development to consider ideas for assessing students' understanding of solving systems of linear functions using the calculator as a tool. Mimics the assessment idea to explain the use of the calculator for solving systems of linear functions by using the trace

function to identify the intersection. Often retests technology questions with paper and pencil questions to be sure that the concept was learned the 'right' way.

Adapting
A: Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).
Ex: Allows use of calculator in an assessment but designs the assessment to focus on gathering students' conceptual understanding of solving systems of linear functions in addition to their procedural understanding.

Exploring
A: Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).
Ex: Designs assessments where students are expected to show their understanding of mathematical ideas using an appropriate technology that extends beyond paper and pencil type questions.

Advancing
A: Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.
Ex: Develops innovative assessments to capture students' understandings of the mathematics embedded in the particular technology.

LEARNING THEME

M: Mathematics learning descriptor Ex: Mathematics example

Recognizing
M: Views mathematics as being learned in specific ways and that technology often gets in the way of learning.
Ex: Mathematical exploration with technology rarely seen.

Accepting
M: Has concerns about students' attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.
Ex: Limits student technology use, particularly during the introduction and development of key topics.

Adapting
M: Begins to explore, experiment and practice integrating technologies as mathematics learning tools.
Ex: Students explore some mathematics topics using technology.

Exploring
M: Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.
Ex: Students explore numerous topics using technology, sometimes ranging outside the topic at hand.

Advancing
M: Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.
Ex: Students explore mathematics topics, integrating various technologies in attempts to better understand mathematical concepts.

LEARNING THEME

C: Conception of student thinking descriptor Ex: Mathematics example

Recognizing
C: More apt to accept the technology as a teaching tool rather than a learning tool.
Ex: Technology is used only outside of normal classroom activities, such as checking homework, calculating large numbers, etc.

Accepting
C: Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.
Ex: Activities that use technology are almost always redone without technology to be certain students really learned the particular concept.

Adapting
C: Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.
Ex: Although students use technology for most topics, assessing student thinking remains mostly technology free.

Exploring
C: Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.
Ex: Technology activities are implemented and evaluated with respect to student learning of mathematics and student attitudes toward mathematics.
Ex: Manages technology-enhanced activities towards directing student engagement and self-direction in learning mathematics.

Advancing
C: Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.
Ex: Engages students in high-level thinking activities (such as project-based and problem solving and decision-making activities) for learning mathematics using the technology as a learning tool.
Ex: Technology is used to develop advanced levels of understanding of mathematical concepts.

TEACHING THEME

M: Mathematics learning descriptor Ex: Mathematics example

Recognizing
M: Concerned that the need to teach about the technology will take away time from teaching mathematics.
Ex: Students use technology on their own and little or no instruction with technology is present.

Accepting
M: Uses technology activities at the end of learning areas, for "days off," or for activities peripheral to classroom instruction.
Ex: Technology-enhanced activities are not used for topics that require more advanced technology skills.

Adapting
M: Uses technology to enhance or reinforce mathematics ideas that students have learned previously.
Ex: Students use technology to reinforce previously teacher-taught concepts.

Exploring
M: Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.
Ex: Teachers share classroom-tested, technology-based lessons, ideas, and successes with peers.

Advancing
M: Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.
Ex: Teacher is seen as a resource as novel ideas for helping students learn mathematics with technology.

TEACHING THEME

I: Instructional descriptor Ex: Mathematics example

Recognizing
I: Does not use technology to develop mathematical concepts.
Ex: Technology, if used in class, is used for menial or rote activities.

Accepting
I: Merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies.
Ex: Introduces the Pythagorean Theorem algorithmically; teacher use of dynamic geometry to verify the Pythagorean Theorem; students find solutions to example problems using paper and pencil.

Adapting
I: Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.

Ex: Technology-based lessons are incorporated that are tailored to students' needs.

Exploring
I: Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.
Ex: Students use technology to explore new concepts as the teacher serves mostly as a guide.

Advancing
I: Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.
Ex: The teacher helps students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics.

TEACHING THEME

E: Environment descriptor Ex: Mathematics example

Recognizing
E: Uses technology to reinforce concepts taught without technology.
Ex: Focus on linear functions where students practice creating graphs by hand to explore different functions. After students have demonstrated competence with linear functions, summarize the knowledge, with a spreadsheet example or a graphing calculator example.

Accepting
E: Tightly manages and orchestrates instruction using technology.
Ex: Technology is directed, in a tightly sequenced, step-by-step process. Skill-based, non-exploratory technology use.

Adapting
E: Instructional strategies with technologies are primarily deductive, teacher-directed in order to maintain control of the how the activity progresses.
Ex: Begins to adapt instructional approaches that allow students opportlearning areaies to explore with technology for part of lessons.

Exploring
E: Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.
Ex: The teacher incorporates a variety of technologies for numerous topics.

Advancing
E: Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics.
Ex: The teacher forms and reforms learning groups where individual and group learning is valued and encouraged.

TEACHING THEME

PD: Professional development descriptor Ex: Mathematics example

Recognizing
PD: Considers attending local professional development to learn more about technologies.
Ex: Attends local workshops that focus on gaining skills with the technology; context of the learning activities is mathematics.

Accepting
PD: Recognizes the need to participate in technology related PD.
Ex: Seeks out technology-related professional development, workshops that are directed at developing the technology in the learning of mathematics.

Adapting
PD: Continues to learn and explore ideas for teaching and learning mathematics using only one type of technology (such as spreadsheets).
Ex: Shares ideas from professional development with other mathematics teachers in the building.

Exploring
PD: Seeks out and works with others who are engaged in incorporating technology in mathematics.
Ex: Organizes teachers of similar mathematics and grade level in investigating the mathematics curriculum to integrate appropriate technologies.

Advancing
PD: Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.
Ex: Engages teachers in the district in evaluating and revising the mathematics curriculum to more seamlessly integrate technology throughout the grades, adjusting the curriculum for a 21st century mathematics curriculum with appropriate technologies.

ACCESS THEME

U: Usage descriptor Ex: Mathematics example

Recognizing
U: Permits students to use technology ‘only’ after mastering certain concepts.
Ex: Mathematical exploration with technology tools is challenged by beliefs about how students need to learn mathematics.

Accepting
U: Students use technology in limited ways during regular instructional periods.
Ex: Student activities with technology are limited to brief tightly controlled situations.

Adapting
U: Permits students to use technology in specifically designed learning areas.
Ex: Access to and use of technology is available for exploration of new topics, usually with the teacher's demonstration.

Exploring
U: Permits students to use technology for exploring specific mathematical topics.
Ex: Access to and use of technology is available and encouraged for mathematics exploration during most class times.

Advancing
U: Permit students to use technology in every aspect of mathematics class.
Ex: Technology is seen as an opportunity to challenge notions of what mathematics students can master.

ACCESS THEME

B: Barrier descriptor Ex: Mathematics example

Recognizing
B: Resists consideration of changes in content taught although it becomes accessible to more students through technology.
Ex: Student access to technology is limited to 'after' they have learned the given concepts using paper and pencil procedures and only for rote activities.
Accepting
B: Worries about access and management issues with respect to incorporating technology in the classroom.
Ex: Students can only use technology in isolated situations or non-important learning situations.
Adapting
B: Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.
Ex: Concepts learned with technology are not assessed with technology.

Exploring
B: Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges.
Ex: Technology is used extensively in assessments. Seeks out ways to obtain technology for classroom use and begins creating methods for technology management issues.

Advancing
B: Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.
Ex: Technology is used to expand the mathematics concepts that can be accessed by students.

ACCESS THEME

A: Availability descriptor Ex: Mathematics example

Recognizing
A: Notices that authentic problems are more likely to involve 'unfriendly numbers' and may be more easily solved if students had calculators.
Ex: Assigns some mathematics problems using school and commlearning areay data but saves then for "extra credit" work if students have calculators.

Accepting
A: Calculators permit greater number of examples to be explored by students.
Ex: Student use calculators to investigate patterns and functions.

Adapting
A: Concepts are taught differently since technology provides access to connections formerly out of reach.
Ex: Students use dynamic geometry software to investigate and make connections between trigonometry functions.

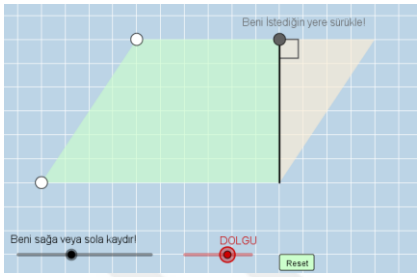
Exploring
A: Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections.
Ex: Simultaneous equations are developed from an authentic situation, solved, and interpreted using graphs, tables, symbols and data.

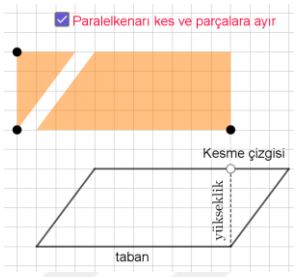
Advancing
A: Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience.
Ex: Using the Internet to find interesting mathematical problems, students investigate the role that technologies can play in finding solutions to the problems.

L. SAMPLE GEOGEBRA ACTIVITIES FROM THE MATHEMATICS COACHING CYCLE 4

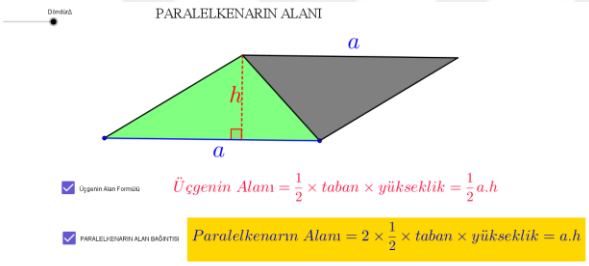
These activities were retrieved from https://www.geogebra.org/materials

1) Area relation of Parallelogram



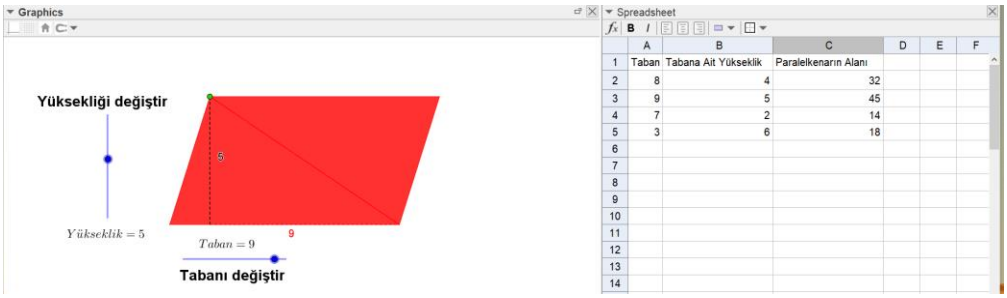


2) Relationship between area of triangle and area of parallelogram

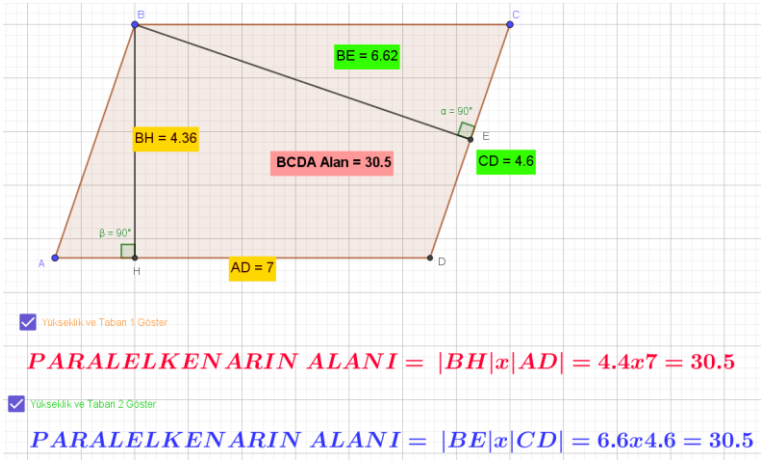




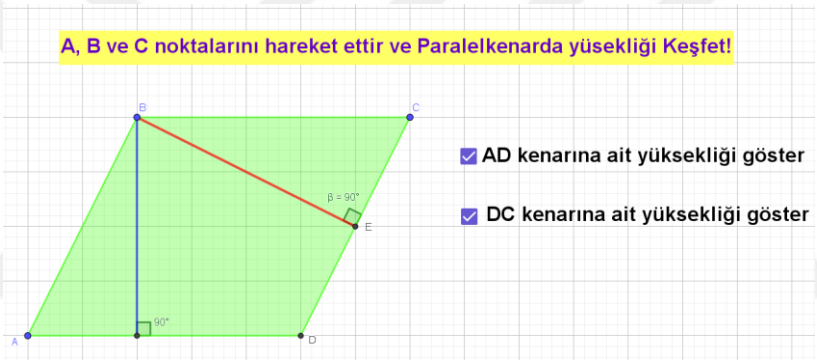
3) Area relation of parallelogram using spreadsheet feature of GeoGebra



