EFFECTS OF SOCIOCULTURAL LEARNING ENVIRONMENTS ON 5TH GRADE STUDENTS' PERFORMANCE ON THE FRACTIONS CONCEPT

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ABSTRACT

EFFECTS OF SOCIOCULTURAL LEARNING ENVIRONMENTS ON 5TH GRADE STUDENTS' PERFORMANCE ON THE FRACTIONS CONCEPT

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The main purpose of this study was to investigate the effects of sociocultural learning environments including social interaction such as collaboration and cultural tools such as language on 5th grade students' mathematics performance on fractions concepts and operations with fractions in their Zone of Proximal Development (ZPD). Besides that, the effect of collaboration on students' attitudes towards mathematics was also investigated.

53 5th grade students from the two classes taught by the same teacher out of six in a private elementary school in Ankara formed the sample of the study. One class was assigned as the experimental group (EG), the other as the control group (CG). Students in CG received the instruction on fractions by the regular conventional teaching method used at the school. EG students were assigned to collaborative small-groups of five in fractions and studied on the materials prepared by the researcher. The topics and the sequence of the instruction were the same in both groups.

Conceptual and Procedural Performance Test (CPPT) on fractions, Mathematics Attitude Scale (AS) and a socioeconomic background and studying habits questionnaire were used in this study. The three instruments were given as pre tests prior to the treatment and CPPT and AS were given as post tests after the treatment. Semi-structured interviews were carried out by six students from EG and four students from CG using the CPPT questions. Besides that, EG and CG classrooms were observed during the treatment on fractions.

CPPT mean scores in EG and CG did not significantly differ in pre and post tests, but AS mean scores were significantly different in favour of CG in pre and post tests. Interviews showed that, EG students built an understanding of fractions in a different way than the CG students. High and low ability students benefited in different ways from collaboration and the materials. The teacher affected students' attitudes towards mathematics differently in EG and CG. The ZPD of EG students improved in terms of the quality of understanding and diversity of ideas rather than the difficulty level of the tasks.

Keywords: Collaborative small-groups, Fractions performance, Zone of Proximal Development (ZPD), Attitude Towards Mathematics

ÖZ

SOSYOKÜLTÜREL ÖĞRENME ORTAMLARININ 5. SINIF ÖĞRENCİLERİNİN KESİRLER KAVRAMINDAKİ PERFORMANSINA ETKİSİ

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Bu çalışmanın amacı, birlikte çalışma anlamında sosyal etkileşim ve kültürel araç anlamında dil içeren sosyokültürel öğrenme ortamlarının, ilköğretim 5. sınıf öğrencilerinin kesirler ve kesir içeren işlemler konularındaki matematiksel performanslarına tahmini gelişim alanlarında etkisini incelemektir. Bunun yanısıra sosyokültürel öğrenme ortamlarının öğrencilerin matematik tutumlarına etkisi de incelenmiştir.

Bu çalışmanın örneklemini Ankara ilindeki bir özel ilköğretim okulunun altı adet 5. sınıfı içinden seçilen aynı matematik öğretmeni tarafından öğretilen iki sınıfındaki toplam 53 öğrenci oluşturmaktadır. Bu iki sınıf deney grubu (EG) ve kontrol grubu (CG) olarak belirlenmiştir. Kontrol grubundaki öğrenciler kesirler konusunu geneleksel öğretim yöntemi ile öğrenmişlerdir. Deney grubundaki öğrenciler ise kesirler konusunda beşli

birlikte çalışma grupları oluşturarak araştırmacı tarafından hazırlanan materyaller üzerinde çalışmışlardır. Her iki grupta konular ve öğretim sıraları aynıdır.

Bu çalışmada Kavramsal ve İşlemsel Performans Sınavı (CPPT), Matematik Tutum Ölçeği (AS) ve Sosyoekonomik Durum ve Çalışma Alışkanlıkları anketi kullanılmıştır. Bu üç araç öğretimden önce öntest olarak verilmiş, CPPT ve AS ise sontest olarak verilmiştir. EG'dan seçilen altı ve CG'dan seçilen dört öğrenci ile CPPT soruları kullanılarak yarı-yapılandırılmış yüz yüze görüşmeler yapılmıştır. Bunun yanında CG ve EG grupları kesirlerin bütün öğretim sürecinde gözlenmiştir.

EG ve CG öğrencilerinin CPPT puanları ortalamalarında ön ve son testlerde belirgin bir fark bulunmamıştır, fakat AS puanları ortalamaları ön ve son testlerde CG lehine belirgin şekilde farklıdır. Yüz yüze görüşmeler, EG öğrencilerinin kesirleri konusunda CG öğrencilerinden daha farklı bir anlama geliştirdiklerini göstermiştir. Yüksek ve düşük yetenek seviyesindeki öğrenciler birlikte çalışma ortamlarından ve materyallerden farklı şekilde yararlanmışlardır. Öğretmen EG ve CG öğrencilerinin matematiğe karşı tutumlarını farklı şekilde etkilemiştir. EG öğrencilerinin tahmini gelişim alanları zorluk düzeyi açısından değil, anlama kalitesi ve fikir çeşitliliği açısından gelişim göstermiştir.

Anahtar Sözcükler: Birlikte Çalışma Grupları, Kesir Performansı, Tahmini Gelişim Alanı, Matematiğe Karşı Tutum

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LIST OF ABBREVIATIONS

ZPD	: Zone of Proximal Development
CPPT	: Conceptual and Procedural Performance Test
AS	: Attitude Scale
CG	: Control Group
EG	: Experimental Group
n	: Sample Size
KR-20	: Kuder-Richardson 20 Formula
F	: F Statistics
p	: Significance Value
η^2	: Effect Size
α	: Significance Level

CHAPTER I

INTRODUCTION

> "The introduction of fractions in the elementary school mathematics curriculum is widely considered to represent a turning point, the point at which serious difficulties with mathematics learning begin" (Sophian and Wood, 1997, p.309).

Fraction is the concept, which is used for denoting the parts of a whole object or parts of a set of objects (Hoffer and Hoffer, 1992). Fractions are taught beginning from the early grades since they form a basis for various concepts in school mathematics (e.g. decimals, rational numbers, ratio, proportion, percentage, measurement and probability) and science (e.g. kinetics in physics and chemical equations in chemistry). But it is reported that many teachers need to review the previous fraction concepts in each grade level (Robitaille, 1992). The previous studies (Aksu, 1997; Booker, 1998; Carpenter, Coburn, Reys and Wilson, 1976; Carpenter et al, 1980; Howard, 1991; Hart, 1993; Leinhardt and Smith, 1984; Newstead and Murray, 1998; Orton and Frobisher, 1996) indicated that students have difficulties in understanding the fundamental concepts in fractions in each grade level. The reasons for difficulties in understanding fractions can be analyzed in two groups: the nature of the fractions and the teaching of the fractions.

The "composite nature of fractions" (Ohlsson, 1988, p.53), which is the combination of two natural numbers in only one number with a different meaning, is the main source of difficulties. There are different contexts in which fractions can be represented such as part-whole and number line

models (Orton and Frobisher, 1996). In the part-whole model, fractions are represented either by a continuous quantity or a set of equal-sized discrete objects (Behr and Post, 1992). The teaching of initial fractions concept is solely based on this model (Novillis-Larson, 1980), and this dominance is claimed to inhibit a general fraction understanding (Kerslake, 1986). Number line model is commonly used to show the equivalence of fractions and converting fractions (Behr and Post, 1992). Placing fractions on number line brings certain difficulties due to the scaling on the number line (Baturo and Cooper, 1999; Novillis-Larson, 1980)

Since fractions have different meanings than natural numbers, students' prior knowledge and experiences with whole numbers do not work in learning fractions (Booker, 1996). In whole numbers, there is a natural context or a group of objects, which supports students with experiences of counting, but in fractions, there is no such context (Behr and Post, 1992; Booker, 1998; Steiner and Stoecklin, 1997). This fact causes difficulties in finding the equivalence of fractions and ordering fractions (Behr, et al, 1984).

In addition to the difficulties in understanding the fundamental concepts in fractions, the new rules introduced in fractions also create difficulties for students. Students cannot comprehend these rules immediately and they try whenever possible to apply the rules of whole numbers to computations with fractions (Hart, 1993). Whole number dominance on both conceptual and computational tasks with fractions is well-documented (Behr, Wachsmucth, Post and Lesh, 1984; Carpenter, Coburn, Reys, and Wilson, 1976; Carpenter et al, 1980; D'Ambrosio and Mewborn, 1994; Howard, 1991; Murray, Olivier and de Beer, 1999; Newstead and Murray, 1998; Oliveira and Ramalho, 1994; Hart, 1993). Students tend to add both the numerators and the denominators separately while adding fractions (Baroody and Huma, 1991; D'Ambrosio and

Mewborn, 1994; Howard, 1991; Newstead and Murray, 1998; Oliveira and Ramalho, 1994; Hart, 1993).

Insufficient instruction of fractions is another main cause for students' difficulties in fractions (Mack, 1990; Kieren, 1988). Limitation of the teaching of the fractions to part-whole construct (Behr, Harel, Post and Lesh, 1992; Prie, Martin and Kieren, 1994), dominance of partitioning tasks (Tzur, 1999), and lack of opportunity for students to involve in more problematic situations with fractions (Murray, Olivier, and Human; 1996) are the underlying reasons of difficulties. Having established these facts, it seems logical to search for effective ways to improve students' understanding of and performance in fractions. There are many ways to assist students in improving their understanding and performance, not only in fractions, but also in other aspects of mathematics and other school subjects. A seemingly promising approach is to enhance mathematics learning by the use of social interaction, which results in discussion, and of cultural tools such as language. Social interaction can be either the cooperative or collaborative type.

In collaborative groups, learning takes place in the groups of students while the group members are studying on a task together by interchanging aids, contradictions and perspectives (Webb, Troper and Fall, 1995). Collaboration helps students in concept learning (van Boxter, van der Linden and Kanselaar, 2000) and generalization of what they have learnt (Mercer, 1996). Collaborative small-groups, which are a tool for constructing the knowledge, make the ideas socially generated (van Boxter, van der Linden and Kanselaar, 2000; Mercer, 1996). In collaborative learning environments, for a given task, students ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive critics and summarize their discoveries in writing (NCTM, 1990).

The effects of sociocultural interactions on individual cognition have been investigated to aid in the developing models of mathematical learning (Teppo, 1998; Steffe, 1996). It was claimed that the interactions between the students in the collaborative small-groups seemed to provide the effect of the help that the students got from their teachers in conventional teaching method. Moreover, in a teaching and learning setting in which sociocultural interactions in the form of collaboration were present, he students' roles like giving directions and asking questions were reversed, which was not the case between a teacher and a student (Forman and Cazden, 1998). Collaboration between the students helped them constructing knowledge and understandings socially by talk and activities related to shared problems or tasks (Driver, et. al., 1998).

The theory behind the collaborative learning depends on the Vygotsky's (1978) Zone of Proximal Development (ZPD) and scaffolding concepts (Edwards and Jones, 1999).

The cultural sources of development, learning and instruction were organized by Vygotsky (1896-1934). In his theory, he claimed that the culture of the individual affects the individual's mental processes (Vygotsky, 1978). The development of the mental processes begins when the individual is in social interaction with the people around him or her. During the social interaction, cultural tools such as language, mathematical systems, art and various signs are used as semiotic mediation (Vygotsky, 1981b). These interactions are transformed from social to individual by internalization and become the functions of the individual's mental ability (Vygotsky, 1960).

Mathematics education has been challenging the researchers on investigating the effect of culture on students' mathematics learning in the recent years. The reason is that, students gain the informal knowledge through cultural activities of daily life before coming to school (Tsai and Post, 1999). For example, addition and subtraction of numbers and

recognition of geometrical shapes were claimed to be culturally shaped (Lin and Tsai, 1999).

Vygotsky also emphasized the need for social interaction for teaching and learning by the concept of Zone of Proximal Development (ZPD). According to him, development of thinking starts from the social and goes through the individual (1978). The child's thinking develops between the level of the problems that he or she can solve alone and the level of the problems that he or she can solve by the help of a more capable peer or adult. The zone between these two levels becomes the child's ZPD (Berger and Thompson, 1995; McCormick and Pressley, 1997; Smith and Cowie, 1991). In ZPD, the less capable students' thinking procedures are scaffolded by the more capable students' explanations and tutoring (Wakefield, 1996).

Among the various studies on the ZPD, most of them were studied by elementary grades, including pre-school and the junior high school ages. Dynamic geometry with 11-14 year old students (Gardiner, Hudson and Povey, 1999) and problem solving in 5^{th} grade mathematics (Zack, 1994) was an example of studies on ZPD within mathematics teaching.

Furthermore, the literature contains mixed findings on the effects of attitude and gender towards mathematics on achievement in mathematics. There has been a considerable effort for investigating the relation between attitude towards mathematics and achievement in mathematics (for a comprehensive review, see Aiken, 1976). For example, some studies have shown that attitude and achievement may have a reciprocal relation (Ma, 1997; Ma and Kishor, 1997). The reason is that, educators started to consider the affective domain in teaching and learning of mathematics as well as the cognitive domain (Aiken, 1976; Shaughnessy, Haladyna and Shaughnessy, 1983). Moreover,certain variables seem to affect the studies investigating attitude towards mathematics such as grade level, gender,

sample size (Ma and Kishor, 1997), teacher, and the learning environment (Shaughnessy, Haladyna and Shaughnessy, 1983). Flexible research methods including interviews besides questionnaires are recommended to understand the attitude towards mathematics in relation with different variables (McLeod, 1992).

Underlying the main points, this study sought to investigate the effects of sociocultural learning environments including social interaction such as collaboration and cultural tools such as language on 5th grade students' mathematics performance on fractions concepts and operations with fractions. The following research questions were addressed in this study:

- Is there a significant difference in the mean performance scores of 5th grade students on fractions, that can be attributed to: (i): treatment, (ii): gender, and (iii): interaction of treatment and gender?
- Is there a significant difference in the mean attitude scores of 5th grade students, that can be attributed to: (i) treatment, (ii): gender, and (iii): interaction of treatment and gender, holding the pre attitude scores constant?

Further investigations involved in this study quest in the following questions:

- What are the students' opinions about teaching and learning environment?
- What are the students' attitudes towards teaching and learning environment?
- What factors affect students' attitude towards mathematics?

The research questions mentioned above were sought by means of an essay type performance test on fractions, a mathematics attitude scale, a questionnaire about students' socioeconomic status and studying habits and the interviews with the students together with the classroom observations.

CHAPTER II

REVIEW OF RELATED LITERATURE

2.1 Learning and Teaching of Fractions

> "Fraction learning involves constructing and even more elaborate, complex, broad and sophisticated fraction world and developing the capacity to function in more complex and sophisticated ways in it." (Martin, Prie and Kieren, 1994, p.248)

Fraction is the first abstract concept in mathematics for the young learners (Booker, 1996). Introduction of fractions to young learners brings serious difficulties with mathematics learning (Sophian and Wood, 1997). The "many related but only partially overlapping ideas that surround fractions" (Ohlsson, 1988, p.53) cause difficulties in understanding fractions. There is a complex relationship among the meanings and the representations of fractions since different meanings can be attached to the fraction concept given in different contexts. However, they are all represented in the same way (Orton and Frobisher, 1996; Robitaille, 1992).

'Part-whole' model is one of the contexts in which continuous quantity or a set of discrete objects is partitioned into equal sized subparts or sets (Behr and Post, 1992; Behr, Lesh, Post, and Silver, 1983; Booker, 1998; Hart, 1993; Orton and Frobisher, 1996; Robitaille, 1992). Continuous quantities refer to length, area or volume (Behr and Post, 1992) and require spatial perception skills (Orton and Frobisher, 1996). Students feel difficulty when there are

equally partitioned different shapes represent the same fraction (Orton and Frobisher, 1996). The comprehensive study, Rational Number Project (RNP), which has been under progress since 1979 have investigated that when the instruction is limited to the continuous part-whole construct, a complete understanding of rational numbers is not adequately developed (Behr, et al., 1992). Kerslake (1986) claimed, "the use of 'part of a whole' model can inhibit the development of more general idea of fraction" (p.248). Moreover, equal sharing is not initially efficient for young learners since family sharing is usually not equal (Murray, Olivier and Human, 1996). Thus, instruction should be extended to include the other constructs (Behr, et al., 1992; Baturo and Cooper, 1999)

Discrete objects refer to the separate identical objects (Behr and Post, 1992). This model can be related to the continuous quantities model as seperating the parts of continuous quantities and showing them as discrete parts. The results in the RNP suggested that, initially discrete representation is easier for students since it is based on students' counting procedures that are already well-developed but unitizing a group of discrete objects is also difficult for the children (Behr, et al., 1992). On the other hand, to overcome the difficulties in learning fractions, which are caused by partitioning activities, iteration tasks are also found to be effective (Murray, Olivier and Human, 1996; Tzur, 1999).

Number line model is another representation of fraction in which fraction is treated as a real number. In this model, fractions are explored to rational numbers. Number line representation helps students in using iterative methods to construct fractions from unit fractions, and is essential in children's learning of order and equivalence of fractions (Behr and Post, 1992). But conceptualizing number line representation of fractions brings cognitive difficulty (Baturo and Cooper, 1999). Students have difficulties when the line

segment is presented in different length and partitioning. When the subdivisions of the unit are not equal to the denominator of the given fraction, students have problems in placing that fraction on the number line, which is related to a good conceptual knowledge of equivalence fractions as well as partitioning or unpartitioning (Bright, Behr, Post and Wachsmuth, 1988; Hart, 1993). Another difficulty with number line representation appears when a number line of length two units is presented to students. The tendency is using the whole line segment as the unit (Novillis-Larson, 1980).

Bright, et al (1988) indicated that more work on translations between different modes of representation might help students generate equivalent fractions both symbolically and graphically on the number line. Two possible teaching techniques were suggested by the researchers for the learning of number line representations of fractions: First, a single fraction might be presented by multiple number line representations and second, fractions might be matched and labeled on the number lines with different subdivisions.

In a study done by Behr, Wachsmuth, Post and Lesh (1984), students were instructed both individually and as a group with emphasize on the use of manipulatives. They investigated that the main difficulty in ordering the fractions was resulted from students' tendencies to consider numerators and denominators as separate whole numbers rather than in relation to one another. Students' explanations for comparing fractions with the same numerators/denominators and different numerators and denominators showed that they used some strategies in comparing fractions: denominators only, numerators only, a reference point, manipulatives and whole number properties in comparing fractions. The whole number dominance is very strong at the beginning of the fraction learning so that students cannot develop a quantitative notion of fractions to understand the order of even unit fractions with small denominators. In ordering unit fractions the fraction with smaller

denominator is considered as the smaller one (Behr and Post, 1992; Hart, 1993). A reason for this behavior can be the fact that fractions do not have a counting algorithm. For instance, we don't know what comes before and after ¼ (Behr and Post, 1992).

It was claimed that even though students can easily carry out the algorithms with fractions, they do not understand of the meanings of such algorithms (Aksu, 1997; Goldin and Passatino, 1996; Wearne-Hiebert and Hiebert, 1983). Many studies (Baroody and Huma, 1991; Carpenter, Coburn, Reys, and Wilson, 1976; Carpenter et al, 1980; D'Ambrosio and Mewborn, 1994; Howard, 1991; Newstead and Murray, 1998; Oliveira and Ramalho, 1994; Hart, 1993) have reported that when students were asked to add two fractions, they added both the numerators and the denominators separately. The reason is that students do not understand fraction as one number with a single value, but they rather understand it as two numbers with distinct values and meanings (Behr and Post, 1992; Newstead and Murray, 1998).

Manipulative materials are reported to be "at least as effective, but perhaps less efficient than other forms of instruction" (Behr et al, 1992, p.121). Actually, they serve as an aid in representing written symbols and abstract theories as concrete real life situations and vice versa (Behr et al, 1992; Orton and Frobisher, 1996). This can be explained by the fact that there can be different manipulative representations of different mathematical concepts for different students even in a single situation such as the same classroom (Behr, and et al, 1992). In other words, a given single manipulative aid can prevent children from constructing their own manipulations and even cause them not to understand the concept since that single manipulation can not be appropriate for each pupil's mathematical understanding (Orton and Frobisher, 1996; Post, 1992).

Post (1992) claimed that, when learning a new concept in mathematics, it is important for students to 'see' the concept in different interpretations or from a different perspective such as pictorial representations and manipulative aids. The main aids in teaching the initial fraction concepts are pictorial and manipulative materials such as regions, folded papers and prepartitioned objects (Behr, Lesh, Post, and Silver, 1983; Booker, 1998; Newstead and Murray, 1998; Robitaille, 1992; Hart, 1993). But these aids also mislead students. Use of region requires spatial perception skills (Orton and Frobisher, 1996) and misconceptions based on region usage were studied by many reseachers (Behr and Post, 1992; Newstead and Murray, 1998; Oliveira and Ramalho, 1994; Orton and Frobisher, 1996; Hart, 1993). When appropriate regions such as prepartitioned square and rectangle are not used, students get confused with showing the required fraction (Hart, 1993; Newstead and Murray, 1998). An example of such confusion appears with triangles, in which students do not consider 'equality' of the partitioning (Newstead and Murray, 1998). Behr, Lesh, Post and Silver (1983) claimed that concrete materials, which represent fractions, might not be appropriate in representing the addition. Use of discrete quantities in addition may easily lead students to add denominators and numerators since the number of both the 'whole' and the 'part' objects are increased.

Previous studies have shown that manipulative aids are only one part of the development of mathematical concepts among the other modes of representation, such as pictorial, verbal, symbolic and real world situations. And to make the mathematical ideas more meaningful for children, the translation betweent the modes of representations should be constructed (Lesh, Landau and Hamilton, 1980).

Different teaching experiments have been carried out in order to see if different methods and materials in teaching fractions are effective in decreasing the frequency of difficulties.

How 6^{th} and 7^{th} grade upper elementary school students perform conceptual and computational tasks with common fractions were investigated before and after they exposed to a teaching material. Sharing situations and variety of fractions and meanings of fractions were emphasized and students were encouraged to create their own representations in a supporting classroom culture. The study showed that the necessity of equal parts in partitioning and the role of the whole should be more emphasized (Newstead and Olivier, 1999).

6^{th} grade students who had serious economic, health and nutrition problems in a rural area school were exposed to teaching practices to construct their fractions concept again. Researchers used fractions other than halves and quarters and allowed students create their own representations for fractions in collaborative learning groups where they can talk about their ideas. Pre-test results showed that students had a weak understanding of fraction concept, in which they used their whole number knowledge and rote algorithm practices so oftenly. But the post-test resulted in a decrease in students' inappropriate understanding and usage of fraction knowledge. Although students had an already formed construction of fraction concept, the teaching experiment was successful at inventing their own correct procedures by the help of a discussion environment (Murray, Olivier and De Bear, 1999).

Maher, Martino and Davis (1994) carried out a constructivist teaching experiment with a 4^{th} grade class to explore and study their development of fraction ideas. They presented problematic situations to the students and observed how they build solutions, develop arguments, construct and compare

models, discuss their ideas and negotiate their conflicts. Teachers facilitated the students' discussion and explored their thinking by questioning that was related to their fraction construction. Researchers found that students used different perspectives in representing fractions on the number line, which was a result of complexity of building fraction ideas. The additivity of measure was not noticed by a student as he placed 1/3 on two different points between 0 and 1 by considering line as a segment.

The teaching suggestions from the literature can be summarized as follows. Fraction learning should not be limited to dominant constructs such as part-whole model and particular strategies such as partitioning (Tzur, 1999). Iteration tasks are also useful for students in constructing a meaningful fraction understanding (Murray, Olivier and Human; 1996; Tzur, 1999). Use of different approaches to the fraction concept or additive dimensions to classroom instruction increases the students' ability in dealing with fractions.

Sharing problems with remainder may be a proper introduction of fractions in the early grades, which encourage students to construct their own idea of fractions (Murray, Olivier and Human, 1996). The role of the whole should be emphasized in different contexts (Newstead and Olivier, 1999). Representing fractions in different contexts makes fraction ideas more meaningful for the students (Behr, et al, 1992). Using appropriate material in appropriate level when needed brings better performances to low achievers (Oliviera and Ramalho, 1994).

Many researchers (Booker, 1998; Maher, Martino and Davis, 1994; Tzur, 1999) have conducted constructivist teaching experiments in order to investigate students' progress in fractions. Working in small groups proved to be more effective in teaching and learning fractions (Booker, 1998; Murray, Olivier and de Bear, 1999). Since "social interaction creates opportunities for

students to talk about their thinking, and this talk encourages reflection" (Murray, Olivier and de Bear, 1999, p.306), it is suggested that students should be encouraged to construct their own knowledge and attempt to establish social procedures like discussion and justification to monitor and improve the nature and quality of those constructions (Newstead and Murray, 1998). Assigning a fascilitator role to the teacher (Booker, 1998) enables students construct their ideas. By this way, the teacher can understand the students' thinking (Yackel, Cobb, Wood, Wheatley and Merkel, 1990).

It was reported that primary school students in Turkey achieved the goals and objectives of mathematics education in the first three grades. They reached 70% achievement, which was accepted as sufficient (MEB, 1997). But they cannot reach such an achievement in the further grades. The traditional teaching method is not found to be efficient in coping with the difficulties in understanding the fractions concept and computation with fractions. So new instructional methods are needed to increase the achievement in the mathematics education. Also, teachers claimed that, the instructional methods described in the Mathematics Curriculum were insufficient for the Mathematics Lessons in the elementary schools (MEB, 1997). Thus the need for new instructional methods is inevitable in Mathematics Education.

2.2 Collaborative Learning

It is known that most of the teachers prefer either to interact with the whole class, or let students work on their own on a textbook (Werry, 1988). Very few teachers in any educational system try to build on their instruction on small group work (Burstein, 1992). But, in recent years teachers have realized that small groups involving students working together is efficient in mathematics learning (Peterson and Swing, 1985; Stacey and Gooding, 1998). The reason might be the significant benefits of small group instruction both for

1998; Yackel, Cobb, Wood, Wheatley and Merkel, 1990). Different ideas appearing in the small groups make the task more likely to be completed or the problem more likely to be solved correctly (Orton and Frobisher, 1996). They "discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive criticism and summarize their discoveries in writing" (NCTM, 1989, p.79). In collaborative small-groups, students realize the need for listening to one another, feel more confident and successful in mathematics and claim that collaboration enables them progress in mathematics faster (Edwards and Jones, 1999).

> "Mathematics education aims to enhance the capabilities of each person to invent and use mahematics both individually and in collaboration with others. The effective combination of both individual and collective aspects of mathematical enquiry is the essence of collaborative group work. Students need to be able to think flexibly and systematically and to be able to communicaite ideas and mathematical strategies to others using informal but clear language." (Neyland, 1994, p.41)

The construction of knowledge in collaborative small-groups is basicly a product of the discussion between the group members (Prie, 1991; Hoyles, Sutherland and Healy, 1991; Davidson, 1990). The effect of discussion on enhancing learning at any level has been a trivial issue for educators (Orton and Frobisher, 1996).

Hoyles, Sutherland and Healy (1991) analyzed the role of discussion in learning in four interrelated aspects: distancing, conflict, scaffolding and monitoring. When there was a need to discuss ideas, students based their ideas on a more conscious plane so that the ideas become more explicit, and this process was defined as distancing. Conflict appearing in a collaborative group was the result of different perspectives on the tasks and was necessary –but not sufficient- component of efficient social interaction. The scaffolding

role of discussion is key to the development of the group members' zone of proximal development. Finally, discussion provides an internal self-regulation of students' own understandings, as a monitore to both their learning processes and the groups' direction, which is also interpreted by Orton and Frobisher (1996). Webb (1989) claimed that discussion in collaborative groups was a scaffold for the understanding of specific tasks or development of cognition.

In mathematics, teachers try to increase the students' reasoning skills and avoid memorizing rules, formulas and some processes. Moreover, they want their students to create their own formulas and rules, which carry their own words, not the statement of the books. Students should understand the definitions or rules in a way that they can restate the definitions and rules in their own words and can make applications by using these definitions and rules (MEB, 1998; Orton and Frobisher, 1996).

The teachers' responsibilities in collaborative learning environments are changed in terms of the basic considerations and the actions. There is a vital role for the teacher in composing collaborative groups and choosing, planning and implementing the tasks (Edwards and Jones, 1999; Orton and Frobisher, 1996). Collaborative design involves students with both higher and lower level of ability (Light and Littleton, 1998). According to Vygotsky (1981b), the more capable group members help the less capable members in constructing meaningful knowledge, in the form of scaffolding. In mixed-ability groups, the explanation receivers benefit from the givers. More capable students understand their less capable friends' approaches to the subjects, problems, rules or definitions better than the teacher and direct their friends' efforts and attention to the relevant issues (Vygotsky, 1981a).

Another collaborative small-group design emphasized the friendship among the students because of several features (Azmitia, 1998; for a review see Zajac and Hartup 1997). The friends in small-groups are more attune to each other's needs and goals, and this makes them construct shared understanding. Friends may be more willing to explain their ideas and challenge each other because of the trust to each other (Nelson and Aboud, 1985). The respect between the friends makes them appreciate each other's ideas, which helps learning (Edwards and Jones, 1999). The equity concern between friends forces them to put them more effort into negotiation of meanings under construction (Azmitia, 1998).

The teacher should decide group size, which results in the most effective learning. It is clear that more groups with small size mean less teacher time for each group. And fewer groups mean more members in each group, which may cause ineffective crowded group and noise (Turner and Furness, 1999). The group size must be determined according to the circumstances in the classroom (Neyland, 1994; Orton and Frobisher, 1996), but it should include 2-6 students (Orton and Frobisher, 1996). Davidson (1990) suggested using groups of four for the mathematics tasks since they are large enough for discussion and absence of a member does not affect the group progress.

The teacher's role collaborative small-groups class is mainly to fascilitate the groups' progress. The teacher supports and guides the groups by means of checking the solutions, giving hints, clarifying and correcting the expression and answering some of the questions (Davidson, 1990). The teacher mediates the group processes by introducing them new contexts so that the students are challenged to learn in new ways. For this, he or she allows and encourages students to extend their abilities and take risks for new experiences and materials (Moll and Whitmore; 1998).

In any of the collaborative small-group design, the explanations of group members to each other are effective when the explanations encourage students to involve in a constructivist activity (Webb, Troper and Fall, 1995). The explanation receiver group members should process the explanations and relate them to their knowledge (Kneser and Rolf, 2001). Hence, the tasks should be prepared in a way that makes students further their knowledge. There are different tasks that are more appropriate for discussion than the others, with the help of concrete manipulatives (Orton and Frobisher, 1996). The tasks should be rich and challenging, and they need to make students realize that working together in collaboration is advantageous (Neyland, 1994b).

Social interaction in collaborative small-group learning has a positive effect on the conceptual learning (van Boxtel, van der Linden and Kanselaar, 2000). Stacey and Gooding (1998) reported that the reduction of misconceptions was one of the most useful results of collaborative group learning in mathematics.

2.3 Development, Learning and Instruction from a Vygotskian Perspective

The study of human development investigates how and why people change, as they grow older and how and why they remain the same. Among the different approaches to human development, sociocultural theory of human development studies the active interaction between the people and the culture around them. Clearly, sociocultural theory seeks to explain the growth of individual knowledge, learning and abilities in terms of guidance, support and structure provided by the surrounding cultural framework.

The culture provides the setting in which learning takes place. It also provides the structure of social functioning that determines the individual psychological functioning. In addition to ways of communication and viewing the world, culture also creates major concepts and ideas. Hence, the child learns to think in the ways that his or her culture provides (Gredler, 1992).

Sociocultural approaches to learning and development were first organized and applied by Russian psychologist Vygotsky (1896-1934). Vygotsky's main interest was on the development of cognitive abilities, such as "the mental processes in which individual thinks, learns and communicates" (Berger and Thompson, 1995, p.3). According to Vygotsky, development of the cognition is the result of interactions between the child and his social environment (John-Steiner and Manh, 1996). Thus, child's development of mental processes can only be recognized by analyzing its social and cultural sources (Barfurth, 1995; Driscoll, 1994; Eliot, Kratochwill, Littlefield and Travers, 1996; Fox 1993; McCormick and Pressley, 1997; Rogoff and Morelli, 1989; Tudge and Putnam, 1997; Wertsch and Tulviste, 1992). This idea is the core of his "general genetic law of cultural development":

> "Any function in the child's cultural development appears twice, or on two planes. First, it appears on the social plane, and then on the psychological plane. First, it appears between people as an interpsychological category, and then within the child as an intrapsychological category. This is equally true with regard to voluntary attention, logical memory, the formation of concepts, and the development of volition...It goes without saying that, internalization transforms the process itself and changes its structure and functions. Social relations or relations among people genetically underlie all higher functions and their relationships" (Vygotsky, 1981a, p.163).

So, it can be said that, culture and social sources of mental processes are not separated explicitly. He expressed the relation between them with the following statements:

> "Above all, in the widest sense of the word, it means that, everything that is cultural is social. Culture is the product of social life and human social activity. That is why just by raising the question of the cultural development of behavior we are directly introducing the social plane of development" (Vygotsky, 1981a, p.164).

In his theory, Vygotsky claimed that the development of child's mental processes "presupposes a specific social nature and a process by which children grow into the intellectual life of those around them" (1978, p.8). It means that, the mental processes are first examined between people, in the interpsychological plane and this constructs a basis for the mastery of mental processes in the child's mind, in the intrapsychological plane. Wertsch and Tulviste (1992) called these two planes intermental and intramental respectively. Hence, child's mental processes are not independent of those around him. Even when these processes are activated or generated in isolation by the child, they are naturally social or sociocultural.

The mental processes in the intrapsychological plane include the socially evolved and organized cultural tools (Wertsch and Tulviste, 1992).

These tools can be named as 'psychological' or 'cultural' (in Wertsch and Tulviste, 1992) tools that are directed internally and appropriated, which means adopted by the individual during the activation of mental processes (John-Steiner and Mahn, 1996).

In this perspective, a key concept, semiotic mediation, will be helpful, before explaining the tools. Semiotic is the way in which people communicate through signs and images. According to Vygotsky, semiotic mediation is a collection of objects, people, language and events that mediates the social and individual processes. By mediation, he meant the situation in which the individual actively reshapes the stimulus, which are the social interactions. But she or he reshapes them as a part of the process of responding to it and builds a mental skill (Cole and Scribner, 1978, p.14). It helps in connecting the external events, people and objects to the individual's internal events, abilities and symbols. Hence, semiotic mediation includes the psychological tools.

> "The following can serve as examples of psychological tools and their complex system: language; various systems for counting; mnemonic techniques; algebraic symbol systems; works of art; writing; schemes, diagrams, maps and mechanical drawings; all sorts of conventional signs; and so on" (Vygotsky, 1981b, p.137).

All these tools are not an invention of the individual alone, but they are the products of sociocultural processes and appropriated by the individual during the mental processes (Wertsch and Tulviste, 1992; John–Steiner and Mahn, 1996), which is the case of general genetic law of cultural development.

Among the cultural tools, the most important one is the language, which is the device for communication and coordination of human activities (Saljö, 1998). According to Vygotsky, language has an important role on the development of higher mental processes (Berger and Thompson, 1995;

Discroll, 1994; Fernyhough, 1997; McCormick and Pressley, 1997; Wakefield, 1996; Wertsch and Tulviste, 1992). Language helps children in constructing a way of thinking and provides a base for teaching and learning (Saljö, 1998) as well as communicating with their social environment (Berger and Thompson, 1995; Edwards and Mercer, 1995; Fernyhough, 1997).

"Children solve practical tasks with the help of their speech, as well as their eyes and hands" (Vygotsky, 1978, p.26). At the beginning, the child uses speech externally, to communicate with those people around him or her. As she or he involves in the social environment, the child starts to imitate the dialogs between the adults and older friends who are more capable in terms of thinking and ability to perform the complex tasks. Social speech is more difficult for the children because it requires the children to be a listener and also a speaker (Craig, 1989). Thus, when children try to perform a task in the social environment, they don't use their imitated dialogs. But they prefer to use their imitations and talk to themselves while trying to perform a task alone. This self-talk is named as "private" or "egocentric" speech. It can also be defined as the tendency of children to talk to themselves about what they are doing (McCormick and Pressley, 1997).

Private speech usually appears when children face a difficult or unfamiliar task since they need a more complex thinking ability. It allows children to use words to control or plan their own behavior and helps in organizing their actions or ideas in order to complete the more complicated tasks. Private speech is a bridge in the way from intermental processes to the intramental ones. It is "the point at which psychological tools, in this case words in natural language, come to mediate mental activity" (Fernyhough, 1997, p.72).

In fact, private speech is a way of the main transformation process, the internalization. Vygotsky explains the transformation of interpsychological processes into intrapsychological processes by the concept internalization. He described internalization as a "set of social relationships transposed inside and having become functions of personality and the forms of its structure" (Vygotsky, 1960, p.198-199). It is thought as a process that the basic cognitive functions of the mind are received from the experiences in social environment (Nuthall, 1999). Hence internalization can be defined as the re-construction of external processes as thinking in individual's mind (Mellin-Olsen, 1987).

Referring to the Vygotsky's idea of semiotic mediation, it can be said that semiotic mediation is fundamental to the internalization concept. The reason is that, internalization is a transformative process, rather than a transmissive one. The intermental processes are not copied as intramental processes but rather, there is a re-construction provided by semiotic mediation (John – Steiner and Mahn, 1996).

Internalization in the child begins with learning to perform activities or a set of behaviors in a social and physical context by using different semiotic mediations like trial and error, imitation and guidance from others. As the child involves in such experiences, he or she becomes competent at the activities and starts to realize the activities and their components separately, independent of the contexts they appear or as a part of a larger activity processes (Nuthall, 1999).

According to Vygotsky, as the child involves in increasingly complex social activities and as he or she involves in increasingly complex cultural tools (such as mathematics), he or she internalizes increasingly complex mental processes and structures (Nuthall, 1999). So the outcomes are "genetically" socio-cultural structures and processes.