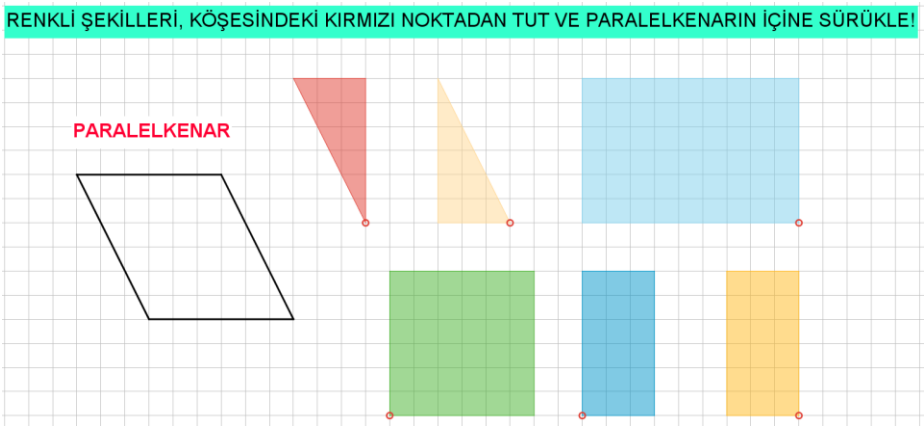
4) Area calculation of Parallelogram using different bases and heights



5) Heights in Parallelogram



6) Geometric shapes inside the parallelogram



M. APPROVAL OF THE METU HUMAN SUBJECTS ETHICS COMMITTEE

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

Sayı :
Konu : Araştırma İzni

ORTA DOĞU TEKNİK ÜNİVERSİTESİ REKTÖRLÜĞÜNE
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2020/2 sayılı Genelgesi.
b)

Üniversiteniz İlköğretim Anabilim Dalı doktora programı öğrencisi Mustafa Buğra AKGÜL'ün **"Ortaokul Matematik Öğretmenlerinin Geometrideki Teknolojik Pedagojik Alan Bilgisi (TPAB) Gelişimi için Matematik Koçluğu"** konulu çalışması kapsamında Gölbaşı ilçemize bağlı ortaokullarda uygulama yapma talebi ilgi (a) Genelge çerçevesinde incelenmiştir. Yapılan inceleme sonucunda, söz konusu araştırmanın Müdürlüğümüzde muhafaza edilen ölçme araçlarının; Türkiye Cumhuriyeti Anayasası, Milli Eğitim Temel Kanunu ile Türk Milli Eğitiminin genel amaçlarına uygun olarak, ilgili yasal düzenlemelerde belirtilen ilke, esas ve amaçlara aykırılık teşkil etmeyecek, eğitim-öğretim faaliyetlerini aksatmayacak şekilde okul ve kurum yöneticilerinin sorumluluğunda, gönüllülük esasına göre uygulanması Müdürlüğümüzce uygun görülmüştür.

Bilgilerinizi ve gereğini rica ederim.

N. CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Akgül, Mustafa Buğra

Nationality:

Date and Place of Birth:

Marital Status:

Phone:

email:

EDUCATION

Degree	Institution	Year of Graduation
MS		
BS		
High School		

WORK EXPERIENCE

Year	Place	Enrollment

FOREIGN LANGUAGES

PUBLICATIONS

A. Conference Proceedings

B. Projects, Seminars, Workshops

HOBBIES

O. TURKISH SUMMARY / TÜRKÇE ÖZET

MATEMATİK KOÇLUĞU YOLUYLA BİR ORTAOKUL MATEMATİK ÖĞRETMENİNİN GEOMETRİDEKİ TEKNOLOJİK PEDAGOJİK ALAN BİLGİSİNİN (TPAB) GELİŞİMİ

1. Giriş

Yıllar içinde hızla gelişen teknoloji, dijital toplulukları dönüştürmüş ve birçok ülkeyi eğitim teknolojisinin öğrenmeyi nasıl geliştirebileceğini keşfetmeye teşvik etmiştir (Timotheou vd., 2023). Eğitim teknolojisi, özellikle Bilgi ve İletişim Teknolojileri (BİT), eğitim kalitesini artırmada ve öğrencilerin ve öğretmenlerin teknolojik becerilerinin geliştirilmesinde önemli bir rol oynamıştır (Ertmer & Ottenbreit-Leftwich, 2010; Higgins vd., 2012). Dünya genelinde hükümetler, Türkiye de dahil olmak üzere, eğitimde BİT entegrasyonuna yatırım yapmış ve FATİH projesi gibi projeler aracılığıyla sınıflara akıllı tahtalar, hızlı internet ve tabletler getirilmiştir (Kurt vd., 2013; Tamim vd., 2015). Türkiye Milli Eğitim Bakanlığı (MEB), 2010-2028 yıllarını kapsayan stratejik planlamalarında dijital içerik geliştirme, bulut tabanlı platformlar, kişiselleştirilmiş öğrenme araçları ve siber güvenlik önlemleri gibi konulara odaklanmıştır (MEB, 2009, 2015, 2019, 2024). Eğitim teknolojisi eğitimde merkezi bir unsur haline geldikçe, öğretmenlerin güçlü teknolojik ve pedagojik beceriler geliştirme ihtiyacı artmıştır (Kaufman, 2014). Uluslararası Eğitimde Teknoloji Derneği (ISTE) gibi uluslararası kuruluşlar, öğretmenlerin dijital okuryazarlık ve teknoloji entegrasyonunda lider rolü üstlenmeleri gerektiğine dikkat çeken standartlar geliştirmiştir (ISTE, 2008, 2017). Öğretmenlerin, teknoloji destekli öğrenme ortamları tasarlaması, öğrenmeyi kişiselleştirmesi ve dijital araçlar kullanarak öğrenci gelişimini değerlendirmesi beklenmektedir (ISTE, 2008, 2017).

Eğitimde teknoloji entegrasyonunu yönlendiren iki önemli çerçeve, Araçsallaştırma Teorisi ve TPAB (Teknolojik Pedagojik Alan Bilgisi) teorik çerçeveleridir. Bu teorik

çerçeveler, teknoloji, içerik ve pedagojinin dinamik ilişkisini vurgulamakta ve öğretmenlerin etkili öğretim için teknolojiyi adapte edip özelleştirmesi gerektiğini belirtmektedir (Angeli & Valanides, 2009; Mishra & Koehler, 2006; Rabardel, 1995; Trouche, 2004). Ancak eğitim teknolojisinin artan önemine rağmen, öğretmenler teknolojiyi etkili bir şekilde derslerine entegre etmekte zorluk çekmektedir (Ertmer & Ottenbreit-Leftwich, 2010; Hew & Brush, 2007). Bu durumun yetersiz mesleki gelişim programlarından kaynaklandığı düşünülmektedir (Desimone, 2009). Kısa süreli atölye çalışmaları ve seminerler gibi geleneksel mesleki gelişim yöntemleri genellikle öğretmenlere pratik ve kişiselleştirilmiş destek sağlamaya odaklı olmadığı için öğretmenlerin öğrendiklerini sınıfta uygulaması zorlaşmaktadır (Darling-Hammond vd., 2017; Niess, Ronau vd., 2009). Öte yandan, iş odaklı mesleki gelişim yöntemleri, koçluk ve mentorluk gibi esnek ve uygulamaya yönelik destek sağlayarak öğretmenlerin öğrendiklerini hemen derslerine yansıtmalarını mümkün kılmaktadır (Knight, 2007, 2008; Loucks-Horsley vd., 2009).

Matematik koçluğu öğretmenlerin TPAB gelişimine katkıda bulunmak için büyük bir fırsat sunmaktadır (McGatha vd., 2018). Araştırmalar koçluğun öğretmenlerin sınıf uygulamaları üzerinde geleneksel mesleki gelişim yöntemlerine kıyasla daha önemli ve kalıcı bir etkiye sahip olduğunu göstermektedir (Campbell & Malkus, 2011; Driscoll, 2008). Matematik koçluğu, derslerin birlikte planlanması, gözlemlenmesi ve değerlendirilmesi süreçlerini içermekte olup, öğretmenlerin GeoGebra gibi teknolojileri matematik derslerine entegre etmek için pratik beceriler geliştirmelerine destek olmaktadır (Hohenwarter & Lavicza, 2007; Hull vd., 2009). Bu çalışma, matematik koçluğunun bir ortaokul matematik öğretmeninin TPAB gelişimi üzerindeki etkisini, sınıfta gerçekleşen gerçek öğretim uygulamaları üzerinden incelemeyi amaçlamıştır.

1.1. TPAB Gelişim Modeli

TPAB Gelişim Modeli, Niess, Ronau ve diğerleri (2009) tarafından geliştirilen, öğretmenlerin Teknolojik Pedagojik Alan Bilgisi (TPAB) gelişimini izlemeye yönelik bir modeldir. Bu model, öğretmenlerin matematik öğretiminde yeni teknolojik araçları benimseme sürecini adım adım tanımlayan Everett Rogers'ın

(1995) yenilik-karar süreci temel alınarak oluşturulmuştur. TPAB Gelişim Modeli dört ana tema üzerine kuruludur (Niess, Ronau vd., 2009):

1. **Müfredat ve Değerlendirme Teması**: Konuya özgü pedagojik yaklaşımlar ve öğrencilerin anlayışlarını değerlendirme süreci.
2. **Öğrenme Teması**: Öğrencilerin nasıl öğrendiğine ve düşünme becerilerini teknoloji aracılığıyla nasıl geliştirdiğine odaklanır.
3. **Öğretim Teması**: Öğretim stratejileri, öğrenme ortamları ve profesyonel gelişim ile ilgilidir.
4. **Erişim Teması**: Teknolojinin sınıfta mevcut olma durumu ve öğretmenlerin teknolojiyi entegre etme süreçlerinde karşılaştıkları engellerin nasıl aşılacağı üzerine yoğunlaşır.

TPAB Gelişim Modeli'nin Müfredat ve Değerlendirme Teması iki ana tanımlayıcıdan oluşmaktadır: (1) *Müfredat tanımlayıcısı* ve (2) *Değerlendirme Tanımlayıcısı*. *Müfredat tanımlayıcısı*, matematiksel içeriklerin ele alınmasını vurgulamakta olup, teknoloji, müfredat ve müfredatın hedefleri doğrultusunda kullanılan materyallerin matematik öğretimi ve öğrenimine nasıl entegre edildiğini ele almaktadır. Diğer taraftan, *Değerlendirme tanımlayıcısı*, öğrencilerin teknoloji aracılığıyla anlama düzeylerinin değerlendirilmesine odaklanmıştır (Niess, Ronau, vd., 2009).

TPAB Gelişim Modeli'nin Öğrenme Teması kapsamında yer alan iki önemli tanımlayıcı bulunmaktadır: (1) *Matematik öğrenimi* ve (2) *Öğrenci düşünme anlayışı tanımlayıcısı*. Bu tema, öğretmenlerin matematik öğretimine teknolojiyi bir öğrenme aracı olarak etkili bir şekilde entegre edip etmediklerini ve teknolojik araçlar aracılığıyla öğrencilerin düşünme becerilerindeki gelişimi ele almaktadır. Aynı zamanda alan bilgisine (matematik konularının anlaşılması) ve öğrencilerin öğrenim süreçlerindeki anlayışına yönelik içgörülerini (örneğin, öğrencilerin düşünme becerilerindeki gelişim) göstermeye odaklanmıştır (Niess, Ronau, vd., 2009).

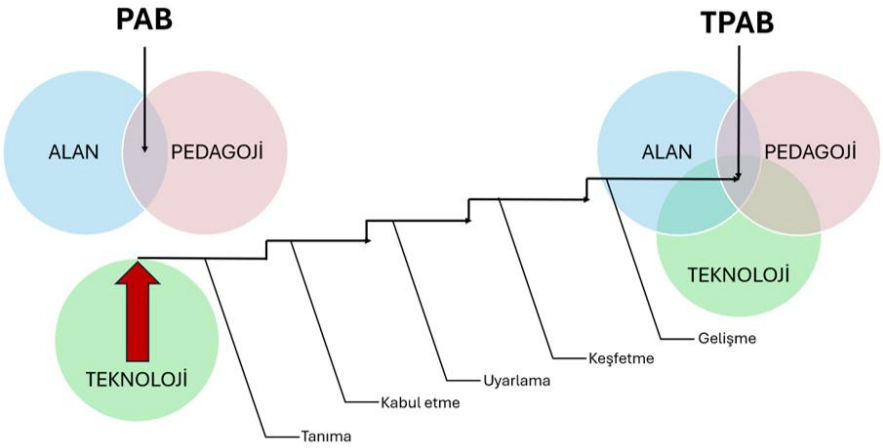
TPAB Gelişim Modeli'nin Öğretim Teması, dört ana tanımlayıcıyı içermektedir: (1) *Matematik Öğrenimi Tanımlayıcısı*, (2) *Öğretimsel Tanımlayıcı*, (3) *Öğrenme ortamı*

tanımlayıcısı ve (4) *Mesleki gelişim tanımlayıcısı*. Bu tema, öğretmenlerin teknolojiye bakış açılarını, mesleki gelişime olan yaklaşımlarını, teknolojiyi içeren öğretim stratejilerini anlama düzeylerini ve bireysel ve işbirlikçi öğrenme ortamlarında teknolojiyi öğrenme aracı olarak kullanarak öğrenme ortamları oluşturma süreçlerini ele almaktadır (Niess, Ronau, vd., 2009).

Erişim teması ise TPAB Gelişim Modeli'nin bir diğer teması olup, (1) *Kullanım*, (2) *Engel* ve (3) *Kullanılabilirlik tanımlayıcılarından* oluşmaktadır. Bu tema, öğrencilerin teknolojiyi kullanmalarına izin verilip verilmediğini, öğretmenlerin teknoloji entegrasyonu sırasında karşılaşabilecekleri olası engelleri ve bu engellerle karşılaşıldığında nasıl başa çıkılacağını, ayrıca öğrencilerin daha ileri düzey matematik keşfi için teknolojiye nasıl eriştiklerini incelemektedir (Niess, Ronau, vd., 2009).

Öğretmenler, TPAB Gelişim Modeli'nde beş seviye boyunca ilerlemektedirler. İlk seviye olan *tanıma* (bilgi) seviyesinde, öğretmenler teknolojinin matematikteki uygulamalarının farkında olsalar da, bu teknolojiyi henüz derslerine entegre etmemektedirler. İkinci seviye olan *Kabul Etme* (ikna) seviyesinde, öğretmenler teknolojiyi öğretimde kullanma konusunda olumlu veya olumsuz tutumlar geliştirmektedirler. Üçüncü seviye olan *Uyarlama* (karar) seviyesinde, öğretmenler teknolojiyi kullanmanın faydalarını fark etmeye başlamakta ve bunu derslerine entegre edip etmeyeceklerine karar vermektedirler. Dördüncü seviye olan *Keşfetme* (uygulama) seviyesinde, öğretmenler teknolojiyi aktif ve etkili bir şekilde öğretimlerine entegre etmekte, müfredatla uyumlu ders materyalleri oluşturmakta ve bunları revize etmektedirler. Son seviye olan *Gelişme* (doğrulama) seviyesinde ise, öğretmenler teknolojiyi tam olarak entegre etmekte, müfredatı gerektiği gibi düzenlemekte ve teknolojinin öğretim ve öğrenme üzerindeki etkilerini değerlendirmektedirler.

Şekil 1'de görüldüğü üzere, öğretmenler bu seviyeler boyunca ilerledikçe, TPAB seviyeleri de gelişmekte ve teknoloji, öğretim uygulamalarında merkezi bir araç haline gelmektedir. Model, teknolojinin potansiyelini tanımaktan, onu derslerin tam anlamıyla ayrılmaz bir parçası haline getirmeye kadar olan süreçteki ilerlemeyi vurgulamaktadır.



Şekil 1. Matematik Öğretmenleri için TPAB Gelişim Modeli
(Niess, Ronau, vd., 2009, s. 10)

Bu çalışmada, Niess, Ronau ve diğerleri (2009) tarafından geliştirilen TPAB Gelişim Modeli, bir ortaokul matematik öğretmeninin GeoGebra kullanarak teknolojiyle zenginleştirilmiş dersler tasarlarken ve uygularken TPAB'inde meydana gelen değişiklikleri ve gelişmeleri incelemek amacıyla teorik çerçeve olarak benimsenmiştir.

1.2. Mesleki Gelişim Yaklaşımı Olarak Matematik Koçluğu

Öğretmenlerin mesleki gelişimlerinin sürdürülebilirliğine yönelik yaklaşımlar, eğitimcilerin en iyi uygulamalar ve stratejiler konusunda güncel kalmalarını sağlamak açısından önemli görülmektedir (Desimone, 2009). Loucks-Horsley ve diğerleri (2009), mesleki gelişim stratejilerini altı kategoriye ayırmıştır: içerik, standartlar ve araştırma, öğrenme ve öğretme, müfredat uyumu ve uygulama ile mesleki gelişim yapısı. Bu kategoriler, öğretmenlerin sürekli gelişimini desteklemek için tasarlanmış on altı stratejiyi içermektedir. Bu stratejiler faydalı olabilirken, günümüz eğitim dünyasının zorluklarına her zaman tam olarak hazırlayamamaktadırlar. Geleneksel mesleki gelişim yaklaşımlarının eksikliklerini gidermek için Joyce ve Showers (1982), eğitim bağlamında koçluğu alternatif bir strateji olarak önermiştir. Önceki yöntemlerin aksine, daha genel ve daha az yoğun olan bu yaklaşımlar, bireyselleştirilmiş ve kurumsal koçluk uygulamalarıyla yakından ilişkilidir. Bu yaklaşım, geleneksel yöntemlerin öğretmenleri değişime veya yeni pedagojik uygulamaları benimsemeye yeterince hazırlamadığını öne süren

araştırmalara yanıt olarak daha da geliştirilmiştir (Showers & Joyce, 1996). Bireyselleştirilmiş mesleki gelişim kavramına dayanarak, matematik koçluğu umut verici bir yaklaşım olarak ortaya çıkmıştır. Matematik koçluğu öğretmenlere yeni yöntemler sunarak öğretim becerilerini geliştirmelerine yardımcı olmaktadır (Bengo, 2013; Gibbons & Cobb, 2017). Koçluk aracılığıyla öğretmenler öğretimlerini değerlendirebilmekte, geri bildirim alabilmekte ve uygulamalarını değiştirebilmekte, bu da özgüvenlerini ve yetkinliklerini artırabilmekte ve sonuç olarak öğrencilerin öğrenimini olumlu yönde etkilemektedir (Liao vd., 2021; Larsen, 2012). İlgili alanyazın, koçluk müdahalelerinin öğretmenlerin uygulamalarını geliştirdiğini çünkü koçluk yapılan öğretmenlerin, yeni bilgi ve pedagojiyi uygulama olasılığının, koçluk yapılmayanlara göre dört kat daha fazla olduğunu ortaya koymaktadır (Knight, 2007; Showers & Joyce, 1996).

Loucks-Horsley ve diğerleri (2009), koçluğu, öğretmenlerin çiftler halinde veya tek başına öğrenmelerine ve fen ve matematik öğretimlerini geliştirmelerine olanak tanıyan mesleki gelişim stratejileri arasında kategorize etmiştir. Bu yaklaşım, öğretmenlerin kendilerini yansıtıcı uygulayıcılar olarak düşünmelerine, mesleki bilgi ve becerilerini geliştirmelerine ve yeni stratejiler benimsemelerine yardımcı olmaktadır (Knight, 2007; Loucks-Horsley vd., 2009; Showers, 1985). Koçluk, öğretmen etkinliğini artırarak öğrenci başarısını yükseltmeyi amaçlayan bütünsel bir iş odaklı öğrenme süreci olarak tanımlanmaktadır. Birçok ülkede okul temelli koçluğun evrimi, bunun sürekli ve etkili bir mesleki gelişim biçimi olduğunu göstermektedir. Koçluk, son yirmi yıldır eğitim sisteminin bir parçası haline gelmiştir ve bilişsel, öğretimsel, içerik-odaklı ve akran koçluğu gibi farklı modellerle devam etmektedir (McGatha, 2008; Grant & Greene, 2004). Showers ve Joyce (1982), iki öğretmenin birbirlerinin öğrenmelerini teşvik etmek amacıyla birlikte çalıştığı akran koçluğu modelini geliştirmiştir ve bu model, günümüzdeki koçluk modellerinin temelini atmıştır. Bilişsel koçluk, öğretmenlerin düşünme süreçlerine ve karar verme yeteneklerine odaklanarak, sorgulamaya dayalı sorular sorarak öğretmenlerin kendini geliştirme sürecini destekleyen ve teşvik eden bir süreçtir (Costa & Garmston, 2002). Öğretim koçluğu, belirli öğretim stratejileri hakkında öğretmenlere rehberlik ve geri bildirim sunarak öğretim ve öğrenme sürecini

geliştirmeyi amaçlayan bir süreç olarak tanımlanmıştır. Bu model, ortaklıklar, hedefler ve verilerden öğrenmeyi geliştirme yollarına odaklanmıştır (Knight, 2007).

West ve Staub (2003) matematik öğretmenlerine yönelik içerik ve pedagojiye odaklanan içerik-odaklı koçluk kavramını geliştirmiştir. İçerik-odaklı koçluk, matematik veya edebiyat gibi belirli konularda öğretim uygulamalarını geliştirmeyi amaçlayan bir koçluk türü olarak tanımlanmıştır. Bu strateji, öğretmenin alan bilgisi ve öğrenci başarısını artırmak için ders planlama, öğretim uygulamasının gözlemi ve uygulanan derse yönelik geri bildirim vermeyi içermektedir (West & Staub, 2003). Bu koçluk türü öğretmenler ve koçların, öğrencilerin performansını geliştirmek amacıyla matematiksel etkinlikler ve öğretim stratejileri tasarlamalarını, planlamalarını ve izlemelerini kapsamaktadır. Gerçek öğretim uygulamalarına dayanan koçuk oturumları koçların öğretmenlerle doğrudan çalışmasını sağlayarak öğretim sürecini iyileştirmeyi amaçlamaktadır (West & Staub, 2003). Farklı koçluk modelleri arasında, içerik-odaklı koçluğun öğretmenlerin belirli konularda öğretimlerini geliştirmelerine en çok yardımcı olan model olduğu düşünülmektedir (Kane & Saclarides, 2023; West, 2009).

Matematik koçluğu, matematik öğretimini geliştirmeyi amaçlayan, içeriğe dayalı koçluğun özel bir türü olarak tanımlanmıştır. Bu, koçun sınıfta dersleri gözlemlediği, geri bildirim sunduğu ve dersleri birlikte tasarladığı mesleki bir işbirliğidir. Amacı öğretmenin alan bilgisini, pedagojik alan bilgisini ve sınıfta matematiksel uygulamaları uygulama yetkinliğini artırarak öğrencilerin matematik anlayışını geliştirmektir (Campbell & Griffin, 2017; Gibbons & Cobb, 2017). İçerik-odaklı koçlukta bir koçluk döngüsü (oturumu) genellikle üç aşamadan oluşmaktadır. Bunlar, dersin uygulanmasından önce dersin birlikte planlanması (ön-konferans), sınıfta gerçekleşen gerçek öğretim (ders gözlemi) ve dersin uygulanmasından sonra dersin değerlendirilmesi (son-konferans) aşamaları olarak belirtilmiştir. Ön-konferans aşamasında koç öğretmenin ders planlamaya yönelik yaklaşımlarına, öğrencilerin matematikle ilgili düşünüşlerine ve inançlarına ve öğretmenin matematik içeriği ve pedagojisi hakkındaki anlayışına dair bilgi edinmektedir (West & Staub, 2003). Bu aşama, koçun öğretmenle birlikte ders planı geliştirmesine ve en iyi öğrenme sonuçlarına ulaşmak için kullanılacak stratejileri belirlemesine olanak

tanımaktadır. Ayrıca, öğretmen ve koç bazen öğrencilerin sahip olabileceği olası kavram yanılgılarını belirlemek ve bu kavram yanılgılarını gidermeye yardımcı olabilecek stratejiler geliştirmek için sınıfta işbirliği yapmaktadırlar (West, 2009; West & Staub, 2003).