The "general genetic law of cultural development" also focuses on the development of higher mental processes and how children are guided and aided in this process (Rogoff and Morelli, 1989). According to Vygotsky, higher mental processes are the more developed forms of lower and more elementary mental processes (Fernyhough, 1997). To understand this relation, the differences between the higher and the lower/elementary mental processes should be analyzed.

Most of the elementary mental processes are "totally and directly determined by stimulation of the environment" (Vygotsky, 1978, p. 39) and some of them are caused by genetic facts. Hence, they are involuntarily, unconscious, unmediated (natural) and isolated individual mental units. There exists no mental network, which combines them to perform complex tasks. The examples of elementary mental processes are perceptual categorization and involuntary attention. Whereas elementary mental functions are biological in origin, higher mental functions are the human capabilities, which have sociohistorical origin.

Vygotsky claimed that higher mental functions included categorical perception, logical memory, abstract thought, and voluntary (selective) attention (1978). They are voluntarily controlled, conscious, semiotically mediated and form a link with some other processes in a wide system of mental processes (Fernyhough, 1997; Subbotsky, 1998). The examples for higher mental processes are voluntary memory, voluntary attention, classification, concept formation, and reasoning the use of social inventions such as language, mathematical systems and memory devices (Fernyhough, 1997; Rogoff and Morelli, 1989).

The transition between the higher and lower mental processes can also be explained by the help of internalization. "All higher mental functions are internalized social relationships" (Vygtosky, 1981a, p.162). Higher mental functions keep at least some of the characteristics of the external,

namely social environment from, which they are derived (Fernyhough, 1997) Thus, higher mental processes are formed during the social interactions involving the cultural tools which are available at that moment, such as language (Rogoff and Morelli, 1989). And they are appropriated simultaneously, especially in the case of interaction with the people in the social environment who have greater competence (Tudge and Putnam, 1997).

There is one more step, which needs to be completed by the children to reach higher mental processes: The internalization of private speech, which results in inner speech. In terms of Vygotsky's "General Genetic Law of Cultural Development", language is first considered as an interpsychological process, since there is already social speech in the child's environment. Then it turns into a semiotic mediation, which is the private/egocentric speech. Thus, in the last step the private speech should be transformed into an intramental process, which is defined as inner speech.

During the egocentric speech, the child guides himself by the modified forms of the speech of more capable ones in the social environment. The re-construction of these speeches as child's own speech, the internalization of private speech, results in inner speech (Wertsch and Tulviste, 1992). To summarize the whole process, it can be said that, children internalize the words that are used in the social environment to address people and their behaviors, and use the re-constructed words to organize their actions (Fernyhough, 1997).

The process of inner speech is one way of developing higher mental functions. The role of social environment in this process is rather natural, and needs hardly any special effort. But with more social interaction in terms of guidance and aid, do higher mental functions develop also.

2.4 Teaching and Learning in the Zone of Proximal Development

> "Learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. Once the process is internalized, they become a part of the child's independent development of mental achievement" (Vygotsky, 1978, p.90)

Vygotsky believed that mental development is basically social and educational process where education includes more than schooling. Development of thinking does not start from individual and goes through socialized but it starts from the social and goes through the individual (Vygotsky, 1988). Thus, the process of development is the point of concentration (Vygotsky, 1978). In particular, development of child is a product of his learning that occurs in the social environment. The central concept of Vygotsky's theory (Wells, 1999), Zone of Proximal Development, was developed to explain how the social environment affects the learning.

> "The zone of proximal development is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance in collaboration with more capable peers" (Vygotsky, 1978, p.86).

Zone of proximal development is the way between what a child can perform without aid and assistance, and what he can potentially perform under aid and assistance (Berger and Thompson, 1995; Fernyhough, 1997; McCormick and Pressley, 1997; Smith and Cowie, 1991; Wakefield, 1996). Operationally, zone of proximal development (ZPD) can be defined as the difference between the child's test performance with assistance and that of without assistance (Wells, 1999). So ZPD draws the top and bottom boundaries of an instruction determined by the social interactions (Discroll, 1994). Between the boundaries of ZPD, there are some mental processes

which have not been developed, but under the process of development. Those processes become more meaningful when a more knowledgeable person enters the child's ZPD.

Vygotsky (1934/1987) believes that, "instruction is only useful when it moves ahead of development" (p.212). Thus, the tasks that the child deals with should just beyond the child's level of competence, or their independent performance (Fernyhough, 1997; Smith and Cowie, 1991; Wakefield, 1996). To perform such tasks, the child needs the intervention of more capable ones, which are in the form of scaffolding.

Scaffolding is providing the aid involving complex set of social interactions and semiotics (Stone, 1998) and which makes the students' mental abilities go further (Wakefield, 1996). It can be in the form of reading a story together, planning a science project and solving a mathematics problem.

Scaffolded instruction can be constructed both by the teachers and more capable peers. In both environments, the teacher's role is a guide providing students with appropriate tasks, learning opportunity and the aid (Wood, Bruner and Ross, 1976).

Scaffolded tasks are those that challenge the less capable students. It should be just beyond the students' level of competence (Fernyhough, 1997; Smith and Cowie, 1991; Wakefield, 1996) and should be broken down into manageable components that are appropriate for the ability of students (Fernyhough, 1997).

Scaffolding and ZPD also describes the basis of collaborative learning environments in which students are engaged in joint learning activities and help each other to master a task (Wells, 1999). In these environments, where all students learn and perform new skills and

concepts, less capable students benefit much. If properly structured, in terms of group members' skills and abilities, and properly managed, in terms of amount of the scaffolding and level of the task to be completed, collaborative learning environments affects the learners' re-construction of knowledge. More capable students will further the cognitive development of the less capable ones by scaffolding (Driscoll, 1994).

In zone of proximal development some points such as the presentation of the task to be performed and the teachers role in the zone should be considered. A task, which is beyond the ability level of a student, may be a very basic task to be performed for another, as a result of the diverse directions of the development of the students (Wells, 1999). So the task should be designed such that, even more capable students get the opportunity to learn more about the concept (Tudge and Rogoff, 1989). In the further applications, according to the abilities of students, there can be different tasks, designed for the different groups, for the same concept to be learnt. In any case, the task should be divided into meaningful components in terms of the skills needed, to prevent the students from being frustrated. The division of the components could be from simple to complex according to the current ability level of the students (Fernyhough, 1991).

Teacher's role in the zone of proximal development is quite different from the traditional teacher's role. Teaching in ZPD requires more than selecting appropriate objectives and assessing whether the students have achieved the objectives or not. The teachers should be aware of the students' current stage of development and assist them to increase their potential of knowledge for future learning (Edwards and Mercer, 1995; Wells, 1999), and should understand the students' needs that are essential for their learning. They should clearly know what students know without any help given. This could be done by building a class discussion on the concept and asking questions, which give feedback about the students' previous knowledge.

Based on students' independent performance levels, teachers should carefully form the groups or peers that will study together during the development of the concept (Driscoll, 1994).

They should carefully design the materials that the formed groups or peers will study. Scaffolding may facilitate learning when it occurs in the context of a total activity (Gredler, 1992). After assigning the tasks to the groups, he should carefully monitor them and intervene when it is needed. For tasks, which are complex even for more capable students, the teacher should carefully help the students to increase their zone of proximal development (Wood, Bruner and Ross, 1976). Even in these cases, the group study is still powerful since more capable students internalize complex tasks quickly and are able to transfer them to the less capable ones. If the more capable ones do not need aid, the teacher should try not to intervene the processes in which less capable ones are scaffolded by the more capable ones in their ZPD. In addition to that, the teacher should encourage less capable students to study the task alone with less help (Gredler, 1992).

The teacher should correct the misunderstandings during the process. How quickly the misunderstood concepts are corrected, then the instruction would be effective. Therefore, teacher's role is to explain, inform, inquire, correct and force the student to perform the task himself by the help of a collaborative learning environment in which more capable students are also involved. What the student internalizes as a result of such learning environment will be used by him in the independent problem solving tasks (Vygotsky, 1934/1937).

2.5 Attitude Towards Mathematics

Mathematics educators have realized that affective domain components may have influence on the achievement in mathematics (Aiken, 1976: Kifer and Robitaille, 1988; Ma and Kishor, 1997; NCTM, 1989; Shaughnessy, Haladayna and Shaughnessy, 1983; Shiomi, 1992). Attitude has been concerned as the most influential (Ma and Kishor, 1997) among the other affective domain components such as beliefs and emotions (McLeod, 1992). Moreover, attitude has become an important issue in mathematics curriculum (MEB, 1998; Meyer and Fennema, 1992; Shaughnessy, Haladayna and Shaughnessy, 1983).

Investigations of the relationship between the attitude towards mathematics and achievement in mathematics indicated that the relationship may be reciprocal (Ma, 1990; Shiomi, 1992; Tsai and Walberg, 1983). Another view of attitude claims "neither attitude nor achievement is dependent on the other; rather; they interact with each other in complex and unpredictable ways" (McLeod, 1992, p.581).

Ma's research investigated the reciprocal relationship between attitudes toward mathematics (ATM) and achievement in mathematics (AIM) (1990). In the study, ATM was defined as "either positive or negative responses in terms of importance, difficulty and enjoyment, when learning algebra, geometry and trigonometry" (p.222). The major findings of the analysis of two achievement tests and one questionnaire indicated that there was a reciprocal relationship between ATM and AIM, and AIM was directly related with the feeling of enjoyment, not the feeling of difficulty. But the feeling of difficulty seemed to shape the feeling of enjoyment. Ma suggested that the difficult mathematical content should be presented in an interesting, attractive and enjoyable way.

However, the reciprocal relationship may not be strong. In the earlier study of Aiken (1970), correlations between attitude and achievement in the elementary school level were reported to be not very large, although statistically significant. In this latter review, he indicated that there was a positive and meaningful relationship between ATM and AIM at the elementary and secondary school level, but this relationship might not have statistical significance (1976). Similar finding was reported by Ma and Kishor (1997), that ATM-AIM relationship in the elementary school level might not have a practical implication. Moreover, they indicated that attitude measures might be least reliable and valid in the elementary school level.

Ma's another findings were related with the effects of parents on ATM and AIM (1990). He found that fathers were more influential on achievement (Ma, 1990), whereas mothers were more influential on attitude (Ma, 1990; Meyer and Fennema, 1992). The effect of parents' attitudes towards mathematics and mathematics education was also reported by Aiken (1972), Bishop and Brew (1996), Meyer and Fennema (1992). On the other hand, parents' influences (Ma, 1990) and students ATM (Ma and Kishor, 1997) are related with the specific culture of the society, and it should be investigated by considering cultural factors. Those investigations are a result of the need to examine the effects of social dimensions in terms of people, on mathematics attitude (Bishop and Brew, 1996). The effects of particular individuals and groups on students' attitudes towards mathematics were investigated by Bishop and Brew (1996). In their study, students' own teachers were found to have the most significant influence on their attitudes. Students' attitudes were also highly influenced by their classmates, a finding also interpreted by Shaughnessy, Haladayna and Shaughnessy (1983).

Gender differences in attitude towards mathematics have been another concern of the mathematics educators. Many studies have been conducted to

investigate the effect of gender on AIM and ATM, and reported both a significant and not significant effect (for a comprehensive review, see Aiken, 1976). Studies involving mixed-gender groups tended to show weaker ATM-AIM relationships than studies with separate gender groups (Ma and Kishor, 1997).

Studies concerning ATM-AIM relationship were recommended to be conducted by a random sampling with a sample size around 300 (Ma and Kishor, 1997). In addition to the quantitative studies, McLeod (1992) suggested that including interviews to attitude investigations may lead to new contributions in mathematics education.

CHAPTER III

RESEARCH DESIGN AND METHOD

This chapter presents the design and the method of the study. The subjects and the instruments together with the scoring criteria are described first. The procedure of the study follows. Then, the treatments for the experimental and the control groups are explained in detail.

3.1 Sample

A total of 53 5th grade students (25 girls and 28 boys) from two classes out of six in a private elementary school in Ankara formed the sample of this study. The classes chosen were the ones that were taught by the same teacher. One group constituted the experimental group (EG) and the other the control group (CG). Each class consisted of 30 students. Students' ages in both groups ranged from 10 to 11. The number of students who took the pre-test and the post-test is shown in Table 3.1. The students who took both the pre-test and the post-test were taken as the subjects of this study. The 27 EG students were composed of 13 girls and 14 boys, whereas the 26 CG students were composed of 12 girls and 14 boys.

Table 3.1: The subjects of the study

Groups	Pre-test	Post-test	Pre-test ∩ Post test
Experimental	28	30	27
Control	28	29	26
Total	56	59	53

3.2 Instruments

3.2.1 Conceptual and Procedural Performance Test on Fractions

An essay type Conceptual and Procedural Performance Test (CPPT) was developed to investigate 5th grade students' conceptual and procedural performance on fractions (See Appendix A). CPPT includes 13 essay type questions, 8 of which include subtasks. The test including 38 tasks altogether was administered to the subjects as a pre-test and post-test, allowing 45 minutes. Pre-test CPPT results yielded a Kuder-Richardson (KR-20) reliability coefficient of internal consistency of 0.77.

The test was based on fraction topics given in the 5th grade:

(i) Types of fractions and their properties,

(ii) Discrete and continuous representations of fractions,

(iii) Relations between mixed and improper fractions,

(iv) Equivalence of fractions,

(v) Comparison of fractions,

(vi) Addition of fractions whose denominators are integers less than 50,

(vii) Subtraction of fractions whose denominators are integers less than 50,

(viii) Multiplication of different types of fractions.

The objectives of each question in CPPT are given in Table 3.2.

Each question in CPPT was analysed by giving 1 for each correct answer and 0 for each incorrect answer, except question 2. In question 2, students are required to find three pairs of equivalent fractions from a group of fractions. That is, the same objective is involved three times in it. Thus, this question was considered as having three subtasks, and each subtask was scored separately.

Table 3.2: Objectives of each question in CPPT

Question	Objectives
1.	a. Write the fraction given on the number line
2.	a. Identify equivalent fractions from a group of fractions
3.	a. Compare two fractions with equal numerator b. Compare two fractions with equal denominator c. Order fractions which have equal numerator or denominator
4.	a. Convert a mixed fraction to an improper fraction
5.	a. Convert an improper fraction to a mixed fraction
6.	a. Write the fraction represented by a continuous figure b. Add three simple fractions with equal denominator c. Represent the fraction on a continuous figure d. Write the addition of simple fractions in the form of multiplication
7.	a. Write the fractions defined by partitioning b. Name the kind of fractions
8.	a. Write the fractions defined by partitioning
9.	a. Recognise whether a given continuous or discrete figure represents a fraction or not
10.	a. Add two different kinds of fractions whose denominators are unequal integers less than 50 and one is not the multiple of another b. Subtract two simple fractions whose denominators are unequal integers less than 50 and one is not the multiple of the other c. Multiply different kinds of fractions d. Subtract two different kinds of fractions whose denominators are unequal integers less than 50 and one is the multiple of the other e. Add three simple fractions whose denominators are unequal integers less than 50 and one is the multiple of other two f. Multiply a mixed fraction by a natural number
11.	a. Write different kinds of fractions
12.	a. Write fractions which are less than or greater than a given fraction
13.	a. Compute the unknown term in a subtraction where different kinds of fractions are subtracted b. Show equivalence of two fractions c. Represent the fraction on a discrete figure

Prior to the main study, the test was piloted on 145 5th grade students in a private elementary school in Ankara during the 1st semester of 1999-2000 school year. The results of the piloting study will be given in more detail in Chapter IV.

3.2.2 Attitude Scale Towards Mathematics

A Likert type attitude scale towards mathematics (AS) developed by Aşkar (1986) was used to investigate students' attitude towards mathematics (See Appendix B). The attitude scale consists of 20 statements about attitudes towards mathematics and mathematics lesson. There are 10 positive and 10 negative statements in the scale, with five possible alternatives: Completely appropriate, appropriate, undecided, inappropriate, and completely inappropriate. Each statement was graded as 4, 3, 2, 1 and 0 for positive statements, and 0, 1, 2, 3 and 4 for negative statements. AS was administered as a pre-test and post-test allowing 15 minutes. Pre-test AS results yielded a Cronbach Alpha reliability coefficient of internal consistency of 0.93.

3.2.3 Questionnaire

A questionnaire including 37 items was developed in order to measure the students' social and economic status together with their studying habits (See Appendix C). The questionnaire includes 24 yes-no, 7 multiple choice and 6 open-ended questions. The questions are related to the social and economic status of the family of the students, physical environment in the students' house, studying habits and leisure activities of the students. The questionnaire was administered prior to the treatment, allowing 20 minutes.

3.3 Procedure

The aim of the study was to investigate the effects of sociocultural learning environments including social interaction such as collaboration and cultural tools such as language on students' zone of proximal development (ZPD) in fractions concepts. In this study, two new instruments (CPPT and Questionnaire) were developed. Conceptual and Procedural Performance Test (CPPT) was developed by taking into consideration the findings from the previous studies, objectives of the National Elementary Mathematics Curriculum (MEB, 1999) and the results of the interviews carried out with the primary teachers and junior high school mathematics teachers. CPPT was pilot tested on 145 5^{th} grade students in a private elementary school in Ankara during 1999-2000 academic year. The details of piloting are given in Chapter IV. While developing the questionnaire, the expert ideas on different areas such as educational sciences, psychology and measurement were taken into consideration. Attitude Scale (AS) developed by Aşkar (1986) was used to collect data on the subjects' attitude towards the mathematics lesson.

The experiment was carried out in both groups at the same time in 2000-2001 academic year. In CG, the teacher taught the fractions according to the regular lesson plans at the school, which were prepared by the 5^{th} grade mathematics coordinator. In EG, the teacher instructed the fractions according to the lesson plans prepared by the researcher. The topics and the sequence of them were the same for both EG and CG. The only difference between EG and CG was that the worksheets prepared by the researcher was used in the former but not in the latter.

In the EG, students were assigned to five-member groups. They were instructed to work as a group, make decisions by consensus, complete the assignments together, ensure that all group members contributed their ideas and suggestions, and seek assistance primarily from

each other. They also sought help from the teacher and the researcher. While the students were working on the tasks given on the worksheet, the teacher as well as the researcher also tried to answer their questions. All the lessons of CG and EG during the treatment on fractions were observed and recorded by the researcher.

The questionnaire developed to measure the students' social and economic status together with their studying habits was given before the instruction on fractions. CPPT and AS were given as pre-test and post-test. The pre-tests were given prior to the instruction on fractions and the operations with fractions and the post-tests upon the completion of the topics on fractions, with an interval of approximately four months. The order of the questions in pre-CPPT was changed in the post-CPPT. Students were not informed for the pre-tests, but they were informed for the post-tests. As the parents of the students reacted negatively to not being informed in the pre-tests, students were informed a week before they were administered. Students were told that they would take a quiz in the coming week but the exact day was not mentioned to them. Upon the completion of the treatment of fractions the interviews were conducted.

Interviews were conducted with 10 students, four from CG and six from EG. Interview subjects were selected by identifying students within each group who performed at various levels on the pre-test scores of CPPT and AS. Considering EG students' observed behaviors in the collaborative small-groups, and both groups' students' pre-test scores from CPPT and AS respectively, students with high-high, high-low, low-high and low-low scores were chosen for the interview. The high attitude and performance scores were the ones in the 4^{th} quartile of pre-AS and pre-CPPT and the low attitude and performance scores were the ones in the 1^{st} quartile of pre-AS and pre-CPPT. Table 3.3 shows the interviewed students' groups, 4^{th} grade mathematics scores and pre and post CPPT and AS scores.

Students' names were not given because of the confidentiality but they are referred by a number.

Table 3.3: Interviewed students' characteristics in terms of genders, groups, 4th grade mathematics scores, pre and post test scores.

Student (Gender)	Group	4th Grade Math Score	Pre AS	Pre CPPT	Post AS	Post CPPT
S1 (F)	Experimental	3	49 (H)	5 (L)	55	30
S2 (M)	Experimental	5	10 (L)	4 (L)	58	24
S3 (M)	Experimental	5	45 (H)	20 (H)	35	28
S4 (M)	Experimental	5	73 (H)	9 (H)	63	30
S5 (M)	Experimental	5	54 (H)	12 (H)	50	30
S6 (F)	Experimental	5	8 (L)	11 (H)	51	35
S7 (F)	Control	4	72 (H)	5 (L)	73	20
S8 (M)	Control	5	56 (L)	5 (L)	38	14
S9 (F)	Control	5	42 (L)	9 (H)	63	35
S10 (F)	Control	5	67 (H)	10 (H)	69	28

Note: "F" represents "Female", "M" represents "Male"

Although the interviews for the study were primarily unstructured, with the interviewer spontaneously reacting to students' descriptions of their solutions, some structure was imposed by preparing interview questions in advance. During the interview, the researcher initially asked to the students to solve certain questions aloud on an empty sheet and later made a general reference to the responses the students had given on the post-test of CPPT. The student was given a few minutes to review the response given on the post-CPPT, then prompted to describe his or her solution and asked to provide justification and clarification to the solutions offered in the interview and in the post-test.

After the students' explanation, the researcher made general inquiries, such as, "explain" or "clarify" and continued to ask more specific questions, if necessary, until a response was elicited or it appeared that all knowledge had been elaborated. This process was repeated for certain questions in CPPT. Interview length varied from 30 to 45 minutes. The interview tone was amiable and non-threatening, and efforts were made to make students comfortable with providing candid response. Analysis of

interview results related to CPPT involves a careful reading of each interview transcript and written solutions, while attempting to identify common student responses and misconceptions.

During the interview, students were also asked to describe what are the things affecting their attitudes towards mathematics and what they thought about the teaching environment. These sections of the interview transcripts were analyzed separately to provide information regarding the factors that influence attitudes towards mathematics and the development of their mathematical abilities, as well as their interest in continuing to study mathematics.

In the study, both the qualitative and the quantitative methods were used to analyze the data. The independent variables of the study included gender and treatment whereas the dependent variables were (i): Post-CPPT scores, and (ii): Post-AS scores. The design of the study was a pretest-posttest CG design involving a 2 x 2 ANOVA model for CPPT and 2 x 2 ANCOVA model for AS.

After scoring each item, in CPPT and AS, group means and standard deviations for each group on CPPT and AS were computed. Besides, frequencies of each item in CPPT were calculated. Group differences in pre-tests were analyzed by independent samples t-test, with an alpha level of $\alpha = 0.05$. Group differences relative to post-CPPT scores were determined by using 2 x 2 ANOVA model by treatment and the gender as independent variables and post-CPPT scores as the dependent variable with an alpha level of $\alpha = 0.05$. Group differences relative to post-AS scores were determined by using 2 x 2 ANCOVA model by treatment and the gender as independent variables, pre-AS scores as the covariate and post-AS scores as the dependent variable with overall alpha level of $\alpha = 0.05$.

The results of the questionnaire were investigated through frequency distribution and percentages of each item for both EG and CG. The results were evaluated in terms of students' social and economic status and their studying habits.

In the qualitative part of the study, written work carried out by the students in the classroom such as group worksheets, classroom observations and the interviews with the students were analyzed in order to investigate the improvement in the students' ZPD. Groups' answers and explanations for a given task in the EG were investigated on group worksheets.

3.4 Treatments for the Experimental and Control Group

The main characteristics of the learning modes to which the experimental and control group students in this study were exposed are presented here.

All students received instruction on topics in their regular mathematics classes by using the conventional teaching approach except on fractions and the operations with fractions. During the teaching of fractions and the operations with fractions, one group was randomly assigned to the "experimental group (EG)" while one group was assigned to the "control group (CG)". These two groups were taught by the same teacher. The treatment in the EG comprised collaborative small-group work, whereas the treatment in the CG was carried out in the same way as before. The main difference between EG and CG was that collaborative small-group approach was used in the former but not in the latter.

The textbook "İlköğretim Matematik 5" (Elementary Mathematics 5) written by Taşkın and Çarhoğlu (1999) was used in both groups together with the 2nd, 3rd and 4th volumes of "Matematik Gezegeni 5. Sınıf"

(Mathematics Planet Grade 5) written by Doğancıoğlu and Çağlar (1999) as exercise books. The homeworks assigned from these books to both groups were the same. The students in two groups took regular common exams and quizzes. The objectives of the lesson plans of the both groups were taken from the National Elementary Mathematics Curriculum (MEB, 1998).

Fractions lasted 6 weeks to be taught. There were 4 mathematics classes in each week, lasting 40 minutes each. In the elementary school, mathematics is taught by the primary teachers till the end of the 3rd grade. From the beginning of the 4th grade, mathematics is taught by the mathematics teachers. The units taught in the 5th grade mathematics till the end of fractions are given in Table 3.4 together with the durations. The topics taught in fractions were given in bold in Table 3.4.

As can be seen in Table 3.4, fractions were not taught continuously. The topics in the fractions were included in different units. The operations on fractions were introduced in the operations unit following the computation on the natural numbers. For example, addition of fractions was given in the addition unit, after the addition of natural numbers.

The approaches used in CG and EG for teaching fractions and the operations with fractions are presented in detail as follows.

3.4.1 Treatment for the Control Group

The class, which constituted the control group, included 30 students. In this group, the teaching materials prepared by the 5th grade mathematics coordinator were used (See Appendix D). The objectives of each topic in fractions covered in the lesson plans are given in Table 3.5. The table also gives the numbers of tasks completed both by the teacher and the students under each objective.

Table 3.4: Topics taught from the beginning of the 5th grade, till the end of the fractions

Unit	Topics	Duration
Sets	Subset, intersection and union of sets, word problems with sets	1.5 Weeks
7, 8 and 9 digit Natural Numbers	Writing and reading natural numbers, number and digit value of natural numbers, ordering of natural numbers, analysis of natural numbers	2.5 Weeks
Fractions	**Kinds of fractions, converting fractions to each other, equivalence of fractions, ordering fractions.**	**2 Weeks**
Decimal Fractions	Writing decimals by using ",", decimals with 3-digit decimal places, comparison of decimals	2 Weeks
Addition	Addition of natural numbers	1 Week
	Addition of fractions	**1 Week**
	Addition of decimal fractions	1.5 Weeks
Subtraction	Subtraction of natural numbers	1.5 Weeks
	Subtraction of fractions	**1 Week**
	Subtraction of decimal fractions	1 Week
Review	Review of the previous units	1 Week
Multiplication	Multiplication of natural numbers	1 Week
	Multiplication of fractions	**1 Week**
	Multiplication of decimal fractions	1.5 Weeks
Review	Review of multiplication unit	
	SEMESTER HOLIDAY	
Division	Division of natural numbers	1.5 Weeks
	Division of decimal fractions	1.5 Weeks
	Relation between multiplication and division in natural numbers, division of natural numbers by 2 and 5	0.5 Week
	Word problems with natural numbers	0.5 Week
	Word problems with fractions	**0.5 Week**
	Word problems with decimal fractions	0.5 Week

Note: Topics on fractions are given in bold.

As mentioned previously, the materials used in this group were prepared by the 5th grade mathematics coordinator. It included teacher notes, handouts and two posters. Teacher notes consisted of the explanation of the topic and the exercises related to them. The handouts were prepared for the students to work during the lesson. The tasks given in the handouts were related to representing the given fractions on continuous figures such as circular region, apple or heart (See Appendix D).

Table 3.5: Objectives and number of tasks for fractions in CG lesson plans

Units	Objectives of the instruction	Number of tasks
Fractions		
Week 1	a) To show the figure of a simple, improper or mixed fraction	a) 9
	b) To write and read a fraction that is given by a figure	b) 9
	c) To convert a mixed fraction to an improper fraction	c) 3
	d) To convert an improper fraction to a mixed fraction	d) 1
	e) To write a natural number as a fraction	e) 2
	f) To expand a fraction by an integer	f) 2
	g) To find the irreducible form of a fraction	g) 3
	h) To select and sign the fraction or fractions that are equivalent to a given fraction from a group of fractions	h) 2
Week 2	a) To write the smallest or greatest fraction of two fractions with equal denominators by using symbol	a) 5
	b) To write the smallest or greatest fraction of two fractions with equal numerators by using symbol	b) 5
	c) To write a fraction that is greater than a given fraction and whose denominator or numerator is equal to the denominator or numerator of the given fraction	c) 2
	d) To write a fraction that is less than a given fraction and whose denominator or numerator is equal to the denominator or numerator of the given fraction	d) 2
	e) To organise at most five fractions with equal denominators in increasing or decreasing order by using symbol	e) 2
	f) To order at most five fractions with equal numerators in increasing or decreasing order by using symbol	f) 2
Addition		
Week 3	a) To add two simple fractions where one's denominator is a multiple of the other	a) 1
	b) To add two improper fractions where one's denominator is a multiple of the other	b) 2
	c) To add two mixed fractions where one's denominator is a multiple of the other	c) 3
	d) To add two simple fractions with one digit co-prime integers as denominators	d) 2
	e) To add two improper fractions with one digit co-prime integers as denominators	e) 1
	f) To add two mixed fractions with one digit co-prime integers as denominators	f) 1
	g) To add three simple, improper or mixed fractions where the denominators of two of them are equal and the denominator of the third is a multiple of the others	g) 3
	h) To write the addition of two fractions given by figures	h) 2
	i) To add a natural number with a fraction	i) 2
	j) To control the correctness of an addition with fractions	j)

Table 3.5: Continued.

Units	Objectives of the instruction	Number of tasks
Subtraction		
Week 4	a) To subtract two simple fractions where one's denominator is a multiple of the other	a) 2
	b) To subtract two improper fractions where one's denominator is a multiple of the other	b) 2
	c) To subtract two mixed fractions where one's denominator is a multiple of the other	c) 2
	d) To subtract two simple fractions with one digit co-prime integers as denominators	d) 2
	e) To subtract two improper fractions with one digit co-prime integers as denominators	e) 2
	f) To subtract two mixed fractions with one digit co-prime integers as denominators	f) 1
	g) To write the subtraction of two fractions given by figures	g) 2
	h) To subtract a fraction from a natural number	h) 3
	i) To control the correctness of a subtraction with fractions	i)
Multiplication		
Week 5	a) To multiply a natural number by a unit fraction	a) 1
	b) To multiply an improper fraction by a natural number	b) 3
	c) To multiply a mixed fraction by a natural number	c) 3
	d) To multiply a fraction by "1"	d) 1
	e) To multiply a fraction by "0"	e) 1
	f) To show the multiplication of two simple fractions by figure	f) 1
	g) To multiply two simple fractions	g) 2
	h) To multiply two improper fractions	h) 1
	i) To multiply two mixed fractions	i) 1

The posters were used as a demonstration in the class by the teacher. First poster, which was prepared by the class teacher, was related with different instruments in an orchestra. The aim here was to explain the comparison of fractions. Here, different musical instruments in the orchestra were represented by the fractions, and these fractions were compared. The last poster was about a farm and the products of the farm such as the crops and the milk. The aim of this poster was to explain addition of fractions with equal denominators, where fractions represent the products of the farm.

The method used in this class was conventional teaching method. In general, the teacher explained the concepts by writing them on the board, and then allowed students to write them on their notebooks. The teacher frequently solved exercises about the topics that were covered in the lesson. Major points that were going to be covered in the topics were initially given by drawing charts. While starting a new concept, she always

wrote the title on the board and used a chart including the subconcepts. An example of a chart is given in Figure 3.1.

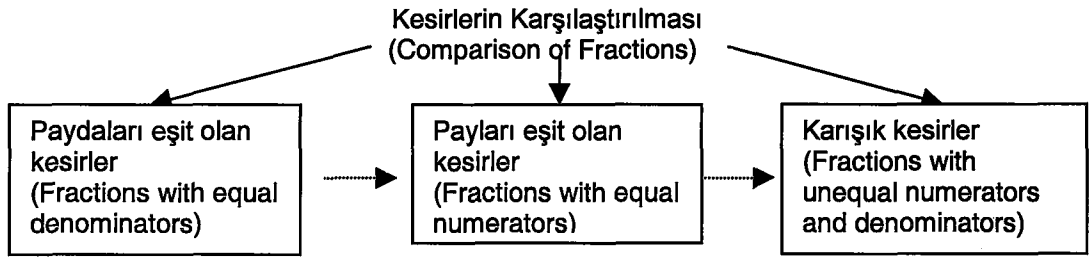


Figure 3.1: An example of chart used in the CG

As can be seen from the chart, teaching was done from the easiest to the difficult topics. The review of the previous topics was done when new topics were related to the previous ones.

In a typical class, the teacher started each lesson by controlling the homeworks given in the previous lesson. Following-up controlling the homework, the review of the previous lesson was done by writing the important rules, definitions or procedures, if the topic was the continuation of the previous lesson. Then, the lesson was carried out either by asking to the student to work on the similar exercises done on the previous lesson or explaining and writing a new rule or a new definition. In order to get students' attention to a new rule, a cloud around the rule with a colored interjection mark was drawn. Then the couple of exercises were solved related to the rule given by the teacher, and similar kinds of exercises were asked to the students to be solved in their own place. Usually the first two exercises were solved by the teacher on the board. After the students worked on the exercises on their pace, one student among the volunteer students was called to come to the board and show the solution of the exercise. The teacher explained the solution of the exercise again after the volunteer student's solution. Then she assigned homework related to the concept covered in that lesson.

As a summary, a map of the typical lesson of CG is given in Figure 3.2.

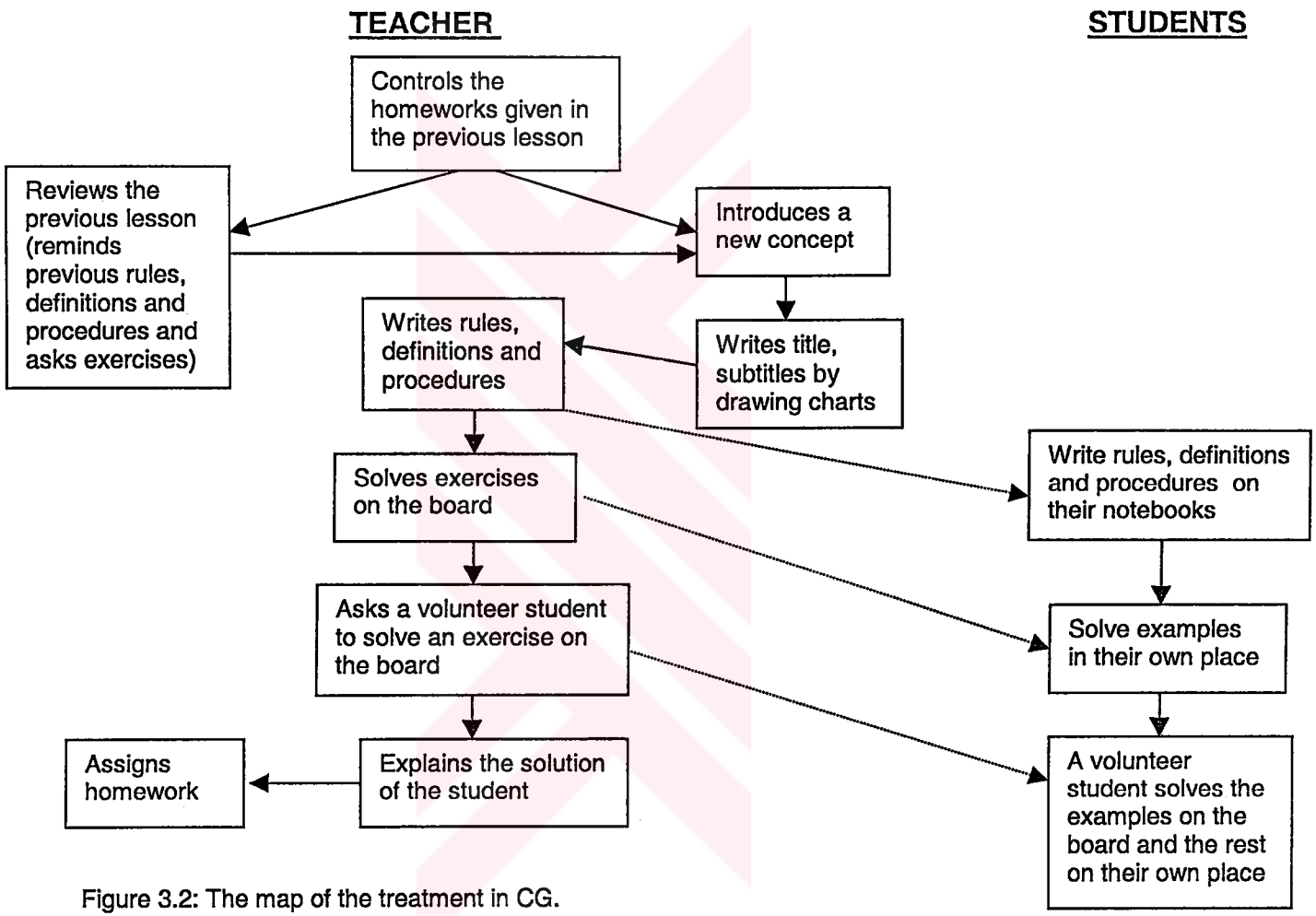


Figure 3.2: The map of the treatment in CG.

3.4.2 Treatment for the Experimental Group

The class, which constituted the experimental group, included 30 students. The method used in this class was collaborative small-group work. Groups of five students were formed in order to work on the worksheets prepared by the researcher.

For the treatment in EG, 13 worksheets were developed by taking into consideration the results of the pilot study of CPPT, the findings of the literature review and the objectives of fractions in the National Elementary Mathematics Education Program. The main aim in these worksheets was to teach fractions concepts and operations with fractions conceptually by using collaborative small-group approach. In these worksheets, students were required to discuss and write the rules related to their conjecture based on their work with manipulatives (See Appendix E). These manipulatives were colorful unfolded cardboards for representing a whole, colorful piece of cardboards for representing fractions in continuous forms, and play dough for representing fractions in discrete forms. The reason for using manipulatives was to make students observe the procedures in concrete situations and check the results of their work. The worksheets used on fractions together with their contents, and the manipulatives used are given in Table 3.6 in the treatment order.