Son-konferans aşaması öğretim uygulaması gözlemlendikten sonra gerçekleştirilmekte ve bu süreçte öğretmen ve koç dersi değerlendirirmekte ve iyileştirilmesi gereken yönleri belirlerlemektedirler (West, 2009). Bu süreçte koç ve öğretmen, dersin uygulanmasını tartışmakta ve öğretim-öğrenme sürecinde, içerik üzerinde daha fazla ayrıntıya girerek veya ders sonuçlarına dayalı olarak öğretim stratejilerini gözden geçirerek ek adımlar atmaktadırlar (West & Staub, 2003). Son-konferans, koç ve öğretmenin öğrencilerin çalışmaları ve sınıf etkileşimlerine yönelik gözlemleri ve değerlendirmeleri üzerine bir sonraki ders için hazırlık yapmayı amaçlamaktadır. Matematik koçları, öğretmenlerin matematik başarısını artırmalarına yardımcı olmak, matematik müfredatını ve materyallerini yönetmek, müfredat yönetimini denetlemek ve öğretmenlere mesleki gelişim fırsatları sunmak gibi çeşitli roller üstlenmektedirler. Koçluk oturumlarındaki koçun rolü öğretim uygulamasına geçiş yapıldığında değişebilmektedir. Ders anlatımı yalnızca öğretmen tarafından, yalnızca koç tarafından veya her ikisi tarafından birlikte yapılabilmektedir (Gibbons & Cobb, 2017). Koç, tüm dersi öğretebilmekte, böylece öğretmen, öğretimin belirli yönlerine, öğrencilerin etkinliklerine veya sınıftaki diğer olaylara odaklanabilmektedir (Haneda vd., 2019).

Öte yandan, koç, derse müdahale etmeden gözlemci olarak kalmaya karar verebilmekte, aynı zamanda öğretmeni tamamlayıcı bir rol üstlenebilmektedir (Gibbons & Cobb, 2016; Nugent vd., 2023). Bu aşamada bile koçun, son-konferans aşamasında tartışılacak önemli konuları öne sürebilecek iyi bir konu geçmişine ve deneyime sahip olması gerekmektedir (Knight, 2019). Öğretmenlere bireysel olarak destek vermenin yanı sıra koçlar öğretmenlerin öğrencilerin matematiksel kavramsal bilgilerini derinleştirmelerine de yardımcı olmaktadırlar. Koçluk, öğretmenlerin işbirlikçi ders planlama, öğretim ve yansıtma süreçlerine dahil edilerek içerik bilgilerini ve pedagojik uygulamalarını geliştirmektedir (Hull vd., 2009). Koçun sorumlulukları arasında bilgi, beceri ve deneyim geliştirme, öğretmenler için

öğrenme toplulukları oluşturma ve öğretim uygulamalarını geliştirmek için sınav sonuçları gibi verileri analiz ederek yorumlama yer almaktadır (Price-Cole, 2018).

Bu çalışmada, içerik-odaklı bir koçluk stratejisi olarak matematik koçluğu uygulanmış ve koçluk oturumları, Gibbons ve Cobb (2017) tarafından tanımlanan odak koçunun planlama süreçlerinin beş temel özelliği üzerine yapılandırılmıştır. Bu unsurlar şunlardan oluşmaktadır: (1) uzun vadeli öğretim hedeflerinin belirlenmesi, (2) bilgi aktarma sürecinin değerlendirilmesi, (3) mevcut öğretim uygulamalarının gelişim yelpazesinde hangi aşamaya ait olduğunun sınıflandırılması, (4) ilgili öğretmenin gelişim planının belirlenmesi, (5) öğretmenin gelişim sürecine fayda sağlayabilecek etkinliklerin saptanması. Sonuç olarak, bu araştırmada, katılımcı öğretmen, beş boyut içerisinde gözlemlenmiş ve öğretim uygulamalarının nasıl analiz edildiği ile koçluk uygulamalarının öğretmenin ders planlamasına nasıl entegre edildiği incelenmiştir (Gibbons & Cobb, 2017).

1.3. Çalışmanın Amacı ve Araştırma Soruları

Bu çalışma, matematik koçluğu mesleki gelişim programına katılan bir ortaokul matematik öğretmeninin geometri alanındaki "Açılar ve Alan Ölçme" öğrenme alanlarında TPAB'ndeki değişiklikleri araştırmayı amaçlamıştır. Bu çalışmada, öğretmenin matematik koçluğuna katılmadan önce ve sonra TPAB düzeyini belirlemek ve katılım sürecindeki TPAB'ndeki değişimlerini izlemek amacıyla Niess, Ronau ve diğerleri (2009) tarafından geliştirilen Matematik Öğretmeni TPAB Standartları ve Gelişim Modeli kullanılmıştır. Çalışmanın bir diğer amacı da öğretmenin matematik koçluğu hakkındaki algısını TPAB gelişimi bağlamında incelemektir. Bu çalışma aşağıdaki araştırma sorularıyla yönlendirilmiştir:

1. Matematik koçluğuna katılmak, bir ortaokul matematik öğretmeninin ‘Açılar ve Alan Ölçme’ öğrenme alanlarındaki TPAB'ndeki gelişimini ne ölçüde etkiler?

1.a Matematik koçluğuna katılan bir ortaokul matematik öğretmeni, TPAB gelişim modeli kapsamında Müfredat ve Değerlendirme Teması, Öğrenme Teması, Öğretim Teması ve Erişim Teması'nda nasıl bir ilerleme kaydeder?

2. Bir ortaokul matematik öğretmeninin matematik koçluğunun TPAB gelişimi üzerindeki etkisine yönelik algısı nedir?

1.4. Çalışmanın Önemi

Teknolojinin K-12 eğitim sistemine entegrasyonu, öğretmenlerin güçlü TPAB bilgisine sahip olmalarını zorunlu kılmaktadır (ISTE, 2008). TPAB teknolojiyi öğretim sürecine entegre ederek öğrencilere olumlu öğrenme deneyimleri sunmak için kritik bir öneme sahiptir (Koehler vd., 2011). Matematik koçluğunun, öğretmenlerin TPAB gelişimine nasıl katkı sağlayabileceği bu çalışmanın odak noktasını oluşturmaktadır. Ayrıca, araçsallaştırma teorisi öğretmenlerin teknoloji entegrasyonu sürecindeki inançları ve algılarını analiz ederken faydalı bir perspektif sunmaktadır (Rabardel & Beguin, 2005). TPAB çerçevesi ile uyumlu olan bu teori, teknolojinin öğretme ve öğrenme süreçlerinde nasıl değişiklik yarattığını anlamaya yardımcı olmaktadır (Angeli & Valanides, 2009; Kushner-Benson vd., 2015). Bu bağlamda, matematik koçluğunun GeoGebra gibi teknolojik araçlarla öğretmenlerin TPAB gelişimine olan etkisi araştırılmaktadır.

Alanyazın matematik öğretmenlerinin TPAB gelişimine dair yeterli sayıda araştırma yapılmadığını göstermektedir (Patahuddin vd., 2016; Zhang ve Tang, 2021). Bu nedenle, bu çalışma, geometri bağlamında bir ortaokul matematik öğretmeninin TPAB gelişimini inceleyerek, bu alandaki boşluğu doldurmayı amaçlamaktadır (Harris & Hofer, 2009). Ayrıca, aday ve hizmet içi öğretmenlerin TPAB gelişiminde farklılıklar olduğu ve hizmet içi öğretmenlerin daha yüksek TPAB seviyelerine sahip olduğu belirtilmektedir (Yeh vd., 2014). Hizmet öncesi öğretmenlerin TPAB'ine odaklanan birçok çalışma olmasına karşın, hizmet içi öğretmenler üzerine araştırmaların sınırlı sayıda olduğu görülmüştür (Jen vd., 2016; Patahuddin vd., 2016). Bu çalışmada deneyimli bir matematik öğretmeninin TPAB gelişimini araştırmak eğitimde teknolojinin etkili kullanımını anlamaya katkı sağlamıştır.

Çoğu TPAB çalışması, akıllı tahtalar ve grafik hesap makineleri gibi eğitim amaçlı dijital araçlara odaklanmıştır ancak GeoGebra gibi uygulamalar üzerine yapılan nitel çalışmalar yeterli sayıda değildir (Voogt vd., 2013). Bu çalışma, GeoGebra'nın

matematik öğretiminde nasıl kullanıldığını inceleyerek geometri bağlamında TPAB üzerindeki etkisini araştırmayı amaçlamaktadır (Hohenwarter & Lavicza, 2007). Çalışmanın bulguları, konuya özgü eğitim teknolojilerinin öğretim süreçlerindeki etkinliğini anlamak için ileri araştırmalara zemin hazırlamıştır.

TPAB çerçevesi öğretmenlerin teknoloji entegrasyonuna dair bilgilerini açıklamada yaygın olarak kullanılan bir modeldir, ancak konuya özgü olmaması nedeniyle eleştirilmiştir (Angeli & Valanides, 2009; Graham, 2011). Niess, Ronau ve diğerleri (2009) tarafından geliştirilen TPAB Gelişim Modeli, konuya özgü tanımlayıcılarla bu sorunu çözmeyi amaçlamaktadır. Bu çalışma, TPAB Gelişim Modeli'ni kullanarak bir ortaokul matematik öğretmeninin geometri alanındaki TPAB gelişimini incelemiştir ve matematik öğretimi için TPAB çerçevesinin geliştirilmesine katkı sağlamayı hedeflemiştir.

Ayrıca, öğretmenlerin gerçek sınıf öğretimi sırasında TPAB gelişimini değerlendiren araştırmaların sınırlı olduğu, çoğu çalışmanın anketler ve mülakatlar üzerinden yapıldığı görülmektedir (Polly & Brantley-Dias, 2009; Patahuddin vd., 2016). Bu çalışma, sınıf gözlemlerini esas alarak öğretmenlerin TPAB gelişimini daha detaylı bir şekilde incelemeyi hedeflemiştir (Ertmer & Ottenbreit-Leftwich, 2010; Saubern vd., 2020).

COVID-19 pandemisi, öğretmenlerin hem eşzamanlı hem de eşzamansız öğrenme ortamlarında ders verme zorunluluğu nedeniyle TPAB'nin önemini daha da artırmıştır (Bozkurt & Sharma, 2020; Trust & Whalen, 2020). Bu çalışma, gelecekteki öğrenme ortamlarında etkili öğretimi sürdürebilmek için öğretmenlerin TPAB gelişimini desteklemeyi ve mesleki gelişim programlarını bilgilendirmeyi amaçlamaktadır (Chai vd., 2013).

Son olarak, hizmet içi öğretmenlerin genellikle düşük TPAB seviyelerine sahip olmaları, teknoloji entegrasyonunu ve uygun pedagojik yaklaşımlarla matematik öğretimini geliştirmek için odaklanmış mesleki gelişim ihtiyacını vurgulamaktadır (Spangenberg & De Freitas, 2019; Niess vd., 2010). Matematik koçluğu, bireyselleştirilmiş ve sürekli destek sağlayarak geleneksel mesleki gelişim

yöntemlerine göre üstünlükler sunmaktadır (Borko, 2004; McGatha, 2009). Bu nedenle, matematik koçluğu, öğretmenlerin TPAB gelişimini destekleyen etkili bir mesleki gelişim modeli olarak değerlendirilmektedir (Desimone & Garet, 2015).

Özetle, bu çalışma, matematik koçluğunun bir öğretmenin TPAB gelişimi üzerindeki etkilerini inceleyerek, etkili mesleki gelişim programlarının tasarlanmasında önemli bulgular sunmuştur (Desimone & Garet, 2015; Fishman & Davis, 2006).

2. Yöntem

2.1. Araştırmanın Deseni

Bu çalışmada bir ortaokul matematik öğretmeninin TPAB gelişimini, matematik koçluğuna katılmadan önce, katılımı sırasında ve sonrasında değerlendirmek ve izlemek için tek durumlu (bütüncül) nitel bir araştırma deseni kullanılmıştır. Çalışmanın odağı, geometri alanında "Açılar ve Alan Ölçme" öğrenme konularına dayanmaktadır. Bu tasarım, öğretmenin TPAB ilerlemesinin zaman içindeki ayrıntılı ve derinlemesine bir incelemesini imkan tanımıştır. Analiz birimi öğretmenin TPAB düzeyidir ve çalışmanın sınırları, öğretmenin uzun yıllara dayanan deneyimi (toplamda 10 yıldan fazla) ve 6. sınıf düzeyindeki öğretimle belirlenmiştir. Bu çalışmada öğretmenin matematik koçluğunun TPAB gelişimi üzerindeki etkisine yönelik algısı da incelenmiştir.

2.2. Çalışmanın Bağlamı

Bu çalışma Türkiye'de özel bir ortaokulda gerçekleştirilmiş olup, ortaokul matematik öğretmeninin öğretim uygulamaları ve MEB (2018) tarafından yayımlanan ortaokul matematik müfredatındaki konularını kapsamıştır. Türkiye Milli Eğitim Bakanlığı (MEB), tüm eğitim hizmetlerini planlama, programlama, ilerletme, izleme ve denetleme dahil olmak üzere yönetmektedir. Türkiye'deki zorunlu eğitim, 12 yıl sürmektedir ve eğitim sistemi okul öncesi, ilkokul, ortaokul, lise ve yükseköğretim düzeylerinden oluşmaktadır. 5-8. sınıf düzeyleri için geçerli olan ilköğretim matematik müfredatında beş ana öğrenme alanı yer almaktadır: veri işleme ve

olasılık, geometri ve ölçme, cebir ve sayılarla işlemler. Müfredatın temel amacı, öğrencilerin matematik okuryazarlığını geliştirmek, problem çözme becerilerini artırmak ve matematik öğrenimine yönelik olumlu tutumlar kazandırmaktır (MEB, 2018). Müfredat, öğrenci merkezli olup matematiğin pratik uygulamalarını, problem çözme becerilerini ve matematik öğrenimini eğlenceli hale getirmeyi vurgulamaktadır. Çalışmanın yapıldığı okulda teknoloji entegrasyonuna güçlü bir vurgu yapılmakta ve öğretmenler için düzenli hizmet içi eğitimler, Eğitim Teknolojileri Müdürlüğü tarafından sağlanmaktadır.

2.3. Katılımcı Seçimi

Çalışmanın katılımcı seçimi iki aşamada gerçekleştirilmiştir. İlk aşamada, araştırmanın yapılacağı özel ortaokul kolayda örnekleme yöntemi ile seçilmiştir. Okul yönetiminin ve matematik öğretmenlerinin, mesleki gelişim programlarını ve bilimsel araştırmaları destekleyici bir tutuma sahip olduğu görülmüştür. İkinci aşamada, araştırmacı amaçlı örnekleme yöntemi kullanarak okuldan bir öğretmen seçmiştir. Tüm matematik öğretmenlerine çalışmanın amacı anlatılmış ve TPAB anketi (Horzum & Öztürk, 2011), Schmidt vd. (2009) tarafından geliştirilen anketin Türkçe versiyonu uygulanmıştır. Öz-algı bakımından TPAB düzeyi en düşük olan öğretmen, çalışmaya gönüllü olarak katılmaya davet edilmiştir. Tecrübeli öğretmenlerin teknoloji entegrasyonuna daha dirençli olabileceği düşüncesiyle (Hermans vd., 2008; Vannatta & Nancy, 2004), bu seçim yapılmıştır. ‘Cem’ rumuzuyla anılan katılımcı, 9 yıllık ortaokul matematik öğretmenliği deneyimine sahiptir. Cem, Fen-Edebiyat Fakültesi Matematik Bölümü'nden mezun olmuş ve pedagojik formasyon sertifikasını üniversiteden almıştır. Lisans eğitimi sırasında pedagojik bir eğitim almamış olmasına rağmen, teknoloji entegrasyonunun önemini kabul eden bir öğretmen olduğu belirlenmiştir. Teknolojiyi öğretimde kullanma konusunda bilgisi sınırlı olmasına rağmen, bu alanda daha fazla bilgi edinme konusunda hevesli olduğu görülmüştür. Bu çalışmaya katılımı öncesinde Cem sınıfta teknoloji kullanımını genellikle video gösterimleri ve çevrimiçi öğretim platformlarıyla sınırlı tutmuştur. Öğretim tarzı ağırlıklı olarak öğretmen merkezli olmakla birlikte, öğrencileri yaparak öğrenme ve soru-cevap yöntemleriyle derse dahil etmeye çalışmıştır.

2.4. Çalışmada Kullanılan Mesleki Gelişim Yaklaşımı: Matematik Koçluğu

Bu çalışmada matematik koçluğu Cem'in teknoloji bilgisi ve derse entegrasyonunu geliştirmek için bir mesleki gelişim stratejisi olarak kullanılmıştır. Koçluk, GeoGebra'nın öğretim ve öğrenme aracı olarak nasıl etkin bir şekilde kullanılabileceği, teknoloji entegrasyonunun öğretim uygulamalarına, değerlendirme yöntemlerine ve müfredatın aktarımına nasıl entegre edilebileceğine odaklanmıştır. Koçluk seanslarından önce, Cem'in TPAB anketi sonuçlarına ve gözlemlerine dayanarak özelleştirilmiş bir TPAB eğitim programı tasarlanmıştır. Bu eğitim, Cem'in TPAB bileşenlerindeki eksiklikleri hedef almış ve tüm bilgi alanlarının geliştirilmesini amaçlamıştır. Çalışmadaki matematik koçluğı döngüsü MKD kısaltmasıyla ile ifade edilmiş ve MKD 1'den MKD 4'e kadar numaralandırılmıştır.

Çalışma boyunca dört koçluk döngüsüne katılan Cem, koçluk döngülerine katılımı dışında çalışmanın başlaması itibariyle iki hafta boyunca haftada beş saat olmak üzere toplam on saat süren eğitimlere katılmıştır. Bu TPAB eğitiminde Cem'e teknoloji entegrasyon modelleri (örneğin, Teknoloji Kabul Modeli, Araçsallaştırma Teorisi), TPAB bileşenleri, öğretim tasarımı, öğretim stratejileri ve teknoloji tabanlı değerlendirme araçları hakkında bilgi verilmiştir. Bu eğitimin büyük bir kısmı, Cem'e GeoGebra'yı nasıl kullanacağı, etkinlik oluşturma, ödev verme ve öğrencilerin ilerlemesini değerlendirme konularında rehberlik edilmesine ayrılmıştır. Ayrıca, geometri alanındaki yaygın kavram yanılgıları, özellikle "Açılar ve Alan Ölçme" ile ilgili olarak ele alınmış ve içerik sunumunu iyileştirme stratejileri sunulmuştur.

Matematik koçluğu süreci, ön-konferans, gözlem ve son-konferans aşamalarından oluşan yapılandırılmış döngüsel bir yaklaşımla yürütülmüştür. Her koçluk döngüsü, Cem'in TPAB'sindeki belirli eksiklikleri belirleyip bunlara yönelik eğitimler sağlamıştır. Örneğin, ders planlama aşamasında Pedagojik Bilgi (PB) eksiklikleri gözlemlendiğinde, Cem'in uygun öğretim stratejileri seçme bilgisini geliştirmek amacıyla ek eğitimler verilmiştir. Koçluk döngüleri boyunca, her TPAB bileşenine yönelik eğitimlerin yoğunluğu Cem'in ilerlemesine göre ayarlanmış ve tüm TPAB bileşenlerinin bütüncül olarak geliştirilmesi sağlanmıştır. Ayrıca, uygulama üzerine

düşünme ve çeşitli teknoloji destekli örnek etkinliklerin gösterimi, koçluk seanslarının önemli bileşenleri arasında yer almıştır.

2.4.1. Ön-konferans

Her matematik koçluk döngüsünün ilk aşaması olan ön-konferanslar, yaklaşık doksanar dakika sürmüştür ve bu aşamada Cem'in ders planlama süreci üzerine yoğunlaşmıştır. Bu oturumlar, koçun Cem'in TPAB düzeyini ve öğretim planlama eğilimlerini anlamasına yardımcı olmuştur. Her ön-konferans öncesinde Cem belirli bir kazanıma yönelik GeoGebra ile zenginleştirilmiş ders planı hazırlamıştır ve bu planları koçla paylaşmıştır. Planlar ön-konferans oturumlarında matematiksel tartışmalarla gözden geçirilip geliştirilmiştir. Bu süreçte önceki dersin son-konferansında ele alınan eksiklikler, sonraki dersin planlama sürecinde dikkate alınmıştır.

Koç, Cem'in TPAB bileşenlerindeki (Teknoloji, Pedagoji, Alan) eksiklikleri ve bu bileşenlerin kesişim noktalarını (TPB, TAB, PAB, TPAB) belirleyip bu alanlara yönelik destek sağlamıştır. Örneğin, MKD 1 ön-konferansında verilen bir açıya eş bir açı oluşturma konusu ele alınmıştır ve GeoGebra kullanılarak bu kazanımın nasıl etkili bir şekilde öğretileceği tartışılmıştır. MKD 2 ön-konferansında komşu, tümler ve bütünler açıların özelliklerine odaklanılmıştır ve GeoGebra etkinlikleri kullanılarak öğrencilerin bu kavramları daha somut bir şekilde anlamaları üzerine çalışılmıştır. MKD 3 ön-konferansında üçgende yükseklik kavramı ve üçgenin alan bağıntısını oluşturma konuları ele alınmıştur ve GeoGebra kullanılarak üçgenlerde yüksekliklerin nasıl çizileceği ve alan hesaplamasının nasıl etkili bir şekilde öğretilebileceği tartışılmıştır. MKD 4 ön-konferansında ise paralelkenarın yüksekliği ve alanı üzerinde çalışılmıştır ve GeoGebra ile bu kavramların nasıl etkili bir şekilde öğretilebileceği tartışılmıştır. Bu ön-konferanslar sırasında koç ile Cem ders planlamasının yanı sıra teknoloji entegrasyonu ve öğrenci merkezli öğrenme stratejileri üzerine de yoğunlaşmışlardır. Ayrıca, koç, derslerde karşılaşılabilecek yaygın kavram yanılgıları ve bu yanılgıların nasıl giderilebileceğine dair alanyazından stratejiler paylaşmıştır. Ders planları bu tartışmalar ışığında yeniden yapılandırılmış, revize edilmiş ve öğretim uygulaması için bir temel oluşturmuştur.

2.4.2. Gözlem

Gözlem aşaması, Cem'in hazırladığı GeoGebra destekli derslerin uygulanması sırasında gerçekleştirilmiştir ve her koçluk döngüsünde iki ila dört saat sürmüştür. Koç, Cem'in öğretim sürecini gözlemlemiştir, öğrencilerin ve Cem'in GeoGebra'yı nasıl kullandığını ve sınıf dinamiklerini not etmiştir. Gözlemler, Cem'in TPAB düzeyini ve TPAB Gelişim Modeli'ndeki ilerlemesini değerlendirmek için önemli veriler sağlamıştır. Öğrencilerin GeoGebra ile yaptığı çalışmalar, sınıf içi tartışmalar ve öğrenci geri bildirimleri ders esnasında kayıt altına alınmıştır. Bu gözlemler, sonraki son-konferanslarda dersin nasıl geliştirilebileceği konusunda önemli ipuçları sunmuştur ve teknoloji entegrasyonunun öğretim sürecine etkilerini değerlendirmek için kullanılmıştır.

2.4.3. Son-konferans

Son-konferans aşaması, ders gözlemlerinden elde edilen bulguların derinlemesine tartışıldığı ve Cem'in ders performansının değerlendirildiği kritik bir aşama olmuştur. Koç, gözlem sırasında aldığı notları Cem ile paylaşarak yapıcı geri bildirimler sunmuş ve Cem'in kendi öğretim pratiği üzerinde düşünmesini sağlamıştır. Bu oturumlar, sonraki derslerin daha etkili bir şekilde planlanması ve TPAB'ın geliştirilmesi için stratejiler oluşturmak amacıyla kullanılmıştır. Örneğin, bazı derslerde Cem'e daha öğrenci merkezli bir yaklaşım benimsemesi önerilmiştir ve öğrencilere bağımsız keşif yapmaları için daha fazla zaman tanıması gerektiği belirtilmiştir. Ayrıca, GeoGebra'nın derslerde daha etkin kullanılması için yeni stratejiler geliştirilmiştir. Sonuç olarak, bu geri bildirimler, Cem'in ders planlarını yeniden gözden geçirmesine ve öğrencilerin aktif katılımını artırmasına yönelik somut adımlar atmasını sağlamıştır. Son-konferanslar, öğretim hedeflerinin gerçekleşme düzeyini değerlendirmek, öğrencilerin katılımını artırmak ve ders planlarını teknoloji entegrasyonuna uygun şekilde düzenlemek amacıyla kullanılmıştır.

2.5. Veri Toplama Araçları

Bu çalışmanın araştırma sorularını yanıtlayabilmek için çeşitli veri toplama araçları, matematik koçluğu öncesinde, süresince ve sonrasında olmak üzere üç farklı zaman

aralığında kullanılmıştır. Bu yöntemler arasında yarı-yapılandırılmış görüşmeler, TPAB gözlem formu ile yapılan gözlemler ve saha notları yer almıştır.

Öğretmenin çalışma öncesindeki TPAB düzeyini belirlemek amacıyla TPAB ön-görüşmesi ve öğretim uygulaması gözlemi (TPAB ön-gözlem) gerçekleştirilmiştir. Koçluk döngüleri boyunca öğretmenin TPAB'ndeki değişim ve gelişimi ön-konferanslar, öğretim uygulaması gözlemleri ve son-konferanslar yoluyla incelenmiştir. Matematik koçluğu sonrasındaki TPAB düzeyini belirlemek için ise TPAB son-görüşmesi ve öğretim uygulaması gözlemi (TPAB son-gözlem) yapılmıştır. Ayrıca, öğretmenin TPAB gelişimi konusundaki matematik koçluğuna yönelik algısını incelemek amacıyla yarı-yapılandırılmış bir matematik koçluğu görüşmesi yapılmıştır.