The objectives of each material and the number of tasks for each objective are given in Table 3.7. Each worksheet was prepared to be worked as a group in order to discuss the tasks. In these tasks, the need for the discussion was emphasized if it was necessary. The tasks on operations (addition, subtraction and multiplication) were also designed to discuss as a group, but the need for the discussion was not emphasized in all activities on operations. The reason was to give more chance to the students for

performing operations with fractions. After discussing the tasks, the groups were asked to write their answers together with their reasons.

Each lesson was started by controlling the homeworks by the teacher. If the tasks in the current lesson were related to the tasks in the previous lesson, a review of the previous lesson was done on the board by the teacher in the same way as described in control group's treatment. If a new concept was to begin, no review was done and the material was distributed to the groups of students.

Table 3.6: Content of the worksheets in the EG.

Worksheets	**Topics covered in the worksheets**	**Manipulatives**
Fractions and We	**Fractions Unit (2 Weeks)**	
1.	Kinds of fractions Area and discrete representations of fractions	
2.	Converting mixed fractions and improper fractions to each other	Play-dough
3.	Expanding and reducing fractions	Cardboards
4.	Equivalent fractions	
5.	Ordering and comparing the fractions with equal denominators and equal numerators	Cardboards
6.	Ordering and comparing the fractions with unequal denominators and numerators	
	Addition Unit (1 Week)	
7.	Addition of fractions with equal denominators	Cardboards
8.	Addition of fractions with unequal denominators where one is a multiple of the other or both are some multiples of the same integer	
9.	Addition of fractions with unequal denominators where one is not a multiple of the other or both are any multiples of the same integer	Cardboards
	Subtraction Unit (1 Week)	
10.	Subtraction of fractions with equal denominators	Cardboards
11.	Subtraction of fractions with unequal denominators where one is a multiple of the other or both are some multiples of the same integer	
12.	Subtraction of fractions with unequal denominators where one is not a multiple of the other or both are not any multiples of the same integer	Cardboards
	Multiplication Unit (1 Week)	
13.	Multiplication of fractions	Cardboards

Table 3.7: The objectives and the number of tasks for each material in EG.

Fractions and We	Objectives of the Material	Number of Tasks
	Fractions Unit	
1.	a) To write different kinds of fractions b) To show different kinds of fractions on figures c) To write the fraction given on a figure d) To discuss the properties of different kinds of fractions	a) 2 b) 5 c) 5 d) 3
2.	a) To write the fraction shown by discrete objects b) To show the fraction by discrete objects c) To discuss a way for converting fractions to each other d) To convert an improper fraction to a mixed fraction e) To convert a mixed fraction to an improper fraction	a) 2 b) 1 c) 2 d) 2 e) 2
3.	a) To show the fraction by discrete objects b) To show the fraction on a continuous figure c) To discuss the properties of equivalent fractions	a) 3 b) 3 c) 4
4.	a) To recognize whether a given pair of fractions are equivalent b) To discuss the properties of equivalent fractions c) To write equivalent fractions d) To represent a fraction on the number line	a) 3 b) 5 c) d) 2
5.	a) To compare two fractions with equal denominator b) To compare two fractions with equal numerator c) To write the fraction shown by objects or figures d) To show the fraction on figure or by objects e) To write a rule for comparing fractions with equal numerators f) To represent a fraction on the number line g) To find the smallest fraction in a given group of fractions h) To find the greatest fraction in a given group of fractions i) To order fractions with equal denominator j) To order fractions with equal numerator k) To modify a rule for a new situation	a) 4 b) 5 c) 2 d) 2 e) 1 f) 5 g) 2 h) 2 i) 2 j) 2 k) 2
6.	a) To discuss the comparison of two fractions with unequal numerators and denominators b) To discuss the comparison of two mixed fractions with equal denominators and equal wholes c) To compare two fractions with unequal numerators and denominators	a) 5 b) 1 c) 5
	Addition Unit	
7.	a) To represent the given fractions by objects b) To write the fraction shown by the objects c) To represent the addition of two fractions with equal denominators by objects d) To state a rule for the addition of two fractions with equal denominators e) To add two fractions with equal denominators	a) 4 b) 2 c) 2 d) 1 e) 6
8.	a) To modify the known procedure to a new situation b) To add two fractions with unequal denominators where one is a multiple of the other or both are some multiples of the same integer c) To add three fractions with unequal denominators where one is a multiple of the other two d) To write a rule for the addition of fractions with unequal denominators where one is a multiple of the other or both are some multiples of the same integer	a) 2 b) 5 c) 1 d) 1

Table 3.7: Continued

Fractions and We	Objectives of the Material	Number of Tasks
9.	a) To check the correctness of a given operation by objects b) To discuss the addition of two fractions with unequal numerators and denominators where one is not a multiple of the other or both are not some multiples of the same integer c) To write a rule for addition of fractions with unequal numerators and denominators where one is not a multiple of the other or both are not some multiples of the same integer d) To add two fractions with unequal numerators and denominators e) To add three fractions with unequal denominators where one is some multiples of the other two	a) 1 b) 1 c) 1 d) 5 e) 1
	Subtraction Unit	
10.	a) To show the fraction by discrete objects b) To write the fraction represented by discrete objects c) To write the operation for a given situation d) To write a rule for subtraction of two fractions with equal denominators e) To subtract two fractions with equal denominators	a) 1 b) 1 c) 1 d) 1 e) 7
11.	a) To modify a known procedure to a new situation b) To write a rule for the subtraction of two fractions with unequal denominators where one is a multiple of the other or both are some multiples of the same integer c) To subtract two fractions with unequal denominators where one is a multiple of the other or both are some multiples of the same integer	a) 2 b) 1 c) 7
12.	a) To check the correctness of a given operation by using objects b) To modify a known procedure to a new situation c) To write a rule for the subtraction of two fractions with unequal denominators and numerators where one is not a multiple of the other or both are not some multiples of the same integer d) To subtract two fractions with unequal denominators and numerators where one is not a multiple of the other or both are not some multiples of the same integer	a) 1 b) 1 c) 1 d) 7
	Multiplication Unit	
13.	a) To analyze the multiplication of a fraction by a natural number from given examples b) To write a rule for multiplication of a fraction by a natural number c) To write the fraction shown by an object or a figure d) To show the fraction on a figure e) To write the multiplication of two fractions represented by a figure or an object f) To write a rule for multiplication of two fractions g) To multiply two fractions	a) 3 b) 1 c) 3 d) 2 e) 2 f) 1 g) 10

Note: In the table, the "figure" refers to the geometric figures on the worksheets, and the "object" refers to the manipulatives such as cardboards and play dough.

There were six groups of five students each. Each group was given a number. In each unit, the groups were formed differently. The formation of the groups in each unit and the reasons for these formations are given below:

1. Fractions Unit

   The groups were formed according to the students' mathematics scores from the teacher-made exams and quizzes. Each group included two students with higher mathematics scores, two students with lower mathematics scores and one student with average mathematics scores. The reason was to see if students with lower and mid ability benefit from the explanations and directions of students with higher ability in mixed ability groups. Two desks were placed face to face and one desk is placed at one end of these desks. The reason for such placement was to make students to see each other.

2. Addition Unit

   In the previous unit, some of the students complained for not being with their close friends during the lesson. Many students wanted to form a group with their friends. Students formed groups with their friends and the teacher and the researcher helped them to organize their groups. Absent students on the day of group formation were placed to the groups with fewer students in the next lesson. The desks were placed in their regular places since the placement in the fractions unit resulted in peer talk unrelated with the lesson in the mathematics classes as well as the other courses. Each group sat in two desks in total, with two students in one desk and three in the other and there were one desk left without students between two groups.

3. Subtraction Unit

   Although the groups worked quite well in the addition unit, the teacher wanted the students to form groups according to their

order of their school number. The reason was to punish students for not being quiet during the lesson. The researcher did not interfere in this process. The sitting plan was the same as in the addition unit.

4. Multiplication Unit

   The groups that were formed in the subtraction unit continued studying together as before.

The duration for working on the worksheet as a group ranged from 10 to 30 minutes. During the group work, students' questions related to the tasks were answered either by the teacher or by the researcher. After completing the tasks, students were asked to hand the worksheet back to the teacher to be checked. Following, each of the group members was given the same empty worksheet to be sticked on the notebook and complete them individually at home. The teacher checked whether the students completed the worksheets at home in the next day. Students who did not complete the worksheets were warned by the teacher.

As a summary, a map of the typical lesson of the EG is given in Figure 3.3.

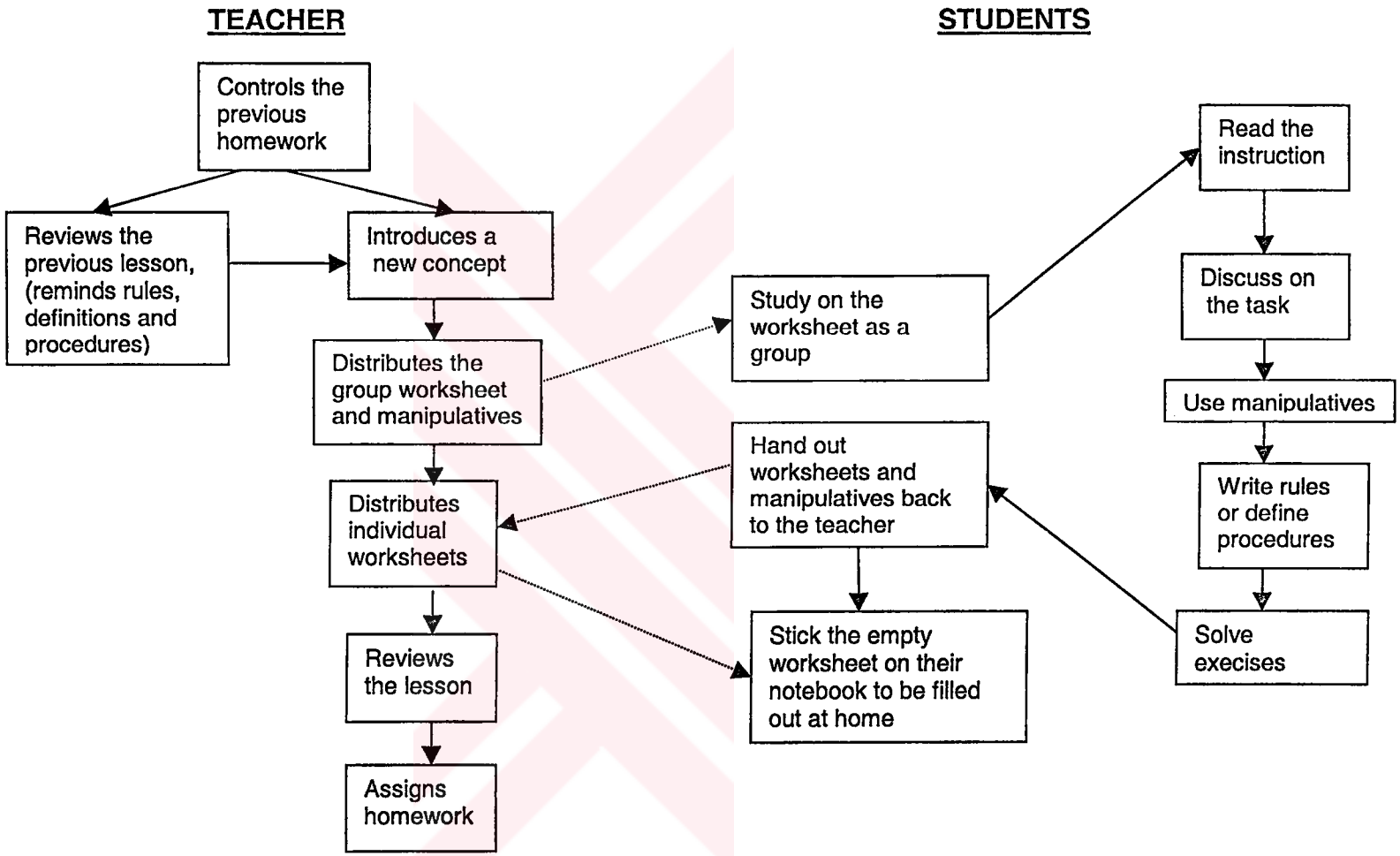


Figure 3.3: The map of the treatment in EG.

CHAPTER IV

PILOT STUDY OF THE CONCEPTUAL AND PROCEDURAL PERFORMANCE TEST ON FRACTIONS

An essay type concept and computation performance test, CPPT, including 14 questions was developed for the study in order to investigate the performances of students in using knowledge and skills on conceptual and computational tasks in the fractions concept (See Appendix F). Eight of these questions had subquestions, hence, the test included a total of 41 questions.

CPPT covered the following fraction concepts:

- Types of fractions and their properties,
- Representations of fractions,
- Relationship between improper and proper fractions,
- Equivalence of fractions,
- Order of fractions,
- Computations with fractions (addition, subtraction, multiplication).

CPPT was developed considering the literature findings, interviews conducted by junior high school mathematics and elementary school class teachers, and the objectives of the National Elementary School Mathematics Curriculum (MEB, 1999). Besides the questions that were similar to those solved in the mathematics lessons and exercise books, questions that could be solved by the knowledge and skills gained in

fractions unit, different from the regular ones were asked. In the latter type of questions, students were expected to relate and use different knowledge and skills about fractions. The objectives of each question were given in Chapter III, in Table 3.2.

The subjects of the pilot study were all the 5th grade students of a private elementary school in Ankara. There were a total of 145 students, attending six classes in the school. Three of these classes were taught mathematics by a teacher, and the rest were taught by another teacher. All students in the pilot study received the same instruction on fractions through the lesson plans prepared by the 5th grade mathematics coordinator.

Each question in CPPT was scored as either 1 or 0. 1 was given to each correct answer and 0 was given to each incorrect answer. SPSS 8.0 computer program was used for the analyses.

In the pilot study, frequency and percentage of each question and that of each objective in questions were computed in order to investigate the students' performances while using their knowledge and skills in the conceptual and computational tasks in fractions. In addition to that, errors in each question were analyzed to investigate how students construct the concepts in fractions. Reliability analysis was conducted, and validity evidences were verified.

4.1 Results

The descriptive statistics of the piloting is given in Table 4.1. The histogram of the pilot study involving the normal curve is given in Figure 4.1. The shape of the distribution was normal for pilot study of the CPPT, as Figure 4.1 shows. The distribution of objectives for each question is given in Table 4.2.

Table 4.1: Descriptive statistics of the pilot study of CPPT

Statistics	
N	145
Mean	22.76
Std. Error of Mean	0.63
Median	23
Mode	22
Std. Deviation	7.64
Variance	58.41
Skewness	-0.307
Range	33
Minimum	5
Maximum	38
Quartile I	18
Quartile II	23
Quartile III	29

Maximum score is 41.

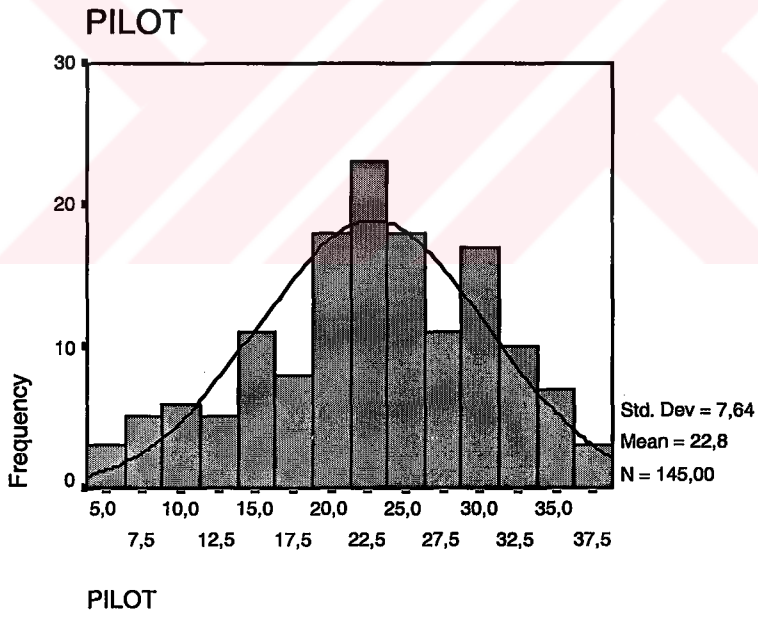


Figure 4.1: Histogram of the pilot study of CPPT.

Table 4.2: Frequency and percentage of students' correct and incorrect answers according to objectives

	1a	2a	3a	3b	3c	4a	5a	6a	6b	6c	6d	7a	7b	8a	9a	10a	10b	10c	10d	10e	10f	11a	12a	13a	13b	13c	C (%)	I (%)
Q1a																						+					106(73)	18(12)
Q1b																						+					114(79)	10(7)
Q1c																						+					46(32)	74(51)
Q2												+	+														33(23)	79(55)
Q3						+							+														29(20)	88(61)
Q4a															+												140(97)	5(3)
Q4b															+												80(55)	63(43)
Q4c															+												112(77)	29(20)
Q4d															+												138(95)	6(4)
Q4e															+												102(70)	39(27)
Q5a							+																				113(78)	25(17)
Q5b							+																				105(72)	30(21)
Q6a						+																					97(67)	29(20)
Q6b						+																					107(74)	18(12)
Q7a		+																									41(28)	23(16)
Q7b		+																									62(43)	11(8)
Q7c		+																									54(37)	20(14)
Q7d		+																									58(40)	18(12)

Table 4.2: Continued

	1a	2a	3a	3b	3c	4a	5a	6a	6b	6c	6d	7a	7b	8a	9a	10a	10b	10c	10d	10e	10f	11a	12a	13a	13b	13c	C (%)	I (%)
Q8a	+																										120(83)	20(14)
Q8b	+																										121(83)	19(13)
Q8c	+																										103(71)	37(26)
Q8d	+																										103(71)	36(25)
Q8e	+																										103(71)	36(25)
Q9a				+																							90(62)	51(35)
Q9b			+																								118(81)	24(17)
Q9c					+																						100(69)	38(26)
Q9d					+																						75(52)	58(40)
Q10a																							+				78(54)	54(37)
Q10b																							+				56(39)	82(57)
Q10c																							+				122(84)	20(14)
Q10d																							+				113(78)	29(20)
Q11a																					+						87(60)	36(25)
Q11b																+											91(63)	41(28)
Q11c																	+										103(71)	30(21)
Q11d																			+								36(25)	63(43)
Q11e																						+					30(21)	68(47)
Q11f																		+									43(30)	63(43)
Q11g																		+									42(29)	60(41)
Q12																								+	+	+	25(17)	56(39)
Q13								+	+	+	+																16(11)	112(77)
Q14																											12(8)	64(44)

The internal consistency reliability estimate of the test was computed by Kuder-Richardson (KR-20) formula as 0.89.

In order to verify the content validity, CPPT questions were reviewed by two researchers studying in the mathematics education area and by four primary teachers in other schools and confirmed to be proper for 5^{th} grade students in terms of length and level. Before the pilot study was conducted, the teachers who were teaching the 5^{th} grade mathematics were interviewed about the questions and the appropriateness of the questions to the topics in the fractions unit. The piloting was done in January 2000, allowing 50 minutes.

Prior to the administration of CPPT, all students took two teacher made quizzes in the fractions concept. The correlation coefficient between the scores from the teacher made quizzes and scores of students from CPPT were computed and given in Table 4.3 where $\alpha = 0.01$.

Table 4.3: Correlation coefficients between CPPT and teacher made quizzes.

Quizes	Correlation Coefficient	N
Quiz 1	0.579	131
Quiz 2	0.670	125

Correlation coefficients indicated that CPPT evaluates the similar behaviors in the fractions concept that other instruments evaluated, which was the concurrent validity evidence.

Common answers and misconceptions of students in each question are given below in detail.

Question 1(Question 11 in pre-CPPT)

Students performed differently in different subquestions of question 1. Although the subquestions involved the same objective, the context of the questions affected students' performances. First two subquestions (1(a)

and 1(b)) were answered correctly to a great extend (73% and 79%, respectively), but the last one, 1(c), had a low correct percentage (32%). Results show that mixed fraction concept was not completely build in students' mind.

Question 2 (Question 7 in pre-CPPT)

There were two objectives involved in question 2. The percentages of the students who performed each objective and the question completely are given in Table 4.4:

Table 4.4: Frequency and percentage of correct and incorrect answers of question 2 in the objective base.

	Correct (%)	**Incorrect (%)**
Objective 1	43(30)	69(48)
Objective 2	74(51)	15(10)
Question	33(23)	79(55)

Most of the students considered four wholes as one whole. So they partitioned the four wholes, each in five equal parts, and crossed out 16 of these equal parts. The resultant fraction was written to represent the 16 of 20 equal parts. Although the students represented the four wholes correctly with figures, they failed to write the fraction that the figures represented. But they wrote the kind of the fraction that was given as an answer correctly.

Question 3 (Question 8 in pre-CPPT)

The low level of performance (20%) in this question was a result of thinking 5 as "a whole". In the figures and the computations that were given in answering this question, students showed 1 as "one whole" assuming that what they showed was 5. Since "one whole" was easier to show than "a few wholes", and the question was different than the questions that the students solved, most of the students could not solve.

Question 4 (Question 9 in pre-CPPT)

When different continuous and discrete representations were given, students had difficulties in deciding whether the representations belong to a fraction or not. The reason was that the equal partitioning was ignored by the students and building a relationship between the representations and fractions were not complete. But in all subquestions in question 4, students performed more than 50%.

Questions 5 and 6 (Question 5 and 4 in pre-CPPT, respectively)

These questions asked the converting of fractions and the performances in both questions were close (78% for 5(a) and 72% for 5(b); and 67% for 6(a) and 74% for 6(b)). Students tried memorized procedures in converting one form of fraction to the other, which showed that they did not understand the meaning of the correct procedure. Hence, they tried some variations of the operations with the numbers given on the fractions in converting improper fraction to mixed fraction:

- To write the tens digit of the two-digit number in the numerator as the whole part and leave the ones digit as the numerator,
- To separate the two-digit number in the numerator into two terms (whose sum will be the two-digit number) so that the greater number will be the whole part and the smaller number will be the numerator,
- To write the quotient to the numerator and the remainder to the whole part, after dividing the numerator by the denominator,

and in converting mixed fraction to the improper fraction:

- To write the whole part of the mixed fraction as the tens digit of the one-digit number in the numerator,
- To multiply the whole part of the mixed fraction by the numerator and the denominator of the same fraction,
- To decrease the numerator and the denominator of the mixed fraction by the whole part of the same fraction.

In some of the cases, although the resultant fraction was not the type of the fraction required, students wrote the result as an answer. These students did not use their fraction knowledge and did not control their results.

Question 7(Question 2 in pre-CPPT)

Equivalence of fractions was not performed well by the students (less than 50% performance). Fractions, which satisfy the following criterion, were considered as equal:

- Only one of the numerators was a multiple of the other or the same,
- Only one of the denominators was a multiple of the other or the same,
- The numerator and the denominator changed place
- The numerator and the denominator of one of the fraction were not the same multiple of the other fractions the numerator and the denominator, respectively.

There were four pairs of equivalent fractions in question 7, and each were considered as a question in the analysis.

Question 8 (Question 1 in pre-CPPT)

The performances of students in writing he fractions given on the number line were generally high. Students performed lower in writing fractions greater than one (71%), than in writing fractions less than one (83%). The reasons were the incomplete understanding of the meaning of the fractions greater than one and the reliability of writing simple fractions. In the correct answers, students tended to write the fractions greater than one as mixed fractions.

Question 9 (Question 3 in pre-CPPT)

Students performed better in ordering fractions with equal denominator (81% for 9(b) and %69 for 9(c)) than in ordering fractions with equal numerator (62% for 9(a) and 52% for 9(d)). In some of the cases, students used wrong symbols in ordering even fractions with equal denominators.

Question 10 (Question 12 in pre-CPPT)

Finding the missing term in an ordered pair of fractions was easier for students when the fractions had equal denominators (84% for 10(c) and 78% for 10(d)). When the numerators and the denominators were not equal, students' performances decreased (54% for 10(a) and 39% for 10(b)). The reason might be the lack of use of additional knowledge and skill, such as equivalent fractions.

Results from the 9^{th} and 10^{th} questions show that students could not completely understand the meanings of symbols in order relationships and when to use these symbols. When students' wrong answers to both questions were considered, it seemed that they could not use the methods that they learnt to use only in one task, in the other similar tasks. When

different tasks required the same methods, they tended to use memorized methods or invent new methods for the tasks.

Question 11(Question 10 in pre-CPPT)

When operations were presented, students' performances in multiplication of fractions and subtraction of different types of fractions were the lowest (less than 30%). The common misconception in addition was that students added the numerators and the denominators separately. Moreover, in equalizing the denominators, they multiply the denominators of the fractions, but not the numerators. Subtraction of simple fractions was easier for students (71% for 11 (c)). But subtracting numerators and denominators separately was also detected. Trying to perform operations with mixed fractions was the reason of low performances in subtraction (25% for 10(d)). In multiplication of mixed fractions with a natural number, students multiplied only the whole part of the mixed fraction with the natural number. Another misconception of students was equalizing the denominators of fractions and multiplying them after this. The errors and misconceptions in reducing fractions such as reducement between numerators and reducement between denominators also led students in wrong results in multiplication.

Question 12 (Question 13 in pre-CPPT)

There are three objectives involved in question 12. The frequencies and the percentages of students who answered the tasks related to each objective and the question in total are given in Table 4.5.

Table 4.5: Frequency and percentage of correct and incorrect answers of question 12 in the objective base.

	Correct (%)	**Incorrect (%)**
Objective 1	51(35)	30(21)
Objective 2	26(18)	14(10)
Objective 3	26(18)	45(31)
Question	25(17)	56(39)

When trying to find the missing term in the subtraction, some of the students wrote 2 as $1\frac{7}{7}$, and converted $\frac{10}{7}$ to $1\frac{3}{7}$. By this way, they could not perform the subtraction correctly. But students, who converted 2 to improper fraction as $\frac{14}{7}$ were able to perform the subtraction correctly. Some of the students who found the missing term as $\frac{4}{7}$ were able to understand that the equivalent fraction of $\frac{4}{7}$ was needed to represent the fraction in the given discrete set. But most of the students who found the missing term correctly represented the fraction on the discrete set by crossing out only 4 circles, which was actually a representation of $\frac{4}{21}$.

Question 13 (Question 6 in pre-CPPT)

In question 13, four objectives were involved in and the frequencies and the percentages of correct and incorrect responses to the tasks related to the objectives are given in Table 4.6.

Table 4.6: Frequency and percentage of correct and incorrect answers of question 13 in the objective base.

	Correct (%)	**Incorrect (%)**
Objective 1	110(76)	16(11)
Objective 2	111(7)	13(9)
Objective 3	53(37)	20(14)
Objective 4	25(17)	22(15)
Question	16(11)	112(77)

As can be seen from Table 4.5, most of the students wrote the fractions represented by the figures. Most of these students and students who wrote the fractions wrong were able to add the fractions that they wrote correctly. But the performance in writing the addition of these fractions with another operation was low. Some of the students answered this question as addition of different fractions, which gave the same result. And the representation of the sum of the fractions was also low. Students failed in

representing an improper fraction by figures and partitioned the figures that they drew according to the numerator, not the denominator. It was seen that, students were not aware of the use of results of the previous step in the current step, and hence, they could not build a relation between the steps of the solution.

Question 14 (Not present in pre and post CPPT)

Computing the unknown term in a multiplication of a natural number with a fraction was hard for students. The performance on this question was very low (%8), which was caused by the misconceptions related to decreasing fractions by a natural number. Moreover, students did not control whether the result satisfied the condition of being less than 7 given in the question.

At a result of the pilot study, 14th question was decided to remove from the test, due to the low performance on this item. Although the performance on the question 1(c) (%32) was lower than the performances of 1(a) (%73) and 1(b) (%79), question 1(b) was decided to remove from CPPT. The reason was that, students write the reverse of the fraction that they wrote in 1(a), in most of the cases, and their answers to the improper fraction was not independent of the simple fraction answers. Moreover, the number of pair of equivalence fractions in question 7 was decreased from four to three for time considerations, by removing the pair of equivalence fractions $\frac{2}{7}$ and $\frac{4}{14}$. In the 11th question, it was seen that students spent much time on computations with the natural numbers during multiplication of fractions. Hence, the fractions were changed in 11(e) and 11(g), with the fractions that were easier to compute.

Finally, CPPT consisted of 38 items, 8 of which included subtasks. The new reliability coefficient after two questions were removed was 0.88.

CHAPTER V

RESULTS

Results of the pre and post tests together with the comparison analyses, are given in detail in this chapter. In addition to that analysis and the interpretations of the qualitative data, such as, the interview the group worksheets are presented.

5.1 Results of the Pre-Tests

Conceptual and Procedural Performance Test (CPPT) and Attitude Scale (AS) were implemented to CG and EG as pre-tests prior to the instruction of fractions.

5.1.1 Conceptual and Procedural Performance Test

The descriptive statistics of the pre-CPPT scores in CG and EG are given in Table 5.1. The Box-and-Whisker plots and the histograms of the pre-CPPT scores in CG and EG in the are given in Figures 5.1, 5.2 and 5.3. respectively.

As can be seen from the Box-and-Whisker plots, the rectangles coincide in the score axis, that is, each group's quartiles, ranges and medians are nearly the same.

As can be seen from the histograms, the shape of the distribution in the CG had a slight negative skew, whereas the distribution of the EG had a slight positive skew.

Table 5.1: Descriptive statistics of the pre-CPPT scores for EG and CG

Statistics	CG	EG
N	26	27
Mean	7.58	8.15
Std. Error of Mean	0.67	0.88
Median	8	8
Mode	6.10	7.8
Std. Deviation	3.43	4.59
Variance	11.774	21.05
Skewness	-0.253	0.604
Range	14	20
Minimum	0	0
Maximum	14	20
Quartile I	5	6
Quartile II	8	8
Quartile III	10	11

Note: Maximum score is 38.

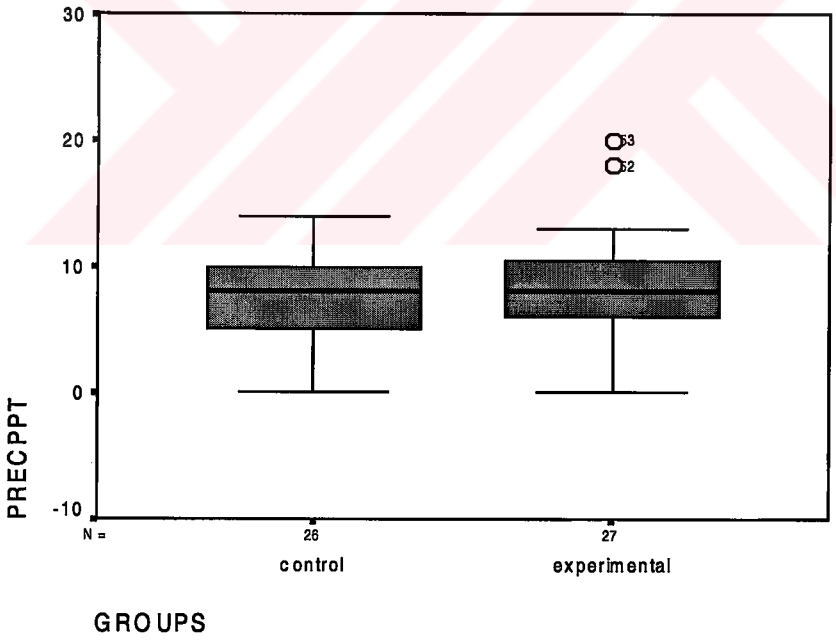


Figure 5.1: Box-and-Whisker plots of pre-CPPT scores for CG and EG

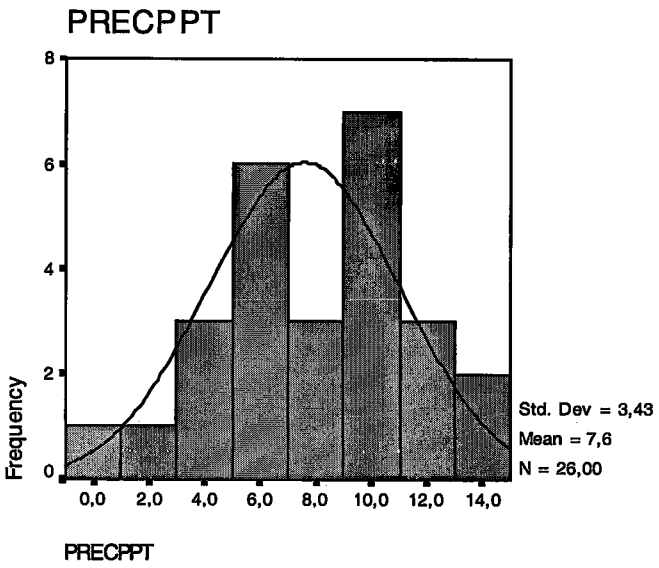


Figure 5.2: Histogram for pre-CPPT scores of the CG

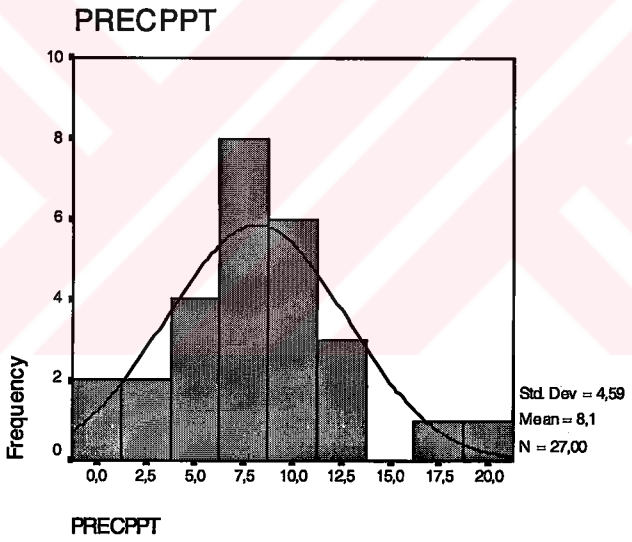


Figure 5.3: Histogram for pre-CPPT scores of the EG

The frequencies of students' correct and incorrect answers for the questions in the pre-CPPT in both groups are given in Table 5.2. First number represents the frequency of the correct answer; second number represents the frequency of the incorrect answers.

Table 5.2: Frequency of correct and incorrect answers of CG and EG students' in pre-CPPT.

Questions	CG (N = 26) Correct/Incorrect	EG (N = 27) Correct/Incorrect
1a	5/21	10/14
1b	6/20	8/16
1c	1/25	3/21
1d	1/25	3/20
1e	1/25	3/20
2a	3/23	1/8
2b	7/19	2/2
2c	6/20	1/5
3a	5/21	1/21
3b	22/4	21/2
3c	11/15	12/10
3d	3/23	1/20
4a	0/26	1/12
4b	0/26	1/12
5a	0/26	1/7
5b	0/26	0/7
6	0/22	0/26
7	1/16	0/11
8	1/22	0/21
9a	19/6	20/7
9b	20/4	20/7
9c	8/16	8/19
9d	18/6	14/12
9e	15/7	12/14
10a	0/5	0/16
10b	0/5	0/14
10c	1/3	7/5
10d	0/3	0/9
10e	0/3	0/12
10f	0/4	0/12
10g	0/4	0/7
11a	11/10	13/6
11b	3/14	5/11
12a	4/15	9/14
12b	1/17	6/17
12c	14/4	19/4
12d	10/9	18/5
13	0/26	0/11

Note: Missing frequencies are not included.

The following observations arise from this table. In most of the questions, students' performances were similar both in CG and EG. There was no much difference in the frequency of correct and incorrect answers in most of the questions. But, in general, more students in the CG than in the EG tried to solve the questions in pre-CPPT, except the questions related to the computation (Question 10) and finding the missing term in an ordered pair of fractions (Question 12). In addition to that, CG students did better in equivalence questions (Question 2), whereas EG students did better in finding the missing term in an ordered pair of fractions (Question 12).

Independent samples t-test was carried out in order to examine whether there was a significant difference between CG and EG prior to the instruction of fractions. The results showed that there was no significant difference between CG and EG with respect to the pre-test prior to the instruction of the fractions ($t = -0.515$, $p = 0.609 > 0.05$).

5.1.2 Attitude Scale (AS)

The descriptive statistics for the pre-AS scores in CG and EG are presented in Table 5.3.

The Box-and-Whisker plots of CG and EG in the pre-tests are given in Figures 5.4. In the Box-and-Whisker plots, the rectangles do not coincide in the scores axis, which indicates that there is a difference between the medians of two groups. The quartiles, ranges, and medians of the groups are different.

The histograms of pre-AS scores for CG and EG are given in Figure 5.5 and Figure 5.6. respectively. The shape of the distribution indicated a slight negative skew in both EG and CG, as can be seen from the histograms of pre-AS.

Table 5.3: Descriptive statistics of pre-AS scores for CG and EG.

Statistics	CG	EG
N	26	27
Mean	64.81	45.63
Std. Error of Mean	1.70	3.40
Median	67	48
Mode	67	45a
Std. Deviation	8.64	17.66
Variance	74.72	311.93
Skewness	-0.973	-0.453
Range	36	66
Minimum	42	8
Maximum	78	74
Quartile I	59	34
Quartile II	67	48
Quartile III	72	57

Note: Maximum score is 80.

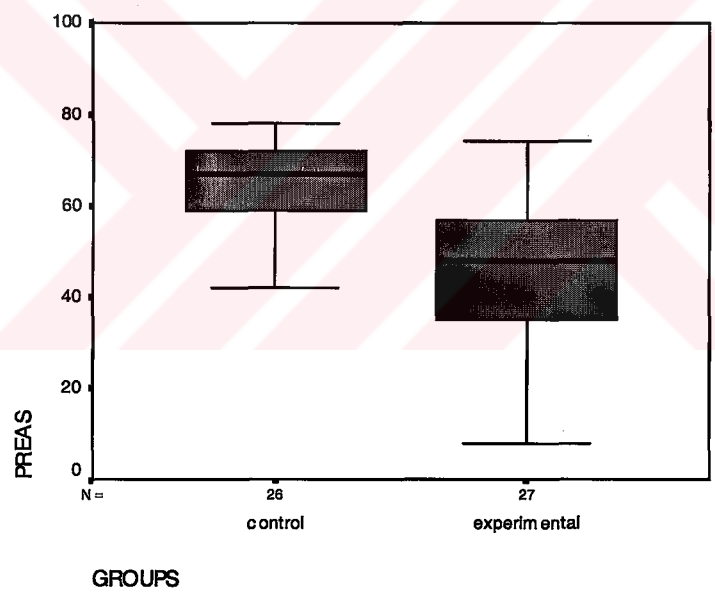


Figure 5.4: Box-and-Whisker plots of pre-AS scores for CG and EG

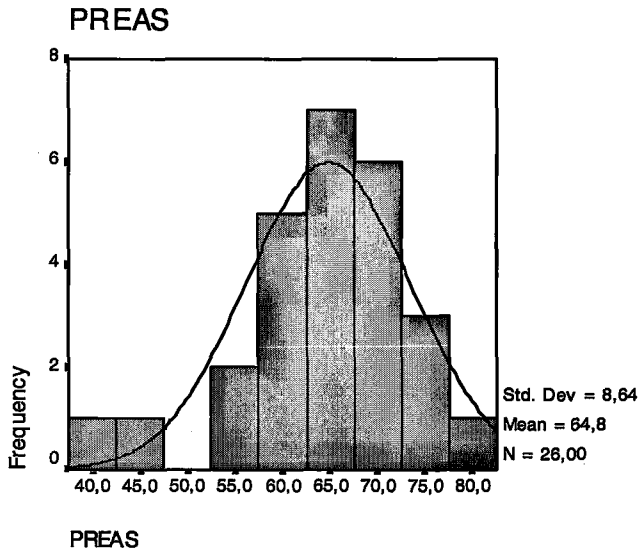


Figure 5.5: Histogram for pre-AS of the CG.

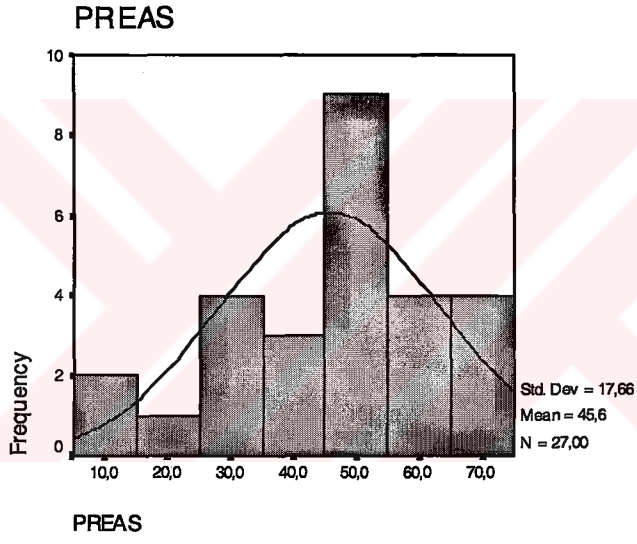


Figure 5.6: Histogram for pre-AS of the EG

Independent samples of t-test was carried out in order to examine whether there was a significant difference between CG and EG with respect to the attitude prior to the instruction of fractions. The results showed that there was a significant difference between CG and EG with respect to the AS prior to the instruction of the fractions ($t = 4.990$, $p = 0.01$). From Table 5.3, it can be seen that the CG's mean is greater than the EG's mean.

5.1.3 Questionnaire

The percentages of the responses given to the questionnaire are presented in Table 5.4, 5.5, 5.6, 5.7 and 5.8. The percentages of missing items are not included in the tables.

The Table 5.4 indicated that, family facts and the economic facts of students in the EG and CG differed only in education of the parents and the mothers' employment status. In the EG, 63% of students' mothers had a university degree, whereas 85% of the mothers of students in the CG had a university degree. Similarly, 67% of fathers of the students in the EG had a university degree, whereas the percentage in the case of CG was 85%. When the mothers' employment status were considered, 74% of the mothers of the students in the EG were working, which was less than the percentage of the mothers of the students in the CG (81%).

Housing facts were quite similar (see Table 5.6), for both group students. The percentages of students who have their own room were almost the same for EG and CG (92% and 96%, respectively).

The students in both EG and CG differed slightly on the studying habits (see Table 5.7). Students in EG compared to the students in CG studied more often when there was a family member in the room (15% for EG and 4% for CG) or during watching TV (33% for EG and 4% for CG). EG students had frequent breaks (56%), whereas CG students did not tend to have frequent breaks (8%). Fewer students in the EG studied everyday (63%) than the CG (92%).

When the leisure time activities of both group students' were compared (see Table 5.8), fewer students in the EG (57%) compared to the students in CG (96%) read books. All students in both groups knew how to use computers.

Table 5.4: Percentages of the answers given to the questions related to the students' family facts in CG and EG.

Family Facts	Control Group	Experimental Group
Mother is alive	Yes (100%)	Yes (96%), No (4%)
Father is alive	Yes (100&)	Yes (96%), No (4%)
Parents live together	Yes (100%)	Yes (85%), No (11%)
Number of sisters or brothers	0 (%31), 1 (54%), 2 (15%)	0 (29%), 1 (63%), 2 (4%), 3 (4%)
Order in the children of the family	1 (69%), 2 (23%), 3 (8%)	1 (48%), 2 (44%), 3 (%4), 4 (4%)
Number of younger sisters or brothers	0 (58%), 1 (34%), 2 (8%)	0 (82%), 1 (18%),
Existence of sisters or brothers who are students	Yes (42%), No (27%)	Yes (52%), No (22%)
Schools of the sisters or brothers	Elementary School (19%), High School (11%), University (11%)	Elementary School (15%), High School (26%), University (15%)
Education of the mother	High School (7%), 2-Year Vocational School (4%), University (85%)	Elementary School (3%), High School (15%), 2-Year Vocational School (11%), University (63%)
Education of the father	High School (4%), 2-Year Vocational School (8%), University (85%)	Elementary School (3%), High School (11%), 2-Year Vocational School (11%), University (67%)
Number of people living in the house	3 (27%), 4 (54%), 5(19%)	2 (7%), 3 (30%), 4 (56%), 5 (7%)

Table 5.5: Percentages of the answers given to the questions related to the economic facts of the family of students in CG and EG.

Economic Facts of the Family	Control Group	ExperimentalGroup
Mother works	Yes (81%), No (19%)	Yes (74%), No (19%)
Father works	Yes (100%)	Yes (93%)
Profession of the mother	Housewife (19%), Secretary (3%), Priciple/V.Principle (3%), Bank Employee (7%), Teacher (19%), Doctor (11%), Engineer (3%), Journalist (3%), Financial Consultant (3%), Chemist (7%), Nutrition Specialist (3%), Lawyer (3%)	Housewife (22%), Secretary (7%), Priciple/V.Principle (11%), Bank Employee (3%), Teacher (18%), Doctor (3%), Engineer (3%), Pharmacist (3%), Architect (11%), Psychologist (3%)
Profession of the father	Architect (11%), Priciple/V.Principle (7%), Doctor (11%), Engineer (19%), Lawyer (3%), Self-employed (15%), Pharmacist (3%), Dentist (7%), Member of the Parliament (7%)	Architect (3%), Priciple/V.Principle (7%), Doctor (3%), Engineer (29%), Lawyer (7%), Self-employed (15%), Bank Employee (3%), Teacher (3%), METU Employee (3%), METU Academician (3%), Cameraman (3%)
Home belongs to family	Yes (89%), no (11%)	Yes (74%), No (%18)

Table 5.6: Percentages of the answers given to the questions related to the housing facts of the family of students in CG and EG.

Housing Facts of the Family	Control Group	Experimental Group
Number of rooms in the house (except the kitchen and the living room)	2 (4%), 3 (46%), 4 (19%), More than 4 (31%)	2 (4%), 3 (44%), 4 (30%), More than 4 (22%)
Existence of own room	Yes (92%), No (8%)	Yes (96%), No (4%)
Sharing the room with sisters or brothers	Yes (4%), No (65%)	Yes (3%), No (67%)

Table 5.7: Percentages of the answers given to the questions related to the students' studying habits in CG and EG.

Studying Habits of the Students	Control Group	Experimental Group
Existence of a family member in the studying room	Yes (4%), No (96%)	Yes (15%), No (85%)
Existence of a family member who helps when studying	Yes (62%), No (38%)	Yes (70%), No (30%)
Helps anyone in the family when studying	Yes (23%), No (77%)	Yes (15%), No (85%)
Study Everyday	Yes (92%), No (8%)	Yes (63%), No (33%)
Approximate time for studying in a day (hour)	0-1 Hour (8%), 1-2 Hours (38%), 2-3 Hours (35%), 3-4 Hours (11%)	0-1 Hour (33%), 1-2 Hours (44%), 2-3 Hours (11%), 3-4 Hours (4%), More than 4 Hours (4%)
Approximate time for studying mathematics in a week (hour)	0-1 Hour (15%), 1-2 Hours (23%), 2-3 Hours (8%), 3-4 Hours (23%), More than 4 Hours (27%)	0-1 Hour (18%), 1-2 Hours (33%), 2-3 Hours (15%), 3-4 Hours (18%), More than 4 Hours (15%)
Has frequent breaks	Yes (8%), No (92%)	Yes (56%), No (44%)
Studies in the TV room	Yes (11%), No (89%)	Yes (59%), No (37%)
Studies during watching TV	Yes (4%), No (96%)	Yes (33%), No (67%)
Studies by listening to music	Yes (58%), No (42%)	Yes (48%), No (52%)

Table 5.8: Percentages of the answers given to the questions related to the students' leisure time activities in CG and EG.

Leisure Time Activities of the Students	Control Group	Experimental Group
Reads books	Yes (96%), No (4%)	Yes (57%), No (33%)
There is a newspaper in the house everyday	Yes (96%), No (4%)	Yes (89%), No (11%)
Reads newspaper everyday	Yes (40%), No (50%)	Yes (33%), No (63%)
Kind of music	Turkish Pop Music (19%), Pop Music (81%)	Turkish Pop Music (4%), Turkish Classical Music (4%), Pop Music (92%)
Makes sports	Yes (100%)	Yes (96%), No (4%)
There is a computer in the house	Yes (81%), No (19%)	Yes (89%), No (11%)
Knows how to use computer	Yes (100%)	Yes (100%)

5.2 Results of the Post-Tests

Conceptual and Procedural Performance Test (CPPT) and Attitude Scale (AS) were implemented to CG and EG as post-tests following the instruction of fractions.

5.2.1 Conceptual and Procedural Performance Test

The descriptive statistics of the post-CPPT scores in CG and EG are given in Table 5.9.

The Box-and-Whisker plot diagrams of CG and EG in the post-tests are given in Figure 5.7. The plot shows that the medians of both groups are almost equal, whereas the ranges differ to a great extent.

EG's range is less than the CG's range, with two outliers.

The histograms of both groups' scores on post-CPPT are given in Figure 5.8 and Figure 5.9. As can be seen from the histograms, the shape of the distribution is left skewed in the CG, but it is close to normal distribution in EG.

Table 5.9: Descriptive statistics of post-CPPT scores for EG and CG.

Statistics	CG	EG
N	26	27
Mean	22.62	22.07
Std. Error of Mean	1.53	1.73
Median	24.5	24
Mode	26, 31	27
Std. Deviation	7.79	9.01
Variance	60.73	81.15
Skewness	-0.807	-1.023
Range	33	31
Minimum	2	4
Maximum	35	35
Quartile I	16.75	18
Quartile II	24.5	24
Quartile III	28.5	27

Note: Maximum score is 36

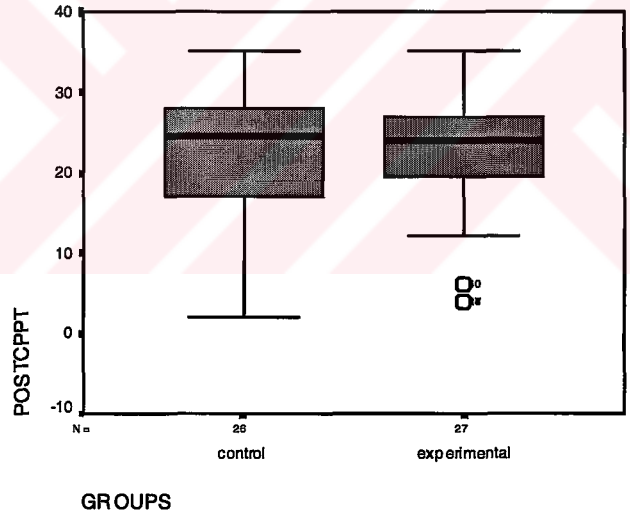


Figure 5.7: Box-and-Whisker plots of post-CPPT scores for CG and EG.

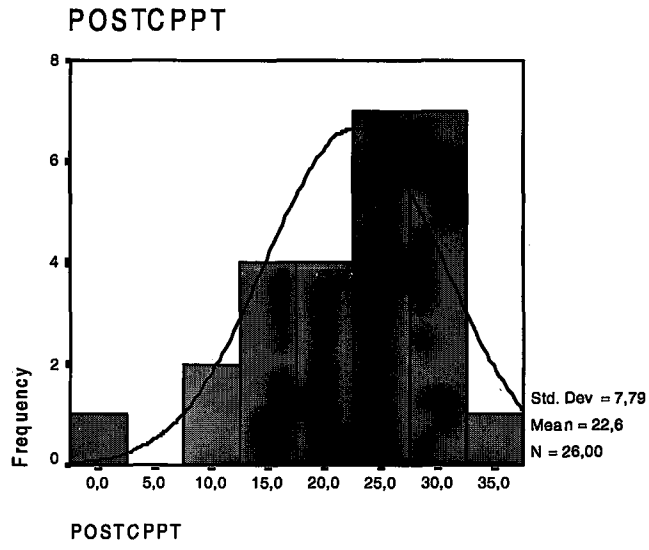


Figure 5.8: Histogram of post-CPPT scores for CG

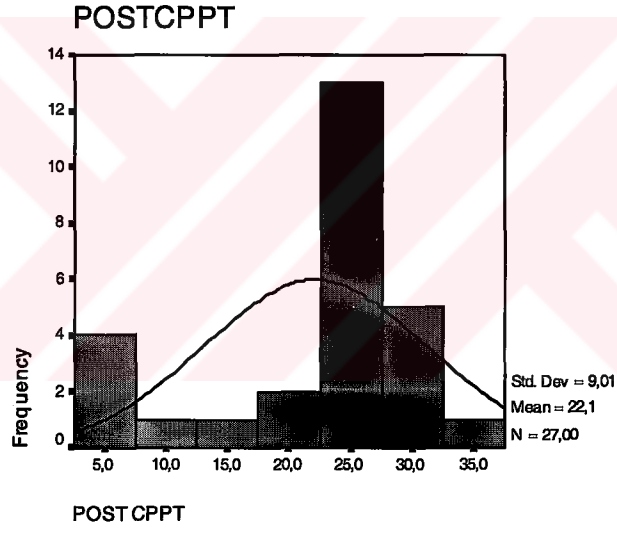


Figure 5.9: Histogram of post-CPPT scores for EG.

The frequencies of students' correct and incorrect answers for the questions in the posttest in both groups are given in Table 5.10. First number represents the frequency of the correct answer; second number represents the frequency of the incorrect answers.

Table 5.10: Frequency of correct and incorrect answers of CG and EG students in post-CPPT.

Questions	Control (N=27) Correct/Incorrect	Experimental (N=28) Correct/Incorrect
1a	18/6	20/7
1b	18/6	20/7
1c	13/11	14/13
1d	14/10	16/11
1e	14/10	15/12
2a	11/6	17/1
2b	20/0	17/1
2c	17/1	18/0
3a	17/8	17/8
3b	23/3	22/3
3c	23/2	20/5
3d	12/12	12/12
4a	19/2	20/5
4b	20/1	20/5
5a	19/6	21/4
5b	21/4	22/3
6	5/20	5/22
7	9/12	4/18
8	1/22	1/21
9a	22/3	24/2
9b	24/2	24/3
9c	18/8	16/10
9d	24/2	20/6
9e	18/8	14/12
10a	7/11	11/13
10b	15/6	11/12
10c	15/6	18/6
10d	10/10	12/9
10e	14/6	12/11
10f	19/6	14/10
10g	12/8	12/10
11a	23/2	21/3
11b	13/11	16/8
12a	12/12	14/12
12b	6/19	8/17
12c	21/5	24/3
12d	21/5	23/4
13	0/14	1/21

Note: Missing frequencies are not included.

The observations that arose from Table 5.10 are as follows. First, students' performances in questions about number line (Question 1), ordering (Question 3) and converting (Questions 5 and 6) were similar in both groups. Second, CG students performed better than the EG students in the questions related to recognizing the incorrect representations of fractions (Question 9). Third, in the computation question (Question 10), students in both groups performed differently in different parts of the question. Last, students in the EG performed better in finding the unknown term in an ordered pair of fractions (Question 12).

5.2.2 Attitude Scale (AS)

The descriptive statistics for post-AS in CG and EG are given in Table 5.11.

The Box-and-Whisker plots of CG and EG scores in post-AS are given in Figure 5.10. The Box-and-Whisker plots of both groups in post-AS indicate that EG students' attitude scores' median is less than the median of the CG students' attitude scores. EG scores have a greater range than the CG scores' range. There is one outlier in each of the groups.

The histograms of both groups' scores on post-AS are given in Figure 5.11 and Figure 5.12. The shapes of the distribution in post-AS scores are left skewed, within different range.

Table 5.11: Statistics of post-AS for CG and EG.

Statistics	CG	EG
N	26	27
Mean	62.88	45.81
Std. Error of Mean	2.02	3.11
Median	62.5	49
Mode	58, 60, 62, 64	51
Std. Deviation	10.28	16.14
Variance	105.63	260.62
Skewness	-0.511	-0.998
Range	42	73
Minimum	38	0
Maximum	80	73
Quartile I	57.75	37
Quartile II	62.5	49
Quartile III	71	57

Note: Maximum score is 80.

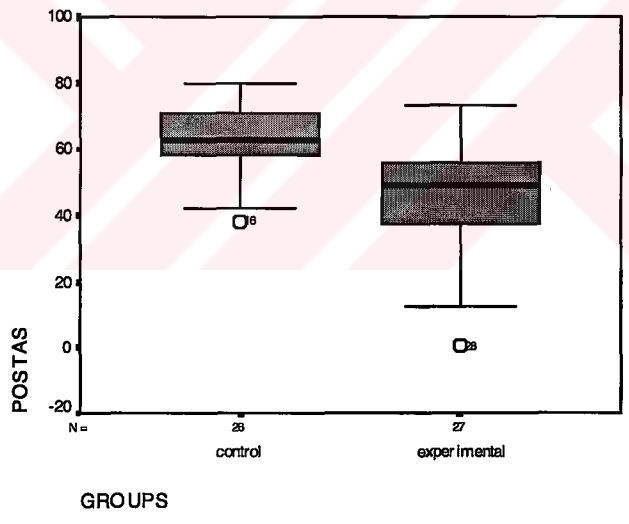


Figure 5.10: Box-and-Whisker plots of post-AS for CG and EG.

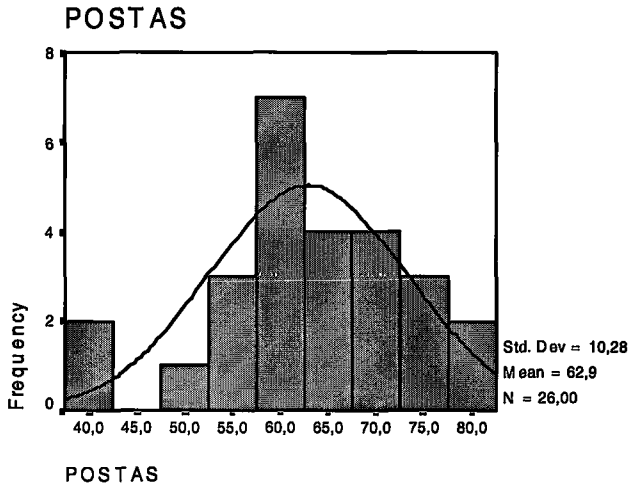


Figure 5.11: Histogram of post-AS scores for CG

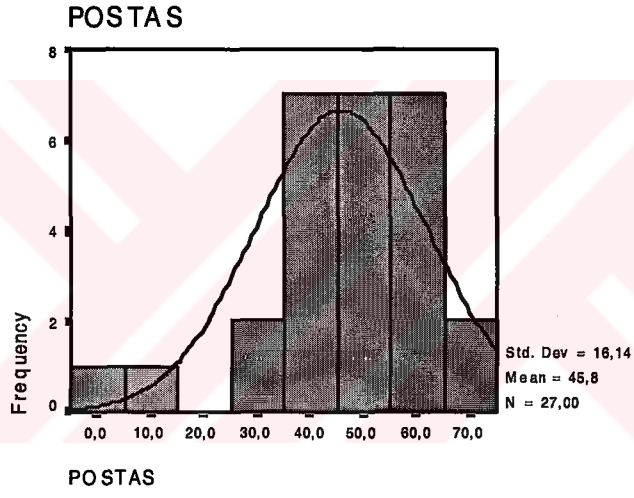


Figure 5.12: Histogram of post-AS for EG.

5.2.3. Comparison of Experimental and Control Groups

In this study, the following research questions were sought:

- Is there a significant difference in the mean performance scores of 5^{th} grade students on fractions, that can be attributed to: (i) treatment, (ii): gender, and (iii): interaction of treatment and gender?

- Is there a significant difference in the mean attitude scores of 5th grade, that can be attributed to: (i) treatment, (ii): gender, and (iii): interaction of treatment and gender, holding the pre attitude scores constant?

For the first question, a 2 x 2 ANOVA was conducted to evaluate the effects of the treatment and the gender on the means post-CPPT scores (dependent variable) of EG and CG students. For the second question, a 2 x 2 ANCOVA was conducted to evaluate the effects of the treatment and the gender on the means of post-AS scores (dependent variable) of EG and CG scores, after the post-AS scores were adjusted for differences in pre-AS scores. In both analyses, the treatment (group) and the gender were the independent variables.

ANOVA was used for the analysis of the post-CPPT scores, since pre-CPPT mean scores did not differ significantly for EG and CG students. However, pre-AS mean scores were significantly different for EG and CG students, in favour of CG students. Hence, ANCOVA was used for the analysis of post-AS scores, with pre-AS scores as a covariate. Table 5.12 gives the correlations between the pre tests and the post tests. As can be seen from the table, the correlations between pre-AS and post-CPPT, and pre-CPPT and post-AS were not significant. Hence, a MANCOVA analysis with post-AS and post-CPPT as dependent variables, treatment and the gender as independent variables and pre-AS and pre-CPPT as covariates, was not conducted.

Table 5.12: The correlations between the pre tests and the post tests.

	Pre-CPPT	Pre-AS	Post-CPPT	Post-AS
Pre-CPPT				
Pre-AS	0.138			
Post-CPPT	0.631*	0.173		
Post-AS	0.076	0.0575*	0.359*	

* Correlation is significant at the 0.01 level (2-tailed)

The descriptive statistics for post-CPPT scores according to the gender and the treatment (groups) are presented in Table 5.14.

Table 5.13: Means and standard deviations of post-CPPT scores of CG and EG.

Groups	Gender	Means	Std. Deviation	N
Control	Female	25.58	5.63	12
	Male	20.07	8.65	14
	Total	22.62	7.79	26
Experimental	Female	21.85	9.81	13
	Male	22.29	8.56	14
	Total	22.07	9.01	27
Total	Female	23.64	8.14	25
	Male	21.18	8.52	28
	Total	22.34	8.36	53

The ANOVA results showed that there was no significant mean difference between the EG and CG students' post-CPPT scores, attributable to the gender and the treatment. The interaction of gender and treatment also did not affect the post-CPPT mean scores significantly. Table 5.14 gives the F, p and partial η^2 values for ANOVA.

Table 5.14: Values of F, p, and η^2 for ANOVA

Effect	F	p	η^2
Group	0.109	0.742	0.002
Gender	1.212	0.276	0.024
Group and Gender	1.669	0.202	0.033

The descriptive statistics for post-AS scores according to the gender and the treatments (groups) are presented in Table 5.15.

Table 5.15: Means and standard deviations of post-AS scores of CG and EG.

Groups	Gender	Means	Std. Deviation	N
Control	Female	65.42	9.53	12
	Male	60.71	10.74	14
	Total	62.88	10.28	26
Experimental	Female	43.23	16.64	13
	Male	48.21	15.89	14
	Total	45.81	16.14	27
Total	Female	53.88	17.55	25
	Male	54.46	14.75	28
	Total	54.19	15.98	53

For ANCOVA, homogeneity of slopes assumption was tested. Results showed that pre-AS scores, as a covariate, was not a function of the independent variables (p (Group and Pre-AS) = 0.860, p (Gender and Pre-AS) = 0.931). Hence, ANCOVA was conducted. ANCOVA results indicated no significant main effect for gender and no significant interaction between the treatment and gender. However, there was a significant main effect of treatment on the mean of post-AS scores of EG and CG students. Table 5.16 gives the F, p and η^2 values for independent variables, and the interaction.

Table 5.16: Values of F, p, and η^2 for ANCOVA

Effect	F	p	η^2
Group	5.798	0.020	0.108
Gender	0.007	0.931	0.000
Group and Gender	1.098	0.300	0.022

The results showed that the treatment explained the 10.8% of the variance of the model. Although the effect of pre-AS scores held constant, EG and CG students' post-AS scores significantly in favour of CG. Hence, it can be said that the difference in attitude could be a result of other factors.

5.3 Results of the Interviews

The interview transcripts were analyzed in order to investigate students' understandings of fractions and their ideas about the teaching and learning environment together with their attitudes towards mathematics and mathematics lessons. Selected students' answers to questions in post-CPPT were analyzed to determine the interview questions related to the conceptual and procedural understanding. These questions were used to understand what difficulties they faced, how they thought during the solution and what affected their solutions. The differences that appeared in students' attitude scores with respect to the pre and post AS were detected and used in questions related to attitude towards mathematics and mathematics lessons. Students' answers to the questionnaire were also considered in order to pose questions during the interview where necessary.

Students S1, S2, S3, S4, S5 and S6 were from EG and students S7, S8, S9 and S10 were from CG. Among these students, S1, S6, S7, S9 and S10 were females where the rest were males.

5.3.1 Conceptual Understanding in Fractions

Questions 2, 3, 6, 8, 9, 10 and 12 that covered the major concepts in fractions were asked to different students and their written and verbal answers were recorded. Here, the results of the interviews will be presented question by question.

Question 2 (Part-Whole and Partitioning)

> 4 bütünün her biri 5 eşit parçaya bölünüp parçalardan 16 tanesi alınmıştır. Oluşan kesri ve çeşidini yazınız. (4 wholes are taken and each of them is divided 5 equal parts. 16 of these parts are taken. What are the fraction and the kind of the fraction that represents these parts?)

Among the four students from EG (S1, S2, S4 and S5) and one student from CG (S7) who were asked to solve question 2 again, three of them (S4, S5 and S7) answered it correctly. These students drew four rectangles that represented four wholes and divided each rectangle into five equal parts. Then crossed out 16 of the 20 equal parts and wrote the fraction as $3\frac{1}{5}$. The following dialog is an example of the verbal explanation of their thinking process:

S4: (After reading the question, he drew four rectangles).... There are four wholes... each of them is divided into five equal parts... (He divided each of the rectangles into five equal parts).... 16 of them are taken ..... 5, 10, 15... and one from here (took one part from the 4th whole) .... And the fraction is ... $3\frac{1}{5}$.

In the post-CPPT, S4 had answered this question as $\frac{16}{20}$, which was the most common wrong answer appeared in the pilot study. S5's answer in the post-CPPT was given according to five wholes and the fraction was written by considering the parts that were not taken: $\frac{9}{5}$. S7 answered this question as $4\frac{16}{20}$, which was a result of misconception that appeared by not distinguishing between the parts and the wholes.

After giving the correct answer in the interviews, students were also posed the following question in order to understand whether the difficulties came out because of the representation of fractions:

Inter: How do you prefer to write or perform computations with the fractions greater than one, by an improper fraction, or by a mixed fraction?

The students claimed that they preferred to write the fractions greater than one in the form of mixed fractions. S4 and S5 also preferred mixed fractions in computations, whereas S7 preferred improper fractions in computations. S5 and S7 told that their incorrect answer in post-CPPT might be due to not reading the question carefully.

S1 and S2 did not answer this question in the post-CPPT. Initially, these students solved the question incorrectly in the interview. Their way of solution was drawing the same figures in the same way as S4, S5 and S7 did, but they wrote the fraction as $\frac{16}{20}$. These students were asked the following question to understand whether the reason for the incorrect answer was a result of a difficulty in representing and writing fractions related to many wholes:

Inter: What does denominator and numerator represent in a fraction?
S1: We are writing the greater one to the denominator.
Inter: Now, how many equal parts did we divide the wholes into?
S1: 5
Inter: So, thinking about all the fractions we know, what do we write as a denominator?
S1: Number of the equal parts ... Here, it is 5.
Inter: What do we write as a numerator?
S1: Numerator.... It should be 16.
Inter: Can you tell me the fraction now?
S1: $\frac{16}{5}$
Inter: Can you tell me the reason for your answer $\frac{16}{20}$?
S1: I thought that I got exactly 16 of 20.
Inter: So, what is the whole here?
S1: These are the wholes (showed the figures). There are four wholes.

...ÖĞRETİM DERNEĞİ
DOKÜMANTASYON MERKEZİ

Her answers were an example of the students' difficulty in representing fractions related to one whole and many wholes. Furthermore, it seemed that S1 did not initially thought about what the denominator and the numerator represented in a fraction. When her preference about the type of fractions while writing the fractions greater than one, S1 said that she preferred to write in the form of improper fractions

At first, S2 answered the question as $\frac{16}{20}$. When he was asked to consider the number of wholes that he crossed out, he said:

S2: Oh, will I tell as a mixed fraction? Then it is $3\frac{1}{5}$.

Inter: You also answered as $\frac{16}{20}$. Which one is correct?

S2: $\frac{16}{20}$

When he taught about the number of equal parts, his answer was $\frac{16}{20}$. But the number of wholes made him answer the question as $3\frac{1}{5}$. He thought that he could write the fraction both as a simple fraction, and as a mixed fraction. Then the interviewer wanted him to represent $\frac{16}{20}$ by figure. At that time, he realized that two fractions differ in their continuous representations. At the end, he decided that the answer should be $3\frac{1}{5}$. S2 preferred to write fractions greater than one in the form of mixed fractions.

All students interviewed about question 2 were able to say the kind of the fractions they wrote correctly, even if their answers were incorrect for writing the fraction. Table 5.17 summarises the interviewed students' nature

of answers in post-CPPT and their initial answers in the interview, for question 2. "C" represents correct answers, and "I" represents incorrect answers.

Table 5.17: Interviewed students' answers given in post-CPPT and the interview for question 2.

	Post-CPPT		Interview	
Group	**C**	**I**	**C**	**I**
CG		S7	S7	
EG		S4, S5	S4, S5	S1, S2

Note: S1 and S2 did not answer the question in post-CPPT

Question 3 (Partitioning)

5 sayısının içinde kaç tane $\frac{1}{5}$ vardır? (How many $\frac{1}{5}$'s are there in 5?)

Three students, S3, S5 and S6, in the EG and one student, S10, in CG were required to solve question 3 again. Students in EG answered the question verbally as "5", and no figures were used by these students. Then the following dialog was carried out between student S3 and the interviewer:

Inter: So, according to your answer, there should be 5 many $\frac{1}{5}$'s in 5.

S3: Yes.

Inter: Can you add them please?

S3: (He added $\frac{1}{5}$ for five times and stopped for a few seconds.)....5... Hmmm...(He suddenly changed his mind, and added $\frac{5}{5}$ for five times.) ...25

During his justification, he realized that sum of 5 many $\frac{1}{5}$'s makes 1, not 5. At that time he built the proportional reasoning that if 1 included $\frac{1}{5}$ for 5 times, 5 included $\frac{1}{5}$ for 25 times, and corrected his answer as 25 in the addition context. His answer in the post-CPPT was 5, without any justification.

In the interview S5 initially gave his answer as "$\frac{5}{5}$ is equal to 5, so the answer is 5". Then, he was asked to control his solution once more, and the following dialog was carried out:

Inter: Can you add $\frac{1}{5}$ for 5 times, please?

S5: I can also multiply $\frac{1}{5}$ by 5 (He multiplies 5 by $\frac{1}{5}$.).... $\frac{5}{5}$... So it is 5.

Inter: Does $\frac{5}{5}$ equal to 5?

S5: Yes

Inter: Please be careful and think again.

S5: Oh.... 1 whole... There are 25..

Inter: Why?

S5: Because...if we make this (5 in the multiplication) 25.... 25 times $\frac{1}{5}$..... makes 5.

S5 corrected his answer in the multiplication context, by using the proportional reasoning, after realizing that 5 many $\frac{1}{5}$'s do not make 5. He did not answer question 3 in post-CPPT.

S6 gave the same incorrect answer, "5", in both post-CPPT and the interview. The justification given in the interview was: "5 many $\frac{1}{5}$'s .. make $\frac{5}{5}$. And it is equal to 1, so the answer is 5". She told that she also thought in this way in post-CPPT, but now she was not sure. Then the reason for her hesitation was asked to investigate her understanding, and the following dialog was carried out:

S6: Actually... I think the same... but....It can also be 25.

Inter: If we control the result, then we can understand which one is true. Do you mean that, when we add $\frac{1}{5}$ for 5 times the result is 5?

S6: It will make $\frac{5}{5}$.... Equals 1 whole... No... So it is $\frac{25}{5}$ ..... But now, $\frac{25}{5}$ ... if 25 is divided by 5.... Again we get a whole number, and it makes 5.

Inter: So what is the answer?

S6: 5 .... No .....25....

Inter: Can you please explain it once more?

S6: 5/1... is a whole number, so ....by enlarging it by 5.. we make it $\frac{25}{5}$. And, when we divide 25 by 5, it makes 5. So the answer is 25.

Inter: Do you mean that $\frac{25}{5}$ is actually the sum of 25 many $\frac{1}{5}$'s?

S6: If we think in that way, we also get the same result. We make it in its simplest form, $\frac{25}{5}$ makes 5.

Inter: Now, how many $\frac{1}{5}$'s in 5?

S6: 5... I cannot explain it, but I am sure there are 5 (She summed 5 many $\frac{1}{5}$'s once more and realized that it made 1).

S6: So it is not 1... So it is 25.

many $\frac{1}{5}$'s. Even when she controlled and saw that the answer 5 was wrong, she was still confused about the other alternative, 25.

Student S10 from CG answered question 3 as "5" with no justification in post-CPPT. She also gave the same incorrect answer in the interview, and justified by drawing a rectangle and showing her solution on the figure by dividing it into five equal parts. Then the following dialog was carried out in order to investigate her understanding:

Inter: Do you mean that when we add $\frac{1}{5}$ for 5 times, we get 5?

S10: Okay...(She added $\frac{1}{5}$ for 5 times).... It makes one whole.... So ...it needs 10 more... I am so confused now... now, if this $(\frac{5}{5})$ is equal to 1 whole.... I am much more confused now.. When I add $\frac{1}{5}$ for 5 times, it does not make 5.

Inter: So there are 5 many $\frac{1}{5}$s in 1.

S10: 5 times 5 ... It is (the answer) 25.

As she drew only one whole (figure) to aid in her thinking, she considered it as 5. But, later as can be seen in the dialog above she realized that it should represent 1. Although, even she realized that she should have taken five wholes (figures), the above misconception "5 many $\frac{1}{5}$'s make 5" seemed not remediated in the students' mind when asked to show the solution on the number line:

Inter: Can you show me $\frac{1}{5}$ on the number line?

seemed not remediated in the students' mind when asked to show the solution on the number line:

Inter: Can you show me $\frac{1}{5}$ on the number line?

S10: Let me try...(She drew a number line, divided it into 5 equal parts, and wrote the numbers from 0 to 5 in their places.)... This one (showed the point 1, as $\frac{1}{5}$.)

Inter: Is that point $\frac{1}{5}$?

S10: I am so confused!!

Inter: Is $\frac{1}{5}$ less than one or greater than one?

S10: Less than one.

Inter: So where should $\frac{1}{5}$ be on the number line?

S10: Here! (She showed the 0-1 interval).

Inter: Can you show me the $\frac{1}{5}$ point?

S10: (She shows correctly.)

Although she corrected her answer in the addition context, she was not able to modify the idea in the number line context.

Table 5.18 summarises the interviewed students' nature of answers in post-CPPT and their initial answers in the interview, for question 3. "C" represents correct answers, and "I" represents incorrect answers.

Table 5.18: Interviewed students' answers given in post-CPPT and the interview for question 3.

	Post-CPPT		Interview	
Group	**C**	**I**	**C**	**I**
CG		S10		S10
EG		S3, S6		S3, S5, S6

Note: S5 did not answer the question in post-CPPT

Question 6 (Relations between improper and mixed fractions)

Aşağıdaki tam sayılı kesirleri bileşik kesre çeviriniz. (Convert the mixed fractions to improper fractions.)

a) $3\frac{2}{15}$ b) $2\frac{3}{22}$

Only S10, a student from CG, could not answer question 6 in the post-CPPT, so question 10 was only posed to her. She answered both subquestions of question 10 correctly in the interview. When she was asked the reason of her missing answer in the post-test, she said that she had some problems with the fractions concept during the period of instruction and the post-test, hence she could not perform well. Moreover, she mentioned that:

S10: I was like... having seen the fractions for the first time. You know, when a person faces a concept for the first time, he or she may get faltered.... I was like I had never seen fractions before...