Her gözlem sırasında alınan saha notları, gözlemsel verileri desteklemek amacıyla kullanılmıştır. Veri toplama süreci Nisan 2023 ile Mayıs 2023 arasında tamamlanmış olup, öğretmenin toplam 17 ders saatini kapsayan öğretim uygulamaları ve yapılan tüm yarı-yapılandırılmış görüşmeler, katılımcının onayı ile görüntü ve ses kaydına alınmıştır. 6. Sınıf matematik dersi kapsamında yer alan 'Açılar ve Alan Ölçme' öğrenme alanlarına ilişkin kazanımlar ve ders gözlem süreleri ve Tablo 1'de özetlenmiştir (MEB, 2018).

Tablo 1. Açılar ve Alan Ölçme öğrenme alanlarına ilişkin gözlem saatleri ve kazanımlar

Süreç	Kazanımlar	Süre (Ders saati)
Ön-gözlem	Açıyı, başlangıç noktaları aynı olan iki ışının oluşturduğunu bilir ve sembolle gösterir.	2
MKD 1	Bir açıya eş bir açı çizer.	2
MKD 2	Komşu, tümler, bütünler ve ters açıların özelliklerini keşfeder; ilgili problemleri çözer.	4
MKD 3	Üçgenin alan bağıntısını oluşturur, ilgili problemleri çözer.	3
MKD 4	Paralelkenarın alan bağıntısını oluşturur, ilgili problemleri çözer.	3
Son-gözlem	Alan ile ilgili problemleri çözer.	3

(MKD: Matematik Koçluğu Döngüsü)

2.6. Pilot Çalışma

Bu çalışmada, aynı özel ortaokulda 21 yıllık öğretmenlik deneyimine sahip tecrübeli bir matematik öğretmeni olan Melis ile bir pilot çalışma gerçekleştirilmiştir. Melis, GeoGebra ve Desmos kullanarak 8. sınıf öğrencilerine eğim kavramını öğretmiştir. Pilot çalışma, ana çalışmada kullanılan prosedürler ve veri toplama araçları ile aynı yöntemler kullanılarak gerçekleştirilmiştir. Ana çalışmadan farkı ise pilot çalışmada yalnızca bir koçluk döngüsü uygulanmış olması, farklı sınıf düzeyi ve öğrenme alanının ele alınmış olmasıdır. Bu pilot çalışma, veri toplama araçlarının, prosedürlerin ve koçluk döngülerinin ana çalışmaya uygunluğunu değerlendirme açısından önemli bir adım olmuştur. Pilot çalışmanın ardından, görüşme soruları, protokoller ve içeriklerde iyileştirmeler yapılmış, TPAB gelişimine katkı sağlamayan kısımlar çalışmadan çıkarılmıştır. Bu süreç, zamanlama ve veri kaynaklarının belirlenmesine yönelik iyileştirmeler sağlamış, ana çalışmanın genel kalitesini ve güvenilirliğini artırmada önemli bir rol oynamıştır.

2.7. Veri Analizi

Nitel araştırmalarda veri toplama ve analiz süreçleri, yansıtıcı ve süreklilik gösteren etkinlikler olarak eş zamanlı bir şekilde yürütülmektedir (Merriam, 1998). Bu çalışmada, veri toplama ve analiz süreci de nitel araştırma yaklaşımına uygun olarak eş zamanlı gerçekleştirilmiştir. Öğretmenin TPAB'ndeki değişim ve gelişimlere odaklanan çalışmanın birinci araştırma sorusunu yanıtlamak amacıyla, koçluk döngülerinde gerçekleşen ön-konferanslar ve son-konferanslar, mülakatların transkripsiyonları, video kayıtları, sınıf gözlemleri ve bu gözlemlere ilişkin alan notları detaylı bir şekilde analiz edilmiştir. Öğretmen Cem'in TPAB seviyesindeki gelişim, tümdengelim analiz yöntemi kullanılarak TPAB Gelişim Modeli (Niess, Ronau vd., 2009) temel alınarak belirlenmiştir. Tümdengelim analizi, verilerin mevcut bir yapıya göre incelendiği durumlarda uygun bir yöntem olup, analizde kullanılan temalar ve kodlar önceden alanyazın, kuramlar ve araştırma sorularına dayanarak seçilmiştir (Patton, 2002). Bu doğrultuda, öğretmenin TPAB gelişimini incelemek için ilgili alıntılar belirlenmiş, TPAB seviyeleri (*tanıma, kabul etme, uyarlama, keşfetme, gelişme*) ve ana temalar (müfredat ve değerlendirme, öğrenme,

öğretim, erişim) sistematik olarak analiz edilmiştir. TPAB Gelişim Modeli, her bir TPAB seviyesi için 11 betimleyici ifade ve bu seviyeleri açıklayıcı örnekler içermektedir (Ek K'ye bakınız). Araştırma sürecinde, Cem'in matematik koçluğu öncesindeki, sırasındaki ve sonrasındaki TPAB seviyeleri; ön gözlem, ön-konferanslar, sınıf gözlemleri, son-konferanslar ve son görüşmelerin analiz edilmesiyle tespit edilmiştir. Bu süreçte, öğretmenin sergilediği davranışlar arasında birden fazla TPAB seviyesine karşılık gelen durumlar gözlemlendiğinde, öğretmenin sergilediği en yüksek seviye davranış temel alınarak TPAB seviyesi belirlenmiştir. Bu sayede, öğretmenin TPAB gelişim süreci tutarlı bir şekilde izlenmiştir.

Öğretmenin TPAB gelişimi açısından matematik koçluğuna yönelik algısına odaklanan çalışmanın ikinci araştırma sorusunu yanıtlamak için, yarı yapılandırılmış görüşmelerin transkripsiyonları ve video kayıtları tümevarımsal analiz yöntemiyle incelenmiştir. Açık kodlama süreci izlenerek görüşmelerde elde edilen veriler düzenlenmiş, kategoriler ve temalar oluşturulmuş, örüntüler belirlenmiş ve veriler kodlanmıştır. Marshall ve Rossman (2011) tarafından önerildiği şekilde, elde edilen bulgular test edilmiş, alternatif açıklamalar araştırılmış ve raporlaştırma süreci gerçekleştirilmiştir. Bu analiz sonucunda, matematik koçluğunun faydaları, diğer mesleki gelişim programlarından ayrıldığı yönler, karşılaşılan zorluklar ve öneriler olmak üzere dört ana tema ortaya çıkarılmıştır. Bu kapsamlı ve sistematik yaklaşım, araştırmacının Cem'in TPAB seviyelerindeki gelişimi izlemesine ve matematik koçluğunun geometri öğretimine katkılarını anlamasına olanak sağlamıştır.

2.8. Araştırmacının Rolü

Matematik koçları, öğretmenlere hedefli destek sağlayarak öğretim uygulamalarını ve öğrenci sonuçlarını iyileştirmede önemli bir rol oynamaktadır (Bengo, 2013). Etkili koçların, matematik içeriği konusunda uzmanlık (Knight, 2007; McGatha vd., 2018), pedagojik beceriler (Campbell & Malkus, 2011; Gibbons & Cobb, 2017), teknolojik yetkinlik (McGatha, 2009) ve güçlü koçluk ve mentorluk yetenekleri (Costa & Garmston, 2002; Knight, 2011; Mangin & Dunsmore, 2015) gibi temel niteliklere sahip olması beklenmektedir. Bu çalışmada, araştırmacı bu nitelikleri, matematik eğitimi alanındaki geçmişiyle ortaya koymuştur. Araştırmacı, ilköğretim

eğitimi alanında lisans derecesine ve teknoloji entegrasyonu alanında yüksek lisans derecesine sahiptir. 11 yıllık öğretmenlik deneyimi ve 13 yıllık GeoGebra gibi eğitim teknolojilerini kullanma geçmişi ile öğretmen Cem'e matematik öğretiminde teknolojiyi entegre etme konusunda etkili rehberlik ve destek sağlamıştır. Ayrıca, araştırmacı koçluk ve mentorluk sertifikası almış, süreç boyunca da güçlü koçluk yetenekleri sergilemiştir (Costa & Garmston, 2002; Knight, 2011). İşbirliğine dayalı ve destekleyici bir ortam oluşturarak, Cem'i yansıtıcı pratiğe teşvik etmiş ve öğretim stratejilerini geliştirmek için kişisel geri bildirimlerde bulunmuştur. Ayrıca, yanlılığı en aza indirmek ve geçerliliği sağlamak amacıyla araştırmacı, çalışma başlamadan önce Cem'in öğretim bağlamını gözlemleyerek hazırlık yapmıştır (Campbell & Malkus, 2011; McGatha vd., 2018). Genel olarak, araştırmacının nitelikleri ve yaklaşımı, etkili bir matematik koçunun özellikleriyle yakından örtüşmekte ve Cem'in öğretim uygulamaları ile profesyonel gelişiminin iyileştirilmesine katkı sağlamaktadır.

3. Bulgular

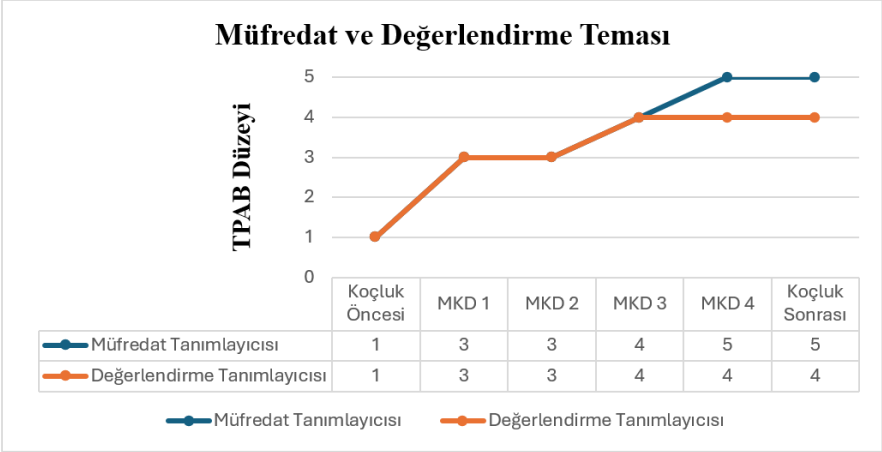
Bu durum çalışmasının temel amacı, geometri odaklı matematik koçluğuna katılan bir ortaokul matematik öğretmeninin TPAB'sındaki değişiklikleri incelemek olmuştur. Toplanan veriler, Niess, Ronau ve diğerleri (2009) tarafından önerilen TPAB Gelişim Modeli kullanılarak analiz edilmiştir. Öğretmenin TPAB'sindeki değişim ve gelişim, matematik müfredatının 6. sınıf seviyesinde öğretilen geometriye ait iki öğrenme alanında incelenmiştir: 'Açılar' ve 'Alan Ölçme'. Matematik Koçluğu Döngüleri (MKD) 1 ve MKD 2, 'Açılar' öğrenme alanında gerçekleştirilirken, MKD 3 ve 4, 'Alan Ölçme' öğrenme alanında gerçekleştirilmiştir.

3.1. Cem'in TPAB Gelişim Modeli'nin Tema ve Tanımlayıcılardaki Genel Gelişimi

3.1.1. Cem'in Müfredat ve Değerlendirme Temasındaki Gelişimi

Aşağıdaki 2'den 5'e kadar olan şekiller, Cem'in matematik koçluğu öncesinden sonrasına kadar TPAB gelişimini, TPAB gelişim modelinin temaları ve ilgili

tanımlayıcılarına göre düzenlenmiş şekilde göstermektedir. TPAB Gelişim Modeli'ndeki seviyeler, en düşük (seviye 1) ile en yüksek (seviye 5) arasında sıralanmıştır: *Tanıma* (seviye 1), *Kabul etme* (seviye 2), *Uyarlama* (seviye 3), *Keşfetme* (seviye 4) ve *Gelişme* (seviye 5).