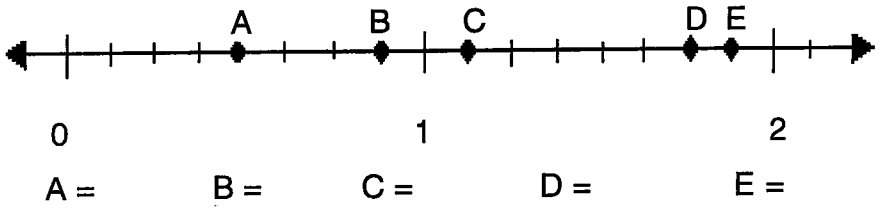
Inter: How did you recover, then?

S10: I studied so hard on the fractions by myself. Now I am very well.

Student's understanding of fractions was inhibited by her anxiety during the instruction on fractions and the post-CPPT. Her pre-CPPT score was among the high level group, but she was not confident about fractions, as mentioned in the interview. However, she was able to overcome this anxiety by studying hard on the fractions concept.

Question 8 (Number Line Representation)

Sayı doğrusu üzerinde verilen A, B, C, D ve E noktalarının gösterdikleri kesirleri yazınız. (Write the fractions that represents the points A, B, C, D and E given on the number line)



Only one student among the interviewed students, S7 from CG, answered this question wrong in the post-CPPT, so the question was posed only to her.

In the interview, S7 initially answered this question as she did in the post-test. The answers for the fractions in 0-1 interval were $\frac{4}{8}$ and $\frac{7}{8}$. And the answers for the fractions in 1-2 interval were $\frac{9}{16}$, $\frac{14}{16}$ and $\frac{15}{16}$. Reviewing her answers, she said "Oh, I did not write the wholes", and she wrote one in front of the first two fractions and two in front of the next three fractions. So the answers became $1\frac{4}{8}$, $1\frac{7}{8}$, $2\frac{9}{16}$, $2\frac{14}{16}$ and $2\frac{15}{16}$. She read the question once more, and finally decided to change the whole parts of the mixed fractions. Her final answers were $\frac{4}{8}$, $\frac{7}{8}$, $1\frac{9}{16}$, $1\frac{14}{16}$ and $1\frac{15}{16}$. Then the reasons for the procedure that she performed and the answers were asked in the following dialog:

Inter: For C, D and E, you decided that the fractions have one whole. Can you tell me the reason for erasing the one whole in A and B?

S7: Because we get these (fractions) from inside (she showed the 0-1 interval).

Inter: So, how many equal parts is one whole divided on the number line?

S7: 2 parts.

Inter: We have two wholes, between 0 and 1, and between 1 and 2. How many equal parts are the wholes divided into?

S7: 8

Inter: If we consider C, we get one whole and some parts from the second whole. How many parts do we get from the second whole?

S7: 1

Inter: In how many parts is the second whole divided?

S7: It is also divided into 8 parts.

Inter: So, can you think this fraction once more. We have a whole, and what do we write to the denominator?

S7: 16..... no.... I think that it is $1\frac{9}{16}$.

Interviewer and S7 discussed it once more, from the beginning. Then, she realized that the number of equal parts in a figure was written as the denominator. So she corrected her answers. S7's main misconception was that she considered the line segment as a whole, and wrote the fractions according to that whole. Moreover, the meaning of mixed fraction was not clear in her mind, and she wrote the whole parts to make the fractions mixed type, considering the intervals that the fractions were in.

Question 9 (Ordering Fractions)

> Aşağıdaki kesirleri < veya > sembollerinden uygun olanını yazarak sıralayınız. (c ve d şıklarında kesirlerin yerlerini değiştirebilirsiniz.) (Order the below fractions by using the one of the symbols < or >. (You can change the place of the fractions in parts c and d.))

a) $\frac{13}{33}, \frac{13}{26}$ b) $\frac{4}{14}, \frac{6}{14}$ c) $\frac{5}{22}, \frac{12}{22}, \frac{3}{22}, \frac{11}{22}$ d) $\frac{7}{5}, \frac{7}{13}, \frac{7}{38}, \frac{7}{21}$

Both S1 and S6 from the EG answered question 9 correctly in post-CPPT and the interview. S6 had written the following hints on her exam paper on post-CPPT, near question 9:

$$\frac{1}{1} > \frac{1}{2} \qquad\qquad \frac{2}{2} > \frac{1}{2}$$

The first one was a hint for the order of fractions with equal numerators and the second one was a hint for the order of fractions with equal denominators. She was asked the reason for writing the hints. She said that it was a "formula" given by the class teacher, and also given by her parents both of who were mathematics teachers. As can be seen in the following episode, she explained that "formula" by giving other examples:

S6: .. When the numerators are equal, if the denominators are less than... like this (Gives $\frac{1}{4} < \frac{1}{2}$ as an example).... I said that the one with the smaller denominator was greater... Then, if denominators are equal, ... the one with the greater numerator is greater (Gives $\frac{5}{2} > \frac{1}{2}$ as an example). This formula is always like this.

S1 was encouraged to write two groups of fractions: one with equal denominators and one with equal numerators. Then, she was required to order them. As she ordered correctly, she was asked the reasons for her answers. She said that she remembered the example of dividing a cake into some number of equal parts and taking some of the parts to eat, which was given by the class teacher. S1 was asked to explain why $\frac{9}{5}$ was the greatest and $\frac{5}{5}$ was the smallest among the fractions $\frac{8}{5}$, $\frac{5}{5}$ and $\frac{9}{5}$ that she wrote.

Inter. Why is $\frac{5}{5}$ the smallest?

S1: Because we divided it into five parts and took all of them.

Inter: Why is 9/5 the greatest?

S1: We divided into five and took nine of them.... It doesn't seem logical to divide a cake into five parts and eat nine of them.... but we did in this way.

The teacher's explanation for ordering fractions with an example of partitioning a cake into some parts and taking some of them was not meaningful for the student if there were improper fractions. Table 5.19 summarizes the interviewed students' nature of answers in post-CPPT and the interview. "C" represents correct answers, and "I" represents incorrect answers.

Table 5.19: Interviewed students' answers given in post-CPPT and the interview for question 9.

	Post-CPPT		Interview	
Group	**C**	**I**	**C**	**I**
CG				
EG	S1, S6		S1, S6	

Question 10 (Order and Equivalence of Fractions)

Aşağıda büyüklük veya küçüklük sırasına göre sıralanmış kesirleri görüyorsunuz. Boşlukları sıralamalar bozulmayacak şekilde uygun doğal sayılarla doldurunuz. (Below, there are ordered fractions in decreasing or increasing order. Fill in the unknown terms with the appropriate natural numbers so that the orders will not change)

a) $\frac{\ }{7} < \frac{13}{14}$ b) $\frac{7}{\ } > \frac{8}{5}$ c) $\frac{\ }{30} < \frac{15}{30}$ d) $\frac{6}{9} > \frac{\ }{9}$

S4 from EG was posed only (a) and (b) parts of question 10. These ones were the ones, which he answered wrong in this question in post-CPPT. His answers were $\frac{12}{7} < \frac{13}{14}$ for (a), and $\frac{7}{6} < \frac{8}{5}$ for (b) in the post-CPPT.

In order to investigate the reasons for S4's incorrect answers for the first two subquestions in question 10, he was asked to solve them again. He answered the first one correctly by using the equivalence of fractions, as in the following dialog:

S4: Now, first we equalize the denominators... When we multiply 7 by 2....(equalized the denominators of the fractions on 14). The (unknown) numerator should be less than 13.... So it is 6.
Inter: Could it be also 4?
S4: It could be 4. I mean it could also be any number less than 6.

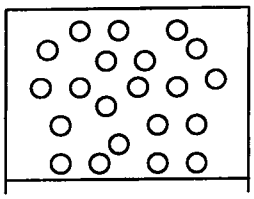
S4 was able to use the knowledge of equivalence to find the unknown term in the ordered pairs of fractions when the denominators were known. But, he answered the second one as he did in the post-CPPT. He thought that the denominator of the greater fraction should be greater than the denominator of the smaller fraction. Then he wrote 6 to the denominator of the greater fraction, where the denominator of the smaller fraction was 5 and later claimed that if he would equalize the denominators, then the fraction would be greater as required. So he was encouraged to equalize the denominators. He realized that the answer was incorrect. Then the following dialog occurred:

Inter: Why did you think that "It should be greater than 5"?
S4: I thought that, when we equalize the denominators, the numerator (of the greater fraction) would be greater.... Oh, it should be less than 5. ... When we do so.... For example 4... The denominators are 20 ....And (pointing out the numerator of the greater fraction) 35. And (pointing out the numerator of the smaller fraction) this is 32 ..... Then it satisfies.

S4 was able to answer the question correctly when the denominators were known. This made him equalize the denominators easily and define an

upper bound for the value of the numerator. However, when one of the denominators was not known, it was hard for him to determine the denominator.

Question 12

$2 -$  $= \frac{10}{7}$

Yandaki çıkarma işleminde çıkan kesir bilinmiyor. Çıkan kesri bulunuz ve verilen kesir üzerinde gösteriniz. (In the subtraction given, the subtracted fraction is not known. Find that fraction and show it on the given set.)

Question 12 was posed to three students: S3 and S6 from EG, and S9 from CG. S3 was the only student who solved this question correctly in post-CPPT. When he solved again in the interview, he computed the unknown fraction correctly and explained the reason of his correct discrete representation as below:

S3: Now, the fraction that I found (4/7) here (on the computation) is not present here (on the discrete set). If it is not there, since equivalent fraction is the same fraction .... The reason for using equivalent fraction is... I count them (the elements of the set),.. there are 21 circles.. . Since $\frac{4}{7}$ is equivalent to $\frac{12}{21}$, I crossed out 12 of the circles.

So it is understood that he was able to combine his understanding related to the discrete representation of fractions and the equivalence of fractions. As he realized that he could not represent the fraction that he found on the given set, he tried to modify the fraction by firstly understanding the properties of the set. Then, he decided that he could get the appropriate fraction by using the equivalence, and got the correct result.

In the post-CPPT, S6 also found "$\frac{4}{7}$" correctly, but she failed to represent the fraction on the discrete set. In the interview, she was able to show $\frac{4}{7}$ correctly on the discrete set. She said that, in the post-test, she was confused about the representation.

S9 said that the question was hard for her in post-CPPT that she had not answered it, and it was still hard for her. So the interviewer explained the question, and S9 could found the missing term correctly. Then, in the representation part, she stated that she did not know how to do it:

Inter: Now, let me remind you by a figure. Here are three circles. I show $\frac{1}{3}$ by crossing out one of the circles.

S9: Oh... I remember now. I should cross out four of the circles (in the question).

Inter: I want to ask what the denominator represents in discrete representation.

S9: .. How?...

Inter: Now, let us review what you have done in the 2nd question. (She answered correctly in the (e) part of 2nd question. When reviewed, she remembered how she represented fractions in discrete sets.)

.....

S9: So I cross 4 of them.

Inter: How many circles do we have in the set?

S9: (counts).... 21.

Inter: Now, what is the denominator of our fraction?

S9: ... So we draw circle around 7 of the small circles and make a set. Then we cross out 4 of them.

Inter: But it says that you should represent in this set.

S9: We cross 4 of them.... Just one minute.... I cross 12 of them.

Inter: Why?

S9: I equalized the denominator of it ($\frac{4}{7}$) to 21.... I mean, I found the numerator when the denominator is 21.

When S9 was required to represent a fraction on a discrete set, she could do when the denominator of the fraction was equal to the number of elements in the discrete set. But when the number of elements is different from the denominator of the fraction, she could not realize what to do. After reviewing what denominator represented in a discrete set, she was able to insert the equivalence concept into the solution.

Table 5.20 summarises the interviewed students' nature of answers in post-CPPT and the interview. "C" represents correct answers, and "I" represents incorrect answers.

Table 5.20: Interviewed students' answers given in post-CPPT and the interview for question 2.

	Post-CPPT		Interview	
Group	**C**	**I**	**C**	**I**
CG				
EG	S3	S6	S3, S6	

S9 did not answer the question in post-CPPT

5.3.2 Procedural Understanding in Fractions

Question 11(Computations with fractions):

Aşağıdaki işlemleri yapınız. (Do the below computations.)

a) $\frac{9}{24}+\frac{1}{8}+\frac{8}{12}=$

b) $3\frac{1}{9}+\frac{7}{6}=$

c) $\frac{1}{4}-\frac{1}{9}=$

d) $4\frac{3}{7}-\frac{49}{14}=$

e) $7\times 4\frac{3}{8}$

f) $\frac{7}{3}\times\frac{4}{21}$

g) $2\frac{1}{4}\times\frac{2}{3}=$

Four students, S3 and S4 from EG and S7 and S8 from CG were asked various subquestions of question 11. All subquestions asked to be solved again were done correctly by the students, except the multiplication subquestions (11(e) and 11(g)) S3 and the addition subquestion 11(b). in the post-CPPT, 11(e) and 11 (g) answered incorrectly by S3 and 11(b) by S7.

S3 answered the multiplication subquestions different from his answers in post-CPPT, which were wrong. In the post-CPPT, he equalized the denominators while multiplying fractions in 11(e), 11(f) and 11(g). During the interview, in 11(e), S3 multiplied the natural number "7" with the denominator, numerator and the whole number of the mixed fraction rather than multiplying only with the numerator and the whole number. In 11(g), he multiplied the numerators and denominators, and wrote in the numerator and denominator of the product fraction respectively, but wrote the whole part of the first fraction in the product fraction as the whole part. After he completed his computations, he explained the interviewer how he solved the questions as seen in the following dialog:

S3: I multiplied the numerators by numerators...and write the result to the numerator (He showed on the computations)... and denominators by denominators and write the result to the denominator and the whole number (He suddenly realized that the whole number was not properly computed).... But I did wrong... There is something wrong here (He showed 11(g))... I did it wrong....(He converted the mixed fraction to improper fraction and computed once more, and found the correct answer).

Inter: You have found two different answers. Which one is the correct one?

S3: This one (he showed the one that he converted the mixed fraction to the improper fraction).

Inter: Okay, so let us turn back to the part (e). What did you do here?

S3: Before that, I will make some computations, then..... (He converted the mixed fraction in 11(e) and then computed again. This time, he found the correct answer).

Inter: Now, let us compare the denominators of the two results that you have found for part (e). The former results' denominator is 56 (as a result of multiplication of 7 and 8), and the latter results' denominator is 8. Why was the former 56?

S3: Because I multiplied (7) by both this (the numerator) and this (the denominator)...

Inter: Does there confusion appear when you do not convert mixed fractions to improper fractions?

S3: In this case (11(e)), it is wrong (in the previous one).

When S3 was required to explain the reason behind his answers in the post-test, he said that he forgot what to do just at the exam duration.

S7 was asked to solve 11 (b) again. In the post-CPPT, her answer to 11 (b) was incorrect. In order to be able to equalise the denominators, she multiplied the whole number in the mixed fraction with the denominator and

wrote as the denominator of the improper fraction. In the interview, she gave the following explanations:

Inter: How would you compute?

S7: First, I would convert this (the mixed fraction), to improper fraction.... (Meanwhile she wrote 27 to the denominator.).... I did wrong.... (She tried to convert the fractions, one more time. As she was convinced that she did correctly, she equalized the denominators).... I did wrong...

Inter: Why did you do wrong?

S7: I tried to equalize these (the denominators of the fractions given), but I did wrong.

She did not repeat the same incorrectness during the interview, but she confused how to convert mixed fraction to improper fraction. But she realized her mistake and corrected it. Moreover, she explained her mistake to the interviewer.

5.3.3 Teaching and Learning Environment

Students in EG were interviewed in order to understand their ideas about teaching and learning in the collaborative small groups. They were asked how they perceived collaborative small-group learning, what advantages and disadvantages they felt, and how they benefit form the manipulatives and the materials. Factors, such as, group formation, group members' behaviors and students' characteristics affecting students' different opinions and feelings about the teaching and learning environment were also investigated. Relationship between the teaching and learning environment and the attitude was analyzed deeply especially for students who had a decrease or increase in the attitude scores. These students were encouraged to explain the reasons for such changes.

Students in CG were asked about the instruction on fractions, whether they would like to learn mathematics with another method, or how should mathematics be taught.

During the interviews students' attitudes towards mathematics was investigated together with the factors affecting the attitude. Students who seemed to be affected by several factors or by a dominant factor were encouraged to explain how they were affected.

Students in EG were posed the following questions:

- During the collaborative small-group works, did you participate in the group work sufficiently? Why?
- Have you ever participated in such group in any other course before?
- What were the things that you liked and disliked during the group work?

- What should be improved in group work?
- What things should not be in the group work?
- Do you think that you learned fractions well with group work?
- Were group worksheets useful for your learning in fractions?
- Were cardboards and play-dough useful for your learning in fractions?
- In general, how do you evaluate the group work?

The following questions were asked to CG students:

- Would it be better for you if mathematics was taught different from how it is taught now?
- How should it be taught? (Related with the previous question)
- What if, for example group work, was used? Do you think that you would learn better?
- Do you remember the posters that your teacher used? Can you tell me what they were related to?

Experimental Group

The responses of EG students given to the questions were grouped according to the categories identified as common.

Participation in Collaborative Small-Group Work

S1 and S2 said that they did not sufficiently participated in group work. S1's main reason for not participating was the group members' behaviours towards her. The following dialog is an explanation of the reason:

Inter: Why do you think, you could not participated sufficiently?
S1: Because, they (group members) always tried to do the tasks.

Inter: Had you ever told them that you also wanted to do?

S1: Yes.

Inter: Then, what did they say?

S1: They said okay, and we studied on the tasks together.

S1 was a kind of student who had difficulty to get on well with her classmates. She usually did not play with any of her classmates, and did not participate in any event that all of her classmates were involved. Moreover, she claimed that she was also not able to work efficiently in the group works in the other courses. She stated that if a particular student would not be in her group, she would have participated more.

S2 said that he did not participated in the group work since he was not willing to do. He explained the reason as in the following dialog:

S2: It was something related with me..... I did not like the group work. I did not want to participate.

During the interview, he wanted to talk about one of the group members in one of the groups that he was involved, and then he decided not to do.

The reason for S1 and S2 for not participating the group work was related with the students' personal characteristics. S1 had problems with her classmates, so she did not try to involve in the group processes, with an additional factor of a particular student. S2 was not willing to work in the groups, and as he mentioned this was related with his personal feelings.

Other students (S3, S4 and S6) claimed that they participated the group work sufficiently by stating "I participated sufficiently".

It seems logical to state that the students' characteristics had an effect in their participation status in the collaborative small-groups.

Advantages and Positive Ideas about Learning in Collaborative Small-Groups

The following explanations of EG students were taken from their interview transcripts. S1, S3, S4 and S6 stated positive ideas about learning in collaborative small-group work. S1 and S6 also stated the advantages of studying in small-groups. The advantages that S1 mentioned were as follows:

S1: If I cannot find a solution to a question, I can ask it to my friends in the group work. I can ask the things that I don't understand to my friends. For example, at the beginning I did not understand ordering the fractions with equal denominators, but then S3 explained me.

Inter. Did he also tell you the reasons?

S1: Yes.

Although S1 claimed that she did not sufficiently participated in the group work, she was able to benefit from the group work, by means of the explanation that she got from one of her group friends. S6 stated the advantage of collaborative small-group work as below:

S6: .... Sometimes my friends know the easiest way to solve a question, and I learn from them. Sometimes I know the easiest way to solve a question and they learn from me. We usually learn in this way by group work. ... It was enjoyable and we found solutions to our questions.

S6 was a student with higher level of performance, but she stated how she and her friends benefit from group work by means of different ideas appeared in the group. S3 and S4 liked to study in the collaborative small-groups. Here, their ideas are given:

S3: There was nothing that I did not like. It was useful. We learned.

S4: Group work was enjoyable. I liked it. There was nothing that I did not like. If I hadn't learned fraction with group-work, I would probably not remember it now.

Moreover, S4 said that, during the group work, everybody explained his or her solution or idea about the given task, and they all decided which one was true. What they wrote on the worksheet was the group's idea. Students benefit from the group work in terms of explanation from a group friend and diversity of ideas.

Disadvantages, Negative Ideas and Suggestions for Learning in Collaborative Small-Groups

Students in EG encouraged stating the disadvantages of learning in collaborative small-groups, even if they claimed advantages and positive ideas. Their suggestions related to working efficiently in a collaborative small-group were asked. Among EG students, S3 and S4 did not claim any disadvantage or negative idea. However, they were asked if they had any suggestion for a better group work, but they did not suggest any idea. S2 only suggested that every worksheet should be supported by manipulatives. S1's claims are given as follows:

S1: Sometimes, the others do your work. When you are studying on the worksheet, they say, "Be quick!!" or "No, it will not be like that! It will be in this way", and I get very angry. If they don't interrupt you, then it will be a good-working group.

Since S1's main reason for her insufficient group experiences was the group members' behaviors, she complained about being interrupted or hindered

by the group members. Another disadvantage related to the group members were stated by S6:

S6: It is not for me.... But, sometimes, when you don't understand a question or a task.... You can't ask it to your group friends. Or sometimes, when you ask, group members say, "We can do it another time".... Therefore you hesitate to ask (This was S6's general idea about the collaborative small-group work).

S6 did not suggest any idea related to her complain. S1 and S6 claimed disadvantages about group dynamics that made them uncomfortable during the group work. Their complains and S1's suggestions were related to the training of group members about desired and undesired group behaviours. S5 both stated the disadvantages he felt and the suggestions for them in the following dialog:

S5: When there is a group work, there are also quarrels among the group members. It takes so long time to solve a question within a group... When there is only one worksheet for the group, group members quarrel for doing the first. Maybe, it would be better if we sit two-by-two and one worksheet is distributed for each pair in the group. By this way, we can also save time.

According to S5, the main disadvantage of group-work was time consumption. He suggested the effective use of time by changing the group structure, not giving more time.

Group Formation in Collaborative Small-Groups

As mentioned previously, the group member in one group was not constant through the treatment. Some of the students were observed that they worked well in one group, but they did not work and hinder the others

in the other groups. EG students interviewed were asked whether they liked or disliked their groups together with the reasons.

In one of the groups that S1 involved, the particular student who mainly hindered S1 during the group processes, was not present. She claimed the following about that group:

S1: We studied well in that group. It wasn't bad.
Inter: Was he (the particular student that S1 mentioned previously) the only factor for insufficient group work?
S1: Yes, probably.

S4 liked to study in all of his groups, but he talked about particular students that talked unrelated to the group process during the lesson. He said that, those students affected the group process and his performance.

S6 was unhappy with the first group and did not participate in the group work, by saying that she already knew the tasks. She wanted to change her first group. After she changed her group, she became more interested in the group work, and studied on the tasks by her group friends. Her reason for changing her first group was as follows:

S6: I did not like it. I was unhappy. I wanted to change it. After I changed, I was quite happy.

Most interesting results in different group formation was related to S5. The group members in two groups complained about S5 for not participating to the group works that they wrote "He did not do any work" on one of the worksheet. The group, which did not complain about him was the one formed with the friends. This group involved the laziest students of the class. Those students, when they were in other groups distracted the other group members. S5 became a leader of this group and tried to direct the

discussion and make others work. He explained the task to the group members and made them solve the questions one by one. He warned them if they solve incorrectly. Even, this group's process was slow during the lesson, the members showed better performance in this group compared to the other groups. When his ideas about this group was asked he said the following:

S5: That group was much better. I was much more happy with this group. ... Group work can be useful with proper group members.

S3 and S2 did not claim much about the groups that they involved. S3 said he was well with all his groups.

Use of Manipulatives and Materials in Collaborative Small-Groups

This study also focused on how use of manipulatives and materials affected students' learning. Thus, what students think about them and what benefits they realized were investigated during the interviews.

S1 explained the benefits of manipulatives in the following dialog. She also described the manipulatives, which showed that she understood what these manipulatives were related to:

S1: Manipulatives were explaining much better... I mean... something was divided into some parts (continuous representation).... Or there were 5 pieces of play-dough, 3 of them were pink, 2 of them were blue (discrete representation).... It was more understandable.

S3 explained the benefit of manipulatives as an aid in completing task, where only thinking was not efficient. Below, he also explained his preference on the type of the aid the benefited much:

S3: When the cartoons are given, we were able to complete the task correctly.... When we tried to do just by thinking, we did some mistakes.... Although I understood more clearly on the figures, most of us understood better by cartoons.

When students receive help from anyone in the family while studying mathematics, it is also important that the intervention of family members is consistent with the classroom experiences. In this sense, S4 liked the manipulatives, and observed that he used efficiently during the group works. He was also able to use manipulatives at home that were similar to those used during the group works, as he stated in the following dialog:

S4: They were useful.... Actually, my father also uses play-dough when we study mathematics together.

S5's ideas were quite similar to his friends:

S5: Learning with play-dough and cartoons was enjoyable.

S6 explained how manipulatives were useful in terms of their functions in the following dialog:

S6: How can I say? ... You make it live... I mean.... For example, we make it live by using the play-dough.... We make it real... And it becomes more meaningful for me... When we do so, it becomes more enjoyable and more understandable.

Students' common idea about the use of manipulatives indicated that manipulatives made them see the fractions in concrete situations. Learning fractions just on a written text was abstract for them, whereas learning with manipulatives made them perceive fractions like something they can hold,

which was more meaningful. Moreover, enjoyment that they felt while using manipulatives was frequently stated in their dialogs.

Students were asked about the materials in terms of their usefulness. Whether the students studied on them at home as required during the treatment was also asked in order to understand the necessity and usefulness of such requirement.

S1 claimed to benefit from the worksheets. Her explanation is given below:

S1: Studying worksheets were useful.... I could not do them in the group work, but I studied them at home.... In the exam (post-CPPT), I tried to remember what I did in the worksheets... When I could not remember, I got confused.

S2 had negative ideas about the worksheets, probably depending on his negative ideas about the group work. He stated his ideas in the following dialog:

S2: I did not like the worksheets, and I did not do them at home. When the worksheets were given with the manipulatives, they were much better.

Although he did not like the group work and the worksheets, he somehow liked the worksheets with the manipulatives than the ones without the manipulatives. The reason may be that, he might felt enjoyment while studying with the manipulatives during the group work.

S3 told that the worksheets were useful, but he only studied on them during the group work. Below, his reason for not studying the worksheets at home is given in the dialog:

S3: I did not study the worksheets at home. Because I knew them when we study them in the class. After I finish my homework, there was not much time to do the worksheets.

S4's ideas for the worksheets were positive, as presented in the following dialog:

S4: Worksheets were useful. I also studied them at home with my father.

S5 did not have positive ideas about group work as the other students had, but he was able to benefit from the worksheets. The following dialog also showed his preference in the work done in the classroom:

S5: I did all of the worksheets at home. They were useful, but not like the things that we do with the class teacher.

S6 did not study the worksheets at home after she studied on them with her group in the class. She explained the reason not studying them at home and what benefits would she got if she had studied on them in the following dialog:

S6: I did not study the worksheets at home... I think that I already studied them in the class... So no need to study again... If I had studied them, I would understand fractions better and solve the questions that I could not do in the exam (post-CPPT).

It can be said that, students generally thought that the worksheets were useful. However, students with high level of performance were not willing to study on the worksheet once more at home like S3 and S6. S5 was also a high level student, however, since he did not involve in the group

processes, he studied the worksheets at home, which was more meaningful for him.

Control Group

CG students' responses to the questions related to the teaching and learning environment in the interview were categorized under the common points.

Another Method of Teaching and Learning Mathematics

Students in CG were not involved in collaborative small-group learning environment in mathematics. The ideas about learning mathematics by a different method in CG were asked. How they perceive learning in small-groups was also investigated.

S7 had problems in understanding mathematics sufficiently in the class. Thus, she was more willing to be taught mathematics by another way as in the following dialog:

Inter: Do you want to learn mathematics by another way?
S7: Yes
Inter: How can it be, can you give an example? How would you want your teacher to teach you mathematics?
S7: She (the teacher) does whatever she can do. Since there are many people in the class that make noise, most of my friends do not understand. If there is less noise, I believe I will understand.
Inter: Have you ever done group work in any course?
S7: Yes. But we did not do in mathematics.
Inter: If it were in mathematics lesson, would you understand better?
S7: Yes, I would understand better... Learning with my friends would be enjoyable.

S7 probably considered the group-work that she involved in other courses, and decided that learning mathematics would be enjoyable and would understand better. Moreover, she stated that the distracter for her learning was the noise in the classroom, which was frequently observed by the researcher during the instruction on fractions.

S8 was a student who was hard to communicate with, during the interview. He was not very willing to talk during the interview, but he gave more answer to the questions related to the teaching and learning environment compared to the questions related to the post-CPPT. He claimed that learning in a group would be better as can be seen in the following dialog:

Inter: How do you want mathematics to be taught?
S8: ..... (no answer given)
Inter: Would you prefer to be taught by another method, such as group work?
S8: It would be good.
Inter: If there were group work, would you be interested in mathematics lessons?
S8: Yes.
Inter: Maybe there would be much noise in the class if there were group work. Would it affect you?
S8: No, noise would not affect me.
Inter: Would you be happier in the group work?
S8: Yes.

S10 had some problems during the instruction of the fractions, and she overcame the problems by studying so hard on the fractions by herself. She stated that it would be better if mathematics was taught by another method. And she was asked the method that she would like to be taught as below:

Inter: What if mathematics was taught by another way?

S10: It would be better.

Inter: How should be the way?

S10: For example... At the beginning of the fractions.... I don't know... It would be better if it were taught in a different way... For example... by figures.... By dramatic things.... I would understand fractions better....

Inter: What if it would be taught by group work?

S10: Yes, it could be... But, it would be better after we learned the concept, just to make it strengthen... Otherwise, I could not understand the concepts, and I could be confused....

S10's perception of learning mathematics in small-groups was different than that of S7 and S8. She suggested the use of pictorial aids in learning mathematics. Her idea was that, group work was useful only when the teacher taught the concept initially.

Materials Used in the Classroom

The teacher used two posters, one about an orchestra and the other about a farm, in the fraction concept. In order to check whether the students remember the posters, the interviewer asked the students the question "Do you remember the posters?" Following responses were given:

S10: There was something like an orchestra... I only remember this.

S8: No, I don't remember.

S7: I think that there was probably an orchestra...

S9: There was an orchestra... There were violins, piano...

None of the students in CG was able to remember the poster related to the farm. Moreover, those who remembered the orchestra had to think for

a while, and were not sure about the existence of the posters except S9. Since farm was not related to the sociocultural background of the students, they did not probably attended to the use of the poster about the farm and did not remember it.

5.3.4 Students' Attitudes Towards Mathematics

In order to investigate the factors affecting the students' mathematics attitude, the interviewer asked questions about whether they liked mathematics, what they felt about mathematics or the reasons behind their score differences in pre and post AS in the flow of interview. The responses of experimental and CG students are presented here.

S1, a student from EG, had some problems during the collaborative small-group work. But, group work did not seem to affect her attitude towards mathematics, as the following dialog gives the clue about a possible factor that may affect her attitude towards mathematics:

Inter. Do you like mathematics?
S1: I do, but sometimes I feel embarrassed.
Inter. How does it make you feel so?
S1: For example... There is a problem that I can't find how to solve.... I ask to the teacher, and she says, "I don't know!", that time I feel so embarrassed.
Inter. Do you think that you liked learning mathematics in groups?
S1: Yes, it was very enjoyable.

Another student who did not like the group work was S2 from EG. He had also a low attitude score (10) in pre-AS and a high attitude score (58) from post-AS. Here, the reason for his negative attitude is given in the dialog:

Inter: Do you like mathematics?

S2: I was very well with mathematics when I was in the 4th grade.... Only my English was not good in the 4th grade. Then...In the 5th grade the concepts became too hard, they were too boring, and they were disgusting! .... And the teacher (gives the name of the mathematics teacher).... She is actually a good teacher...but not in the class.... If she makes us laugh, or makes jokes in the lesson ..... Then she will be a good teacher.

It seems that, S1 and S2's attitudes were shaped by the mathematics teacher to a great extend. The situation that S1 described was a frequent behavior of the teacher when students in EG asked questions. What S2 wanted to state was also similar to S1's explanation, in the sense that the teacher made the students grow negative feelings and attitudes towards the mathematics lessons in EG. In addition to that, in S1's post-CPPT exam paper, there was a picture that she drew a king and the slaves of the king, which needed further investigation by school counselors.

Similar to S2's ideas about the concepts, S7's (from EG) attitude was affected by the concepts that she learned. The following dialog shows the concepts that affected her attitude towards mathematics:

Inter: Why do you like mathematics?

S7: There are good concepts in it, but I don't like the ones that I can't do.

Inter: Which ones can't you do?

S7: I could not come to class in the first week of the school... I can't do the sets.... I am studying but... I forgot after I learn...

It seems that S7's attitude towards mathematics varies according to the concept learned. Some concepts that she could not perform made S7 develop an attitude towards mathematics concept by concept. When she

performed well, she liked mathematics otherwise, she did not like mathematics.

S8's (from CG) score on AS were higher in pre test (56), but it was lower in the post test (38). The following are the detailed explanation given when the reason for the increase in AS score was asked:

Inter: Do you remember the survey that I distributed to you? .....(Explained the purpose of the survey)... Do you like mathematics? Or when do you like mathematics and when do you dislike it?

S8: I like mathematics, but not that much.

Inter: Your score in the second one (post-AS) is less than the first one (pre-AS). I wonder the reason for this decrease in your score. What affects your like or dislike?

S8: The teaching of the lesson.

Then the interview asked whether there was something wrong with the teaching of the mathematics. He answered that, he could not specify anything with the current teaching of mathematics.

There was a decrease in S4's (from EG) post-AS score (63) when compared with the pre-AS score (73). When the reason for this difference was asked, he did not give a clear answer, but he said that it was not related with the group work and he insisted that he liked mathematics.

S6 had a great increase in her post-AS score (51) when compared with the pre-AS score (8). When the reason for her pre-AS score asked, she answered as follows:

S6: At first, I did not like it so much because.... Everywhere is mathematics... My mother.... With her students... mathematics....

My father has lots of mathematics books... My mother also has many mathematics books.... And... I was bored with mathematics....

S6's parents were both mathematics teacher, which affected her attitude in the negative way. However, she mentioned that studying other courses besides mathematics made her like mathematics again. Moreover, she claimed that group work had a positive influence on her attitude.

A similar increase in AS scores was also detected with S9. She could not explain the reason for the increase but she added, "I've always liked mathematics". S10 in the CG also stated that she liked mathematics so much.

5.4 Results of the Students' Written Works

Students' written works on the group worksheets completed during the treatment were analysed. Different groups' selected written explanations to different tasks related to writing a procedure, a law or a reason are presented here. Since groups were not constant during the study, no group number is given. There were six groups in the class. In the parentheses near the answers, the orders of the tasks in the worksheet are given.

Fractions and We - V (Ordering of Fractions with Equal Denominators or Equal Numerators)

The groups were asked to write a rule for the ordering of the fractions with equal numerators by the help of the figures, in task 3. Five groups completed this task correctly. The following examples are the rules that were written correctly on the worksheets by two of the groups:

> "When the numerators of two fractions are equal, if one of the fraction's denominator is greater, then that fraction is smaller." (Task 3, the part related to writing the rule)

> "For the fractions with equal numerators and unequal denominators: the one with the smaller denominator is greater." (Task 3, the part related to writing the rule)

In this task, five groups were able to write the rule for ordering of the fractions with equal numerators correctly. One group did not give any answer to this task. .

Fractions and We - VI (Ordering of Fractions with Unequal Denominators and Unequal Numerators)

Given two fractions, $\frac{4}{6}$ and $\frac{3}{7}$, the greater one was asked with the reason (without any emphasise on equalising the denominators). All groups completed this task correctly. The following are the answers of two groups:

> "$\frac{4}{6}$ is greater. Because the whole is divided into bigger parts and 4 of them are taken." (Task 1, the part related to finding the greater fraction and the reason)

> "$\frac{4}{6}$ is greater, because if we divide a whole into 6 parts, we get bigger parts" (Task 1, the part related to finding the greater fraction and the reason)

Students were able to comprehend the relation between the number of equal parts and the magnitude of the parts and answered the question correctly without using the equivalence. Selected answers for task 2 (comparing $\frac{13}{4}$ and $\frac{6}{8}$ without any computation) and task 3 (comparing $5\frac{1}{4}$ and $2\frac{5}{12}$) of a group are given below:

> "$\frac{13}{4}$ is greater than one whole, and $\frac{6}{8}$ is less than one whole. So $\frac{13}{4}$ is greater than $\frac{6}{8}$." (Task 2)

"In the fraction $5\frac{1}{4}$ there are five wholes. But in the fractions $2\frac{5}{12}$, there are two wholes. Hence, $5\frac{1}{4}$ is greater than $2\frac{5}{12}$." (Task 3)

The relative magnitude of the fractions in task 2 and the magnitudes of the whole parts of the fractions in task 3 were used by the students to order the fractions without using the equivalence. Five groups were able to answer task 2 and task 3 correctly. The group, which gave the incorrect answer, wrote the following on their worksheet for both questions:

"We can not compare these fractions, because we can not understand whether they are equivalent without any computation."

It seems that this group did not develop a good understanding of equivalence and comparison of fractions. In this worksheet, they tried to complete every task by computation, although they were not wanted. No manipulatives were used in this worksheet.

Fractions and We – VII (Addition of Fractions with Equal Denominators)

Groups were asked to write a rule for the addition of fractions with equal denominators in task 6. Manipulatives were also used in this worksheet. All groups completed this task correctly, and the following are the examples of their answers:

"When adding the fractions with equal denominators, we add the numerators. But we shouldn't add the denominators." (Task 6)

"The denominator is the same. The numerators and the whole numbers are added." (Task 6)

> "During the addition of the fractions with equal denominators, numerators are added, and the denominator is the same." (Task 6)

Although the groups were not able to write the rule exactly, they were aware of the fact that the resultant fraction's denominator was the same as the added fractions, and the numerator of the resultant fraction was the sum of the numerator of the added fractions.

Fractions and We – IX (Addition of Fractions with Unequal Denominators)

By using the given manipulatives, students realised that the addition of the fractions was not just adding the numerators and adding the denominators in the first task given in this worksheet. In task 5, groups were asked to write a rule for addition of fractions with unequal denominators, and all of them answered it correctly. The following are examples of their answers:

> "When performing computations with these kinds of fractions we equalise the denominators. We don't forget to multiply the numerators by the same number" (Task 5)

> "We add the fractions with unequal denominators, by equalising their denominators and then adding as if they are fractions with equal numerators." (Task 5)

The groups realised that the denominators of the fractions should be equal to add them. Similarly, they were able to write the rules for the subtraction tasks.

Fractions and We – XIII (Multiplication of Fractions)

In the first task of this worksheet, students were wanted to remember the multiplication of a fraction with a natural number, which they learnt in the 4^{th} grade mathematics course, by analysing the given multiplications. In task two, they were asked to write a rule for the multiplications given in the first task. All groups answered it correctly, and some examples of the groups' answers are as follows:

> "The natural number is multiplicated by the numerator. The denominator is the same" (Task 2)

> "Only the numerator is multiplicated, the denominator does not change." (Task 2)

The groups were able to remember the initial multiplication experiences by writing the rule for the multiplication of natural numbers with the fractions.

CHAPTER VI

DISCUSSION, CONCLUSION AND IMPLICATIONS

6.1 Discussion

The main purpose of this study was to investigate the effects of sociocultural learning environments including social interaction such as collaboration and cultural tools such as language on 5th grade students' mathematics performance on fraction concepts and operations with fractions. Besides that, the effects of learning in collaborative small-groups on students' attitudes towards mathematics was sought.

Three instruments were used in the study. Conceptual and Procedural Performance Test (CPPT) and Mathematics Attitude Scale (AS) were administered to both EG and CG as pre and post tests. Pre-CPPT results yielded that the EG and CG students' mean scores on CPPT did not differ significantly. Thus, it can be said that EG and CG students were equal in terms of performance on fraction concepts and computations with fractions, prior to the treatment on fractions. However, students' mean scores on pre-AS were significantly different in favour of CG students. Students in CG had more positive attitude towards mathematics, compared to the students in EG. Although these two groups received the same instruction from the same teacher, their mean scores in pre-AS were considerably different, prior to the instruction.

Questionnaire developed was used in order to understand the students' sociocultural background together with their studying habits. The

results showed that the two groups were almost equal in terms of family, economic, housing facts and leisure time activities. There was a slight difference in students' studying habits such as studying everyday and having frequent breaks during studying.

The treatments on both groups were conducted simultaneously in the fractions concept and operations with fractions. EG students were instructed by being involved in collaborative small-groups formed by five students. Materials including worksheets and manipulatives that were prepared by the researcher were used in EG. In CG, students were instructed by the conventional teaching method according to the lesson plans prepared by the 5th grade coordinator. Following the instruction on fractions, both groups were implemented the post-CPPT and post-AS.

In order to investigate the effects of treatment and gender on students' mean scores in post-CPPT, ANOVA was conducted. The results showed that there were no significant main effects of gender and treatment, and no significant interaction effect of gender and treatment on 5th grade students' conceptual and procedural performance. EG students performed almost as well on the post-CPPT as CG. Their performance was remarkably suggested that changing the teacher role from explanation giver to facilitator did not make any changes on the students' performance. However, students from EG class had the chance to build the concepts by themselves rather than receiving concepts directly from the teacher.

ANCOVA was conducted to investigate the effects of treatment and gender on students' means scores in post-AS in both groups, holding the pre-AS scores constant. The results showed that there was no significant main effect of the gender and no significant interaction effect of the gender and the treatment on 5th grade students' attitude towards mathematics, when pre-AS scores were held constant. However, there was a significant main effect of treatment on students' mean post-AS scores in both groups,

when pre-AS scores were held constant. Although this difference seemed not to arise from the pre-AS scores, there appeared other factors that affect students' attitudes negatively in EG, during the interview.

Students' development in ZPD was investigated through semi-structured interviews, by using the questions in CPPT. The results of the interviews showed that there were evidences of the existence of group members' effect in the ZPD of EG students, attributable to the social interaction in collaborative small groups. This finding was an indication of the internalization process, that group members internalized the processes, which they were involved in the class, during collaboration (Cole and Scribner, 1978; Mellin-Olsen, 1987; Nuthall, 1999; Vygotsky, 1960). Moreover, it was found that students used language effectively as a cultural tool in discussion, disagreement and agreement situations during the collaborative small-group work, consistent with the literature findings (Maher, Martino and Davis, 1994; Murray, Olivier and de Bear, 1999). In addition, the results showed that, social interaction and use of language were effective for students in building fractions concepts, consistent with the literature for the mathematics education in general (Stacey and Gooding, 1998). However, when students were asked to solve certain conceptual and procedural questions in post-CPPT, some difficulties and misconceptions in partitioning, number line and discrete representation of fractions, ordering of fractions and operations with different types of fractions were detected.

This study showed that when the partitioning questions were given in the context of "many wholes", students had difficulties in solving the questions. Existence of "many wholes" confused the students' minds, and they were not able to use their understandings of the meaning of fractions in the mentioned contexts. The role of the "whole" in different contexts should be emphasised (Newstead and Olivier, 1999), probably with more handouts and the manipulatives. Although this study involved iteration tasks as recommended by the literature (Murray, Olivier and Human, 1996; Tzur,

1999), those tasks were not enough for students in developing a more general fraction idea. Dominance on partitioning (Tzur, 1999) and part-whole (Behr, Harel, Post and Lesh, 1992; Prie, Martin and Kieren, 1994) constructs from the beginning of the teaching of fractions in the earlier grades might be the reason for this.

However, when students realised their mistakes they were able to use proportional reasoning and solved the questions by first studying on "one whole" and then modifying the results to the "many wholes". In addition to that, students had a good conception of the meaning of partitioning as they always stated the equality of partitioned parts during the interview. The necessity of equal parts for partitioning (Newstead and Olivier, 1999) seemed useful for students, for a basic understanding of fractions. Rectangle was the most frequently used figure in representing fractions with continuous figures.

The number line representation of fractions was found to be difficult for the students (Baturo and Cooper, 1999). The basic misconception appeared in number line representation was considering the number line segment as the whole. Students tended to ignore the numbers on the number line and were not able to recognise them as the wholes, which was consistent with Novillis-Larson's (1980) findings. Moreover, this study showed that when the number line had more than one number, it was difficult to write the fractions greater than one.

In ordering the fractions, there were certain "formulas" that were used by the students. These "formulas" were either given by the teacher or explained by a group member. However, difficulties appear in tasks that involved ordering of fractions with equal numerators and whole number dominance, reported by the literature (Behr, Wachsmuth, Post and Lesh, 1984) was detected during the interview. This study also showed that

students were likely to make errors that were caused by the incomplete equivalence knowledge, in the ordering tasks.

The results of this study showed that representing a given fraction on a discrete set was more difficult for students, compared to writing the fraction represented on a discrete set. This might be because of the fact that the latter needed only counting skill (Behr, et al., 1992), whereas the former might need other skills such as finding an equivalence. But, EG students were more able to use their knowledge and skills in other concepts of fractions, like equivalence, when representing the fractions on discrete sets during the interview.

In this study, it was found that, operations with different kinds of fractions were more difficult for students than the operations with the same kinds of fractions. Mixed fractions were the most problematic kind of fractions in the operations. Errors that were caused by the incorrect converting knowledge and skills were also detected in the operations with mixed fractions. However, students were aware of the fact that operations with improper fractions yielded in correct results.

The results of the EG students' written works in the group worksheets showed that students were able to write the rules related to particular tasks and procedures in fractions. They built an understanding of fractions by studying on the worksheets in collaborative small-groups and learned the rules by using the manipulatives as scaffolds. This kind of invention of own correct procedures by the help of discussion environment was also emphasised by Maher, Martino and Davis (1999), and Newstead and Murray (1998). In CG, however, these rules were written on the table by the teacher and students only wrote the rules on their notebooks and memorised them.

Perhaps, the most important results of this study, which were gained by the interviews, were the students' ideas and suggestions about collaborative small-groups. First, this study showed that students with certain personal characteristics, such as feeling difficulty in building friendship with the classmates, were not willing to participate during the collaborative small-groups. Moreover, in the classroom observations, students who asserted that their answers were the correct one, made their group friends dislike collaborative small-group work. These students slowed down the group's processes in all the groups that they involved, by rejecting every idea other than their ideas. Students who were not interested in the lesson were also distracters during the group study. They also reduced the group members' attention to the tasks and interfered with the discussion in the group. On the other hand, collaborative small groups involving the close friends were found to be better working than the other forms of groups, which was consistent with the literature findings (Azmitia, 1998; Edwards and Jones, 1999; Nelson and Aboud, 1985).

Second, most students benefited from the collaborative small-groups, even if they did not participate sufficiently. In addition to that, collaborative small-groups was found to be useful in learning by the students. The benefit for the low-level students was mainly the explanations that hey received from a group member while studying on a task. Hence, their ZPD was developed by the effects of more capable group members in terms of the difficulty of the tasks that they could perform alone. High level students, on the other hand, benefited from the diversity of ideas that appeared during the discussion. These students realised alternative, and sometimes easier ways (compared to their ways) to solve questions, their ZPD was developed in terms of diversity and quality of ideas, during the collaboration. The above findings of this study were all consistent with Vygotsky's (1981) claims about the role of collaboration in learning.

Figure 6.1 shows how EG students ZPD develop during the collaborative small-group learning. Before treatment, students in EG were in their current ZPD, and were able to solve and understand the tasks between the points E (easiest) and the point M (mid-difficulty). Theoretically, collaboration enables students solve and understand the tasks in the DT point (difficult task in theory). Their ZPD was a thin line before the treatment, since they only had a one-way relation in the teaching and learning environment in terms of receiving explanation (the teacher's instruction). After the treatment, they could not reach the DT point but were able to solve and understand the tasks in the DR point (difficult task in real). However, since they had a reciprocal relation in terms of both receiving and giving explanations (discussion within the group) in the teaching and learning environment, their ZPD became thicker and had a variety of ideas shown by the grey color.

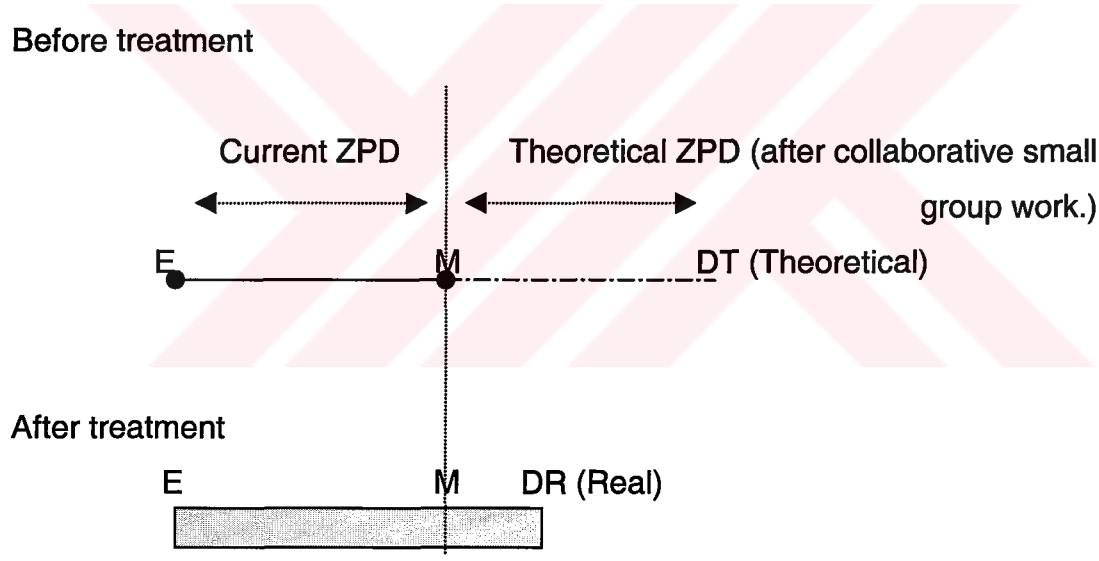


Figure 6.1: EG students' ZPD before and after the treatment.

However, some disadvantages related to the effectiveness of collaboration were also reported during the interviews. When the students were not trained for collaborative small-groups skills, group members and especially who had certain personal characteristics as mentioned before, had problems participating in the group processes. An interviewed high level student from CG stated that she sometimes did not give opportunity to her

group friends who had low level ability. Shyness was also a drawback for some students in the groups, since they hesitated and did not ask questions to the group members in most of the cases. Moreover, having been untrained about collaboration skills, almost all groups quarrelled for sometime during the lesson, either short or long. Ashman and Gillies (1997) indicated that when students were trained for working in groups, the interaction and the use of language in groups and the achievement of learning outcomes were more in those groups, compared to the groups with untrained students. These findings resulted that students should have been trained for the collaborative small-group work skills prior to the implementation of the method.

In addition to that, groups of four were suggested being more efficient (Davidson, 1990) than groups of five, in the form that students sat two by two on a desk and see each other during the study. At this point, group formation also becomes an important factor in effective collaborative small-group work. In this study, students reported to be more happy and good working in the groups that they formed with their close friends, compared to the other groups. During the class observations, some students seemed to have the role of the leader, and they were more influential on group decision and interfered the other students' processes especially during computations with fractions, where students performed the computations one by one after a certain amount of group work. When these students could not have the leader role in a group, they had participation problems.

In this study, instructional materials were developed such as 13 worksheets and some manipulatives with cardboard and play-dough. The results showed that the manipulatives together with the group members were a scaffold in students' thinking by making the fractions meaningful for the students. This findings support the findings of Lesh, Landau and Hamilton (1980). Moreover, it was enjoyable for the students to study on the tasks by manipulatives. The worksheets were useful for students especially when supported by the manipulatives. The figures drawn related to the

manipulatives as well as the manipulatives were useful for students. Some students benefited more from the manipulatives, while some students benefited from the figures drawn. If all the worksheets were prepared including the manipulatives, it was more likely to get better results from the treatment. More emphasis on studying worksheets also at home should be done.

In addition to that, in the tasks that involved manipulatives, the figures that represented the manipulatives were also given on the worksheets. Hence, students who understood better by manipulatives and students who understood better by figures benefited from the materials equivalently, in different contexts. As a result, each student in the group had the opportunity to learn during the lesson, using different aids, which was suggested by the literature (Tudge and Rogoff, 1989; Wood, Bruner and Ross, 1976).

In this study, the ideas of the parents of EG students about the use of collaborative small-groups were also gained in a regular parent meeting of the school. They claimed that they were able to observe changes in their children's progress in mathematics related to the treatment and the materials. They claimed that their children were able ask the questions, which they hesitated to ask to the teacher, to their friends in the collaborative small-groups comfortably. Moreover, they found that the use of manipulatives were significantly useful for their children, especially for those who were slower than their classmates and could not find the chance to go with the class during the conventional teaching. The parents suggested that the teacher should summarise the concept that the students learned in the collaborative small-groups, at the end of the lesson.

The results of the interviews conducted with CG students showed that students were willing to learn mathematics by a method other than the conventional teaching method. Moreover, when group-work was given as an alternative, the students were more sure about its' effects on their learning.

There were two posters used by the teacher during the instruction of the fractions in CG, one about an orchestra, and the other about a farm. Students were not able to remember the poster with the farm, and could remember the orchestra only with little detail. This result showed that, students could remember the materials, which were more close to their sociocultural background. This finding was consistent with Vygotsky's (1981a) "General Genetic Law of Cultural Development" in the sense that, students were able to remember the orchestra poster since an orchestra was an existing image on their sociocultural background, whereas a farm did not exist.

In the same parent meeting mentioned above, parents of CG students claimed they worried about their students' learning of mathematics. They were not happy with their children's progress and suggested that the teacher should teach mathematics better. After this parent meeting, the teacher was observed to be more patient for the CG students' behaviours like loud noise and walking around the classroom during the lesson, which she defined as misbehaviours in EG and warned and punished EG students.

Interview results also showed that students' attitudes towards mathematics were affected by many factors. EG students' attitudes were affected in the negative sense by many factors unrelated to the treatment on fractions. The most important factor appeared as the teacher's negative behaviour towards EG students, which was also detected by the researcher during the classroom observations. The teacher answered fewer questions in EG, warned the students unpleasantly for misbehaviour and punished them during the lesson in less noisy situations, compared to the CG. Hence, this study showed that teacher's behaviour towards students was influential on EG students' behaviour, which was also reported as the most significant effect in the literature (Bishop and Brew, 1996). However, in this study, there were positive effects of learning with collaborative small-groups on the attitude of EG students towards mathematics.

Another factor that affected students' attitude towards mathematics was the effect of parents as reported by Aiken (1972). When parents' tried to be more influential on their children's achievement on mathematics, these children were likely to develop negative attitude towards mathematics. In this study, it was also found that the topics in mathematics were effective in developing positive attitude towards mathematics.

In this study, each lesson of CG and EG in fractions was observed and recorded by the researcher. The advantage of the observation was that the conditions of both EG and CG classes were analysed and the uncontrolled variables in the study were carefully detected. Working with the same teacher during the study was a threat for the study, since the teacher's treatment on CG was likely to be affected by the treatment on EG. It was observed that the teacher was more willing to prepare materials for the teaching of fractions in the control group during the study. Although the materials for 5^{th} grade students were prepared by the coordinator, the teacher started preparing materials for the control group students such as the poster related with the orchestra. During the study, certain difficulties related with the teacher's behaviour were appeared in the treatment of EG. Although the students were working more efficiently in their groups that they formed with their close friends, the teacher formed new groups for the last two weeks because of the loud noise in EG class when she and the researcher entered in the class at the beginning of the lesson. On the other side, she did not warn the students in CG for their misbehaviours even when they continued during the lesson. Working with two teachers would, however, also have a threat such as the effect of different teacher on the students' understandings and attitudes in different classes.

Another issue in the research design was the use of interviews. The development in students' ZPD and the factors that affected students' attitudes towards mathematics were investigated by the interviews as suggested by McLeod (1992).

6.2 Conclusion

Statistical analysis showed that there was no significant mean difference in EG and CG students' post-CPPT scores attributable to the gender, the treatment and the interaction of gender and treatment. There was also no significant mean difference found in EG and CG students' post-AS scores attributable to the gender and the interaction of gender and the treatment. Results, however, showed that there was a significant mean difference in EG and CG students' post-AS scores attributable to the main effect of the treatment. The significant differences in students' AS scores prior and completion of the treatment on fractions in favour of CG students, were found to be caused mainly by the teacher's behaviour.

Although there was no significant mean difference in students' post-CPPT scores in both groups, interviews conducted with students from both EG and CG showed that students built the fractions concepts and computations with fractions in different ways. Each student in a collaborative small-group was able to enhance his or her ZPD in terms of either the difficulty of the tasks, like low ability students did, or the quality and the diversity of the ideas like the high ability students. In this process, manipulatives scaffolded the learning of especially the low ability level students. Manipulatives helped all level of students in understanding the fractions and the procedures that they were involved in better. CG students, however, enhanced their ZPD in terms of the difficulty of the tasks or number of correct answers given to the questions.

The desire for the use of different methods in teaching and learning of fractions were reported by the students who were involved in conventional teaching environment. It seems that students would like to be involved in different activities while learning mathematics.

The results showed that 5th grade students felt difficulties even on the basic concepts of fractions like part-whole and partitioning. Discrete and number line representations were found to be confusing and fractions greater than one were difficult to be represented in these ways. In addition to that, mixed fractions were difficult to compute than the improper fractions. Evidences of whole number dominance were detected more in conceptual tasks then in the procedural tasks.

Attitudes of students towards mathematics were affected by many factors such as the teacher, the parents, the concepts and the teaching method. Among these factors, the teacher affected the CG students' attitudes in the positive way, whereas a negative effect was observed for the EG students' attitude caused by the teacher. However, EG students felt an enjoyment of studying fractions.

It can be concluded that, the sociocultural learning environments including social interaction such as collaboration and cultural tools such as language had effects on 5th grade students' conceptual and procedural performance on fractions performance. The most important effect can be seen in students ZPD, more in the form of quality than the difficulty. Students were able to build fractions concepts by using various ideas and by being scaffolded by the materials and their group friends, hence many misconceptions that were likely to appear was eliminated by scaffolding. Moreover, students were able to understand that mathematics could be enjoyable when the certain method and the materials were used.

Although no significant effect was found by the statistical analysis, EG students were more confident and accurate in explaining their ideas and feelings, compared to CG students during the interviews. EG students presented more ideas about any task during the interview, than the CG students.

6.3 Implications

One of the main aims of the mathematics education is to enhance students' creativity and critical thinking skills by considering their cognitive development level. In the elementary school level, collaborative small-groups should be used with the teacher as a mediator in order to reach the aim mentioned above (MEB, 1999). Students from different ability levels benefit from collaboration and use of language in discussion in different ways, such as, high level ability students benefit much from the variety of ideas where, low level ability students benefit much from the explanations during the group works. This shows that collaboration creates different opportunities for different level of students. At the end, all students get the most proper idea for their learning. However, only collaboration may not be sufficient in certain conditions. Students with different ability levels need the manipulatives as scaffolds in the form of, which help them equally benefit from the collaborative small-groups.

Collaboration, on the other hand, gives more than a shared understanding of school concepts. Listening and respecting each others' ideas, reaching a common decision through discussion and working with a group member that one do not know much are also performed in collaborative small-groups, which in turn, helps students communicate ideas and live with other people in the society. By this way, not only instruction, but also education is given.

Considering the mathematics education, teachers should be aware of all ability level students in their classes, and try to develop certain activities which involves manipulatives that may scaffold the students with low level ability, while the high ability level students benefit from the activity by other forms of scaffolds such as discussion.

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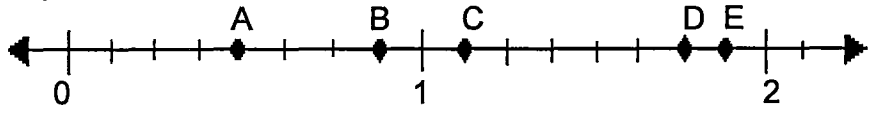
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APPENDIX A

CONCEPTUAL AND PROCEDURAL PERFORMANCE TEST (CPPT)

AD, SOYAD:
NUMARA: **SINIF:**

1.) Sayı doğrusu üzerinde verilen A, B, C, D ve E noktalarının gösterdikleri kesirleri yazınız.



A = B = C = D = E =

2.) Aşağıdaki kesirlerden birbirine denk olan kesirleri gösteriniz.

$$\frac{21}{18}, \frac{4}{9}, \frac{10}{4}, \frac{9}{7}, \frac{7}{9}, \frac{5}{2}, \frac{7}{6}, \frac{8}{18}$$

3.) Aşağıdaki kesirleri < veya > sembollerinden uygun olanını yazarak sıralayınız. (c ve d şıklarında kesirlerin yerlerini değiştirebilirsiniz.)

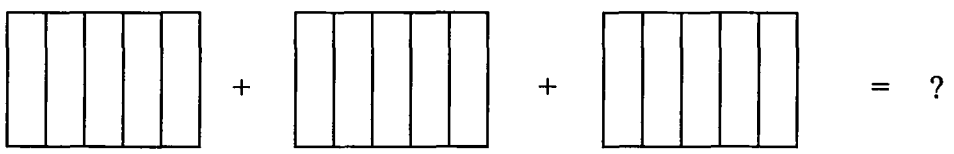
a) $\frac{13}{33}, \frac{13}{26}$ b) $\frac{4}{14}, \frac{6}{14}$ c) $\frac{5}{22}, \frac{12}{22}, \frac{3}{22}, \frac{11}{22}$ d) $\frac{7}{5}, \frac{7}{13}, \frac{7}{38}, \frac{7}{21}$

4.) Aşağıdaki tam sayılı kesirleri bileşik kesre çeviriniz.

a) $3\frac{2}{15} =$ b) $2\frac{3}{22} =$

5.) Aşağıdaki bileşik kesirleri tam sayılı kesre çeviriniz.

a) $\frac{23}{9} =$ b) $\frac{37}{7} =$

6.)  = ?

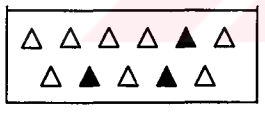
__ + __ + __ = ?

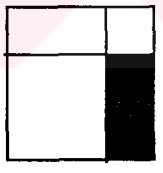
Yukarıda şekiller ile gösterilen işlemin sonucunu bulunuz. Bu işlemi farklı bir işlem kullanarak ifade ediniz. Sonucu şekille gösteriniz.

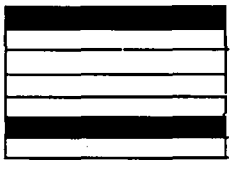
7.) 4 bütünün her biri 5 eşit parçaya bölünüp parçalardan 16 tanesi alınmıştır. Oluşan kesri ve çeşidini yazınız.

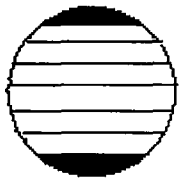
8.) 5 sayısının içinde kaç tane 1/5 vardır?

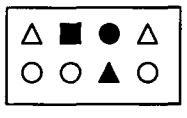
9.) Aşağıda verilen şekil ve çokluklardaki taralı kısımları kesir ile ifade ediniz. Eğer ifade edemiyorsanız sebebini açıklayınız.

a)  = __

d)  = __

b)  = __

e)  = __

c)  = __

10.) Aşağıdaki işlemleri yapınız.

a) $3\frac{1}{9}+\frac{7}{6}=$

b) $\frac{1}{4}-\frac{1}{9}=$

c) $\frac{7}{3}\times\frac{4}{21}$

d) $4\frac{3}{7}-\frac{49}{14}=$

e) $2\frac{1}{4}\times\frac{2}{3}=$

f) $\frac{9}{24}+\frac{1}{8}+\frac{8}{12}=$

g) $7\times4\frac{3}{8}$

11.) Pay ve paydasının toplamı 19 olan

a) Bir basit kesir yazınız.

b) Bir tamsayılı kesir yazınız.

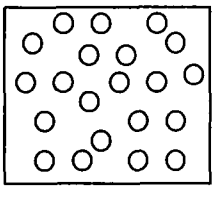
12.) Aşağıda büyüklük veya küçüklük sırasına göre sıralanmış kesirleri görüyorsunuz. Boşlukları sıralamalar bozulmayacak şekilde uygun doğal sayılarla doldurunuz.

a) $\frac{\quad}{7}<\frac{13}{14}$

b) $\frac{7}{\quad}>\frac{8}{5}$

c) $\frac{\quad}{30}<\frac{15}{30}$

d) $\frac{6}{9}>\frac{\quad}{9}$

13.) $2-$  $=\frac{10}{7}$

Yandaki çıkarma işleminde çıkan kesir bilinmiyor. Çıkan kesri bulunuz ve verilen küme üzerinde gösteriniz.

APPENDIX B

MATHEMATICS ATTITUDE SCALE (AS)

Adınız Soyadınız:..... Cinsiyetiniz:.....

Okulunuzun İsmi:..... Sınıfınız:.....

MATEMATİK DERSİNE KARŞI TUTUM ÖLÇEĞİ

Genel Açıklama: Aşağıda öğrencilerin matematik dersine ilişkin tutum cümleleri ile her cümlenin karşısında "Tamamen Uygundur", "Uygundur", "Kararsızım", "Uygun Değildir ve "Hiç Uygun Değildir" olmak üzere beş seçenek verilmiştir. Lütfen cümleleri dikkatli okuduktan sonra her cümle için kendinize uygun olan seçeneklerden birini işaretleyiniz.

	Tamamen Uygundu	Uygundur	Kararsızım	Uygun Değildir	Hiç Uygun Değildir
1. Matematik sevdiğim bir derstir.	O	O	O	O	O
2. Matematik dersine girerken büyük sıkıntı duyarım.	O	O	O	O	O
3. Matematik dersi olmasa öğrencilik hayatı daha zevkli olur.	O	O	O	O	O
4. Arkadaşlarımla matematik tartışmaktan zevk alırım.	O	O	O	O	O
5. Matematiğe ayrılan ders saatlerinin fazla olmasını dilerim.	O	O	O	O	O
6. Matematik dersi çalışırken canım sıkılır.	O	O	O	O	O
7. Matematik dersi benim için gereksizdir.	O	O	O	O	O
8. Matematikten hoşlanırım.	O	O	O	O	O
9. Matematik dersinde zaman geçmez.	O	O	O	O	O
10. Matematik dersi sınavından çekinirim.	O	O	O	O	O
11. Matematik benim için ilgi çekicidir.	O	O	O	O	O
12. Matematik bütün dersler içinde en korktuğum derstir.	O	O	O	O	O
13. Yıllarca matematik okusam bıkmam.	O	O	O	O	O
14. Diğer derslere göre matematiği daha çok severek çalışırım.	O	O	O	O	O
15. Matematik beni huzursuz eder.	O	O	O	O	O
16. Matematik beni ürkütür.	O	O	O	O	O
17. Matematik dersi eğlenceli bir derstir.	O	O	O	O	O
18. Matematik dersinde neşe duyarım.	O	O	O	O	O
19. Derslerin içinde en sevimsizi matematiktir.	O	O	O	O	O
20. Çalışma zamanımın çoğunu matematiğe ayırmak isterim.	O	O	O	O	O

APPENDIX C

QUESTIONNAIRE

1. Anneniz hayatta mı?
   O evet O hayır

2. Babanız hayatta mı?
   O evet O hayır

3. Anne ve babanız birlikte mi yaşıyorlar?
   O evet O hayır

4. Kaç kardeşiniz var?.....

5. Siz ailenin kaçıncı çocuğusunuz? .....

6. Sizden küçük kaç kardeşiniz var? .....

7. Kardeşlerinizden öğrenci olan var mı?
   O evet O hayır

8. Kardeşleriniz hangi okullarda okuyorlar? (birden fazla seçenek işaretleyebilirsiniz)
   O İlköğretim 1. Kademe O İlköğretim 2. Kademe O Lise O Üniversite

9. Anneniz çalışıyor mu?
   O evet O hayır

10. Babanız çalışıyor mu?
    O evet O hayır

11. Annenizin mesleği nedir? .....

12. Babanızın mesleği nedir? .....

13. Annenizin öğrenim durumu nedir?
    O İlkokul O Ortaokul O Lise O Yüksekokul O Üniversite

14. Babanızın öğrenim durumu nedir?
    O İlkokul O Ortaokul O Lise O Yüksekokul O Üniversite

15. Oturduğunuz evde kiracı mısınız?
    O evet O hayır

16. Oturduğunuz ev sobalı mı?
    O evet O hayır

17. Evinizde toplam kaç kişi yaşıyor? .....

18. Evinizde salon ve mutfak dışında kaç oda var?
O 1 O 2 O 3 O 4 O 4'ten fazla

19. Sadece kendinize ait bir odanız var mı?
O evet O hayır

20. Odanızı kardeşlerinizle mi paylaşıyorsunuz?
O evet O hayır

21. Ders çalıştığınız odada başka aile bireyleri de olur mu?
O evet O hayır

22. Ailenizde ders çalışırken size yardımcı olan bir kişi var mı?
O evet O hayır

23. Ailenizde ders çalışmasına yardımcı olduğunuz bir kişi var mı?
O evet O hayır

24. Her gün ders çalışır mısınız?
O evet O hayır

25. Günde yaklaşık olarak toplam kaç saat ders çalışırsınız?
O 1 saatten az O 1-2 saat O 2-3 saat O 3-4 saat O 4 saatten fazla

26. Haftada yaklaşık olarak toplam kaç saat matematik çalışırsınız?
O 1 saatten az O 1-2 saat O 2-3 saat O 3-4 saat O 4 saatten fazla

27. Ders çalışırken sık sık ara verir misiniz?
O evet O hayır

28. Televizyonun olduğu odada ders çalışır mısınız?
O evet O hayır

29. Televizyonun karşısında ders çalışır mısınız?
O evet O hayır

30. Boş zamanlarınızda kitap okur musunuz?
O evet O hayır

31. Evinize her gün gazete alınır mı?
O evet O hayır

32. Her gün gazete okur musunuz?
O evet O hayır

33. Ne tür müzik dinlersiniz? .....

34. Müzik ile ders çalışır mısınız?
O evet O hayır

35. Spor yapar mısınız?
O evet O hayır

36. Evinizde bilgisayar var mı?
O evet O hayır

37. Bilgisayar kullanmayı biliyor musunuz?
O evet O hayır

APPENDIX D

TEACHING MATERIALS USED IN THE CONTROL GROUP

DERS: Matematik **TARİH:** 09-10-2000 / 13-10-2000

SINIF: 5

KONU: Kesirler

HEDEF: Kesirleri kavrayabilme

DAVRANIŞLAR: H_1: $D_1 \rightarrow D_8$

YÖNTEM ve TEKNİKLER: Soru – Cevap, Problem Çözme, Keşfetme

KAYNAKLAR: İlköğretim Matematik 5 (Salih Zeki TAŞKIN, M.Abbas ÇARHOĞLU), Matematik Gezegeni 3. Sayı

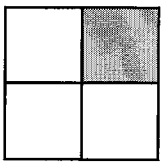
ÖĞRETMEN:

SÜRE: 4x40'

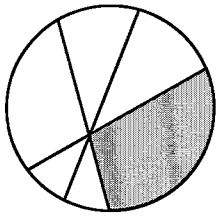
KESİRLER

Öğrencilere " Kesir neydi? " sorusu sorularak, bir önceki yıldan bilgilerini hatırlamaları sağlanır.

Bir bütünün eş parçalara bölünerek, aralarından bazılarının belirlenmesiyle yazılan sayılara **kesir** denir.



Bir bütün 4 eşit parçaya ayrılmış ve bu eş parçalardan 1 tanesi belirlenmiş. Taralı kısmı gösteren kesirsayısı $\frac{1}{4}$'tür.



Bütün eş parçalara ayrılmadığı için taralı kısmı gösteren kesir sayısı $\frac{1}{6}$ değildir.

Pay → Eş parçalardan kaç tanesinin belirlendiğini gösterir
____ → Kesir çizgisi
Payda → Bir bütünün kaç eş parçaya ayrıldığını gösterir

$\frac{3}{5}$ kesri "üç bölü beş" veya "beşte üç" diye okunur.

$\frac{1}{2}$ yarım

$\frac{1}{4}$ çeyrek

KESİR ÇEŞİTLERİ

1-) Basit Kesir

Babam akşam eve gelirken bir yaş pasta almış, annem de pastayı 8 parçaya ayırıyor ve herkese birer parça veriyor. Ailece pastanın ne kadarını yemiş olduk?

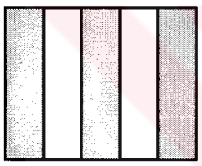
Pastanın $\frac{3}{8}$ ini yemiş olduk.

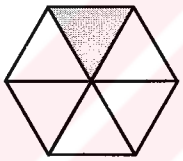
Payı paydasından küçük olan kesirler basit kesirdir.

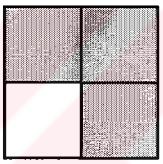
Basit kesirler bir bütünden küçüktür.

Örneğin; $\frac{3}{7}, \frac{4}{9}, \frac{8}{15}$ gibi...

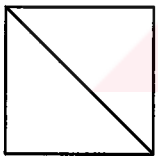
Soru: Verilen şekillerin boyalı kısımlarını gösteren kesirleri yazınız.



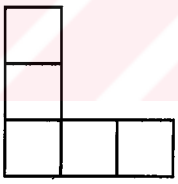




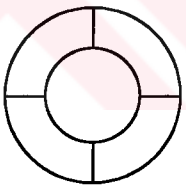
Soru: Verilen şekillerin istenilen kesir kadarını bulunuz.



$\frac{1}{2}$



$\frac{2}{5}$



$\frac{1}{4}$

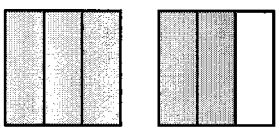
2-) Bileşik Kesir

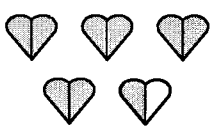
Aynı büyüklükte 2 tane karton aldım. Kartonların her birini 3 eş parçaya ayırdım. Eş parçaların 5 tanesini kullandım. Kullandığım kartonları kesir olarak yazarsam $\frac{5}{3}$ olur.

Payı paydasına eşit ya da payı paydasından büyük olan kesirler bileşik kesirdir.

Örneğin; $\frac{4}{9}, \frac{15}{11}, \frac{5}{5}$ gibi...

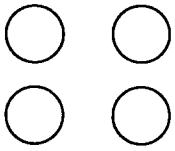
Soru: Verilen şekillerin boyalı kısımlarını gösteren kesirleri yazınız.







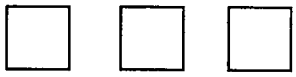
Soru: Verilen şekillerin istenilen kesir kadarını bulunuz.