Şekil 2. Cem'in Müfredat ve Değerlendirme Temasındaki Gelişimi

Cem'in Müfredat ve Değerlendirme Teması kapsamındaki öğretim uygulamaları, matematik koçluğu döngüleri boyunca önemli bir gelişim göstermiştir. Koçluk öncesinde Cem, hem *müfredat* hem de *değerlendirme tanımlayıcılarında tanıma* seviyesinde (seviye 1) yer almıştır. Teknoloji kullanımı oldukça yetersiz görülmüştür ve bu kullanımı büyük ölçüde temel sunum araçları ile sınırlı kalmıştır. Değerlendirme konusunda ise, teknolojiyi entegre etmeden geleneksel yöntemlere (yazılı sınavlar ve gözlem gibi) başvurmuştur. Koçluk sürecine katıldıkça Cem'in öğretim uygulamaları gelişmeye başlamıştır. MKD 1 ve MKD 2'de *uyarlama* seviyesine (seviye 3) ilerlemiştir. Bu aşamada, Cem teknolojiyi derslerine daha aktif bir şekilde entegre etmeye başlamıştır ve GeoGebra gibi araçları kullanarak öğrencilerin geometri kavramlarını, özellikle açıları, daha iyi anlamalarını sağlamıştır. Öğretim uygulamaları daha yapılandırılmış hale gelmiştir ve teknoloji öğrencilerin öğrenmesini desteklemek için kullanılmıştır. Bu sayede öğrencilere matematiksel fikirleri keşfetme fırsatları sunulmuştur. MKD 3'te Cem, *müfredat tanımlayıcısında keşfetme* seviyesine (seviye 4) ilerlemiştir. Bu döngüde, öğrencilerin teknoloji ile daha derinlemesine etkileşimde bulunmalarını teşvik eden dersler tasarlamaya başlamıştır. GeoGebra gibi araçları kullanarak, öğrencilerin alan

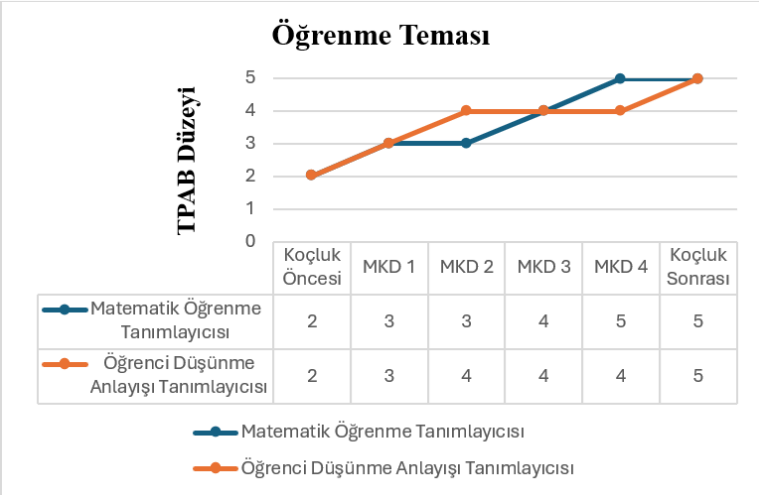
ölçümü ve geometrik şekiller (üçgenler ve paralelkenarlar gibi) ile ilgili problemleri görselleştirmelerine ve teknoloji desteğiyle çözmelerine yardımcı olmuştur. Öğretim yöntemleri daha etkileşimli ve sorgulamaya dayalı hale gelmiştir ve teknoloji ile desteklenen öğrenme deneyimlerine odaklanılmıştır. MKD 4'te ve koçluk sürecinin sonunda Cem *müfredat tanımlayıcısında gelişme* seviyesine (seviye 5) ilerlemiştir. Cem çalışma sonunda teknolojiyi derslerine tam anlamıyla entegre etmiş ve teknolojiyi derslerin merkezine konumlandırmıştır. Örneğin, GeoGebra'yı sadece geometrik kavramları göstermek için değil, aynı zamanda öğrencilerin şekilleri manipüle etmeleri ve karmaşık özellikleri dijital araçlarla anlamaları için kullanmıştır. Öğretim stratejileri daha dinamik hale gelmiş ve teknoloji öğrencilerin kavrama ve problem çözme becerilerini geliştirmede önemli bir rol oynamaya başlamıştır. *Değerlendirme tanımlayıcısında* da Cem sürekli bir ilerleme göstermiştir. Başlangıçta *tanıma* seviyesinde (seviye 1) iken, MKD 1 ile *uyarlama* seviyesine (seviye 3) ilerlemiştir ve bu aşamada geleneksel yöntemlerin yanında teknoloji tabanlı değerlendirmeler kullanmaya başlamıştır. MKD 3'te ise *keşfetme* seviyesine (seviye 4) ilerlemiştir ve dijital araçları entegre eden daha etkileşimli değerlendirmeler geliştirmiştir. Bu değerlendirmeler, öğrenci öğrenmesini ve anlayışını daha kapsamlı bir şekilde değerlendirmeye olanak tanımıştır. Ancak, öğrencilerin öğrenmelerini değerlendirmek için tam teknoloji kullanımı gerektiren teknoloji-tabanlı değerlendirme uygulamalarını öğretime entegre etmediği için Cem *değerlendirme tanımlayıcısında keşfetme* seviyesinde (seviye 4) çalışmayı tamamlamıştır

Sonuç olarak, Cem'in öğretim uygulamaları Müfredat ve Değerlendirme Teması kapsamında önemli bir ilerleme göstermiştir. Koçluk süreci boyunca teknoloji kullanımında sınırlı bir yaklaşımdan, öğrenci öğrenimini ve katılımını artıran, daha gelişmiş ve tamamen entegre edilmiş bir yaklaşıma geçiş yapmıştır.

3.1.2. Cem'in Öğrenme Temasındaki Gelişimi

Cem'in Öğrenme Teması kapsamındaki öğretim uygulamaları, matematik koçluğu döngüleri boyunca belirgin bir gelişim göstermiştir. Koçluk öncesinde Cem, hem *matematik öğrenme* hem de *öğrenci düşünme anlayışı tanımlayıcılarında kabul etme*

seviyesinde (seviye 2) yer almıştır. Bu aşamada, teknoloji kullanımını sınırlı tutuyor ve geleneksel öğretim yöntemlerine ağırlık vermiştir. Teknolojiyi, öğrencilerin kavramsal düşünme ve *keşfetme* süreçlerinde aktif bir araç olarak kullanmamıştır. Koçluk sürecine katıldıkça Cem'in öğretim uygulamaları gelişmeye başlamıştır. MKD 1'de Cem, *uyarlama* seviyesine (seviye 3) ilerlemiştir. Bu döngüde, derslerinde teknoloji kullanımını daha sistemli bir şekilde entegre etmeye başlamıştır ve GeoGebra gibi dijital araçları, öğrencilerin matematiksel kavramları keşfetmelerine ve daha derin bir anlayış geliştirmelerine olanak tanıyan bir araç olarak kullanmaya başlamıştır. Öğrencilerin öğrenme süreçlerinde daha aktif rol almasını sağlayan bu değişiklik, Cem'in öğretim yöntemlerini de daha öğrenci merkezli hale getirmiştir.



Şekil 3. Cem'in Öğrenme Temasındaki Gelişimi

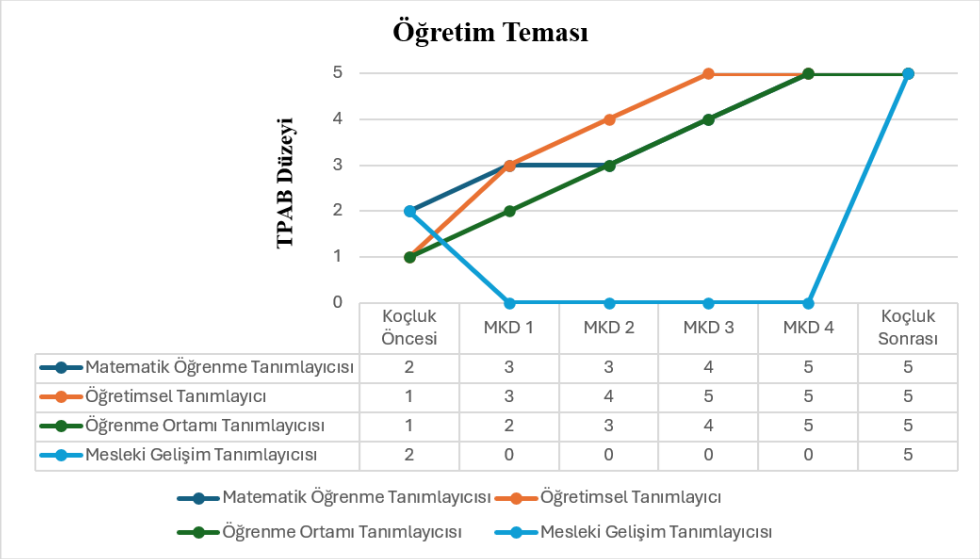
MKD 3'te Cem, *matematik öğrenme tanımlayıcısında keşfetme* seviyesine (seviye 4) ilerlemiştir. Bu noktada, öğrencilerin matematiksel kavramları teknoloji aracılığıyla keşfetmelerini teşvik eden dersler tasarlamıştır. Öğrencilere daha karmaşık problemlerin çözümü için fırsatlar sunan bir öğretim yaklaşımı benimseyerek teknoloji ile desteklenen derslerde öğrenci katılımını artırmıştır. Koçluk sürecinin sonunda, MKD 4'te Cem, *matematik öğrenme tanımlayıcısında gelişme* seviyesine (seviye 5) ilerlemiştir. Bu döngüde, teknolojiyi derslerinde tam anlamıyla entegre etmiş ve öğrencilerin matematiksel düşünme süreçlerini güçlendiren bir öğretim yaklaşımı benimsemiştir. Öğrencilerin bireysel öğrenme süreçlerini destekleyen ve

onların problem çözme becerilerini geliştiren etkinlikler düzenleyerek dersleri daha etkileşimli hale getirmiştir.

Öğrenci düşünme anlayışı tanımlayıcısında da Cem benzer bir gelişim göstermiştir. Koçluk öncesinde *kabul etme* seviyesinde (seviye 2) olan Cem, MKD 1 ile *uyarlama* seviyesine (seviye 3) ilerlemiştir ve teknoloji kullanımının öğrencilerin kavramsal düşünme süreçlerine nasıl katkı sağlayabileceğini fark etmeye başlamıştır. MKD 2'de *keşfetme* seviyesine (seviye 4) ilerlemiştir ve bu seviyede öğrencilere daha derinlemesine düşünme ve keşfetme fırsatları sunan dersler tasarlamıştır. MKD 3 ve MKD 4'te bu seviyede devam eden Cem, teknoloji aracılığıyla öğrencilerin düşünme süreçlerini zenginleştiren etkinlikler düzenlemiştir. Koçluk sürecinin sonunda, bu tanımlayıcıda *gelişme* seviyesine (seviye 5) ilerlemiştir.

Sonuç olarak, Cem'in öğretim uygulamaları Öğrenme Teması kapsamında önemli bir gelişim göstermiştir. Başlangıçta teknoloji kullanımını sınırlı tutarken, koçluk süreci boyunca öğrenci merkezli, etkileşimli ve teknoloji destekli bir öğretim anlayışını benimsemiş ve bu sayede öğrencilerin matematik öğrenme süreçlerini derinlemesine destekleyen bir yaklaşım geliştirmiştir.

3.1.3. Cem'in Öğretim Temasındaki Gelişimi



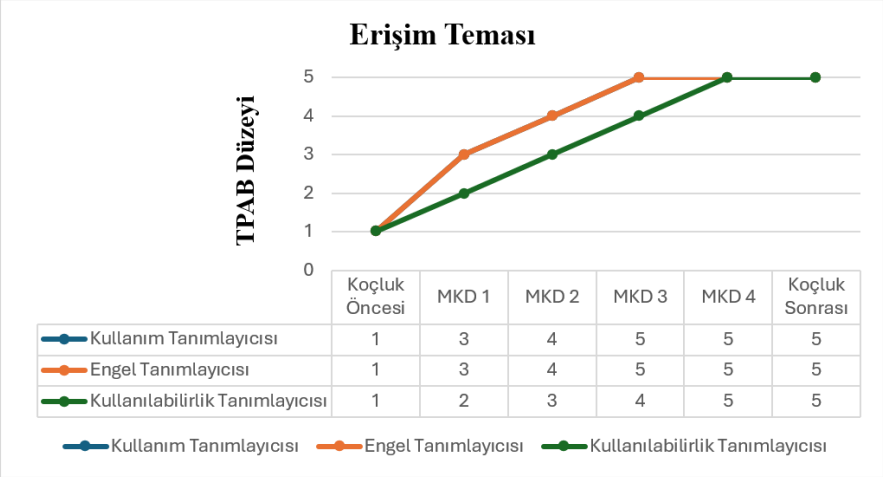
Şekil 4. Cem'in Öğretim Temasındaki Gelişimi

Cem'in Öğretim Teması kapsamındaki öğretim uygulamaları, matematik koçluğu döngüleri boyunca önemli ölçüde gelişim göstermiştir. Koçluk öncesinde Cem, *matematik öğrenme, öğretimsel* ve *öğrenme ortamı tanımlayıcılarında tanıma* ve *kabul etme* seviyelerinde yer almıştır. Teknolojiyi sınırlı bir şekilde kullanmıştır ve derslerini daha çok geleneksel yöntemlerle yürütmüştür. Teknoloji, öğretimde merkezi bir rol oynamamıştır ve öğrencilere yönelik uygulamalarda aktif olarak yer almamıştır.

Koçluk sürecine katıldıkça Cem'in öğretim uygulamaları giderek gelişmiştir. MKD 1'de Cem, *uyarlama* seviyesine (seviye 3) ilerlemiştir ve teknoloji kullanımını derslerinde daha aktif hale getirmiştir. Öğrencilere teknolojiyi kullanarak daha anlamlı ve kavramsal öğrenme fırsatları sunmaya başlamıştır. Örneğin, GeoGebra gibi dijital araçlarla derslerini zenginleştirerek, öğrencilerin kavramları görselleştirmelerine ve anlamalarını sağlamaya yönelik adımlar atmıştır. MKD 2'de, Cem, *öğretimsel* ve *öğrenme ortamı tanımlayıcılarında keşfetme* seviyesine (seviye 4) ilerlemiştir. Bu aşamada, öğrencileri daha fazla araştırmaya ve keşfetmeye teşvik eden etkinlikler planlamıştır. Derslerde teknoloji, öğrenci katılımını artırmak için entegre bir araç haline gelmiştir ve öğrenciler derslerde daha aktif roller almaya başlamıştır. Bu değişiklik, derslerin daha etkileşimli ve öğrenci merkezli bir yaklaşıma kaymasını sağlamıştır. Koçluk sürecinin ilerleyen aşamalarında, MKD 3'te Cem, *öğretimsel, öğrenme ortamı* ve *matematik öğrenme tanımlayıcılarında gelişme* seviyesine (seviye 5) ilerlemiştir. Bu noktada, teknoloji tam anlamıyla derse entegre edilerek, öğrencilerin daha yüksek düzeyde düşünme ve problem çözme becerilerini geliştirmelerine olanak tanıyan dersler tasarlamaya başlamıştır. Öğretim uygulamaları daha yapılandırılmış ve sistematik hale gelmiştir. Cem her aşaması dikkatle planlanmış ve teknolojiyle desteklenmiş dersler oluşturmaya başlamıştır. *Mesleki gelişim tanımlayıcısında* ise Cem, koçluk öncesinde *kabul etme* seviyesinde (seviye 2) yer almıştır. Koçluk döngüleri süresince, bu tanımlayıcıda da sürekli bir gelişim göstererek *gelişme* seviyesine (seviye 5) ilerlemiştir. Mesleki gelişim sürecinde kazandığı bilgi ve beceriler, öğretim uygulamalarında önemli bir değişim yaratmıştır ve koçluk süreci boyunca edindiği deneyimlerle öğretim yöntemlerini daha etkili ve yapılandırılmış bir şekilde geliştirmiştir.

Sonuç olarak, Cem'in öğretim uygulamaları Öğretim Teması kapsamında önemli bir gelişim göstermiştir. Koçluk süreci boyunca teknoloji kullanımında başlangıçta sınırlı bir yaklaşımdan, teknoloji destekli ve öğrenci merkezli bir öğretim stratejisine geçiş yapmıştır. Cem, öğrencilere daha anlamlı ve etkileşimli öğrenme deneyimleri sunan dersler tasarlayarak, öğretim uygulamalarını en üst seviyeye taşımıştır.

3.1.4. Cem'in Erişim Temasındaki Gelişimi



Şekil 5. Cem'in Erişim Temasındaki Gelişimi

Cem'in Erişim Teması kapsamındaki öğretim uygulamaları, matematik koçluğu döngüleri boyunca önemli ölçüde gelişmiştir. Koçluk öncesinde Cem, *kullanım, engel* ve *kullanılabilirlik tanımlayıcılarında tanıma* seviyesinde (seviye 1) yer almıştır. Teknolojiye sınırlı erişim ve bilgi eksikliği, derslerinde teknolojiyi etkili kullanmasını engellemiştir. Koçluk öncesinde öğrenciler derste teknolojiyi kullanmamaktaydı ve Cem teknoloji çok sınırlı bir araç olarak kullanılıyordu.

Matematik koçluğu süreciyle birlikte Cem önemli ilerlemeler kaydetmiştir. MKD 1'de, *kullanım* ve *engel tanımlayıcılarında uyarlama* seviyesine (seviye 3) ilerlemiştir. Bu aşamada, teknolojiye daha olumlu yaklaşıp, öğrencilere teknoloji kullanma fırsatı sunmuştur. Teknolojiyi planlı şekilde kullanarak, GeoGebra gibi dijital araçları öğrencilerin kavramları keşfetmeleri için kullanmıştır. MKD 2'de Cem *keşfetme* seviyesine (seviye 4) ilerlemiştir. Teknolojiyi öğretim sürecine entegre etmekle kalmayıp, dersleri daha etkileşimli ve öğrenci merkezli hale getirmiştir.

Teknoloji ile öğrencilerin kavramları keşfetmelerini ve problem çözmelerini sağlamıştır. Teknoloji derslerin merkezi bir parçası olmuştur ve Cem teknolojiyi dersin her aşamasında kullanarak öğrencilerin katılımını artırmıştır. MKD 3'te Cem *gelişme* seviyesine (seviye 5) ilerlemiştir. Hem *kullanım* hem de *engel tanımlayıcılarında* teknolojiyi öğretimin her aşamasına entegre etmiştir. Teknolojiyi sadece bir araç olarak değil, öğrencilerin daha karmaşık düşünme becerilerini geliştirmek için kullanmıştır. Ayrıca teknoloji entegrasyonuna engel olan durumları ortadan kaldırma konusunda daha proaktif hale gelmiştir. Cem, *kullanılabilirlik tanımlayıcısında* da ilerlemeler kaydetmiştir. MKD 1'de *kabul etme* seviyesine (seviye 2), MKD 2'de ise *uyarlama* seviyesine (seviye 3) ilerlemiştir. MKD 3 ve MKD 4'te ise *keşfetme* ve *gelişme* seviyelerine (seviye 4 ve 5) ilerleyerek, öğrencilerin daha aktif katılım göstermelerini sağlamıştır. Cem matematik koçluğu sürecini erişim temasının tüm tanımlayıcılarında en üst seviyede tamamlamıştır.

Sonuç olarak, Cem, teknoloji kullanımında sınırlı bilgiye sahipken koçluk süreciyle birlikte teknolojiyi etkin şekilde derslerine entegre etmeyi öğrenmiştir. Teknoloji entegrasyonuna engel olan durumları ortadan kaldırarak, derslerini daha etkileşimli ve öğrenci merkezli hale getirmiştir.

3.2. Cem’in Matematik Koçluğuna Yönelik Algısı

Cem'in matematik koçluğuna dair algısı, matematik koçluğu görüşmesinin analiziyle incelenmiştir. Bulgular, öğretmenin matematik koçluğu ve bunun TPAB gelişimi, öğretim uygulamaları ve mesleki gelişimi üzerindeki etkilerine ilişkin algısını ortaya koymuştur. Cem, matematik koçluğu sayesinde TPAB, öğretim yöntemleri, öğrenci motivasyonu, katılımı ve başarısında önemli gelişmeler kaydettiğini belirtmiştir. Koçluk sürecinin, teknolojiyi derslerine etkili bir şekilde entegre etmesine yardımcı olduğunu ve koçluğun iş birliğine dayalı yapısının öğretim uygulamalarını geliştirdiğini vurgulamıştır. Ayrıca Cem, matematik koçluğu ile geleneksel hizmet içi eğitim arasındaki farklara dikkat çekmiş ve koçluğun daha pratik ve kalıcı faydalar sağladığını ifade etmiştir. Süreci oldukça faydalı bulmasına rağmen, ders hazırlığı için gereken zamanın önemli bir zorluk olduğunu belirtmiştir. Matematik koçluğunu geliştirmeye yönelik önerileri arasında koçluk süresinin uzatılması, daha

geniş bir matematik ünite yelpazesinde uygulanması, ek konuya özgü teknolojilerin entegre edilmesi ve yapay zeka araçları ile oyunlaştırmanın kullanılması yer almıştır; böylece hem öğretim hem de öğrenme deneyimlerinin daha da zenginleştirilebileceğini ifade etmiştir.

4. Sonuç

Bu çalışma, deneyimli bir ortaokul matematik öğretmeninin matematik koçluğundan önce, sırasında ve sonrasında yürütülen araştırmada birkaç önemli bulguyu ortaya koymuştur. TPAB Gelişim Modeli'ne dayanan matematik koçluğu sürecine katılım sayesinde öğretmen, Pedagojik Alan Bilgisi'nden (PCK) Teknolojik Pedagojik Alan Bilgisi'ne (TPAB) doğru bir ilerleme kaydetmiş ve içerik merkezli yaklaşımdan, içerik ve teknoloji merkezli yaklaşımları dengeleyen bir yönteme geçiş yapmıştır. Başlangıçta, öğretmen TPAB Gelişim Modeli'nin dört teması ve bu temalara ait tanımlayıcılar kapsamında düşük seviyelerde TPAB gelişimi sergilemiştir. Ancak, koçluk süreci boyunca öğretmen tüm temalarda daha yüksek seviyelere doğru kayda değer bir ilerleme göstermiştir. Koçluk programının sonunda öğretmen, Müfredat ve Değerlendirme temasındaki *Değerlendirme Tanımlayıcısı* hariç tüm temalar ve tanımlayıcılarda TPAB Gelişim Modeli'ndeki en yüksek seviyeye ulaşmıştır. Sınıfın teknolojik altyapısı ve tüm öğrencilerin teknolojiye eşit erişiminin olmaması değerlendirmelerde teknoloji entegrasyonu zorlaştırmıştır.

Öğretmenin matematik koçluğuna dair algısı, TPAB'i geliştirme, öğretim uygulamalarını iyileştirme, öğrenci motivasyonu ve başarıyı artırma ve mesleki gelişim araştırmalarını teşvik etme konularında olumlu etkiler yaratmıştır. Ayrıca öğretmen, matematik koçluğunu geleneksel hizmet içi eğitimle karşılaştırarak öğrenci öğrenme çıktılarında, öğretmen uygulamalarındaki kalıcı ve dönüştürücü etkilerde ve teori ile pratiğin etkili entegrasyonunda önemli farklar olduğunu belirtmiştir. Bu olumlu sonuçlara rağmen, öğretmen özellikle ders hazırlığı için gereken sürenin önemli bir zorluk olduğunu vurgulamıştır. Cem matematik koçluğunun uygulanmasının iyileştirilmesi için koçluğun okullarda yaygınlaştırılmasını, sürenin uzatılmasını, farklı matematik ünitelerinde uygulanmasını ve yapay zeka araçları ile oyunlaştırma stratejilerinin entegre

edilmesini önermiştir. Ayrıca, koçluğun meslektaşlar arasında iş birliğini teşvik etmek için uygulanmasının faydalarını vurgulamıştır.

Bu çalışmanın bulguları koçluk programlarının öğretmen liderleriyle iş birliği yaparak koçların etkisinin okul geneline yayılmasını sağlayan modelleri içermesi gerektiğini önermektedir. Bu sayede öğretim uygulamalarında ve TPAB gelişiminde daha tutarlı ve sürdürülebilir iyileşmeler ortaya çıkabileceği düşünülmektedir. Çalışma aynı zamanda koçluğun en verimli şekilde yürütülebilmesi için okulların gerekli teknolojik kaynakları sağlaması ve koçları öğretim uygulması dışındaki sorumluluklardan muaf tutmaları gerektiğini vurgulamaktadır. Koçların tam anlamıyla planlama, gözlem ve geri bildirim süreçlerine odaklanabilmesinin koçluk sürecin etkinliğini artırabileceği düşünülmektedir. Sonuç olarak, öğretmenlerin bireysel ihtiyaçlarına göre uyarlanmış ve iyi donanımlı bir okul ortamı tarafından desteklenen koçluğun, öğretim uygulamalarında ve teknoloji entegrasyonunda önemli gelişmelere yol açabileceği vurgulanmaktadır (Kraft vd., 2018).

Bu çalışmanın sınırlılıkları doğrultusunda, gelecekteki araştırmalar için şu öneriler sunulmaktadır: Matematik koçluğu stratejisinin daha uzun süreli uygulanması ve diğer matematik alanları (örneğin cebir, veri analizi) ile dinamik geometri yazılımlarının (Geometry Expressions, Geometer's Sketchpad) incelenmesi önerilmektedir. Ayrıca, Türkiye’de öğretmenlerin mesleki gelişimine matematik koçluğunun dahil edilmesi, teorik ve pratik bilgi arasındaki boşluğu kapatabilir. Son olarak, bu stratejinin farklı eğitim bağlamlarında test edilmesi ve Türkiye'deki öğretmen eğitimine uyarlanması önerilmektedir .

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