$\frac{7}{2}$



$\frac{3}{3}$



$\frac{9}{4}$

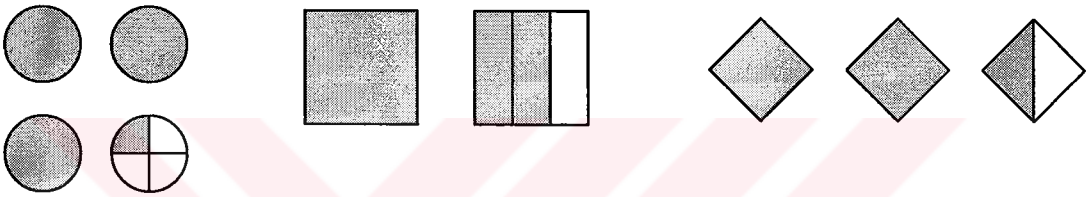
3-) Tam Sayılı Kesir

Bir manav, pazar günü elindeki domateslerin 4 kasasının tamamını, beşinci kasanın da üçte birini satmıştır. Satılan domatesleri kesir ile yazarsak $4\frac{1}{3}$ olur. Burada $4\frac{1}{3}=4+\frac{1}{3}$ tür.

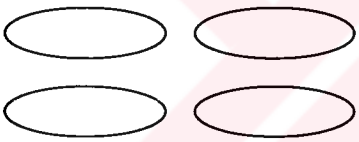
Bir sayma sayısı ile basit kesrin oluşturduğu kesirler tam sayılı kesirdir.

Örneğin; $2\frac{1}{2}$, $5\frac{3}{7}$, $3\frac{2}{5}$ gibi...

Soru: Verilen şekillerin boyalı kısımlarını gösteren kesirleri yazınız.



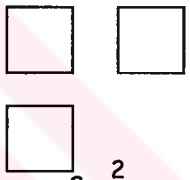
Soru: Verilen şekillerin istenilen kesir kadarını bulunuz.



$3\frac{3}{4}$



$1\frac{1}{2}$

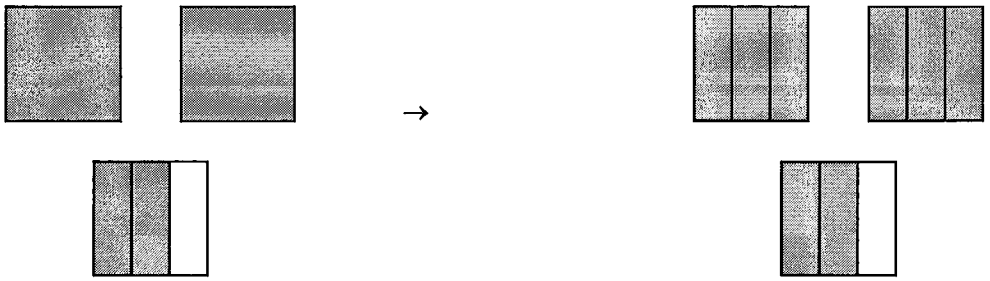


$2\frac{2}{3}$

Kesirlerin Birbirine Çevrilmesi

a) Tam Sayılı Kesrin Bileşik Kesre Çevrilmesi

Cem maket yapmak için aldığı resim kağıtlarının $2\frac{2}{3}$'ünü kullandı. Cem'in kullandığı resim kağıtlarını bileşik kesir olarak gösterelim.



$2\frac{2}{3}$ → $\frac{8}{3}$

Bir tam sayılı kesri, bileşik kesre çevirmek için tam sayı ile kesrin paydası çarpılır. Çarpım pay ile toplanıp bileşik kesrin payına yazılır. Payda aynen yazılır.

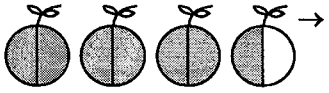
$$\text{Tam Sayı}\ \frac{\text{Pay}}{\text{Payda}} = \frac{(\text{Tam Sayı} \times \text{Payda}) + \text{Pay}}{\text{Payda}}$$

Soru: Verilen tam sayılı kesirleri bileşik kesir olarak yazınız.

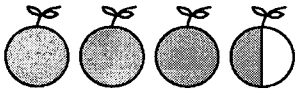
$2\frac{3}{10}$ $3\frac{3}{5}$ $4\frac{5}{7}$

b) Bileşik Kesrin Tam Sayılı Kesre Çevrilmesi

Annem, elmalı pasta yapmak için $\frac{7}{2}$ tane elma kullandığını söyledi. Annemim elmalı pasta yaparken ne kadar elma kullandığını tam sayılı kesir olarak bulabilir misiniz?

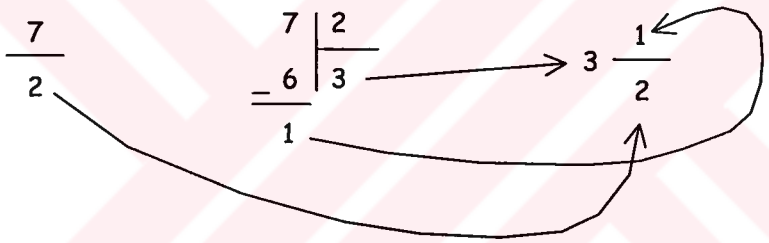


$\frac{7}{2}$



$3\frac{1}{2}$

Bir bileşik kesri, tam sayılı kesre çevirmek için pay paydaya bölünür. Bölme işlemindeki bölüm, tam sayı; kalan da pay olarak yazılır. Payda aynen yazılır.



Doğal Sayıların Kesir Olarak Yazılması

7 doğal sayısının 1'e bölümü yine kendisine eşit midir? EVET...

O halde; 5 doğal sayısını kesir olarak, $\frac{5}{1}$ şeklinde yazarız.

Her doğal sayı, paydası 1 olan kesir olarak yazılabilir.

Soru: 15 ve 1071 doğal sayılarını kesir olarak yazınız.

$15 = \frac{15}{1}$ $1071 = \frac{1071}{1}$

Kesirleri Genişletme

Öğrencilerden defterlerine birbirinin aynısı 3 tane dikdörtgen (12 × 5 lik) çizmeleri istenir.

Sırasıyla $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ kesirlerini bu dikdörtgenlerde göstermeleri istenir.

Bütünler aynı olduğu zaman bu kesirlerin aynı şeyi ifade ettikleri belirtilir.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$ olduğu söylenir

Burada 2.kesrin ilk kesrin pay ve paydasının 2 ile çarpılmasıyla elde edildiği, 3.kesrin ise yine ilk kesrin pay ve paydasının 3 ile çarpılmasıyla elde edildiği belirtilir.

Bir kesrin, pay ve paydasını aynı sayma sayısı ile çarpma işlemine kesri genişletme denir.

Kesrin genişletilmesi sırasında kesrin değeri değişmez.

Soru: Verilen kesirleri 3 ve 5 ile genişletiniz.

$\frac{6}{5}$ $\frac{4}{4}$

Kesirleri Sadeleştirme

3 ile genişleterek elde ettiğimiz $\frac{18}{15}$ kesrinin, genişletilmemiş halini tekrar elde edebilir miyiz?

Genişletilmiş kesrin pay ve paydasını 3 e bölelim:

$\frac{18:3}{15:3}=\frac{6}{5}$ bulunur.

$\frac{6}{5}$ kesri $\frac{18}{15}$ kesrinin genişletilmemiş biçimidir.

Bir kesrin, pay ve paydasını aynı sayma sayısına bölme işlemine kesri sadeleştirme denir.

Kesrin sadeleştirilmesi sırasında kesrin değeri değişmez.

Soru: Verilen kesirleri en sade şekilde yazınız.

$\frac{36}{48}$ $\frac{12}{16}$ $\frac{34}{51}$

Denk Kesirler

Bir bütünün eş kısımlarını gösteren kesirler denktir.

Kesirlerde genişletme ve sadeleştirme yaparak birbirine denk kesirler elde edilir.

Soru: Verilen kesre denk olan kesri, karşısındaki kesirler arasından seçip daire içine alınız.

$\frac{25}{40}$ $\frac{5}{9}$ $\frac{10}{16}$ $\frac{5}{20}$

$\frac{7}{2}$ $\frac{5}{1}$ $\frac{10}{5}$ $\frac{49}{14}$

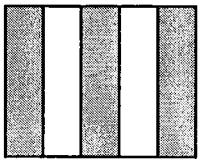
Soru: $\frac{20}{12}$ kesri $\frac{15}{9}$ kesrine denk midir?

Soru: $\frac{6}{12}=\frac{?}{4}$ eşitliğinin doğru olabilmesi için "?" yerine ne yazılmalıdır?

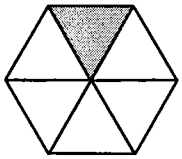
ÖDEV:

Matematik Ders Kitabı Sayfa 41, 44, 45, 49 Alıştırmalar

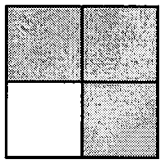
Soru: Verilen şekillerin boyalı kısımlarını gösteren kesirleri yazınız.



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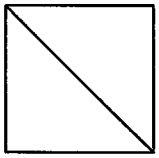


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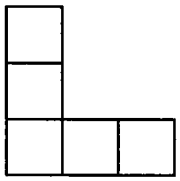


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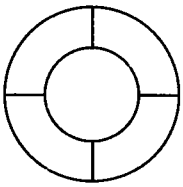
Soru: Verilen şekillerin istenilen kesir kadarını bulunuz.



$\frac{1}{2}$

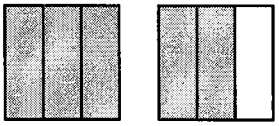


$\frac{2}{5}$

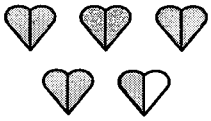


$\frac{1}{4}$

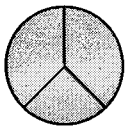
Soru: Verilen şekillerin boyalı kısımlarını gösteren kesirleri yazınız.



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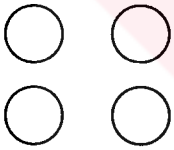


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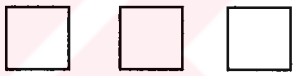
Soru: Verilen şekillerin istenilen kesir kadarını bulunuz.



$\frac{7}{2}$

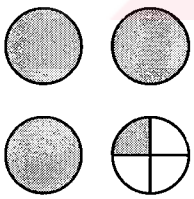


$\frac{2}{2}$

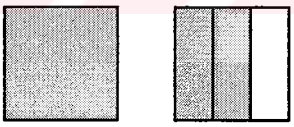


$\frac{9}{4}$

Soru: : Verilen şekillerin boyalı kısımlarını gösteren kesirleri yazınız.



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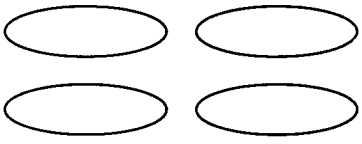


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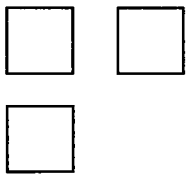
Soru: Verilen şekillerin istenilen kesir kadarını bulunuz.



$3\frac{3}{4}$



$1\frac{1}{2}$



$2\frac{2}{3}$

DERS: Matematik **TARİH:** 16-10-2000 / 20-10-2000

SINIF: 5

KONU: Kesirler

HEDEF: Kesirleri karşılaştırabilme

DAVRANIŞLAR: H_2: $D_1 \rightarrow D_6$

YÖNTEM ve TEKNİKLER: Soru – Cevap, Problem Çözme, Keşfetme

KAYNAKLAR: İlköğretim Matematik 5 (Salih Zeki TAŞKIN, M.Abbas ÇARHOĞLU), Matematik Gezegeni 3. Sayı

ÖĞRETMEN:

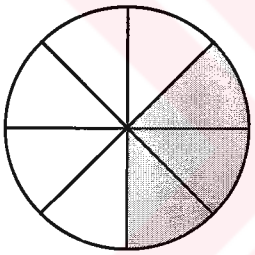
SÜRE: 4x40'

KESİRLERİN KARŞILAŞTIRILMASI

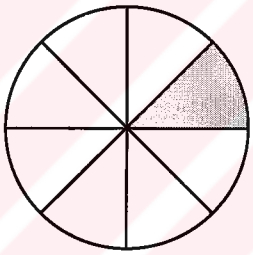
Paydaları Eşit Olan Kesirleri Karşılaştırma:

Dün akşam pizzacıda ben 3 dilim pizza yedim, kardeşim ise 2 dilim yedi. Hangimiz daha fazla pizza yedik?

Dilimlerin büyüklükleri eşit olduğu ve ben daha çok dilim yediğim için daha fazla pizza yemiş oldum. Burada dilimlerin eş büyüklükte olduğu vurgulanır.

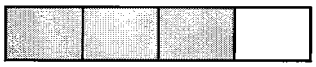


Benim yediğim pizza



Kardeşimin yediği pizza

Dünyamızın yaklaşık $\frac{3}{4}$'ünün sularla, $\frac{1}{4}$'ünün karalarla kaplı olduğunu öğrenmiştik. Bu bilgilere göre dünyamızda sular mı yoksa karalar mı daha fazladır?



Sular



Denizler

Bitinler eş parçalara bölünmüş ise parça sayısı fazla olan büyüktür, az olan küçüktür.

Yani; **paydası eşit olan iki kesirden payı büyük olan kesir büyüktür.**

Soru: Verilen kesirlerin arasına < veya > işaretlerinden uygun olanını yazınız.

$\frac{7}{8}$ ..... $\frac{11}{8}$ $\frac{5}{5}$ ..... $\frac{4}{5}$ $\frac{18}{17}$ ..... $\frac{9}{17}$

Soru: Verilen kesirleri küçükten büyüğe doğru sıralayıp sembol kullanarak yazınız.

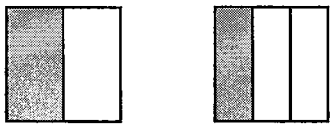
$\frac{11}{9}$ $\frac{2}{9}$ $\frac{8}{9}$

Soru: Verilen kesirleri büyükten küçüğe doğru sıralayıp sembol kullanarak yazınız.

$\frac{17}{5}$ $\frac{8}{5}$ $\frac{11}{5}$ $\frac{3}{5}$ $\frac{5}{5}$

Payları Eşit Olan Kesirleri Karşılaştırma:

Marketten aldığım 2 paket Ülker kare çikolatadan birini kardeşim ve ben, diğerini de annem, babam ve büyükbabam aramızda eşit olarak paylaştık. Acaba annem mi daha fazla çikolata yedi yoksa ben mi?



Ben Annem

$\frac{1}{2} > \frac{1}{3}$

Annemler daha fazla kişi oldukları için çikolata daha fazla parçaya bölündü, parçalar daha küçük oldu.

Eşit büyüklükte iki kekten 4 dilime ayrılmış olandan 3 dilim mi yersem yoksa 8 dilime ayrılmış olandan 3 dilim mi yersem daha fazla yemiş olurum?



$\frac{3}{4} > \frac{3}{8}$

Birinci kek daha az dilime ayrıldığı için parçalar daha büyük oldu. İkinci kekte parçalar daha küçüktü. Her ikisinden de üçer dilim yiyebileceğim için az sayıda parçaya bölünmüş olandan yemem daha mantıklı...

Payları eşit olan iki kesirden paydası küçük olan daha büyüktür.

Soru: Verilen kesirlerin arasına < veya > işaretlerinden uygun olanını yazınız.

$\frac{3}{5}$ ..... $\frac{3}{2}$ $\frac{8}{9}$ ..... $\frac{8}{11}$ $\frac{5}{4}$ ..... $\frac{5}{5}$

Soru: Verilen kesirleri küçükten büyüğe doğru sıralayıp sembol kullanarak yazınız.

$\frac{7}{3}$ $\frac{7}{20}$ $\frac{7}{11}$

Soru: Verilen kesirleri büyükten küçüğe doğru sıralayıp sembol kullanarak yazınız.

$\frac{6}{5}$ $\frac{6}{19}$ $\frac{6}{6}$ $\frac{6}{9}$ $\frac{6}{13}$

Soru: $\frac{13}{5}$ kesrinden büyük ve küçük olan birer kesir yazınız.

Paydayı sabit tutarsak ----> $\frac{12}{5} < \frac{13}{5} < \frac{14}{5}$

Payı sabit tutarsak ----> $\frac{13}{6} < \frac{13}{5} < \frac{13}{4}$

Soru: Karşılaştırmalar doğru olacak şekilde ? yerine yazılabilecek en küçük doğal sayıyı bulunuz.

$\frac{13}{10} < \frac{?}{10}$ $\qquad$ $\frac{3}{5} > \frac{3}{?}$

Soru: Karşılaştırmalar doğru olacak şekilde ? yerine yazılabilecek en büyük doğal sayıyı bulunuz.

$\frac{7}{17} > \frac{?}{17}$ $\qquad$ $\frac{8}{11} < \frac{8}{?}$

Payları ve Paydaları Eşit Olmayan Kesirleri Karşılaştırma:

Ne yapabiliriz?

Neler biliyoruz?

Soru: $\frac{3}{8}$ ile $\frac{1}{4}$ kesirlerini karşılaştıralım.

$\frac{1}{4} = \frac{2}{8}$ olduğu için $\frac{3}{8} > \frac{2}{8}$ ve $\frac{3}{8} > \frac{1}{4}$ olur.

Soru: $\frac{6}{7}$ ile $\frac{3}{5}$ kesirlerini karşılaştıralım.

$\frac{3}{5} = \frac{6}{10}$ olduğu için $\frac{6}{10} < \frac{6}{7}$ ve $\frac{3}{5} < \frac{6}{7}$ olur.

Soru: $\frac{7}{10}$ ile $\frac{8}{15}$ kesirlerini karşılaştıralım.

$\frac{7}{10} = \frac{21}{30}$ ve $\frac{8}{15} = \frac{16}{30}$ olduğu için $\frac{21}{30} > \frac{16}{30}$ ve $\frac{7}{10} > \frac{8}{15}$ olur.

Soru: $\frac{2}{5}$ ile $\frac{2}{3}$ kesirlerini karşılaştıralım.

$\frac{2}{5} = \frac{6}{15}$ ve $\frac{2}{3} = \frac{10}{15}$ olduğu için $\frac{6}{15} < \frac{10}{15}$ ve $\frac{2}{5} < \frac{2}{3}$ olur.

Payları eşit olan kesirleri karşılaştırırken paydası küçük olanın büyük olacağını zaten öğrenmiştik.

ÖDEV:

Matematik Ders Kitabı Sayfa 54, 55 Alıştırmalar

DERS: Matematik **TARİH:** 13-11-2000 / 17-11-2000

SINIF: 5

KONU: Toplama İşlemi

HEDEF: Paydaları en çok iki basamaklı olan kesirlerle toplama işlemini yapabilme

DAVRANIŞLAR: H_3: $D_1 \rightarrow D_{10}$

YÖNTEM ve TEKNİKLER: Soru – Cevap, Anlatım,

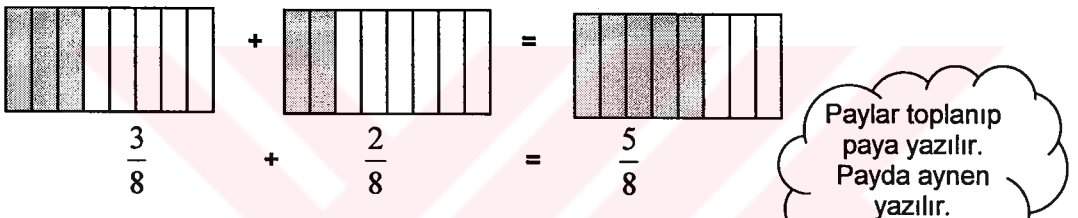
KAYNAKLAR: İlköğretim Matematik 5 (Salih Zeki TAŞKIN, M.Abbas ÇARHOĞLU), Matematik Gezegeni 3. Sayı

ÖĞRETMEN:

SÜRE: 4×40'

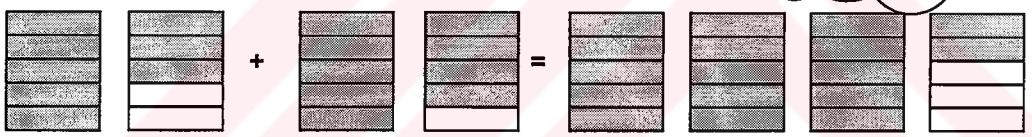
KESİRLERLE TOPLAMA İŞLEMİ

Kesirlerle toplama işleminin nasıl yapıldığını hatırlayalım mı?

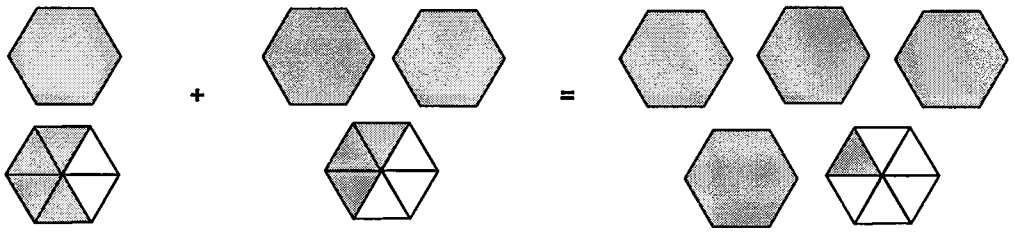


$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

Paylar toplanıp paya yazılır. Payda aynen yazılır.

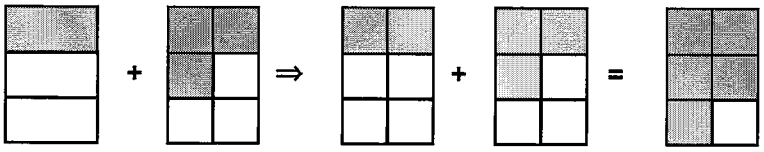


$\frac{8}{5} + \frac{9}{5} = \frac{17}{5}$



$1\frac{4}{6} + 2\frac{3}{6} = 4\frac{1}{6}$

Tam sayılı kesirler bileşik kesre çevrilip toplama işlemi yapılır.



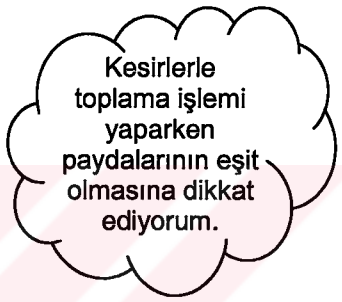
$\frac{1}{3} + \frac{3}{6} \Rightarrow \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

$\frac{1}{3}$ ve $\frac{2}{6}$ kesirleri birbirine denk oldukları için $\frac{1}{3}$ yerine $\frac{2}{6}$ yazabildik. Daha sonra ise payları toplayıp paya yazdık, ortak olan paydayı da aynen aldık.

$$\frac{7}{10}+\frac{3}{5}=\frac{7}{10}+\frac{6}{10}=\frac{7+6}{5}=\frac{13}{10}=1\frac{3}{10}$$

$$\frac{3}{2}+\frac{13}{12}=\frac{18}{12}+\frac{13}{12}=\frac{18+13}{12}=\frac{31}{12}=2\frac{7}{12}$$

$$\frac{23}{12}+\frac{5}{4}=\frac{23}{12}+\frac{15}{12}=\frac{23+15}{12}=\frac{38}{12}=\frac{19}{6}=3\frac{1}{6}$$



$$1\frac{2}{3}+2\frac{1}{6}=\frac{5}{3}+\frac{13}{6}=\frac{10}{6}+\frac{13}{6}=\frac{10+13}{6}=\frac{23}{6}=3\frac{5}{6}$$

$$2\frac{2}{5}+3\frac{8}{15}=\frac{12}{5}+\frac{53}{15}=\frac{36}{15}+\frac{53}{15}=\frac{36+53}{15}=\frac{89}{15}=5\frac{14}{15}$$

$$3\frac{3}{4}+\frac{11}{8}=\frac{15}{4}+\frac{11}{8}=\frac{30}{8}+\frac{11}{8}=\frac{30+11}{8}=\frac{41}{8}=5\frac{1}{8}$$

ALIŞTIRMALAR

1. $\frac{3}{5}+\frac{4}{7}=\frac{21}{35}+\frac{20}{35}=\frac{21+20}{35}=\frac{41}{35}=1\frac{6}{35}$

2. $\frac{2}{9}+\frac{1}{4}=\frac{8}{36}+\frac{9}{36}=\frac{8+9}{36}=\frac{17}{36}$

3. $\frac{13}{6}+\frac{7}{5}=\frac{65}{30}+\frac{42}{30}=\frac{65+42}{30}=\frac{107}{30}=3\frac{17}{30}$

4. $\frac{3}{4}+\frac{9}{7}=\frac{21}{28}+\frac{36}{28}=\frac{21+36}{28}=\frac{57}{28}=2\frac{1}{28}$

5. $2\frac{1}{3}+1\frac{2}{5}=\frac{7}{3}+\frac{7}{5}=\frac{35}{15}+\frac{21}{15}=\frac{35+21}{15}=\frac{56}{15}=3\frac{11}{15}$

6. $\frac{1}{2}+3\frac{5}{9}=\frac{1}{2}+\frac{32}{9}=\frac{9}{18}+\frac{64}{18}=\frac{9+64}{18}=\frac{73}{18}=4\frac{1}{18}$

- Kesirleri toplarken payları toplayıp paya, paydaları toplayıp paydaya yazsak olur mu acaba?

(Sınıfta tartışılacak ve şekille bunun olmayacağı gösterilecek.)

ALIŞTIRMALAR

1. $\frac{2}{5}+\frac{1}{5}+\frac{3}{10}=\frac{3}{5}+\frac{3}{10}=\frac{6}{10}+\frac{3}{10}=\frac{6+3}{10}=\frac{9}{10}$

2. $\frac{7}{4}+\frac{17}{12}+\frac{6}{4}=\frac{21}{12}+\frac{17}{12}+\frac{18}{12}=\frac{21+17+18}{12}=\frac{56}{12}=\frac{14}{3}=4\frac{2}{3}$

3. $1\frac{2}{3}+2\frac{1}{3}+1\frac{5}{6}=\frac{5}{3}+\frac{7}{3}+\frac{11}{6}=\frac{12}{3}+\frac{11}{6}=\frac{24}{6}+\frac{11}{6}=\frac{24+11}{6}=\frac{35}{6}=5\frac{5}{6}$

4. $2+\frac{5}{4}=\frac{2}{1}+\frac{5}{4}=\frac{8}{4}+\frac{5}{4}=\frac{8+5}{4}=\frac{13}{4}=3\frac{1}{4}$

5. $3+\frac{1}{5}=3\frac{1}{5}$

6. $4+2\frac{1}{5}=5\frac{1}{5}$

Kesirlerle yaptığım bir toplama işleminden sonra

- Paydalar eşit mi?
- Paylar toplamını paya yazdım mı?
- Payda aynen duruyor mu?

Sorularının üçünü de EVET diye cevaplıyorsam yaptığım işlem doğrudur.

ÖDEV:

Matematik Ders Kitabı Sayfa 86 3. Soru

Matematik Gezegeni 3. Sayı 120, 126, 127, 129 ve 131. Sayfalar

DERS: Matematik **TARİH:** 11-12-2000 / 15-12-2000

SINIF: 5

KONU: Çıkarma İşlemi

HEDEF: Paydaları en çok iki basamaklı olan kesirlerle çıkarma işlemini yapabilme

DAVRANIŞLAR: H_3: $D_1 \rightarrow D_9$

YÖNTEM ve TEKNİKLER: Soru – Cevap, Keşfetme

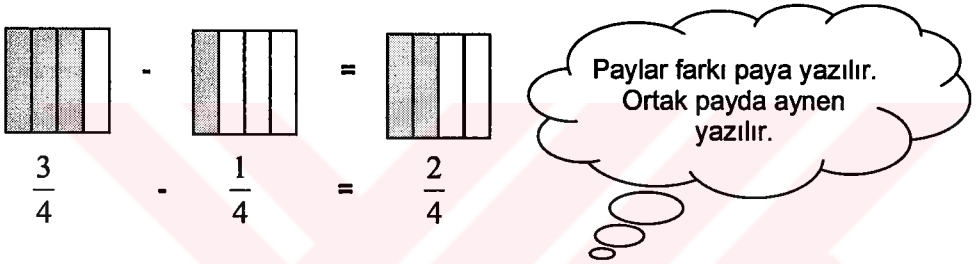
KAYNAKLAR: İlköğretim Matematik 5 (Salih Zeki TAŞKIN, M.Abbas ÇARHOĞLU) Matematik Gezegeni 3. Sayı

ÖĞRETMEN:

SÜRE: 4×40'

KESİRLERLE ÇIKARMA İŞLEMİ

Kesirlerle çıkarma işleminin nasıl yapıldığını hatırlayalım mı?

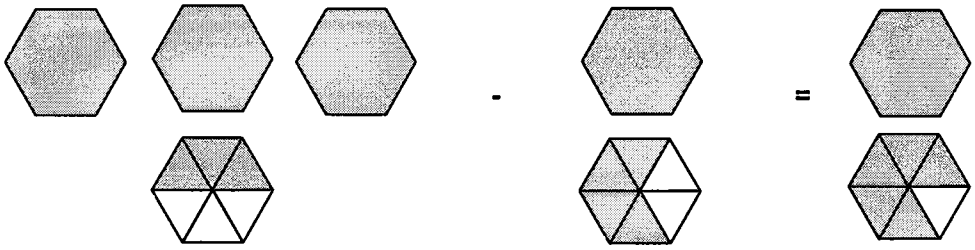


$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

Paylar farkı paya yazılır. Ortak payda aynen yazılır.

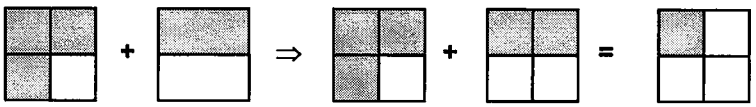


$\frac{7}{3} - \frac{5}{3} = \frac{2}{3}$



$3\frac{3}{6} - 1\frac{4}{6} = 1\frac{5}{6}$

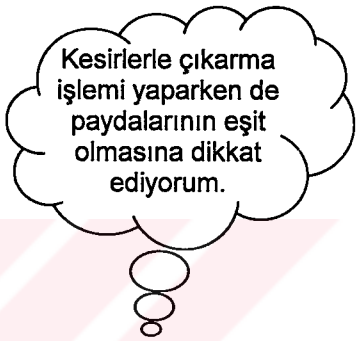
Tam sayılı kesirler bileşik kesre çevrilip çıkarma işlemi yapılır.



$\frac{3}{4} - \frac{1}{2} \Rightarrow \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

$\frac{1}{2}$ ve $\frac{2}{4}$ kesirleri birbirine denk oldukları için $\frac{1}{2}$ yerine $\frac{2}{4}$ yazabildik. Daha sonra ise payları çıkarıp paya yazdık, ortak olan paydayı da aynen aldık.

ALIŞTIRMALAR



1-) $\frac{9}{10} - \frac{3}{5} = \frac{9}{10} - \frac{6}{10} = \frac{9-6}{10} = \frac{3}{10}$

2-) $\frac{3}{4} - \frac{1}{12} = \frac{9}{12} - \frac{1}{12} = \frac{9-1}{12} = \frac{8}{12} = \frac{2}{3}$

3-) $\frac{15}{7} - \frac{25}{21} = \frac{45}{21} - \frac{25}{21} = \frac{45-25}{21} = \frac{20}{21}$

4-) $\frac{61}{18} - \frac{4}{3} = \frac{61}{18} - \frac{24}{18} = \frac{61-24}{18} = \frac{37}{18} = 2\frac{1}{18}$

5-) $5\frac{7}{36} - 2\frac{2}{9} = \frac{187}{36} - \frac{20}{9} = \frac{187}{36} - \frac{80}{36} = \frac{187-80}{36} = \frac{107}{36} = 2\frac{35}{36}$

6-) $3\frac{15}{22} - 1\frac{4}{11} = \frac{81}{22} - \frac{15}{11} = \frac{81}{22} - \frac{30}{22} = \frac{81-30}{22} = \frac{51}{22} = 2\frac{7}{22}$

7-) $4\frac{7}{15} - \frac{8}{5} = \frac{67}{15} - \frac{24}{15} = \frac{67-24}{15} = \frac{43}{15} = 2\frac{13}{15}$

ALIŞTIRMALAR

1. $\frac{3}{5} - \frac{4}{7} = \frac{21}{35} - \frac{20}{35} = \frac{21-20}{35} = \frac{1}{35}$

2. $\frac{1}{4} - \frac{1}{9} = \frac{9}{36} - \frac{4}{36} = \frac{9-4}{36} = \frac{5}{36}$

3. $\frac{13}{6} - \frac{7}{5} = \frac{65}{30} - \frac{42}{30} = \frac{65-42}{30} = \frac{23}{30}$

4. $\frac{23}{7} - \frac{3}{4} = \frac{92}{28} - \frac{21}{28} = \frac{92-21}{28} = \frac{71}{28} = 2\frac{15}{28}$

5. $2\frac{1}{3}+1\frac{5}{7}=\frac{7}{3}-\frac{12}{7}=\frac{49}{21}-\frac{36}{21}=\frac{49-36}{21}=\frac{13}{21}$

6. $4\frac{7}{9}-\frac{2}{5}=\frac{43}{9}-\frac{2}{5}=\frac{215}{45}-\frac{18}{45}=\frac{215-18}{45}=\frac{197}{45}=4\frac{17}{45}$

ALIŞTIRMALAR

1-) $4-\frac{3}{5}=\frac{4}{1}-\frac{3}{5}=\frac{20}{5}-\frac{3}{5}=\frac{20-3}{5}=\frac{17}{5}=3\frac{2}{5}$

2-) $7-\frac{9}{2}=\frac{7}{1}-\frac{9}{2}=\frac{14}{2}-\frac{9}{2}=\frac{14-9}{2}=\frac{5}{2}=2\frac{1}{2}$

3-) $3-2\frac{1}{6}=\frac{3}{1}-\frac{13}{6}=\frac{18}{6}-\frac{13}{6}=\frac{18-13}{6}=\frac{5}{6}$

4-) $6\frac{3}{8}-2=4\frac{3}{8}$

KONTROL

- Paydaları eşitledim mi?
- Payların farkını alıp paya yazdım mı?
- Payda aynen duruyor mu?

Sorularının üçüne de EVET cevabını veriyorsam yaptığım çıkarma işlemi doğrudur.

ÖDEV:

Matematik Ders Kitabı Sayfa 101 Alıştırmalar

Matematik Gezegeni 3. Sayı Sayfa 142, 145, 148

Matematik Gezegeni 3. Sayı Sayfa 149, 150, 151, 152 (Hafta sonu için)

DERS: Matematik **TARİH:** 08-01-2001 / 12-01-2001

SINIF: 5

KONU: Çarpma İşlemi

HEDEF: Kesirlerle çarpma işlemini yapabilme

DAVRANIŞLAR: H_3: $D_1 \rightarrow D_9$

YÖNTEM ve TEKNİKLER: Soru – Cevap, Anlatım

KAYNAKLAR: İlköğretim Matematik 5 (Salih Zeki TAŞKIN, M.Abbas ÇARHOĞLU) Matematik Gezegeni 3. Sayı

ÖĞRETMEN:

SÜRE: 4x40'

KESİRLERLE ÇARPMA İŞLEMİ

Çiftçi Ahmet her gün tarlasının $\frac{1}{4}$'ünü sürerse 3.günün sonunda tarlanın kaçta kaçı sürülmüş olur?



1.gün 2.gün 3.gün 3 günün sonunda

3 tane $\frac{1}{4} = \frac{3}{4}$ veya $3 \times \frac{1}{4} = \frac{3 \times 1}{4} = \frac{3}{4}$ olur.

Soru: Verilen işlemleri yapınız.

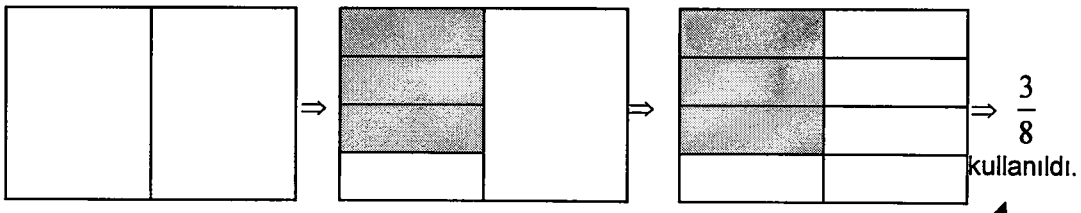
$7 \times \frac{1}{9} =$ $\quad 4 \times \frac{2}{5} =$ $\quad 3 \times \frac{7}{4} =$

$2 \times \frac{9}{2} =$ $\quad 6 \times 2\frac{3}{5} =$ $\quad 4 \times 3\frac{5}{8} =$

$7 \times 1\frac{5}{7} =$ $\quad 1 \times \frac{7}{5} =$ $\quad 0 \times 3\frac{4}{9} =$

Soru: Can aldığı renkli kartonun $\frac{1}{2}$'sinin $\frac{3}{4}$'ünü maket yapmak için kullandı. Can maket yaparken bütün kartonun kaçta kaçını kullanmıştır?

Can'ın yaptığı işlemleri karton üzerinde biz de yapalım.



$$\frac{1}{2}\times\frac{3}{4}=\frac{1\times3}{2\times4}=\frac{3}{8}$$

Soru: Verilen işlemleri yapınız.

$\frac{4}{5}\times\frac{3}{7}=$ $\quad$ $\frac{9}{15}\times\frac{7}{8}=$ $\quad$ $\frac{4}{5}\times\frac{6}{5}=$

$\frac{15}{8}\times\frac{4}{9}=$ $\quad$ $2\frac{3}{5}\times3\frac{1}{2}=$ $\quad$ $5\frac{1}{4}\times\frac{12}{7}=$

- Çarpma işlemi yapılırken çapraz olarak sadeleştirme yapılabileceğinde bahsedilecek.
- Tam sayılı kesirlerle çarpma işlemi yaparken bileşik kesre çevirip işlem yapıyoruz.

ÖDEV:

Matematik Gezegeni 3. Sayı170, 171, 172, 173 ve 174.Sayfalar

APPENDIX E

TEACHING MATERIALS USED IN THE EXPERIMENTAL GROUP

KESİRLER VE BİZ – I

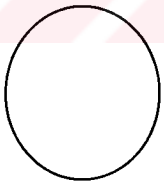
9-10-2000
40'

GRUP NO:
AD, SOYAD:

1. Aşağıda verilen şekiller üzerinde sizden istenen kesirleri gösteriniz ve yazınız.
   a) Paydası 9 olan bir BASİT KESİR yazıp, kare üzerinde gösteriniz (gerektiğinde cetvel kullanabilirsiniz).

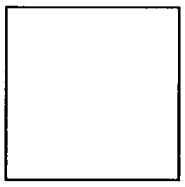


   b) Payı 5 olan bir BASİT KESİR yazıp daire üzerinde gösteriniz.



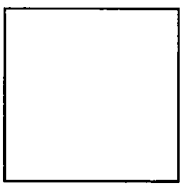
2. Aşağıda verilen şekiller üzerinde sizden istenen gösterimleri yapınız ve kesirleri yazınız.
   a) Aşağıda verilmiş şekilleri 4 EŞİT parçaya bölünüz. İlk bütünün tamamını ve ikinci bütünün de bir basit kesir kadarını karalayınız. Karaladığınız kısımların ifade ettiği kesri her şeklin altına yazınız.

I



_____

II



_____

b) Şimdi ise yukarıdaki şekil üzerinde gösterdiğimiz parçaları kesir olarak ifade edeceğiz.
   - İki şekilde toplam kaç parça karaladınız? _____ (PAY)
   - Bir şekilde toplam kaç parça var? _____ (PAYDA)

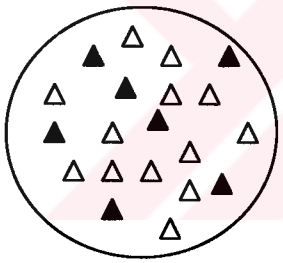
$$\frac{PAY}{PAYDA} = \text{_____}$$

Oluşturduğunuz kesrin çeşidi nedir? Bu kesrin özelliklerini aranızda tartışınız.

3. Az önce elde ettiğiniz kesirleri şimdi de tamsayılı kesir olarak ifade edeceğiz.
   a) 2. sorunun a) şıkkında kaç BÜTÜN karaladınız? ____ (TAMSAYI)
   2. sorunun a) şıkkında ikinci bütünde oluşturduğunuz basit kesir nedir? ____ (BASİT KESİR)

$$TAMSAYI \ \underline{(BASİT\ KESİR)} = \text{_____}$$

4. Aşağıda eş parçalardan oluşan bir küme verilmiştir. Bu kümenin elemanları olan küçük üçgenlerin bazıları sizin için karalanmıştır. Bu kümenin elemanlarıyla bir kesir oluşturacağız.



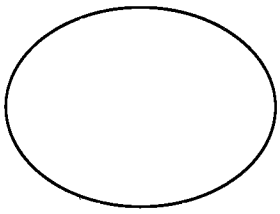
Kümede kaç üçgen karalanmıştır? _____ (PAY)

Kümede TOPLAM kaç üçgen vardır? _____ (PAYDA)

$$\frac{PAY}{PAYDA} = \text{_____}$$

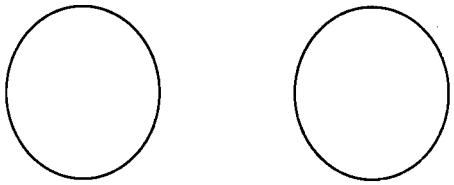
Oluşturduğunuz kesrin çeşidi nedir? Bu kesrin özelliklerini aranızda tartışınız.

5. 1. sorunun a) şıkkında gösterdiğiniz basit kesri şimdi de küme kullanarak gösteriniz.



6. 2. sorunun b) şıkkında oluşturduğunuz bileşik kesri hatırlayın. Bir bütün elde etmeniz için şeklin tamamını karalamıştınız. Şimdi ise bir bütün elde etmeniz için bir kümenin tamamını karalamanız gerekiyor. Buna göre

   - 2. sorunun b) şıkkında oluşturduğunuz kesri kümeler ile gösteriniz.



Sizce tamsayılı kesirleri de bu şekilde gösterebilir miyiz? Tartışınız.

KESİRLER VE BİZ – II

11-10-2000
40'

GRUP NO:
AD, SOYAD:

1. Elinizde iki farklı renkte oyun hamurlarından yapılmış EŞİT BÜYÜKLÜKTE küçük toplar var. Bu hamurlardan aynı renkli olan 5 tanesini elinizdeki plastik tabaklardan birine koyunuz. Bir küme oluşturdunuz. Şimdi, farklı renkte bir top daha koyunuz.

   Toplam kaç topunuz oldu? _____

   Bunlardan biri farklı renkte. Bu farklı renkte olan topun karalanmış olduğunu düşünün. Bu kümeyi bir kesir olarak ifade ediniz.

   Şimdi en son koyduğunuz renkte iki top daha koyunuz. Oluşan kesri yazınız.

2. Elinizdeki plastik tabakların küme olduğunu düşününüz. Bu kümelerin içine hamurdan yapılmış küçük toplardan uygun sayıda koyarak aşağıdaki kesri gösteriniz. Yukarıda uyguladığınız yöntemle karalanmış parçaları farklı renkte hamurla gösterebilirsiniz.

   $2\frac{5}{7}$

   Oluşturduğunuz kümelere bakarak bu kesri bileşik kesir olarak yazabilir misiniz? Neden?

   Eğer yazabiliyorsanız kesri belirtiniz.

   $\frac{16}{7}$

   Yukarıdaki kesri tamsayılı kesre çevirmek için neler yapmanız gerekiyor, aranızda tartışınız.

3. Aşağıdaki çevirmeleri yapınız ve yolunuzu anlatınız.

   a) $3\frac{3}{8}$ kesrini bileşik kesre çeviriniz.

   b) $\frac{32}{9}$ kesrini tamsayılı kesre çeviriniz.

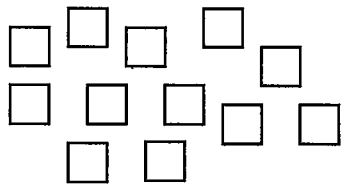
KESİRLER VE BİZ - III

12-10-2000
40'

GRUP NO:
AD, SOYAD:

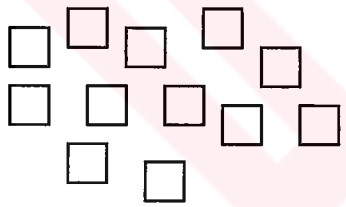
1. a) Elinizdeki karton parçalarının tamamını kullanarak $\frac{8}{12}$ kesrini ifade ediniz.

Aynı kesri aşağıdaki şekil üzerinde gerekli miktarda parçayı karalayarak gösteriniz.



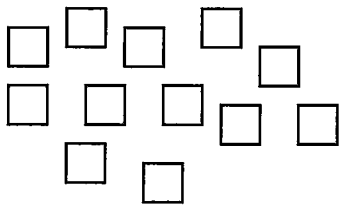
b) Şimdi yine karton parçalarının tamamını kullanarak $\frac{4}{6}$ kesrini ifade etmeye çalışınız.

Aynı kesri aşağıdaki şekil üzerinde gerekli miktarda parçayı karalayarak gösteriniz.



c) Aynı şekilde parçaların hepsini kullanarak $\frac{2}{3}$ kesrini gösteriniz.

Aynı kesri aşağıdaki şekil üzerinde gerekli miktarda parçayı karalayarak gösteriniz.



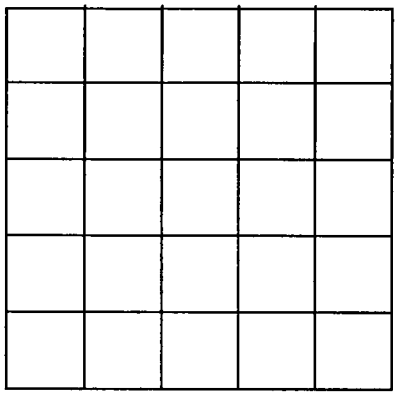
Her şıkta, toplam kaç parça vardı ve ne kadarını karaladınız?

ŞIK	TOPLAM	KARALANMIŞ
a)	_____	_____
b)	_____	_____
c)	_____	_____

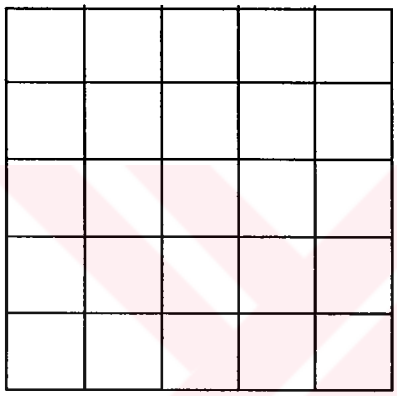
Bu üç kesrin belirttiği miktarlar arasında nasıl bir ilişki vardır? Tartışınız.

Peki bu kesirler arasında sizce ne tip bir ilişki vardır? Tartışınız.

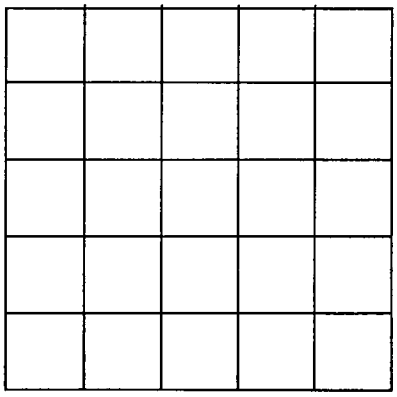
2. a) Aşağıdaki şekli kullanarak $\frac{2}{5}$ kesrini karalayarak gösteriniz.



b) Aynı şekil üzerinde $\frac{4}{10}$ kesrini karalayarak gösteriniz.



c) Şimdi de aynı küme üzerinde $\frac{6}{15}$ kesrini karalayarak gösteriniz.



Bu üç kesrin belirttiği miktarlar arasında nasıl bir ilişki vardır? Tartışınız.

Peki bu kesirler arasında sizce ne tip bir ilişki vardır? Tartışınız.

KESİRLER VE BİZ - IV

16-10-2000
40'

GRUP NO:
AD, SOYAD:

1. Bir önceki etkinlikte bazı kesirlerin aynı miktarı gösterdiğini gördük. Bu kesirlerin ortak özellikleri nedir? Tartışınız.

2. Aşağıdaki kesirlerin denk olup olmadığını tartışınız ve sebeplerini yazınız.

   a) $\frac{2}{7} \equiv \frac{4}{21}$

   b) $\frac{16}{12} \equiv \frac{4}{3}$

   c) $\frac{8}{3} \equiv \frac{8}{9}$

3. Aşağıda iki küme verilmiştir. Bu kümelerden A kümesinin elemanları sadece paydası belli olan kesirler, B kümesinin elemanları ise sadece payı belli olan kesirlerdir.

   A kümesinden seçeceğiniz bir kesrin B kümesinden seçeceğiniz bir kesre denk olması gerekiyor. Bunun için pay ve paydadaki bilinmeyen sayıları bulunuz.

   Buna göre, farklı kesirleri kullanmaya çalışarak bulabildiğiniz kadar denk kesir çifti bulunuz. Bir kesri birden fazla defa kullanabilirsiniz.

   A KÜMESİ

   $\frac{\square}{7}, \frac{\square}{3}, \frac{\square}{16},$

   $\frac{\square}{8}, \frac{\square}{6}, \frac{\square}{14}$

   B KÜMESİ

   $\frac{15}{\square}, \frac{5}{\square}, \frac{2}{\square},$

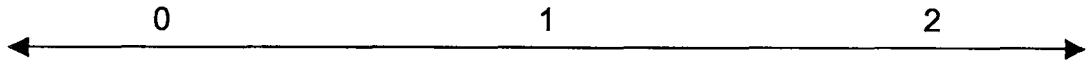
   $\frac{4}{\square}, \frac{9}{\square}, \frac{21}{\square}$

   Örneğin:
   A kümesinden $\frac{\square}{14}$, B kümesinden $\frac{9}{\square}$ alalım. Bu kesirlerin birbirine denk olabilmesi için pay kısmına 18 ve payda kısmına 7 sayılarının yazılması gerekir. Bu durumda, ortaya çıkan denk kesir çifti aşağıdaki gibidir.

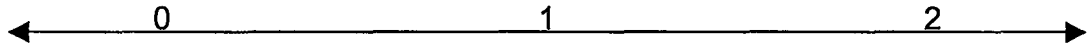
   $$\frac{18}{14} \equiv \frac{9}{7}$$

4. Aşağıda iki sayı doğrusu verilmiştir. I. sayı doğrusu üzerinde 1 ile 2 arasında bir kesir gösteriniz. Bu kesre denk bir kesri de II. sayı doğrusu üzerinde gösteriniz.

I.



II.



Kesirlerin sayı doğrusu üzerinde belirttiği uzunluklar birbirinden farklı mıdır? Tartışınız.

KESİRLER VE BİZ – V

18-10-2000
40'

GRUP NO:
AD, SOYAD:

1. Aşağıda verilen paydası eşit kesir çiftlerini < ya da > işaretlerini kullanarak sıralayınız.

   a) $\frac{11}{5} \quad \frac{2}{5}$ b) $\frac{6}{13} \quad \frac{13}{13}$

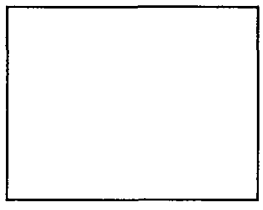
2. Bir bütünü 9 parçaya bölüyorum. Aynı büyüklükte başka bir bütünü de 4 parçaya bölüyorum. Elinizdeki kartonlar yardımıyla aşağıdaki soruları cevaplandırınız:

   $\frac{1}{9}$ kesri ile $\frac{1}{4}$ kesrini karşılaştırınız. Hangisi daha büyüktür? _____

   Her iki parçadan 3 tane alınız. Hangi kesirleri oluşturmuş oldunuz?

   Son oluşturduğunuz kesirleri kartonların da yardımıyla < ya da > işaretlerini kullanarak sıralayabilir misiniz?

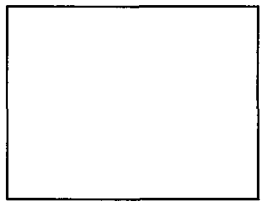
3. $\frac{4}{5}$ ve $\frac{4}{8}$ kesirlerini aşağıda verilen şekiller üzerinde tarayarak gösteriniz.

   = _____

   Hangi kesirde taranan alan daha büyüktür?

   _____

   = _____

   Sizce hangi kesir daha büyüktür? Tartışınız.

   _____

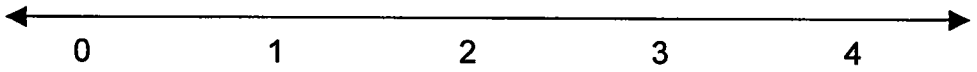
   Buna göre, $\frac{4}{8}$ ve $\frac{4}{5}$ kesirlerini < ya da > işaretlerini kullanarak sıralayınız.

Payı aynı, paydası farklı kesirlerin büyüklüklerine göre sıralanması için bir kural yazabilir misiniz? Tartışın.

4. Aşağıdaki kesir çiftlerinin arasına < veya > işaretlerinden uygun olanını yazarak sıralayınız.

a) $\frac{8}{12}$ $\frac{8}{7}$ b) $\frac{21}{15}$ $\frac{21}{5}$ c) $\frac{8}{3}$ $\frac{14}{3}$ d) $\frac{16}{16}$ $\frac{28}{16}$

5. $\frac{7}{3}, \frac{1}{3}, \frac{10}{3}, \frac{4}{3}, \frac{11}{3}$ kesirlerini sayı doğrusu üzerinde gösteriniz



En büyük kesir? _____

En küçük kesir? _____

Paydası eşit, payı farklı kesirleri sıralarken iki kesir sıralamıştınız. Aynı kural ile yukarıdaki kesirleri < ya da > işareti kullanarak sıralayınız.

6. Payları eşit, paydaları farklı iki kesir arasındaki büyüklük küçüklük ilişkisini hatırlayınız. Buna göre aşağıdaki kesirleri büyüklüklerine göre sıralarken nasıl bir yol izleriz? Tartışınız.

$$\frac{5}{12}, \frac{5}{7}, \frac{5}{4}, \frac{5}{13}, \frac{5}{2}$$

En küçük kesir? _____

En büyük kesir? _____

Payı eşit, paydası farklı kesirleri sıralarken iki kesir sıralamıştınız. Buna göre yukarıdaki 5 kesri < ya da > işareti kullanarak sıralayınız.

7. Aşağıdaki örneklerde verilen kesirleri arkadaşlarınızla birlikte işaret kullanarak sıralayınız.

a) $\frac{11}{7}, \frac{11}{12}, \frac{11}{4}, \frac{11}{14}, \frac{11}{28}$

b) $\frac{4}{13}, \frac{7}{13}, \frac{8}{8}, \frac{11}{13}, \frac{1}{13}$

KESİRLER VE BİZ – VI

19-10-2000
40'

GRUP NO:
AD, SOYAD:

1. $\frac{3}{7}$ ve $\frac{4}{6}$ kesirlerinin hangisinin büyük olduğunu anlamak için ne yaparsınız? Tartışınız.

Sizce hangisi büyüktür? _____

Neden?

Daha önce
- Payı aynı, paydası farklı
- Paydası aynı, payı farklı

kesirler için karşılaştırma yolları öğrenmiştiniz. Bu yolları kullanabilir misiniz? Neden?

2. $\frac{13}{4}$ kesri ile $\frac{6}{8}$ kesrini herhangi bir işlem kullanmadan karşılaştırabilir misiniz? Nasıl?

3. $5\frac{1}{4}$ kesri ile $2\frac{5}{12}$ kesrini herhangi bir işlem kullanmadan karşılaştırabilir misiniz? Nasıl?

4. $2\frac{3}{8}$ kesri ile $2\frac{7}{8}$ kesrini herhangi bir işlem kullanmadan karşılaştırabilir misiniz? Nasıl?

5. Aşağıdaki kesirleri **<** ya da **>** işaretlerini kullanarak sıralayınız.

a) $\frac{4}{9}$ $\frac{15}{6}$

b) $\frac{9}{15}$ $\frac{11}{12}$

KESİRLER VE BİZ - VII

20-11-2000
40'

GRUP NO:
AD, SOYAD:

1. Elinizdeki karton parçalarından uygun sayıda yan yana getirerek aşağıdaki kesirleri oluşturunuz.

   $\frac{3}{9}$ ve $\frac{4}{9}$

2. Karton parçalarıyla oluşturduğunuz kesirleri birleştiriniz. Hangi kesri elde ettiniz?

3. $\frac{3}{9}+\frac{4}{9}$ işleminin sonucunu bulunuz.

4. Yukarıdaki işlemi kartonları kullanarak gösteriniz.

5. Şimdi de elinizdeki kartonları kullanarak $1\frac{3}{4}+2\frac{2}{4}$ işlemini yapınız ve sonucunu yazınız.

6. Yukarıda yaptığınız işlemlere göre, paydası birbirine eşit olan kesirlerin toplanması için bir kural yazınız.

7. Aşağıdaki toplama işlemlerini yapınız.

   a) $\frac{7}{15}+\frac{8}{15}=$

   b) $\frac{14}{5}+\frac{2}{5}=$

   c) $2\frac{3}{7}+3\frac{5}{7}=$

   d) $\frac{3}{11}+\frac{7}{11}+\frac{12}{11}=$

KESİRLER VE BİZ – VIII

22-11-2000
40'

GRUP NO:
AD, SOYAD:

1. Aşağıdaki toplama işlemini yapmaya çalışınız. Sonucunu yazınız.
$$\frac{2}{5}+\frac{3}{10}=$$

2. Görüldüğü üzere yukarıdaki kesirlerin paydaları birbirine eşit değil. Paydası birbirine eşit olan kesirlerin nasıl toplandığını biliyoruz. Bu kesirleri daha önce öğrendiğiniz kesir toplamasına benzeterek toplamak istersek ne yapmamız gerekir?

3. Şimdi $\frac{2}{5}+\frac{3}{10}=$ işleminin sonucunu tekrar bulmaya çalışınız.

4. Şimdi de aşağıdaki toplama işlemini inceleyiniz ve sonucu bulmaya çalışınız.
$$3\frac{4}{15}+1\frac{1}{10}=$$

5. Bu kesirlerin toplanmasını da paydası eşit kesirlerin toplama işlemine benzetmek istesek, önce ne yapmamız gerekir?

6. Buna göre işlemi tekrar yapınız ve sonucunu yazınız.
$$3\frac{4}{15}+1\frac{1}{10}=$$

7. Paydası birbirinin katı ya da aynı sayının farklı katları olan kesirlerin toplanması için bir kural yazınız.

8. Aşağıdaki toplama işlemlerini yapınız.

a) $\frac{12}{21}+\frac{5}{7}=$

b) $4+\frac{3}{7}=$

c) $5\frac{1}{7}+3\frac{5}{21}=$

d) $\frac{2}{6}+\frac{1}{6}+\frac{5}{12}=$

KESİRLER VE BİZ – IX

23-11-2000
40'

GRUP NO:
AD, SOYAD:

1. Aşağıdaki toplama işleminin doğru mudur? Elinizdeki karton parçalarını kullanarak inceleyiniz. Cevabınızı ve sebebini yazınız.

   $$\frac{2}{4}+\frac{1}{3}=\frac{3}{7}$$

2. Paydası birbirine eşit olan kesirlerin nasıl toplandıklarını biliyorsunuz. Paydası birbirine eşit olmayan $\frac{2}{4}$ ve $\frac{1}{3}$ gibi kesirleri toplamak istersek önce ne yapmamız gerekir?

3. Buna göre toplama işlemini nasıl yaparsınız?

   $$\frac{2}{4}+\frac{1}{3}=$$

4. Şimdi de aşağıdaki toplama işlemini inceleyiniz ve sonucu bulmaya çalışınız.

   $$2\frac{3}{6}+4\frac{5}{8}=$$

5. Paydası birbirine eşit olmayan ve aynı sayının farklı katları olmayan kesirlerin toplanması için bir kural yazabilir misiniz?

6. Aşağıdaki toplama işlemlerini yapınız?

   a) $\frac{12}{5}+\frac{2}{7}=$

   b) $1\frac{2}{4}+\frac{5}{3}=$

   c) $6\frac{2}{8}+2\frac{1}{3}=$

   d) $\frac{5}{6}+\frac{1}{3}+1\frac{1}{2}=$

KESİRLER VE BİZ – X

11-12-2000
40'

GRUP NO:
AD, SOYAD:

1. Elinizdeki karton parçalarını kullanarak $\frac{7}{9}$ kesrini oluşturunuz.
2. Şimdi, yukarıda oluşturduğunuz kesirden $\frac{3}{9}$ kesrini ayırınız. Geriye kalan parçaların oluşturduğu kesir kaçtır?

3. 1. ve 2. bölümde yaptıklarınızı şimdi de işlem kullanarak gösteriniz.

4. Şimdi de aşağıdaki çıkarma işlemini yapmaya çalışınız. Tamsayılı kesirleri bileşik kesirlere çevirmeyi unutmayınız. Dilerseniz kartonlardan da yararlanabilirsiniz.

$$3\frac{1}{4}-1\frac{3}{4}=$$

5. Yukarıda yaptığınız işlemlere göre paydası birbirine eşit kesirler arasındaki çıkarma işlemleri için bir kural yazınız.

6. Aşağıdaki çıkarma işlemlerini yapınız.

a) $\frac{23}{14}-\frac{9}{14}=$

b) $5\frac{7}{8}-3\frac{2}{8}=$

c) $\frac{4}{45}-\frac{1}{45}=$

d) $4\frac{3}{11}-\frac{4}{11}=$

e) $\frac{17}{5}-2\frac{1}{5}=$

KESİRLER VE BİZ – XI

13-10-2000
40'

GRUP NO:
AD, SOYAD:

1. Aşağıdaki çıkarma işlemini yapmaya çalışınız.
$\frac{4}{7}-\frac{1}{21}=$

2. Görüldüğü üzere yukarıdaki kesirlerin paydaları birbirine eşit değil. Paydası birbirine eşit olan kesirler arasındaki çıkarma işleminin nasıl yapıldığını biliyoruz. Yukarıdaki kesirleri daha önce öğrendiğiniz kesir çıkarmasına benzeterek çıkarmak istersek ne yapmamız gerekir?

3. Şimdi $\frac{4}{7}-\frac{1}{21}=$ işleminin sonucunu tekrar bulmaya çalışınız.

4. Şimdi de aşağıdaki çıkarma işlemini inceleyiniz ve sonucunu bulmaya çalışınız.
$7\frac{1}{6}-4\frac{5}{9}=$

5. Bu kesirlerin çıkarılması işlemini de paydası eşit kesirlerin çıkarılması işlemine benzetmek istesersek, ne yapmamız gerekir?

6. Buna göre işlemi tekrar yapınız ve sonucu yazınız.
$7\frac{1}{6}-4\frac{5}{9}=$

7. Paydası birbirinin katı ya da aynı sayının farklı katları olan kesirler rasındaki çıkarma işlemi için bir kural yazınız.

8. Aşağıdaki çıkarma işlemlerini yapınız.

a) $\frac{1}{2}-\frac{1}{8}=$

b) $\frac{21}{12}-\frac{2}{3}=$

c) $4\frac{2}{5}-\frac{12}{15}=$

d) $\frac{22}{6}-\frac{4}{4}=$

e) $8\frac{2}{7}-2\frac{9}{14}=$

KESİRLER VE BİZ – XII

14-12-2000
40'

GRUP NO:
AD, SOYAD:

1. Elinizdeki karton parçalarını kullanarak aşağıdaki işlemin doğru olup olmadığını gösteriniz. Cevabını yazınız.

$$\frac{8}{9}-\frac{3}{5}=\frac{5}{4}$$

2. Paydaları birbirine eşit olan kesirler arasındaki çıkarma işleminin nasıl yapıldığını biliyorsunuz. Paydaları birbirine eşit olmayan ve birinini katı olmayan $\frac{8}{9}$ kesrinden $\frac{3}{5}$ kesrini çıkarmak istersek önce ne yapmamız gerekir?

3. Buna göre çıkarma işlemini nasıl yaparsınız?

$$\frac{8}{9}-\frac{3}{5}=$$

4. Şimdi de aşağıdaki çıkarma işlemini yapınız. Tam sayılı kesirelri bileşik kesre çevirmeyi unutmayınız.

$$5\frac{1}{2}-3\frac{4}{15}=$$

5. Paydaları birbirine eşit olmayan ve birbirinin katı olmayan kesirler arasındaki çıkarma işlemi için bir kural yazabilir misiniz?

6. Aşağıdaki toplama işlemlerini yapınız?

a) $\frac{1}{2}-\frac{1}{7}=$

b) $\frac{18}{8}-1\frac{2}{3}=$

c) $4\frac{5}{9}-3\frac{6}{10}=$

d) $4-\frac{3}{13}=$

e) $8\frac{2}{5}-\frac{19}{3}=$

KESİRLER VE BİZ – XIII

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GRUP NO:
AD, SOYAD:

1. Bir kesir ile bir doğal sayının çarpılması işlemini 4. sınıfta öğrenmiştik. Şimdi, aşağıdaki örnekleri inceleyerek çarpma işlemini hatırlayınız.

a) $\frac{4}{7} \times 5 = \frac{20}{7} = 2\frac{6}{7}$ b) $9 \times \frac{6}{5} = \frac{54}{5} = 5\frac{4}{5}$

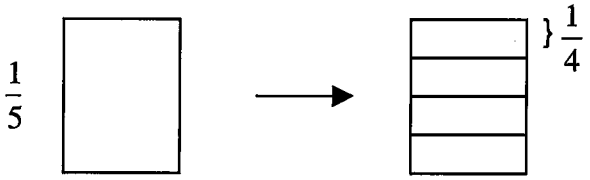
c) $3\frac{1}{3} \times 4 = \frac{10}{3} \times 4 = \frac{40}{3} = 13\frac{1}{3}$

2. Yukarıdaki işlemleri gözönüne alarak bir kesir ile bir doğal sayının çarpımını belirten kuralı yazınız.

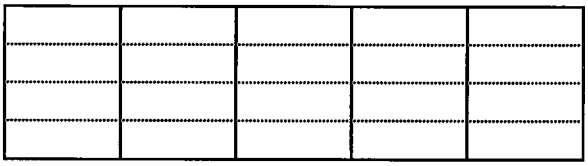
3. Şimdi de iki kesir arasındaki çarpma işlemini öğreneceğiz. Bunun için grubunuzla birlikte elinizdeki karton parçalarını kullanarak aşağıdaki işlemi hepbirlikte yapacağız.

$$\frac{1}{5} \times \frac{1}{4} =$$

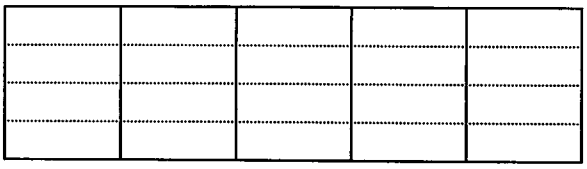
a) Grubunuzun her üyesinin elinde $\frac{1}{5}$ kesrini gösteren karton parçaları var. Şimdi, grubunuzun her üyesi elindeki karton parçasını üstüste 2 defa katlayarak 4 eşit parçaya bölsün ve ortaya çıkan parçalardan birini karalasın. Bulduğunuz parça hangi kesri ifade ediyor?



b) Elimizdeki parçalar $\frac{1}{5}$ kesrini gösteriyordu. Şimdi, bu kesrin ait olduğu bütünü oluşturacaksınız. Diğer grup üyeleri ile birlikte elinizdeki parçaların **karalamadığınız taraflarını** yanyana getirerek bütünü oluşturun. Bütünü toplam kaç parçaya ayırmış oldunuz?



c) Birleştirdiğiniz parçalardan sadece bir tanesini ters çevirin. Böylece bütün üzerinde bir parça karalamış oldunuz. Bu parça bütünün hangi kesrini ifade etmektedir? Gösterilen yere yazınız.



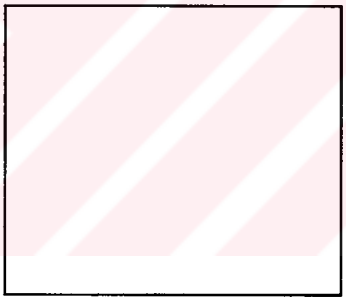
d) Yaptığımız işlemleri gözden geçirelim:

- Önce, bir bütün 5 eşit parçaya bölünmüştü.
- Sonra, bu 5 parçanın herbirini 4'er eşit parçaya böldük.
- Oluşan parçalardan bir parçasını karaladık.
- Oluşan parçanın bütünün hangi kesrini ifade ettiğini bulduk.

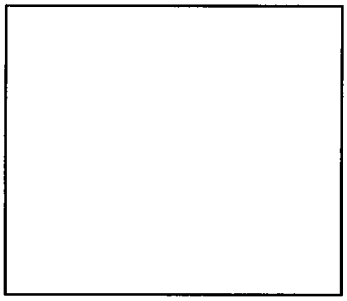
e) $\frac{1}{5}$ kesrinin $\frac{1}{4}$'ünü bulmuş olduk. Yaptıklarınızı bir de çarpma işlemi ile gösteriniz.

2. Şimdi de benzer şekilde $\frac{2}{3} \times \frac{4}{5}$ işlemini şekil üzerinde yapacağız.

a) Aşağıdaki şeklin $\frac{2}{3}$'ünü kurşun kalemle karalayarak gösteriniz.



b) Karaladığınız kısmın $\frac{4}{5}$'ini de kırmızı kalemle karalayarak gösteriniz.



c) Şekli toplam kaç parçaya bölmüş oldunuz?

d) Kırmızı kalemle karaladığınız kısmı bütün şeklin hangi kesrini ifade ediyor?

e) $\frac{2}{3}$ kesrinin $\frac{4}{5}$ 'ini bulmuş oldunuz. Yaptıklarınızı bir de çarpma işlemi ile gösteriniz.

3. Bulduğunuz sonuçları gözönüne alarak iki kesrin çarpımı için bir kural yazınız.

4. Aşağıdaki çarpma işlemlerini yapınız.

a) $\frac{1}{3} \times \frac{5}{7} =$

b) $\frac{4}{11} \times \frac{8}{3} =$

c) $\frac{3}{4} \times 2\frac{4}{5} =$

d) $1\frac{1}{6} \times 3\frac{2}{3} =$

e) $\frac{1}{12} \times \frac{1}{10} =$

f) $1\frac{2}{9} \times \frac{1}{3} =$

g) $\frac{5}{8} \times \frac{5}{4} =$

h) $2\frac{2}{11} \times \frac{9}{2} =$

i) $\frac{1}{8} \times \frac{8}{1} =$

j) $\frac{15}{4} \times \frac{3}{7} =$

APPENDIX F

CONCEPTUAL AND PROCEDURAL PERFORMANCE TEST (CPPT) USED IN THE PILOT STUDY

AD, SOYAD:
NUMARA: SINIF:

1.) Pay ve paydasının toplamı 19 olan

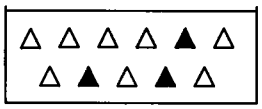
a) Bir basit kesir yazınız. b) Bir bileşik kesir yazınız.

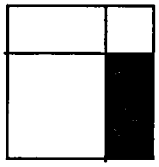
c) Bir tamsayılı kesir yazınız.

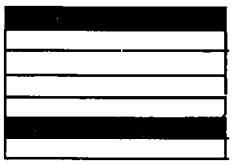
2.) 4 bütünün her biri 5 eşit parçaya bölünüp parçalardan 16 tanesi alınmıştır. Oluşan kesri ve çeşidini yazınız.

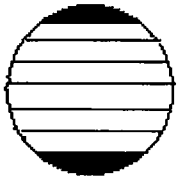
3.) 5 sayısının içinde kaç tane 1/5 vardır?

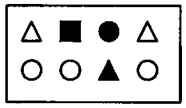
4.) Aşağıda verilen şekil ve çokluklardaki taralı kısımları kesir ile ifade ediniz. Eğer ifade edemiyorsanız sebebini açıklayınız.

a)  = ___

d)  = ___

b)  = ___

e)  = ___

c)  = ___

5.) Aşağıdaki bileşik kesirleri tam sayılı kesre çeviriniz.

a) $\frac{23}{9} =$ b) $\frac{37}{7} =$

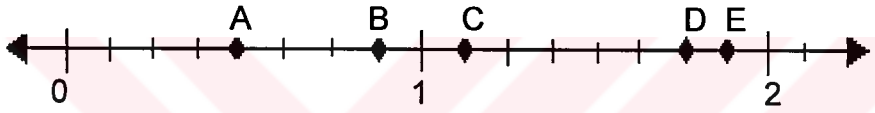
6.) Aşağıdaki tam sayılı kesirleri bileşik kesre çeviriniz.

a) $3\frac{2}{15} =$ b) $2\frac{3}{22} =$

7.) Aşağıdaki kesirlerden birbirine denk olan kesirleri gösteriniz.

$$\frac{21}{18}, \frac{4}{9}, \frac{10}{4}, \frac{2}{7}, \frac{9}{7}, \frac{7}{9}, \frac{5}{2}, \frac{7}{6}, \frac{8}{18}, \frac{4}{14}$$

8.) Sayı doğrusu üzerinde verilen A, B, C, D ve E noktalarının gösterdikleri kesirleri yazınız.



A = B = C = D = E =

9.) Aşağıdaki kesirleri < veya > sembollerinden uygun olanını yazarak sıralayınız. (c ve d şıklarında kesirlerin yerlerini değiştirebilirsiniz.)

a) $\frac{13}{33}, \frac{13}{26}$ b) $\frac{4}{14}, \frac{6}{14}$ c) $\frac{5}{22}, \frac{12}{22}, \frac{3}{22}, \frac{11}{22}$ d) $\frac{7}{5}, \frac{7}{13}, \frac{7}{38}, \frac{7}{21}$

10.) Aşağıda büyüklük veya küçüklük sırasına göre sıralanmış kesirleri görüyorsunuz. Boşlukları sıralamalar bozulmayacak şekilde uygun doğal sayılarla doldurunuz.

a) $\frac{\quad}{7} < \frac{13}{14}$

b) $\frac{7}{\quad} > \frac{8}{5}$

c) $\frac{\quad}{30} < \frac{15}{30}$

d) $\frac{6}{9} > \frac{\quad}{9}$

11.) Aşağıdaki işlemleri yapınız.

a) $3\frac{1}{9}+\frac{7}{6}=$

b) $\frac{1}{4}-\frac{1}{9}=$

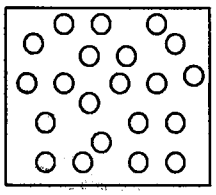
c) $\frac{7}{3}\times\frac{4}{21}$

d) $4\frac{3}{7}-\frac{49}{14}=$

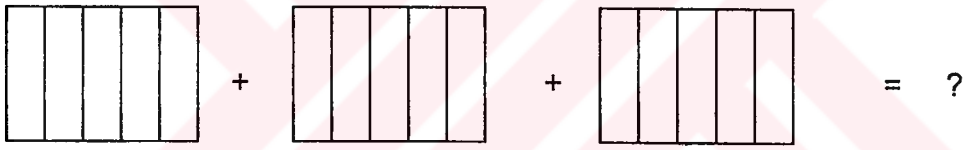
e) $2\frac{1}{4}\times\frac{2}{3}=$

f) $\frac{9}{24}+\frac{1}{8}+\frac{8}{12}=$

g) $7\times4\frac{3}{8}$

12.) $2-$  $=\frac{10}{7}$

Yandaki çıkarma işleminde çıkan kesir bilinmiyor. Çıkan kesri bulunuz ve verilen küme üzerinde gösteriniz.

13.) 



Yukarıda şekiller ile gösterilen işlemin sonucunu bulunuz. Bu işlemi farklı bir işlem kullanarak ifade ediniz. Sonucu şekille gösteriniz.

14.) $\square \times \frac{15}{12}$ işleminden elde edilen sayının 7'den küçük olması için boş kutuya hangi sayı konmalıdır.